APS1022 Project

Part a

Load data

```
data = readtable('monthly_prices_dec_to_june.csv');
dates = data{:,1};
prices = data{:,2:end};
returns = diff(log(prices)); % log returns
```

Estimate parameters

```
mu = mean(returns); % monthly mean return
Q = cov(returns); % monthly covariance matrix
mu_annual = mu * 12;
Q_annual = Q * 12;
```

Obtain risk free rate

```
% Retrieve the actual U.S. risk-free rate for June 2011,
% using the 3-month Treasury Bill rate from FRED, which is
% a standard proxy in portfolio analysis.
c = fred;
data = fetch(c, 'TB3MS', '06/01/2011', '06/30/2011'); % 3-month T-bill
rf = data.Data(end,2) / 100; % convert % to decimal
```

Compute market mean return using shares and prices data from yfinance

```
% We assume 20 stocks is the market. Since yfinance doesn't provide
historical market caps
% directly, we estimate it by using formula market cap = shares outstanding
* close price,
% where shares and close price are downloaded from yfinance.

% We will use latest shares outstanding with June 2011 prices (given that
float doesn't
% change much month-to-month) because yfinance only gives current shares
outstanding.

T = readtable('price_and_shares_2011.csv');

prices = T.Price_2011_06_30;
shares = T.Shares_Outstanding;
tickers = T.Ticker;
```

```
% Compute Market Capitalizations and Weights
market_caps = prices .* shares;
weights_mktcap = market_caps / sum(market_caps);

r_mkt_series = returns * weights_mktcap; % full time series of market
return
mean_mkt = mean(r_mkt_series) * 12; % annualized mean
var_mkt = var(r_mkt_series) * 12; % annualized variance

lambda_idzorek = (mean_mkt - rf) / var_mkt;
fprintf('lambda = %.4f\n', lambda_idzorek);
```

lambda = 0.9588

Construct MVO

```
% Assume:
% - mu_annual: 1×n vector of annualized expected returns
% - Q annual: n×n annualized covariance matrix
% - lambda idzorek: scalar
% - n = number of assets
n = length(mu annual);
% Define MVO objective function
mvo_obj = Q(x) lambda_idzorek * (x' * Q_annual * x) - mu_annual * x;
% Constraints
Aeq = ones(1, n);
beq = 1;
lb = [];
ub = [];
x0 = ones(n,1)/n;
% fmincon solver
opts = optimoptions('fmincon', 'Display', 'none', 'Algorithm', 'sqp');
[x_mvo, fval] = fmincon(mvo_obj, x0, [], [], Aeq, beq, lb, ub, [], opts);
% Display weights
disp('MVO Portfolio Weights:')
```

MVO Portfolio Weights:

```
disp(array2table(x_mvo', 'VariableNames', tickers'))
```

F	CAT	DIS	MCD	K0	PEP	WMT	С	WFC	JPM
1.2787	-2.3102	6.2093	-2.6341	0.26781	7.7859	0.064815	0.1239	-5.5249	-0.272

```
% Convert weights into table
```

```
x_mvo_table = array2table(x_mvo, 'VariableNames', {'MVO Portfolio
Weights'}, 'RowNames', tickers);
disp(x_mvo_table)
```

MVO Portfolio Weights

F	1.2787
CAT	-2.3102
DIS	6.2093
MCD	-2.6341
К0	0.26781
PEP	7.7859
WMT	0.064815
С	0.1239
WFC	-5.5249
JPM	-0.27272
AAPL	2.5689
IBM	10.703
PFE	-0.21225
JNJ	-7.0389
XOM	1.0108
COP	-7.9527
ED	-0.83034
T	0.27239
VZ	1.663
NEM	-4.1725
	111/23

Construct robust MVO with CI = 90%

```
T_obs = size(returns, 1); % Number of time periods (e.g., 43)
months)
alpha = 0.90;
                                  % 90% confidence level
epsilon = sqrt(chi2inv(alpha, n)); % sqrt of chi-squared critical value
Theta = diag(diag(Q_annual)) / T_obs;
% Set Up Optimization Problem
robust_obj_90 = @(x) lambda_idzorek * (x' * Q_annual * x) ...
                - mu annual * x ...
                + epsilon * norm(sqrtm(Theta) * x, 2);
% Define Constraints
Aeq = ones(1, n); % sum(x) = 1
beq = 1;
b = []; % x >= 0
          % no upper bound
ub = [];
x0 = ones(n, 1) / n; % initial guess: equal weights
% Solve Using fmincon
opts = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');
[x_robust_90, fval] = fmincon(robust_obj_90, x0, [], [], Aeq, beq, lb, ub,
[], opts);
```

```
% Display weights
disp('Robust MVO Weights (ellipsoidal, 90%):')
```

Robust MVO Weights (ellipsoidal, 90%):

```
disp(array2table(x_robust_90', 'VariableNames', tickers'))
```

F	CAT	DIS	MCD	К0	PEP	WMT	C	WFC
0.21046	-0.48048	0.47218	-0.281	0.1399	0.83847	0.11128	0.83594	-0.64041

```
% Convert weights into table
x_robust90_table = array2table(x_robust_90, 'VariableNames', {'Robust MV0
Weights (ellipsoidal, 90%)'}, 'RowNames', tickers);
disp(x_robust90_table)
```

Robust MVO Weights (ellipsoidal, 90%)

F	0.21046
CAT	-0.48048
DIS	0.47218
MCD	-0.281
К0	0.1399
PEP	0.83847
WMT	0.11128
С	0.83594
WFC	-0.64041
JPM	-0.14037
AAPL	0.14656
IBM	2.3092
PFE	-0.076363
JNJ	-1.0619
XOM	0.068733
COP	-0.76324
ED	-0.19051
T	0.10558
VZ	0.32539
NEM	-0.92939

Construct robust MVO with CI = 95%

```
Aeq = ones(1, n);  % sum(x) = 1
beq = 1;
lb = [];  % x >= 0
ub = [];  % no upper bound
x0 = ones(n, 1) / n; % initial guess: equal weights

% Solve Using fmincon
opts = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

[x_robust_95, fval] = fmincon(robust_obj_95, x0, [], [], Aeq, beq, lb, ub,
[], opts);

% Display weights
disp('Robust MVO Weights (ellipsoidal, 95%):')
```

Robust MVO Weights (ellipsoidal, 95%):

```
disp(array2table(x_robust_95', 'VariableNames', tickers'))
```

F	CAT	DIS	MCD	К0	PEP	WMT	C	WFC
0.1869	-0.41518	0.37303	-0.24307	0.12842	0.71352	0.097129	0.76853	-0.50911

```
% Convert weights into table 
x_robust95_table = array2table(x_robust_95, 'VariableNames', {'Robust MV0 
Weights (ellipsoidal, 95%)'}, 'RowNames', tickers); 
disp(x_robust95_table)
```

Robust MVO Weights (ellipsoidal, 95%)

F	0.1869	
CAT	-0.41518	
DIS	0.37303	
MCD	-0.24307	
K0	0.12842	
PEP	0.71352	
WMT	0.097129	
C	0.76853	
WFC	-0.50911	
JPM	-0.11898	
AAPL	0.12869	
IBM	1.9892	
PFE	-0.061326	
JNJ	-0.87527	
XOM	0.044912	
COP	-0.62815	
ED	-0.16204	
T	0.078995	
VZ	0.29756	
NEM	-0.79376	

Construct Risk Parity Portfolio

```
% define objective function
```

```
% Q annual: covariance matrix (n×n)
n = size(0 annual, 1);
% Objective function: sum of squared deviations from \theta
risk_parity_obj = @(z) sum((z(1:n) * (Q_annual * z(1:n)) - z(end)).^2);
% set constraints
Aeq = [ones(1, n), 0]; % sum(x) = 1
beq = 1;
lb = [zeros(n, 1); -Inf]; % x \ge 0, \theta unrestricted
ub = [];
                           % no upper bound
x0 = [ones(n, 1)/n; 0.01]; % initial guess for [x; <math>\theta]
% solve Using fmincon
opts = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');
[z_opt, fval] = fmincon(risk_parity_obj, x0, [], [], Aeq, beq, lb, ub, [],
opts);
x_risk_parity = z_opt(1:n); % extract weights
% display portfolio weights
disp('Risk Parity Portfolio Weights:')
```

Risk Parity Portfolio Weights:

```
disp(array2table(x_risk_parity', 'VariableNames', tickers'))
```

F	CAT	DIS	MCD	K0	PEP	WMT	С	WFC
0.036936	0.013179	0.020277	0.030853	0.036434	0.10024	0.016082	0.061993	0.05834

```
% Convert weights into table
x_rp_table = array2table(x_risk_parity, 'VariableNames', {'Risk Parity
Portfolio Weights'}, 'RowNames', tickers);
disp(x_rp_table)
```

Risk Parity Portfolio Weights

F	0.036936
CAT	0.013179
DIS	0.020277
MCD	0.030853
K0	0.036434
PEP	0.10024
WMT	0.016082
C	0.061993
WFC	0.058343
JPM	0.034072
AAPL	0.054386
IBM	0.070394
PFE	0.05557
JNJ	0.059023

```
XOM 0.050083

COP 0.058521

ED 0.056296

T 0.025291

VZ 0.088359

NEM 0.073662
```

Construct a portfolio where weights are based on market capitalizations

```
% the weights are already computed above in the section of
% computing market mean return.
% Convert weights into table
x_mkt_table = array2table(weights_mktcap, 'VariableNames', {'Market Cap
Portfolio Weights'}, 'RowNames', tickers);
disp(x_mkt_table)
```

Market Cap Portfolio Weights

F	0.023085
CAT	0.02146
DIS	0.030081
MCD	0.025841
K0	0.062069
PEP	0.041388
WMT	0.060743
C	0.033334
WFC	0.039137
JPM	0.048765
AAPL	0.076743
IBM	0.065331
PFE	0.047626
JNJ	0.068599
MOX	0.15032
COP	0.031014
ED	0.0082216
T	0.073217
VZ	0.067279
NEM	0.025746

Compute metrics for each portfolios using July data

```
% load file
P = readtable('monthly_prices_july.csv');
P.Date = datetime(P.Date); % convert string to datetime

% extract prices
price_0630 = P{P.Date == datetime(2011,7,1), 2:end}; % June 30 prices
price_0731 = P{P.Date == datetime(2011,8,1), 2:end}; % July 29/30 prices

% compute july log returns
r_july = log(price_0731 ./ price_0630)'; % returns as column vector (n×1)

% obtain rf again
% Retrieve the actual U.S. risk-free rate for July 2011,
```

```
% using the 3-month Treasury Bill rate from FRED, which is
% a standard proxy in portfolio analysis.
c = fred;
rf_data = fetch(c, 'TB3MS', '07/01/2011', '07/31/2011'); % 3-month T-bill
rf_july = rf_data.Data(end, 2) / 100; % convert % to decimal
% define a function to evaluate portfolios
function [ret, var_, std_, sharpe] = eval_portfolio(w, r, rf, Q)
    ret = w' * r;
    var_{-} = w' * 0 * w;
    std_ = sqrt(var_);
    if std > 0
        sharpe = (ret - rf) / std_;
    else
        sharpe = NaN;
    end
end
% evaluate portfolios
[ret mvo, var mvo, std mvo, sr mvo] = eval portfolio(x mvo, r july,
rf_july, Q_annual);
[ret_rob90, var_rob90, std_rob90, sr_rob90] = eval_portfolio(x_robust_90,
r july, rf july, Q annual);
[ret_rob95, var_rob95, std_rob95, sr_rob95] = eval_portfolio(x_robust_95,
r_july, rf_july, Q_annual);
[ret_rp, var_rp, std_rp, sr_rp] = eval_portfolio(x_risk_parity, r_july,
rf_july, Q_annual);
[ret_mkt, var_mkt, std_mkt, sr_mkt] = eval_portfolio(weights_mktcap,
r_july, rf_july, Q_annual);
% display results
strategy names = {'MVO', 'Robust MVO 90%', 'Robust MVO 95%', 'Risk Parity',
'Market Cap'};
results = table(strategy_names', ...
    [ret_mvo; ret_rob90; ret_rob95; ret_rp; ret_mkt], ...
    [var mvo; var rob90; var rob95; var rp; var mkt], ...
    [std_mvo; std_rob90; std_rob95; std_rp; std_mkt], ...
    [sr_mvo; sr_rob90; sr_rob95; sr_rp; sr_mkt], ...
    'VariableNames', {'Strategy', 'Return', 'Var', 'StdDev',
'SharpeRatio'});
disp('Portfolio Evaluation for July 2011:')
```

Portfolio Evaluation for July 2011:

disp(results)

Strategy		Return	Var	StdDev	SharpeRatio
{'MV0'	}	1.3445	2.3241	1.5245	0.88167
{'Robust MVO	90%'}	0.21134	0.12179	0.34899	0.60443

0.30843

0.58996

0.095128

Part b

%

Load data (Jan 2011 to Nov 2011)

0.18236

```
prices = readtable('monthly_prices_jan_to_nov.csv'); % must include dates
from Jan 2011 to Dec 2011
T = readtable('price_and_shares_2011.csv');
prices.Date = datetime(prices.Date); % ensure datetime format
```

Rolling window loop

{'Robust MVO 95%'}

```
test months = 7:11; % July to November
results = [];
c = fred; % Create FRED connection once
prev_rf = 0.0004; % Default in case of missing data
for m = test_months
    % Define estimation window: previous 6 months
    start_idx = m - 6;
    end_idx = m - 1;
    % Get estimation window log returns
    price_window = prices{start_idx:end_idx, 2:end}; % exclude date column
                                                      % (5 x 20)
    log_returns = diff(log(price_window));
    mu = mean(log_returns);
                                 % monthly mean return
    Q = cov(log returns);
                                 % monthly covariance
    mu_annual = mu * 12;
    0 \text{ annual} = 0 * 12;
    % Market cap weights for this month
    % Use price at end of estimation window (i.e., end of month m - 1)
    price_latest = prices{end_idx, 2:end};
                                                    % 1×20 row vector
    shares = T.Shares_Outstanding;
                                                     % 20×1 vector
    market_caps = price_latest .* shares';
                                                     % 1×20 .* 1×20
(element-wise)
   weights_mktcap = market_caps / sum(market_caps); % normalize to sum to 1
    % Market return and variance
    r_mkt_series = log_returns * weights_mktcap';
    mean_mkt = mean(r_mkt_series) * 12;
    var_mkt = var(r_mkt_series) * 12;
```

```
% Get risk-free rate for month m-1
    estimation_date = datestr(prices.Date(end_idx), 'mm/dd/yyyy'); % e.g.,
'06/30/2011'
    start_rf = datestr(prices.Date(end_idx - 1), 'mm/dd/yyyy');
    end_rf = datestr(prices.Date(end_idx), 'mm/dd/yyyy');
    rf_data = fetch(c, 'TB3MS', start_rf, end_rf);
    if isempty(rf_data.Data)
        warning('Missing FRED data for %s. Using previous rate.',
estimation_date);
        if m > 7
            rf = prev_rf;
        else
            rf = 0.0004; % fallback if July has no data
        end
    else
        rf = rf_data.Data(end, 2) / 100; % convert % to decimal
        prev rf = rf;
    end
    % compute lambda
    lambda_idzorek = (mean_mkt - rf) / var_mkt;
    % === Solve all 4 portfolios using previous code blocks ===
   % 1. MVO --> x_mvo
   % Assume:
   % - mu_annual: 1×n vector of annualized expected returns
   % - Q annual: n×n annualized covariance matrix
   % - lambda idzorek: scalar
    % - n = number of assets
    n = length(mu annual);
    e = ones(n, 1);
    mu_vec = mu_annual';
    % Define MVO objective function
    mvo_obj = Q(x) lambda_idzorek * (x' * Q_annual * x) - mu_annual * x;
   % Constraints
    Aeq = ones(1, n);
    beq = 1;
    lb = zeros(n,1);
    ub = [];
   x0 = ones(n,1)/n;
   % fmincon solver
    opts = optimoptions('fmincon','Display','off','Algorithm','sqp');
    [x_mvo, fval] = fmincon(mvo_obj, x0, [], [], Aeq, beq, lb, ub, [],
opts);
```

```
% 2. Robust MVO with CI=90% --> x robust 90
   T_obs = size(log_returns, 1);
                                            % Number of time periods
(e.g., 43 months)
    alpha = 0.90;
                                         % 90% confidence level
    epsilon_rob90 = sqrt(chi2inv(alpha, n)); % sqrt of chi-squared
critical value
    Theta = diag(diag(Q_annual)) / T_obs;
    % Set Up Optimization Problem
    robust obj 90 = @(x) lambda idzorek * (x' * Q annual * x) ...
                     - mu_annual * x ...
                     + epsilon rob90 * norm(sqrtm(Theta) * x, 2);
    % Define Constraints
    Aeq = ones(1, n); % sum(x) = 1
    bea = 1:
   lb = zeros(n, 1); % \times >= 0
ub = []; % no upper bound
    x0 = ones(n, 1) / n; % initial guess: equal weights
    % Solve Using fmincon
    opts = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');
    [x_robust_90, fval] = fmincon(robust_obj_90, x0, [], [], Aeq, beq, lb,
ub, [], opts);
    % 3. Robust MVO with CI=95% --> x robust 95
    alpha = 0.95;
                                         % 95% confidence level
    epsilon_rob95 = sqrt(chi2inv(alpha, n)); % sqrt of chi-squared
critical value
    Theta = diag(diag(Q_annual)) / T_obs;
    % Set Up Optimization Problem
    robust obj_95 = @(x) lambda_idzorek * (x' * Q_annual * x) ...
                     - mu annual * x ...
                     + epsilon_rob95 * norm(sqrtm(Theta) * x, 2);
    % Solve Using fmincon
    [x_robust_95, fval] = fmincon(robust_obj_95, x0, [], [], Aeq, beq, lb,
ub, [], opts);
    % 4. Risk Parity --> x_risk_parity
   % define objective function
   % Q annual: covariance matrix (n×n)
    n = size(Q_annual, 1);
   % Objective function: sum of squared deviations from \theta
```

```
risk_parity_obj = Q(z) sum((z(1:n) * (Q_annual * z(1:n)) - z(end)).^2);
    % set constraints
    Aeq_rp = [ones(1, n), 0]; % sum(x) = 1
    beg rp = 1;
    lb_rp = [zeros(n, 1); -Inf]; % x \ge 0, \theta unrestricted
    ub rp = [];
                                  % no upper bound
    x0_{rp} = [ones(n, 1)/n; 0.01]; % initial guess for [x; <math>\theta]
   % solve Using fmincon
    [z_opt, fval] = fmincon(risk_parity_obj, x0_rp, [], [], Aeq_rp, beq_rp,
lb rp, ub rp, [], opts);
    x_risk_parity = z_opt(1:n); % extract weights
   % 5. Market Cap --> weights_mktcap
    % the weights are already computed above
    weights_mktcap = weights_mktcap';
   % ==== Evaluate on next month ====
    price start = prices{m, 2:end};
    price end = prices{m+1, 2:end};
    r_test = log(price_end ./ price_start)';
    % Store performance
    [r1, v1, s1, sh1] = eval_portfolio(x_mvo, r_test, rf, Q_annual);
    [r2, v2, s2, sh2] = eval_portfolio(x_robust_90, r_test, rf, Q_annual);
    [r3, v3, s3, sh3] = eval portfolio(x robust 95, r test, rf, Q annual);
    [r4, v4, s4, sh4] = eval_portfolio(x_risk_parity, r_test, rf, Q_annual);
    [r5, v5, s5, sh5] = eval_portfolio(weights_mktcap, r_test, rf,
Q_annual);
    results = [results;
        r1, v1, s1, sh1;
        r2, v2, s2, sh2;
        r3, v3, s3, sh3;
        r4, v4, s4, sh4;
        r5, v5, s5, sh5];
end
```

Display results

```
methods = repmat(["MVO"; "Robust MVO 90% CI"; "Robust MVO 95% CI"; "Risk
Parity"; "Market Cap"], 5, 1);
months = repelem(["July", "August", "September", "October", "November"]',
5);

T_result = array2table(results, ...
    'VariableNames', {'Return', 'Variance', 'StdDev', 'Sharpe'});
T_result.Method = methods;
```

```
T_result.Month = months;

T_result = movevars(T_result, {'Month', 'Method'}, 'Before', 'Return');
format shortG
disp(T_result)
```

Month	Method	Return	Variance	StdDev	Sharpe
 "July"	"MV0"	0.026428	0.0004283	0.020695	1.2577
"July"	"Robust MVO 90% CI"	0.030605	0.00020532	0.014329	2.1079
"July"	"Robust MVO 95% CI"	0.030588	0.00020231	0.014223	2.1224
"July"	"Risk Parity"	-0.00030617	0.0010824	0.032899	-0.021465
"July"	"Market Cap"	-0.011711	0.0056892	0.075427	-0.16056
"August"	"MV0"	-0.022324	0.0015681	0.0396	-0.57385
"August"	"Robust MVO 90% CI"	-0.018736	0.0012116	0.034808	-0.54975
"August"	"Robust MVO 95% CI"	-0.018601	0.0012015	0.034662	-0.54816
"August"	"Risk Parity"	-0.043215	0.0013885	0.037263	-1.1705
"August"	"Market Cap"	-0.054255	0.004846	0.069613	-0.78512
"September"	"MV0"	0.064476	0.060139	0.24523	0.2621
"September"	"Robust MVO 90% CI"	0.055696	0.00084714	0.029106	1.9067
"September"	"Robust MVO 95% CI"	0.055696	0.00084714	0.029106	1.9067
"September"	"Risk Parity"	0.053925	0.00018172	0.01348	3.9855
"September"	"Market Cap"	0.072202	0.0048688	0.069777	1.0319
"October"	"MVO"	-0.10668	0.078635	0.28042	-0.3808
"October"	"Robust MVO 90% CI"	-0.10668	0.078635	0.28042	-0.3808
"October"	"Robust MVO 95% CI"	-0.10668	0.078635	0.28042	-0.3808
"October"	"Risk Parity"	0.0085666	0.00097998	0.031305	0.27046
"October"	"Market Cap"	0.0056897	0.0041564	0.06447	0.086703
"November"	"MVO"	-0.13785	0.04649	0.21561	-0.64028
"November"	"Robust MVO 90% CI"	0.021506	0.0055173	0.074279	0.28683
"November"	"Robust MVO 95% CI"	0.021901	0.0053437	0.0731	0.29686
"November"	"Risk Parity"	0.033902	0.0043648	0.066066	0.51013
"November"	"Market Cap"	0.0051989	0.024016	0.15497	0.032257

Part c

Extract returns and Sharpe ratios

```
% Extract monthly returns (reshape to 5 portfolios x 5 months)
returns = reshape(results(:,1), [5, 5]);  % 5 portfolios x 5 months
(July—Nov)
stds = reshape(results(:,3), [5, 5]);  % corresponding StdDevs

% Compute cumulative wealth over June—Nov
% Start from June 30 prices → apply July return first
W0 = ones(5, 1);  % Initial wealth = 1 for all portfolios
wealth = zeros(5, 6);
wealth(:,1) = W0;

for t = 1:5
    wealth(:,t+1) = wealth(:,t) .* (1 + returns(:,t));  % recursively
multiply
end

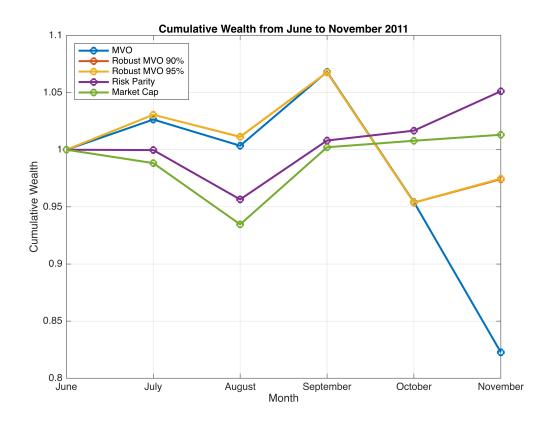
% Compute Sharpe ratios
```

```
sharpe = returns ./ stds;
```

Plot cumulative wealth

```
% Plot cumulative wealth from June to November
months = ["June", "July", "August", "September", "October", "November"];
methods = ["MVO", "Robust MVO 90%", "Robust MVO 95%", "Risk Parity",
"Market Cap"];

plot(wealth', '-o', 'LineWidth', 2)
legend(methods, 'Location', 'northwest')
xlabel('Month')
ylabel('Cumulative Wealth')
xticks(1:6)
xticklabels(months)
title('Cumulative Wealth from June to November 2011')
grid on
```



Plot Sharpe ratios

```
plot(sharpe', '-o', 'LineWidth', 2)
legend(methods, 'Location', 'northwest')
xlabel('Month')
ylabel('Sharpe Ratio (no rf)')
xticks(1:5)
xticklabels(["July", "August", "September", "October", "November"])
```

