MIE1622H: Assignment 1 - Mean-Variance Portfolio Selection Strategies

Shuman Zhao, 1005843991

Part 1: Implement investment strategies in Python

I'm using five strategies to do the portfolio re-balancing:

1. "Buy and Hold"
2. "Equally Weighted"
3. "Minimum Variance":

$$\min_{w} w^T Q w$$
$$s.t. \sum_{i} w_i = 1$$
$$w \geq 0$$

Where variable w_i is the weight (proportion of total funds invested in asset i), and variable Q is the variance-covariance matrix (PSD). The objective function is the variance of the resulting portfolio (which we want to minimize). The first constraint forces the sum of weight (proportion) equal to 1, which is reasonable. The second constraint means short selling is not allowed.

4. "Maximum Expected Return":

$$\max_{w} \mu^T w$$
$$s.t. \sum_{i} w_i = 1$$
$$w \geq 0$$

Where variable μ is the vector of expected returns.

5. "Maximum Sharpe Ratio":

$$\min_{y \in R^n, k>0} y^T Q y$$
$$s.t. \sum_{i=1}^{n} (\mu_i - r_f) y_i = 1$$
$$\sum_{i=1}^{n} y_i = k$$
$$y \geq 0$$

After solving for y* and k*, the optimal portfolio weights are covered as:

$$w^* = \frac{y^*}{k^*}$$
$$Max\ sharpe\ ratio = \frac{1}{\sqrt{(y^*)^T Q y^*}}$$

This is transforming problem from $\max_{w} \frac{\mu^T w - r_f}{\sqrt{w^T Q w}}$ because the Sharpe ratio is homogeneous of degree zero. The variable y and k are introduced to satisfy $w^* = \frac{y^*}{k^*}$. Variable r_f is the

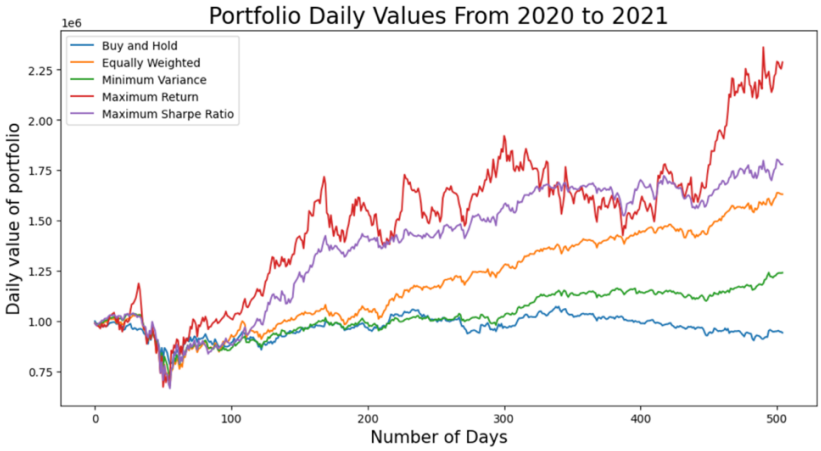
risk-free rate. The first constraint forces the portfolio's expected return to be $\frac{1}{k}$. The second constraint is transformed from $\sum_i w_i = 1$, and third constraint is transformed from $w \geq 0$.

Part 2: Analyze the results

The output for the 12 periods is produced, and period 1 result is posted here:

```
Period 1: start date 01/02/2020, end date 02/28/2020
  Strategy "Buy and Hold", value begin = $ 1000016.96, value end = $ 887595.87, cash account = $0.00
  Strategy "Equally Weighted Portfolio", value begin = $ 990731.02, value end = $ 904019.44, cash account = $1318.91
  Strategy "Minimum Variance Portfolio", value begin = $ 990213.07, value end = $ 906937.18, cash account = $622.14
  Strategy "Maximum Expected Return Portfolio", value begin = $ 990066.86, value end = $ 917076.82, cash account = $63.59
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990068.97, value end = $ 919097.51, cash account = $487.11
```

Here is the chart that illustrates the daily value of the portfolio (for each trading strategy) over the two years using daily adjusted closing prices provided:



Strategies Comparison based on graphs (other 3 graphs are in the Appendix):

1. "Buy and Hold":
   Pro: It saved the transaction cost since there are no transactions and has low fluctuations.
   Cons: It has a high risk because only two stocks, "HOG" and "VZ", are hold. If market declines, the investor will lose a lot. It also has the lowest daily values.
2. "Equally Weighted":
   Pro: It fully diversified across all stocks since the weight for all stocks is $\frac{1}{30}$.
   Cons: This may lead to non-optimal returns.
3. "Minimum Variance":
   Pro: Friendly to risk-averse investors since it focuses on reducing risk (low fluctuations)
   Cons: Only considering minimizing risk will cause non-optimal returns (second-lowest daily values from graph).
4. "Maximum Expected Returns":
   Pro: Can help investor get highest possible returns (highest daily values from graph). Friendly to investors with high risk-tolerance.
   Cons: High risk might lead to huge losses. (High fluctuations in daily value graph). The performance of this strategy in the "dynamic changes in portfolio allocations" graph is bad because there are no diversification constraints. It naturally allocates 100% of the portfolio to the single best-performing stock at each rebalancing period.

5. “Maximum Sharpe Ratio”:
   Pro: It ensures enough diversification, which prevents only concentration in one stock.
   Cons: Can’t achieve optimal returns because it adds limits.

To manage my own portfolio, I’ll select the “Maximum Sharpe Ratio” strategy because I want a high enough return without having a high risk. According to the daily values graph and dynamic changes graphs, this strategy has a high daily value of the portfolio (compared to minimum variance, equally weighted, and buy and hold) while maintaining a high diversification across all stocks (compared to maximum return strategy).

Then I compute the risk measures for the strategies:

	Variance	Max Drawdown	Sharpe Ratio
Min Variance	0.000124	0.274480	0.036402
Max Return	0.001823	0.446808	0.069892
Max Sharpe	0.000614	0.387374	0.064164

The Max Return strategy has the highest variance, which supports my statement before. Then, focusing on “Max Drawdown”, where the drawdown is the measurement of the largest peak-to-trough loss experienced by the portfolio over time. The Max Drawdown value of the Max Return strategy is the highest, which means at some point the portfolio experienced large losses (almost 45% of its value). Besides, the Min Variance strategy has the lowest Max Drawdown value, which shows a low-risk performance.

However, the sharpe ratio value for the Max Sharpe strategy is not the largest. This is because the Max Sharpe strategy is designed to optimize risk-adjusted return, not necessarily maximize total returns. This means it has lower risk than the Max Return strategy, which leads to lower returns and so lower sharpe ratio (from its formula).

Part 3: Discuss possible improvements to the trading strategies

In this section, I test two new strategies: “1/n Buy and Hold” and “Maximum Expected Return with Sector Constraints”.

The “1/n Buy and Hold” strategy selects a “1/n” portfolio at the beginning of period 1 and holds it till the end of period 12 to save a large amount of transaction costs. The risk measures for this strategy are: 0.000274 variance, 33.6% Max Drawdown, and 0.069829 Sharpe ratio. Compared to other strategies, it has a lower variance than Max return and Max Sharpe, a lower Max Drawdown than Max return and Max Sharpe, and a higher Sharpe Ratio than Max Sharpe and Min Variance strategies. It achieves a better result since it has lower risk, lower Drawdown, and higher Sharpe Ratio.

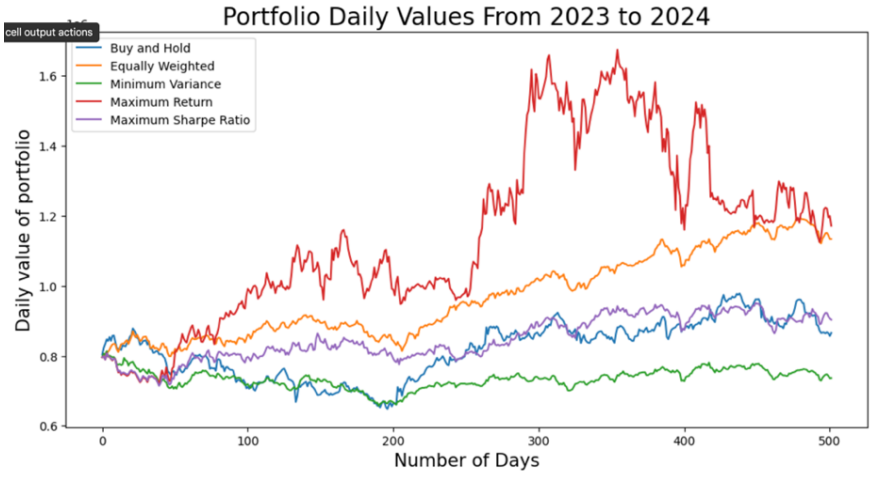
What’s more, I have some suggestions for the improvement of the five strategies that I have implemented.

1. “Buy and Hold”:
   Because holding only two stocks produces a high-risk level, expanding the stock selection to 10-15 diversified stocks across different sectors can be a good improvement.

2. “Equally Weighted”:
   Each stock performs differently. It’s better to adjust the weights based on volatility instead of having equal weights. For example, have higher weight on good-performing stocks and lower weight on bad-performing ones. Besides, some sectors contain many stocks, but others do not. This causes higher weights on some specific sectors and will increase the risk.
3. “Minimum Variance”:
   Focusing only on minimizing variance makes the investor cannot achieve high returns (because of the trade-off between risk and return. Instead, optimize for risk-adjusted returns (e.g. target volatility approach).
4. “Maximum Return”:
   Focusing only on maximizing returns makes a high-risk portfolio. To solve this, add some constraints like sector diversification constraints.
5. “Maximum Sharpe ratio”:
   Trying the Sortino Ratio can make an improvement since it penalizes downside risk.

Then let’s look at the new strategy, the “Maximum Expected Returns with Sector Constraints”, which is an improvement on the “Max Return” strategy. The new risk measures are: 0.000429 variance, 38.09% Max Drawdown, and 0.082351 Sharpe Ratio. It has a lower variance than the “Max Return” and “Max Sharpe”, a lower drawdown than the “Max Return” and “Max Sharpe”, and a higher Sharpe ratio than all the strategies. This is a huge improvement.

Finally, I re-tested my strategies for the 2023-2024 time period. Below is the portfolio daily value graph.



The Max return strategy of the portfolio from 2023 to 2024 shows more fluctuation, especially a large drawdown that happened after 300 days. This means this strategy was highly sensitive to market downturns. The overall daily value of a portfolio of different strategies from year 2020 to 2021 is higher than from year 2023 to 2024. This is because in 2020 the market re-opened with monetary stimulus from the bad influence of COVID-19. The risk-free rate (interest rate) in the year 2020-2021 is 1.5%, which is lower than in the year 2023-2024. This encouraged risky investments like the Max Return Strategy. From the year 2023 to 2024, strategies were more volatile (especially the Max return strategy). The higher interest rate also created more conservative investor behavior. This time, lower-risk strategies perform better than Max Return.

AI-Generated One-Page Report:

Executive Summary: Portfolio Modeling and Performance Analysis

Introduction

This report presents an analysis of various portfolio selection strategies applied to stock market data from **2020-2021** and **2023-2024**. Using **quantitative optimization techniques**, we evaluate risk-return trade-offs, portfolio allocations, and overall investment performance. The strategies tested include:

- **Buy and Hold** (Fixed holdings of two stocks)
- **Equally Weighted Portfolio** (Diversified across all assets)
- **Minimum Variance Portfolio** (Risk minimization)
- **Maximum Expected Return Portfolio** (Return maximization)
- **Maximum Sharpe Ratio Portfolio** (Return-risk efficiency)
- **1/n Buy and Hold (No Rebalancing)** (Fixed equal allocation)

Key Findings

1. Performance Across Strategies

- The **Maximum Expected Return strategy** yielded the highest portfolio value but was extremely volatile due to concentrated holdings.
- The **Minimum Variance strategy** effectively reduced risk but generated lower returns.
- The **Maximum Sharpe Ratio strategy** balanced risk and return but underperformed the pure return-maximizing approach.
- **Buy and Hold** (two-stock portfolio) significantly underperformed due to lack of diversification.
- The **Equally Weighted Portfolio** achieved moderate risk-adjusted returns.
- The **1/n Buy and Hold strategy** demonstrated robustness, avoiding excessive risk while maintaining competitive returns.

2. Risk Measures and Drawdowns

- The **Maximum Expected Return strategy** had the highest variance and largest drawdowns, reflecting its aggressive nature.
- The **Minimum Variance strategy** showed the lowest volatility and smallest drawdowns.
- The **Maximum Sharpe Ratio strategy** balanced return and risk but had lower Sharpe ratios than expected.
- **1/n Buy and Hold** achieved **lower drawdowns** than the Maximum Return strategy while keeping risk under control.

3. Market Conditions: 2020-2021 vs. 2023-2024

- **2020-2021:** Market volatility due to COVID-19 shocks led to high drawdowns in aggressive strategies.

- **2023-2024:** More stable but **rising interest rates (4.5%) impacted performance**, benefiting diversified portfolios.
- The **Maximum Return strategy was more effective in bull markets, while risk-averse strategies performed better in volatile conditions.**

Conclusion & Recommendations

- **For High Returns**: Maximum Expected Return is ideal but very risky.
- **For Risk Management**: Minimum Variance is preferable for stability.
- **For Balanced Growth**: Maximum Sharpe Ratio offers a mix of return and risk control.
- **For Practical Use**: The **1/n Buy and Hold** strategy demonstrated strong stability, making it an attractive option.

Future improvements may include sector constraints, transaction cost considerations, and dynamic rebalancing strategies to enhance portfolio efficiency.

Discussion:

Clarity: The summary is well-structured and captures the key results concisely.

Accuracy: The summary is almost accurate except forgetting to mention the “Max Return with Sector Constraint” strategy.

Important points it has missed:

1. Fail to mention “Max Return with Sector Constraint” strategy
2. Lacks discussion on diversification constraints and why some strategies failed in certain periods.

APPENDIX

Three charts in Python for strategy 3, 4, and 5 to show dynamic changes in portfolio allocations: