MIE 1622H: Assignment 3 – Credit Risk Modeling and Simulation

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Part 1. Output

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======= Credit Risk Model with Credit-State Migrations =======
============== Monte Carlo Scenario Generation ===============


 Number of out-of-sample Monte Carlo scenarios = 100000
 Number of in-sample Monte Carlo scenarios = 5000
 Number of counterparties = 100


Portfolio 1:

Out-of-sample: VaR 99.0% = $16586775.72, CVaR 99.0% = $24924774.02
In-sample MC1: VaR 99.0% = $37157125.00, CVaR 99.0% = $44585575.95
In-sample MC2: VaR 99.0% = $37205997.16, CVaR 99.0% = $44738986.15
In-sample No: VaR 99.0% = $10697314.55, CVaR 99.0% = $11962727.15
In-sample N1: VaR 99.0% = $26169819.11, CVaR 99.0% = $29055129.69
In-sample N2: VaR 99.0% = $26265644.88, CVaR 99.0% = $29162957.82

Out-of-sample: VaR 99.9% = $36324206.11, CVaR 99.9% = $47018608.95
In-sample MC1: VaR 99.9% = $53653413.04, CVaR 99.9% = $60726265.15
In-sample MC2: VaR 99.9% = $53870879.64, CVaR 99.9% = $61910930.15
In-sample No: VaR 99.9% = $13549852.12, CVaR 99.9% = $14583709.02
In-sample N1: VaR 99.9% = $32673987.65, CVaR 99.9% = $35031320.16
In-sample N2: VaR 99.9% = $32796869.59, CVaR 99.9% = $35164008.17


Portfolio 2:

Out-of-sample: VaR 99.0% = $21454034.38, CVaR 99.0% = $31009403.61
In-sample MC1: VaR 99.0% = $27328612.72, CVaR 99.0% = $33370474.01
In-sample MC2: VaR 99.0% = $27384298.11, CVaR 99.0% = $33521314.15
In-sample No: VaR 99.0% = $15536880.08, CVaR 99.0% = $17185860.30
In-sample N1: VaR 99.0% = $21109302.85, CVaR 99.0% = $23275103.97
In-sample N2: VaR 99.0% = $21177846.08, CVaR 99.0% = $23351474.27

Out-of-sample: VaR 99.9% = $44623790.06, CVaR 99.9% = $55298156.25
In-sample MC1: VaR 99.9% = $40786445.60, CVaR 99.9% = $47059831.89
In-sample MC2: VaR 99.9% = $41136877.25, CVaR 99.9% = $48020278.96
In-sample No: VaR 99.9% = $19254069.28, CVaR 99.9% = $20601305.42
In-sample N1: VaR 99.9% = $25991527.88, CVaR 99.9% = $27761012.74
In-sample N2: VaR 99.9% = $26077715.17, CVaR 99.9% = $27853594.84
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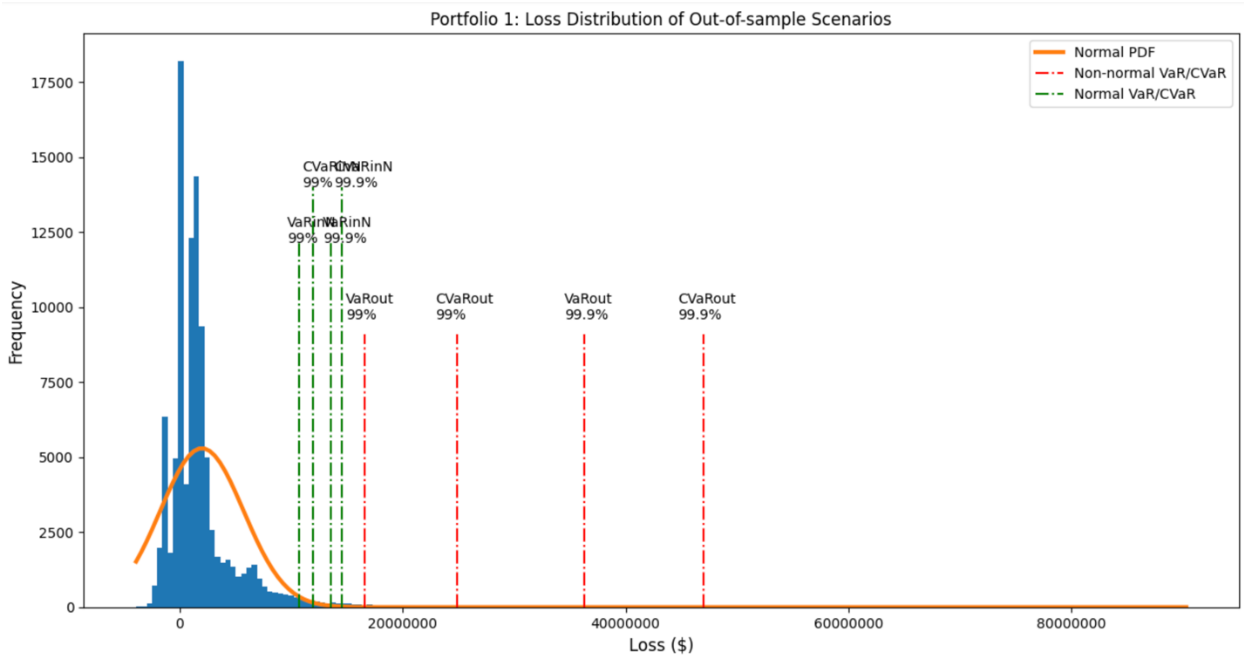
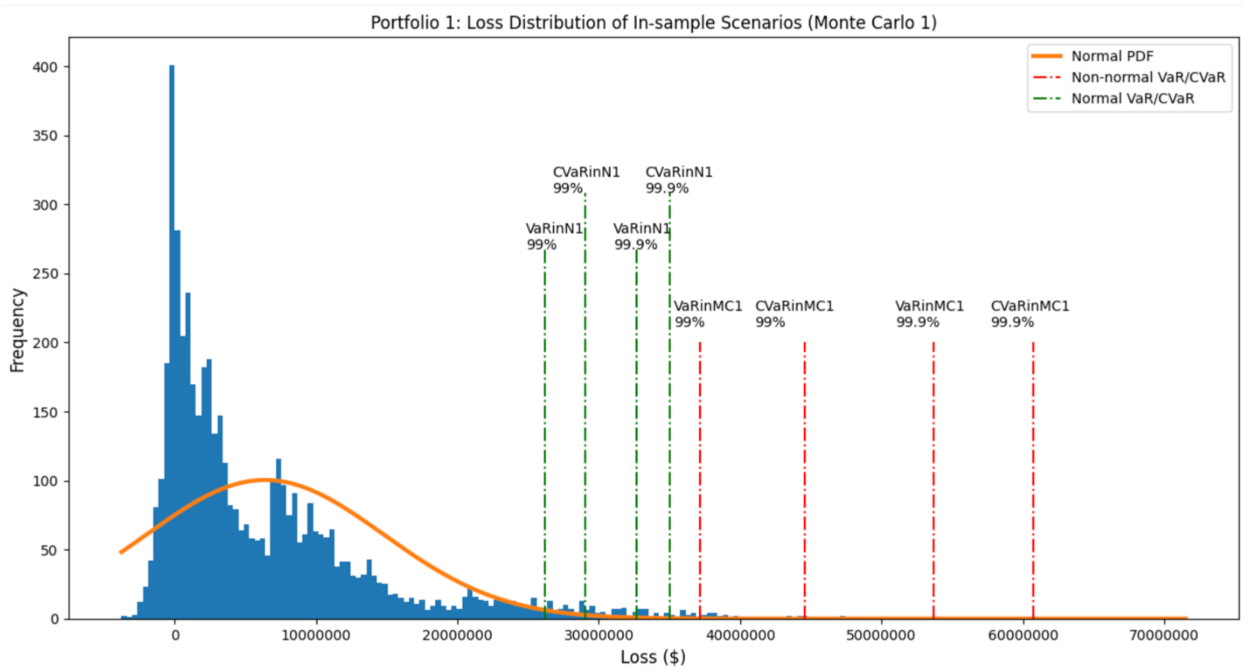
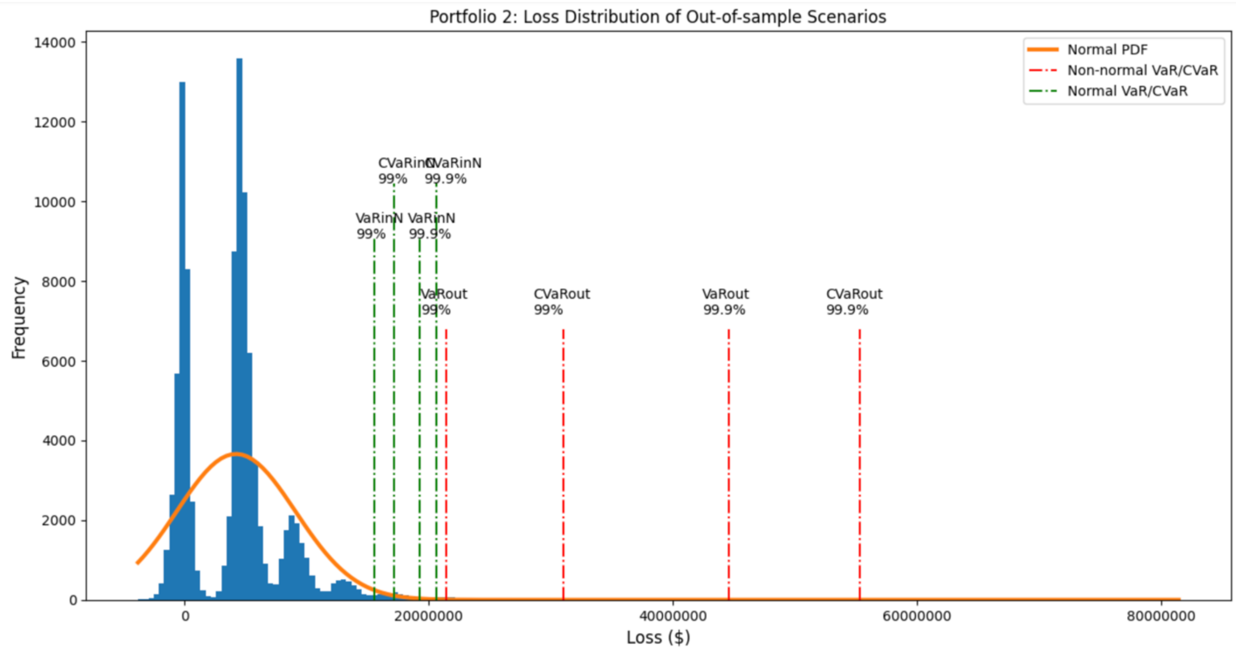
Part 2. Plots

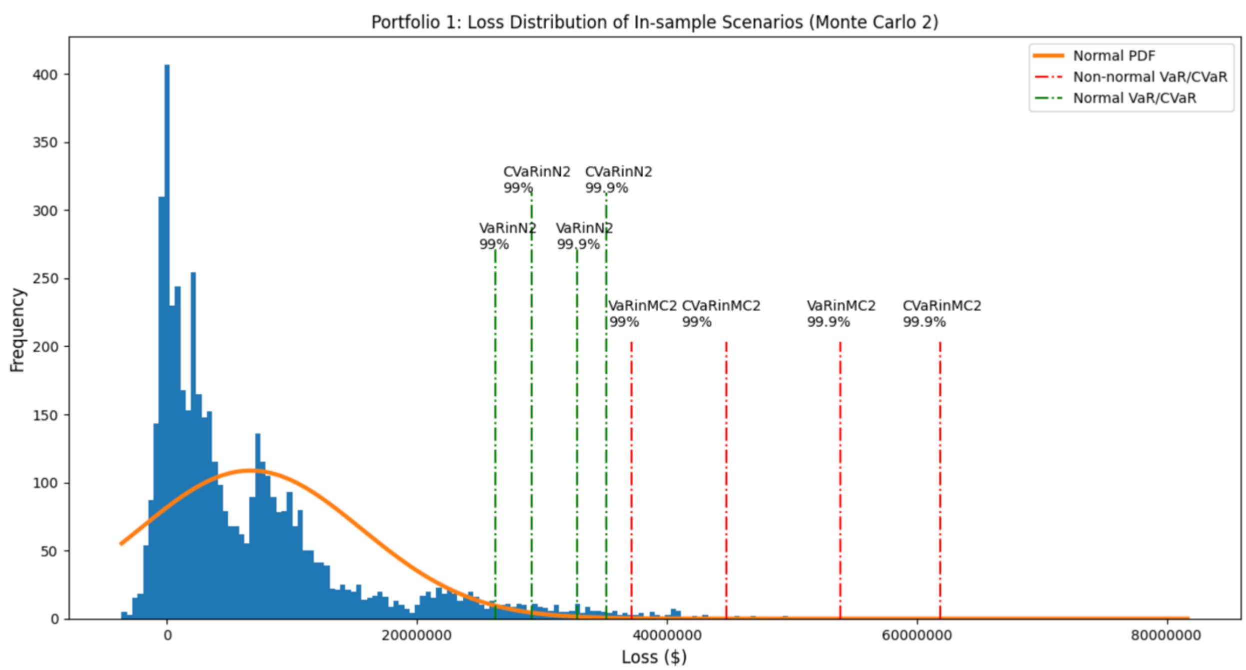


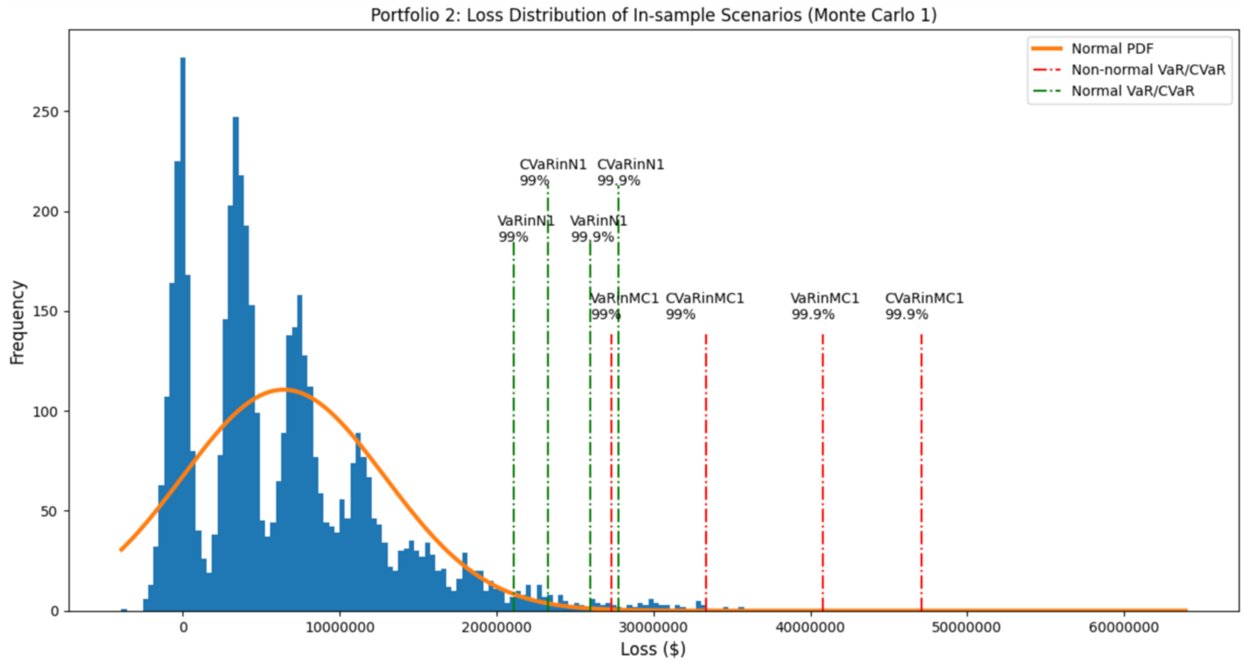
Portfolio 1: Loss Distribution of Out-of-sample Scenarios
Normal PDF
Non-normal VaR/CVaR
Normal VaR/CVaR
CVaRinN 99%
CVaRinN 99.9%
VaRinN 99%
VaRinN 99.9%
VaRout 99%
CVaRout 99%
VaRout 99.9%
CVaRout 99.9%
Frequency
Loss ($)

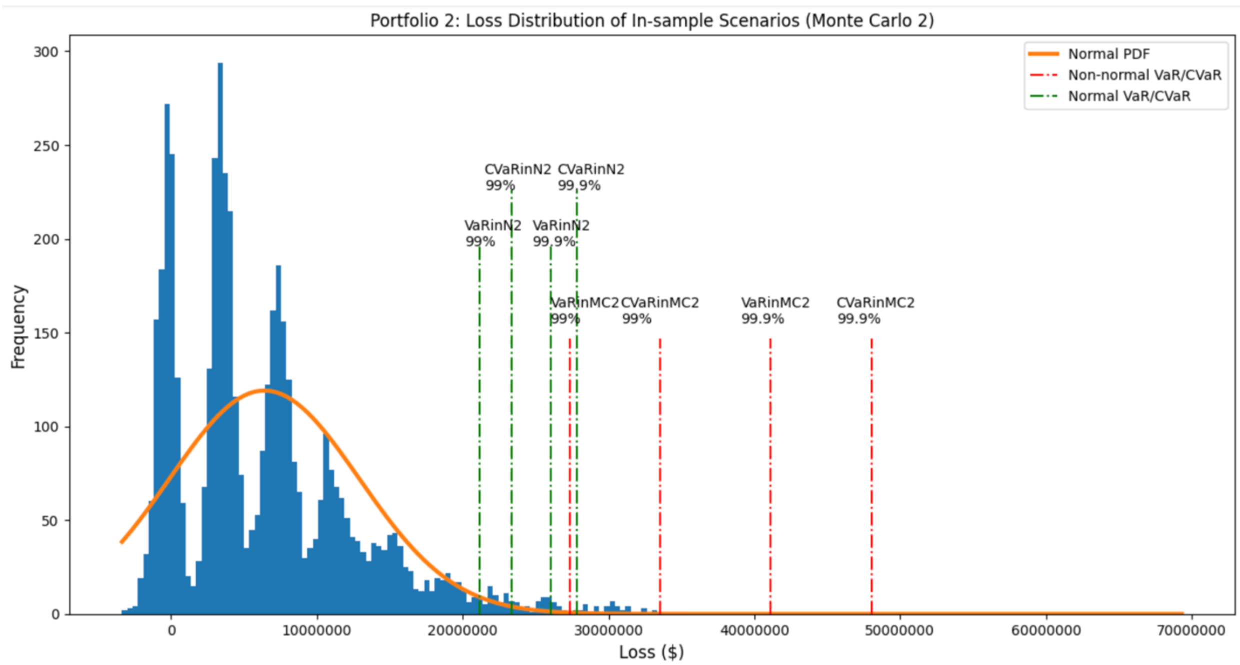


Portfolio 1: Loss Distribution of In-sample Scenarios (Monte Carlo 1)
Normal PDF
Non-normal VaR/CVaR
Normal VaR/CVaR
CVaRinN1 99%
CVaRinN1 99.9%
VaRinN1 99%
VaRinN1 99.9%
VaRinMC1 99%
CVaRinMC1 99%
VaRinMC1 99.9%
CVaRinMC1 99.9%
Frequency
Loss ($)


Portfolio 2: Loss Distribution of Out-of-sample Scenarios
Normal PDF
Non-normal VaR/CVaR
Normal VaR/CVaR
CVaRinN 99%
CVaRinN 99.9%
VaRinN 99%
VaRinN 99.9%
VaRout 99%
CVaRout 99%
VaRout 99.9%
CVaRout 99.9%
Frequency
Loss ($)
0
2000
4000
6000
8000
10000
12000
14000
20000000
40000000
60000000
80000000


Portfolio 1: Loss Distribution of In-sample Scenarios (Monte Carlo 2)
Normal PDF
Non-normal VaR/CVaR
Normal VaR/CVaR
CVaRinN2 99%
CVaRinN2 99.9%
VaRinN2 99%
VaRinN2 99.9%
VaRinMC2 99%
CVaRinMC2 99%
VaRinMC2 99.9%
CVaRinMC2 99.9%
Frequency
Loss ($)
0
50
100
150
200
250
300
350
400
20000000
40000000
60000000
80000000





Part 3. Sampling Error and Model Error

3.1 Mean and Standard Deviation of Losses

	Mean	Standard Deviation
Out of sample, Portfolio 1	$2,010,143.53	$3,734,234.11
MC1, Portfolio 1	$6,410,275.18	$8,386,405.34
MC2, Portfolio 1,	$6,696,558.92	$8,865,993.64
Out-of-sample, Portfolio 2	$4,216,482.87	$4,866,142.65
MC1, Portfolio 2	$6,421,765.88	$6,447,904.82
MC2Portfolio 2	$6,410,945.89	$6,493,104.97

3.2 Sampling Error and Model Error

	In-sample value	True distribution value	Sampling Error (% difference)
99% VaR, MC1, Portfolio 1	$37,157,125.00	$16,586,775.72	124.02%
99% CVaR, MC1, Portfolio 1	$44,585,575.95	$24,924,774.02	118.53%
99% VaR, MC2, Portfolio 1	$37,205,997.16	$16,586,775.72	124.31%
99% CVaR, MC2, Portfolio 1	$44,738,986.15	$24,924,774.02	119.46%
99.9% VaR, MC1, Portfolio 1	$53,653,413.04	$36,324,206.11	47.71%
99.9% CVaR, MC1, Portfolio 1	$60,726,265.15	$47,018,608.95	37.74%
99.9% VaR, MC2, Portfolio 1	$53,870,879.64	$36,324,206.11	48.31%
99.9% CVaR, MC2, Portfolio 1	$61,910,930.15	$47,018,608.95	41.00%
99% VaR, MC1, Portfolio 2	$27,328,612.72	$21,454,034.38	27.38%
99% CVaR, MC1, Portfolio 2	$33,370,474.01	$31,009,403.61	11.01%
99% VaR, MC2, Portfolio 2	$27,384,298.11	$21,454,034.38	27.64%
99% CVaR, MC2, Portfolio 2	$33,521,314.15	$31,009,403.61	11.71%
99.9% VaR, MC1, Portfolio 2	$40,786,445.60	$44,623,790.06	-8.60%
99.9% CVaR, MC1, Portfolio 2	$47,059,831.89	$55,298,156.25	-18.46%
99.9% VaR, MC2, Portfolio 2	$41,136,877.25	$44,623,790.06	-7.81%
99.9% CVaR, MC2, Portfolio 2	$48,020,278.96	$55,298,156.25	-16.31%

By looking at the sampling error table above, we conclude that portfolio 1 has much larger sampling error with even some of them exceed 100% compared to portfolio 2. This means the model for portfolio 1 overestimates VaR and CvaR of the true distribution while the 99.9% (higher quantiles) model for portfolio 2 underestimates the VaR and CvaR of the true distribution.

	Normal model value	True distribution value	Model Error (% difference)
99% VaR, N1, Portfolio 1	$26,169,819.11	$16,586,775.72	57.78%
99% CVaR, N1, Portfolio 1	$29,055,129.69	$24,924,774.02	24.90%
99% VaR, N2, Portfolio 1	$26,265,644.88	$16,586,775.72	58.35%
99% CVaR, N2, Portfolio 1	$29,162,957.82	$24,924,774.02	25.55%
99.9% VaR, N1, Portfolio 1	$32,673,987.65	$36,324,206.11	-10.05%
99.9% CVaR, N1, Portfolio 1	$35,031,320.16	$47,018,608.95	-33.00%
99.9% VaR, N2, Portfolio 1	$32,796,869.59	$36,324,206.11	-9.71%
99.9% CVaR, N2, Portfolio 1	$35,164,008.17	$47,018,608.95	-32.64%
99% VaR, N1, Portfolio 2	$21,109,302.85	$21,454,034.38	-1.61%
99% CVaR, N1, Portfolio 2	$23,275,103.97	$31,009,403.61	-36.05%
99% VaR, N2, Portfolio 2	$21,177,846.08	$21,454,034.38	-1.29%
99% CVaR, N2, Portfolio 2	$23,351,474.27	$31,009,403.61	-35.69%
99.9% VaR, N1, Portfolio 2	$25,991,527.88	$44,623,790.06	-41.75%
99.9% CVaR, N1, Portfolio 2	$27,761,012.74	$55,298,156.25	-61.71%
99.9% VaR, N2, Portfolio 2	$26,077,715.17	$44,623,790.06	-41.56%
99.9% CVaR, N2, Portfolio 2	$27,853,594.84	$55,298,156.25	-61.50%

By looking at the model error table above, we conclude that the 99.9% Normal model for portfolio 1 underestimates the VaR and CVaR of the true distribution while the 99% Normal model overestimates the VaR and CVaR of the true distribution. Besides, for portfolio 2, all models have an underestimation. The level of overestimation of 99% Normal model for portfolio 1 is lower here compared to the sampling error table above.

Part 4. Discuss possible strategies for minimizing impacts of sampling and model errors

If I report the estimated result to decision-makers, the consequences will vary between overestimate and underestimate of VaR and CVaR. If the result is an overestimate, the team will become conservative and has high capital reserves to deal with the risk, which reduces bank's capital efficiency. If the result is an underestimate, the bank will face liquidity crisis and insufficient capital buffers.

To minimize sampling error, we should first have a large scenario size (more systemic and idiosyncratic paths), which reduces the variance. Besides, we can use the stratified sampling method to divide the input space into strata, which also reduces the variance. Applying importance sampling also reduces the error by getting more samples on the tail events.

To minimize model error, we should use tools like EVT to model the distribution of the tail events (extreme losses) beyond a threshold. Hybrid model method is also useful by combining Monte Carlo simulation with analytical stress scenarios.