MIE 1622H: Assignment 4 – Asset Pricing

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Part 1. Pricing Functions Results

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Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.0413073720776
One-step MC price of an European put option is 7.8826861104748644
Multi-step MC price of an European call option is 7.998946194407457
Multi-step MC price of an European put option is 7.912290243831642
One-step MC price of an Barrier call option is 7.816279692038638
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.952106272791315
Multi-step MC price of an Barrier put option is 1.189165523964626
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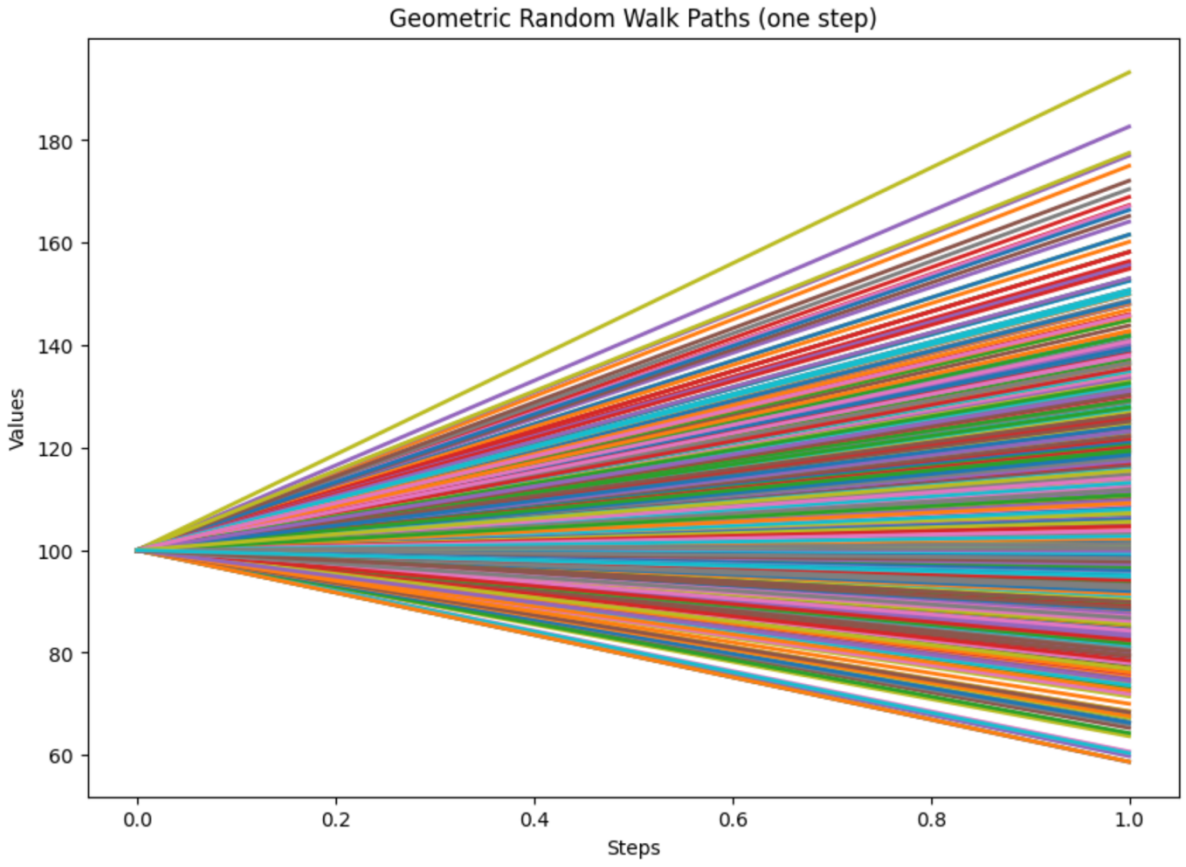
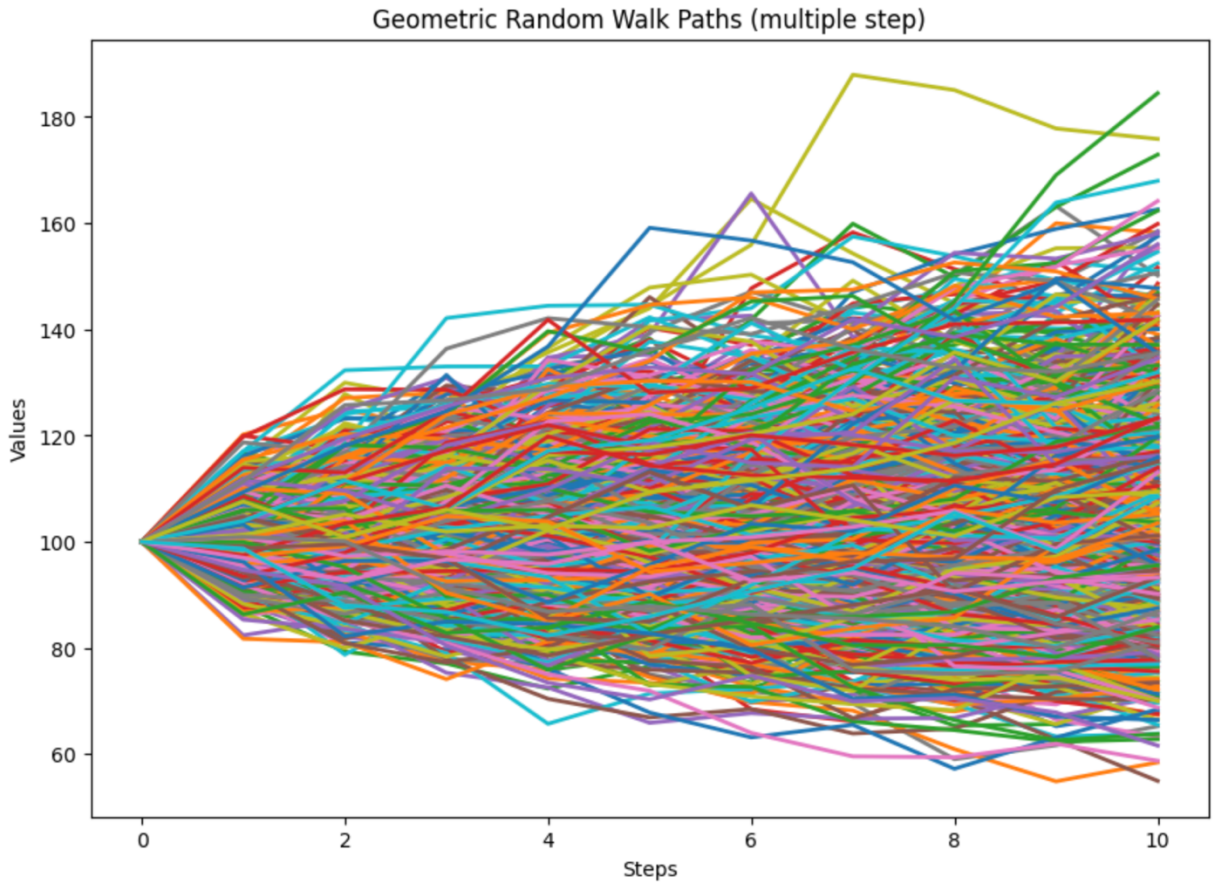
Part 2. Analyze Results





2.1 Compare three pricing strategies for European option and discuss their performance relative to each other

The Black-Scholes strategy only works on European options while assumes continuous time and no path dependency. It also requires none pay of dividend. This method is accurate and fast because it only uses terminal price.

The Monte Carlo strategy can not only work on European options. It uses GMB function to simulate the distribution of the price. The one-step method matches the Black-Scholes strategy the most closely because it simulates the final asset price directly using the distribution. It is more efficient and accurate for European options than the multi-step method. This is because the multi-step method simulates full asset price paths across multiple time steps, which is more suitable to path-dependent options like Asian options and Barrier options.

To compare one-step and multi-step methods for European options, we can find evidence in the path plots. Since the one-step method only uses terminal price, the path lines are straight. The path lines of the multi-step method are fluctuant because it is path dependent.

2.2 Explain the difference between call and put prices obtained for European and Barrier options

The Barrier options are a modified form of standard European option with a strike price determines the payoff at expiration. The knock-in option we focused on for this assignment let the option becomes a standard option if the barrier was crossed some time before expiration. It will then pay if it ends up in-the -money.

The difference between European and Barrier options can be found from the pricing function result outputs. The result of One-step MC price of a Barrier put option is 0.0 whenever the One-step MC price of a European put option is 7.893989875118342. This is because the barrier was not crossed before expiration and didn't exist. Besides, European options always have higher payoffs compared to Barrier because they are always active. In the result output, the call price for the barrier option is lower than the European call, because it only pays off if the price crosses $110 (which is above the strike). The put price for the barrier option is also lower because a put option benefits from prices below the strike. If the stock never rises high enough to breach the up barrier, the put is never activated.

Part 3. Volatility Increased and Decreased by 10%

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One-step MC price of an Barrier call option with volatility increased by 10% is 8.629321145338059
One-step MC price of an Barrier put option with volatility increased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility increased by 10% is 8.766073436055725
Multi-step MC price of an Barrier put option with volatility increased by 10% is 1.4957437612777778
One-step MC price of an Barrier call option with volatility decreased by 10% is 7.004450192196351
One-step MC price of an Barrier put option with volatility decreased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility decreased by 10% is 7.135727744048845
Multi-step MC price of an Barrier put option with volatility decreased by 10% is 0.9180588177098236
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Call and Put options both have higher price with higher volatility because the asset has more chance to cross the barrier. However, the one-step Put options remain 0 because barrier must be crossed on a single jump, which is very hard (a 10% increase in volatility is not enough).

Besides, the Call and Put options both have lower price with decrease in volatility because the asset has less chance to cross the barrier.

Part 4. Discuss possible strategies to obtain the same prices from two procedures

Choosing a number of time steps and a number of scenarios in Monte Carlo pricing for European option to get the same price (up to the cent) as given by the Black-Scholes formula:

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Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
Closest MC price of call within 0.01 is 8.0172614674741 using number of paths 30000
Closest MC price of put within 0.01 is 7.89288456733141 using number of paths 700000
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The time step is set to be 1 because Black-Scholes is the strategy only uses the terminal price. Setting the time step equals to 1 makes the MC strategy also only uses the terminal price, which is the closest way to the Black Scholes. After running the code, the optimal path number for the MC price of call option is 30000 and for the MC price of put option is 700000.