11- . The Rockwell Electronics Corporation retains a service crew to repair machine break downs that occur on an average of λ = 3 per day (approximately Poisson in nature).The crew can service an average of μ = 8 machines per day, with a repair time distribution that resembles the exponential distribution.

a) What is the utilization rate of this service system?
b) What is the average downtime for a machine that is broken?
c) How many machines are waiting to be serviced at any given time?
d) What is the probability that more than one machine is in the system? Probability that more than two are broken and waiting to be repaired or being serviced? More than three? More than four?

a. The utilization rate, ρ,

$$\rho = \frac{\lambda}{\mu} = \frac{3}{8}$$

$$= 0.375$$

b. The average down time, W,

$$W = \frac{1}{\mu - \lambda} = \frac{1}{8-3} = 0.2 \text{ day}$$

c. The number of machines waiting to be served, L_q, is, on average,

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{3^2}{8(8-3)} = 0.225 \text{ machine waiting}$$

d. Probability that more than one machine is in the system

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1} \quad P_{n>1} = \left(\frac{3}{8}\right)^2 = \frac{9}{64} = 0.141$$

Probability that more than two machines are in the system:

$$P_{n>2} = \left(\frac{3}{8}\right)^3 = \frac{27}{512} = 0.053$$

$$P_{n>3} = \left(\frac{3}{8}\right)^4 - \frac{81}{4,096} = 0.020$$

$$P_{n>4} = \left(\frac{3}{8}\right)^5 = \frac{243}{32,768} = 0.007$$

12. From historical data, Harry's Car Wash estimates that dirty cars arrive at the rate of 10 per hour all day Saturday. With a crew working the wash line, Harry figures that cars can be cleaned at the rate of one every 5 minutes. One car at a time is cleaned in this example of a single-channel waiting line. Assuming Poisson arrivals and exponential service times, find
(a) Average number of cars in line.
(b) Average time a car waits before it is washed.
(c) Average time a car spends in the service system.
(d) Utilization rate of the car wash.
(e) Probability that no cars are in the system

$\lambda = 10$ cars/hour, $\mu = 12$ cars/hour.

a. The average number of cars in line, L_q,

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{12(12-10)} = \frac{10^2}{(12)(2)} = 4.167 \text{ cars}$$

b. The average time a car waits before it is washed, W_q, is given by

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} - \frac{10}{12(12-10)} = \frac{10}{(12)(2)} = 0.4167 \text{ hour}$$

c. The average time a car spends in the service system, W, is given by

$$W = \frac{1}{\mu-\lambda} = \frac{1}{12-10} = \frac{1}{2} \text{ hour}$$

d. The utilization rate, ρ, is given by

$$\rho = \frac{\lambda}{\mu} = \frac{10}{12} = 0.8333$$

e. The probability that no cars are in the system, P_0, is given by:

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho = 1 - 0.8333 = 0.1667$$

13-Mike Dreskin manages a large Los Angeles movie theater complex called Cinema I, II, III and IV. Each of the four auditoriums plays a different film; the schedule is set so that starting times are staggered to avoid the large crowds that would occur if all four movies started at the same time. The theater has a single ticket booth and a cashier who can maintain an average service rate of 280 movie patrons per hour. Service times are assumed to follow an exponential distribution. Arrivals on a typically active day are Poisson distributed and average 210 per hour. To determine the efficiency of the current ticket operation, Mike wishes to examine several queue operating characteristics.

a) Find the average number of moviegoers waiting in line to purchase a ticket.
b) What percentage of the time is the cashier busy
c) What is the average time that a customer spends in the system?
d) What is the average time spent waiting in line to get to the ticket window?
e) What is the probability that there are more than two people in the system? More than three people? More than four?

$\lambda = 210$ patrons/hour; $\mu = 280$ patrons/hour.

a. The average number of patrons waiting in line, L_q, is given by

$$Lq = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{210^2}{280(280-210)} = \frac{44{,}100}{280(70)} = \frac{44{,}100}{19{,}600} = 2.25 \text{ patrons in line}$$

b. The average fraction of time the cashier is busy, ρ, is given by

$$\rho = \frac{\lambda}{\mu} = \frac{210}{280} = 0.75$$

c. The average time a customer spends in the ticket-dispensing system, W, is given by

$$W = \frac{1}{\mu-\lambda} = \frac{1}{280-210} = \frac{1}{70} = 0.0143 \text{ hour in the line}$$

d. The average time spent by a patron waiting to get a ticket, W_q, is given by

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{210}{280(280-210)} = \frac{210}{280(70)} = 38.6 \text{ seconds}$$

e. The probability that there are more than two people in the system, $P_{n>2}$, is given by

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1} \quad P_{n>2} = \left(\frac{210}{280}\right)^3 = 0.422$$