Local Search

PREPARED BY

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Outlines

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- Terminologies
- Hill Climbing Algorithm
- Examples
- Hill Climbing Pros/Cons
- Hill Climbing Types
- Simulated Annealing

Local Search and Optimization Problems

- **Local Search** algorithms operate by searching from a start state to neighbouring states, without keeping track of the paths, nor the set of states that have been reached.
- might never explore a portion of the search space where a solution actually resides.
- solve **optimization problems**, in which the aim is to **find the best state** according to an objective function.

Key advantages:

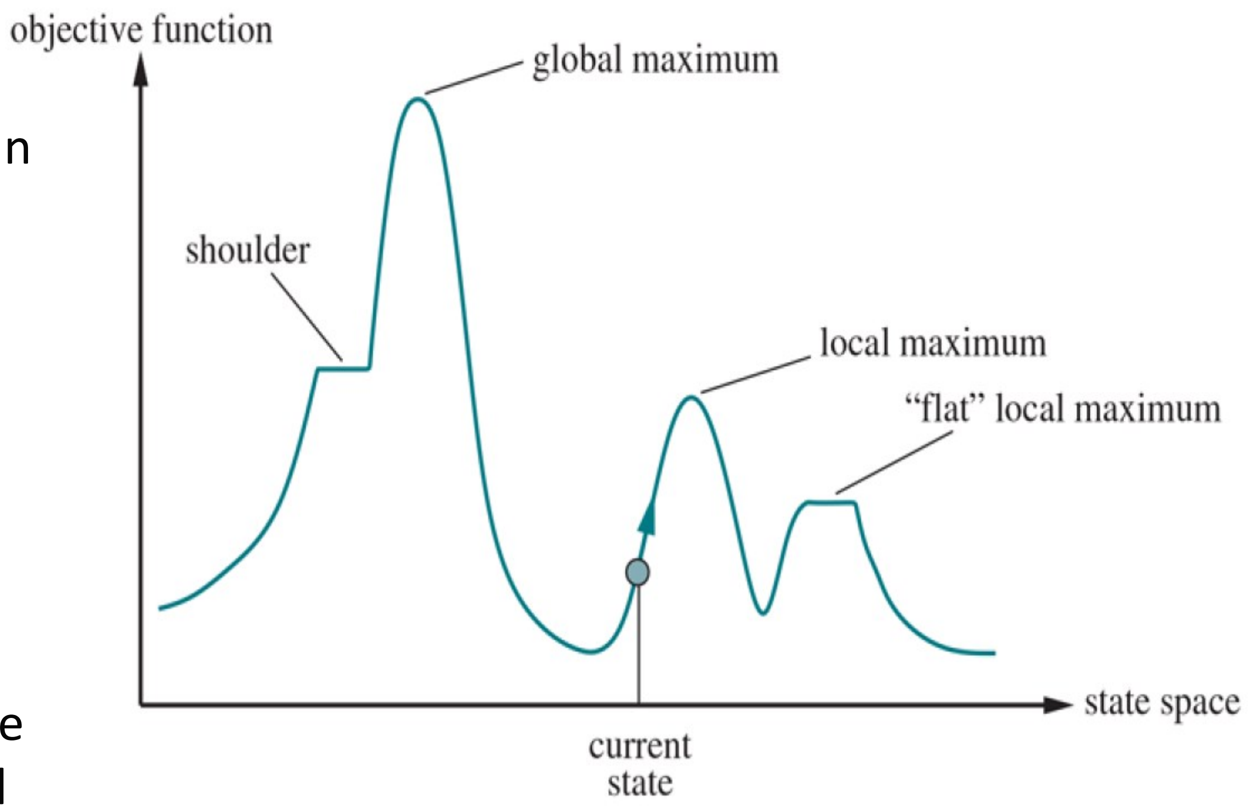
1. use very little memory
2. often find reasonable solutions in large or infinite state spaces for where systematic algorithms are unsuitable.

Local Search

- Each point (state) in the landscape has an "elevation," defined by the value of the objective function.

- If elevation corresponds to an objective function, then the aim is to find the highest peak—a global maximum—and we call the process **hill climbing**.

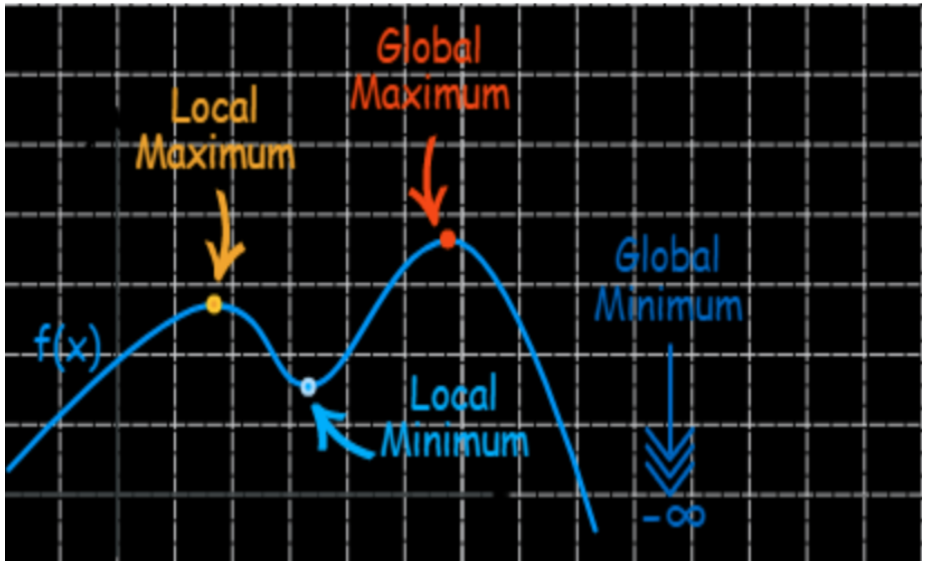
- If elevation corresponds to cost, then the aim is to find the lowest valley—a global minimum—and we call it **gradient descent**.



A one-dimensional state-space landscape in which elevation corresponds to the objective function. The aim is to find the global maximum.

Terminologies

- A high point is called a **maximum (plural maxima).**
- A low point is called a **minimum (plural minima).**
- We say **local maximum (or minimum)** when there may be higher (or lower) points elsewhere but not nearby.
- The **maximum or minimum** over the entire function is called an "**Absolute" or "Global" maximum or minimum.**



Example

Find the maxima and minima for a function:

$$y = 5x^3 + 2x^2 - 3x \qquad (1)$$

<u>**Solution:**</u>

1) Apply first derivative to equation (1)

   $y' = 15x^2 + 4x - 3 \qquad (2)$

2) Find the value of x

   $15x^2+9x-5x-3=0$

   $3x(5x+3)-1(5x+3)=0$ using factorization

   $(3x-1)(5x+3)=0$

   $x=1/3, \quad x=-3/5$

   <u>At $x=1/3$:</u>

   $y'=15(1/3)^2 +4(1/3) - 3 = 0$

   <u>At $x=-3/5$:</u>

   $y'= 15(-3/5)^2 +4(-3/5) - 3 = 0$

Example

3) Apply derivative once more to equation (2), the second derivative

$y'' = 30x + 4$ (3)

<u>At $x = -3/5$:</u>

$y'' = 30(-3/5) + 4 = -14$

it is less than 0, so **−3/5 is a local maximum**

<u>At $x = +1/3$:</u>

$y'' = 30(+1/3) + 4 = +14$

it is greater than 0, so **+1/3 is a local minimum**

Hill-Climbing Search

- The hill-climbing search algorithm keeps track of one current state and on each iteration moves to the neighbouring state with highest value.
- heads in the direction that provides the **steepest ascent**.
- terminates when it reaches a “peak” where no neighbour has a higher value.
- does not look ahead beyond the immediate neighbours of the current state.
- one way to use hill-climbing search is to use the negative of a heuristic cost function as the objective function; that will climb locally to the state with smallest heuristic distance to the goal.
- sometimes called greedy local search because it grabs a good neighbour state without thinking ahead about where to go next.

Hill-Climbing Search

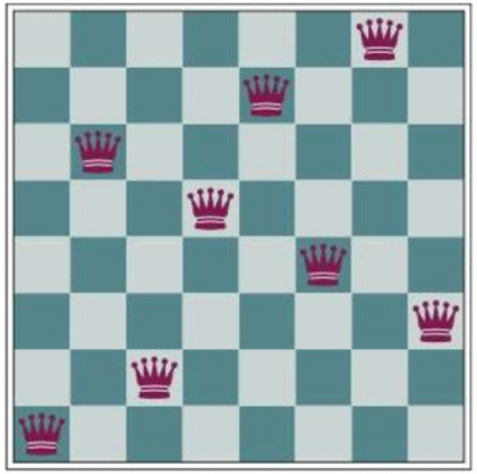
The hill-climbing search algorithm, which is the most basic local search technique. At each step the current node is replaced by the best neighbour.

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
    current ← *problem*.INITIAL
    while *true* **do**
        neighbor ← a highest-valued successor state of *current*
        if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
        current ← *neighbor*

Example

8 Queens

- The goal is to minimize the number of queens attacking each other.
- Heuristic cost estimate, h=number of queens (Q) attacking each others directly or indirectly.
- Successors are all possible states
  moving one Q in the same column.
- 8 * 7= 56 possible successors.
- (a) h=1
- (b) h= 17



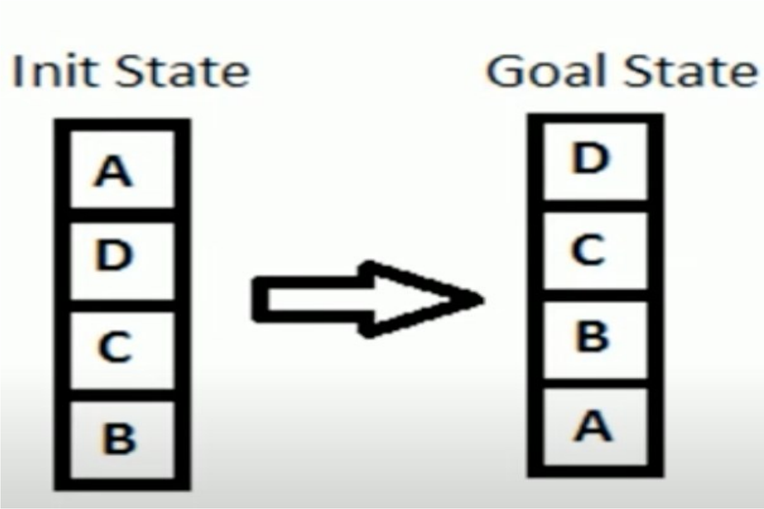
(a)

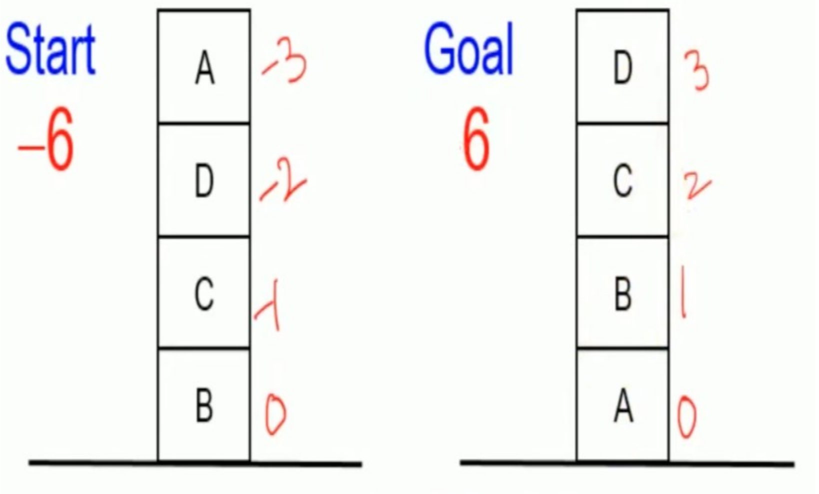
(b)

Example

4-Blocks: global heuristic function

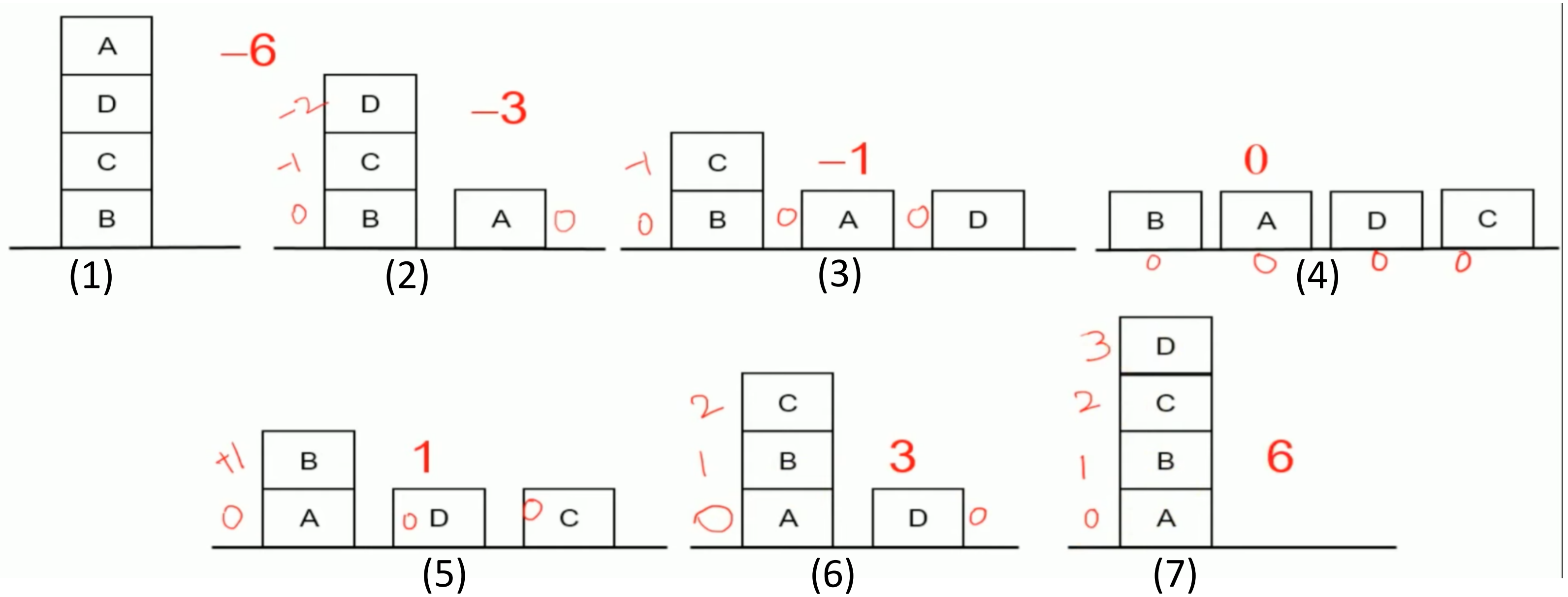
- h= +1 for all the blocks in the support structure if the block is correctly positioned
- otherwise h= -1





Example

Start with h= -6

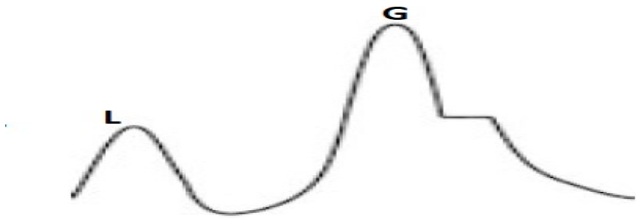


End with h=-6

Hill-Climbing Search Pros/Cons

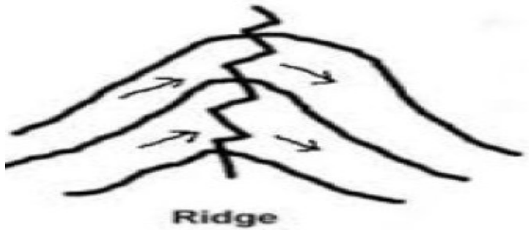
Pros:

- It can make rapid progress toward a solution because it is usually quite easy to improve a bad state.
- No backtracking



Cons:

- **LOCAL MAXIMA**: a peak that is higher than each of its neighbouring states but lower than the global maximum.
- **PLATEAUS**: a flat area of the state-space landscape. It can be a flat local maximum, from which no uphill exit exists, or a shoulder, from which progress is possible.
- **RIDGES**: result in a sequence of local maxima that is very difficult for greedy algorithms to navigate.



Hill Climbing Types

1. Stochastic hill climbing
   - chooses at random from among the uphill moves
   - the probability of selection can vary with the steepness of the uphill move
   - converges more slowly than steepest ascent
   - finds better solutions in some state
2. First-choice hill climbing
   - implements stochastic hill that generates successors randomly until one is generated that is better than the current state
   - a good strategy when a state has many (e.g., thousands) of successors.
3. Random-restart hill climbing
   - conducts a series of hill-climbing searches from randomly generated initial states, until a goal is found
   - complete with probability 1, because it will eventually generate a goal state as the initial state

Simulated Annealing

- Key Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- Take some uphill steps to escape the local minimum
- Instead of picking the best move, it picks a random move
- If the move improves the situation, it is executed. Otherwise, move with some probability less than 1.
- Physical analogy with the annealing process:
  - Allowing liquid to gradually cool until it freezes
- The heuristic value is the energy, E
- Temperature parameter, T, controls speed of convergence.

Simulated Annealing

- **Basic inspiration**: What is annealing?
  - In mettallurgy, annealing is the physical process used to temper or harden metals or glass by heating them to a high temperature and then gradually cooling them, thus allowing the material to coalesce into a low energy cristalline state.
    - **Heating then slowly cooling a substance to obtain a strong cristalline structure.**
- **Key idea:** Simulated Annealing combines Hill Climbing with a random walk in some way that yields both efficiency and completeness.
- Used to solve VLSI layout problems in the early 1980

Simulated Annealing

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow$ 1 **to** ∞ **do**
    $T \leftarrow$ *schedule*[t]
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Simulated Annealing

Temperature T

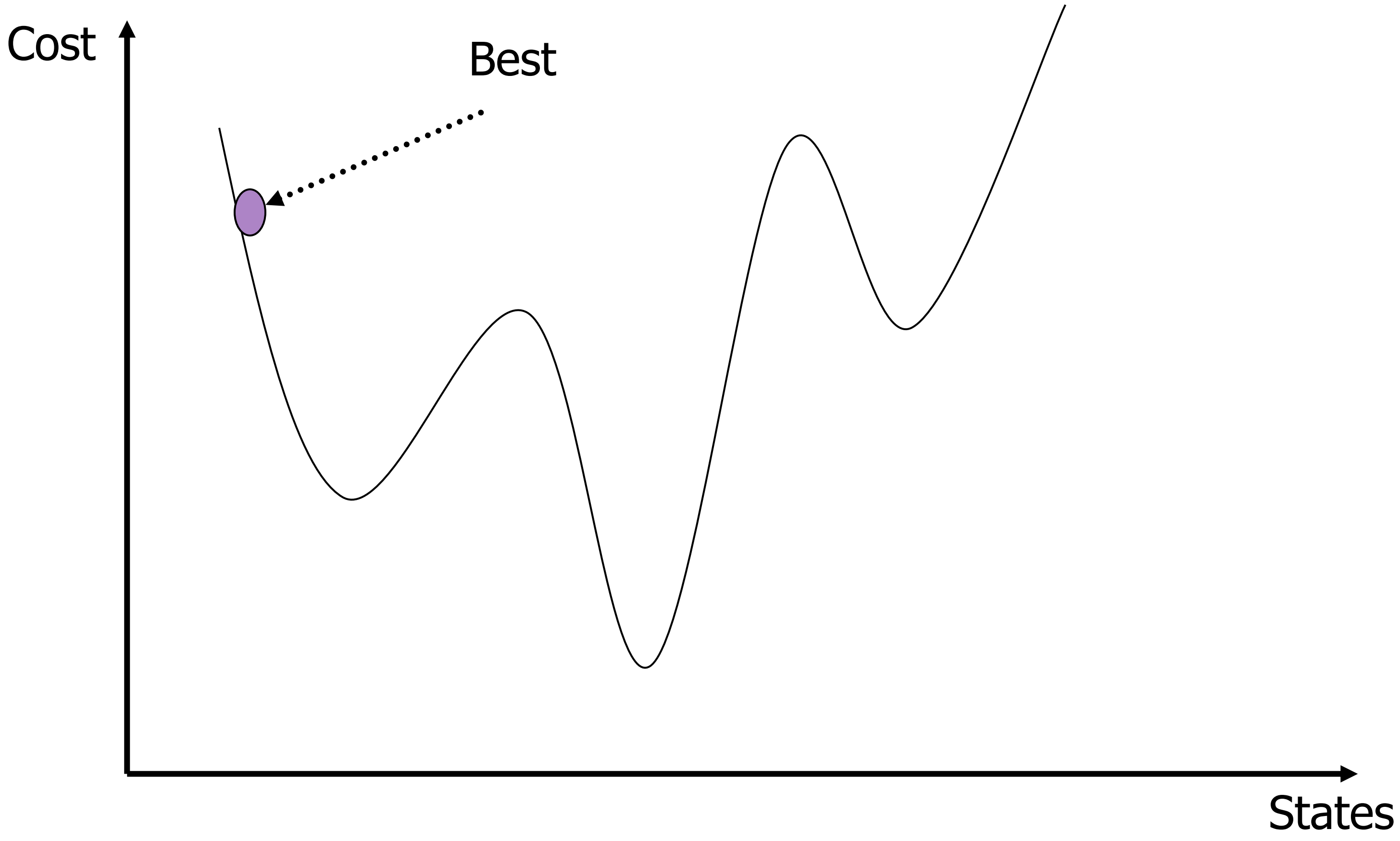
- Used to determine the probability
- High T : large changes
- Low T : small changes

Cooling Schedule

- Determines rate at which the temperature T is lowered
- Lowers T slowly enough, the algorithm will find a global optimum

In the beginning, aggressive for searching alternatives, become conservative when time goes by

Simulated Annealing



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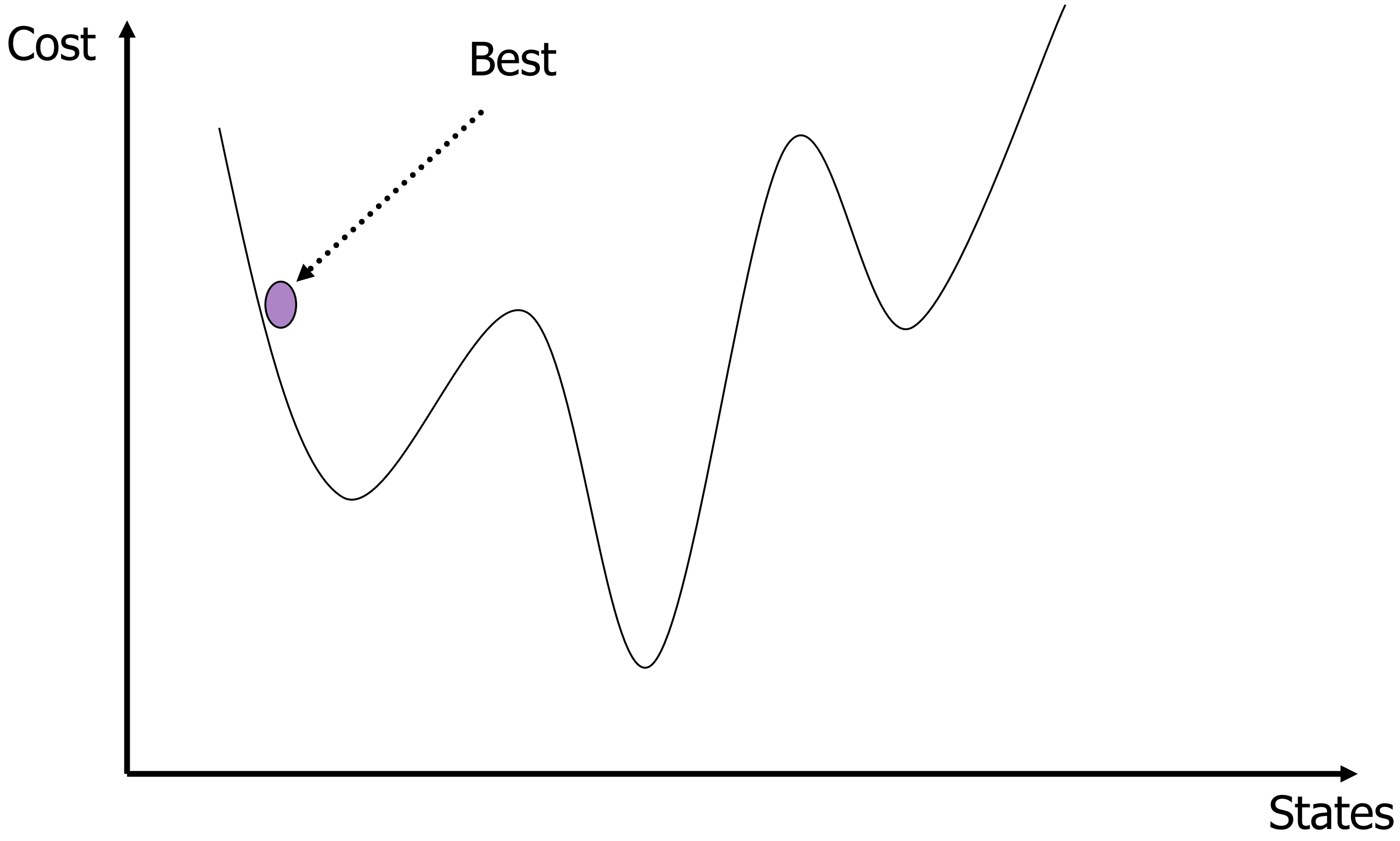

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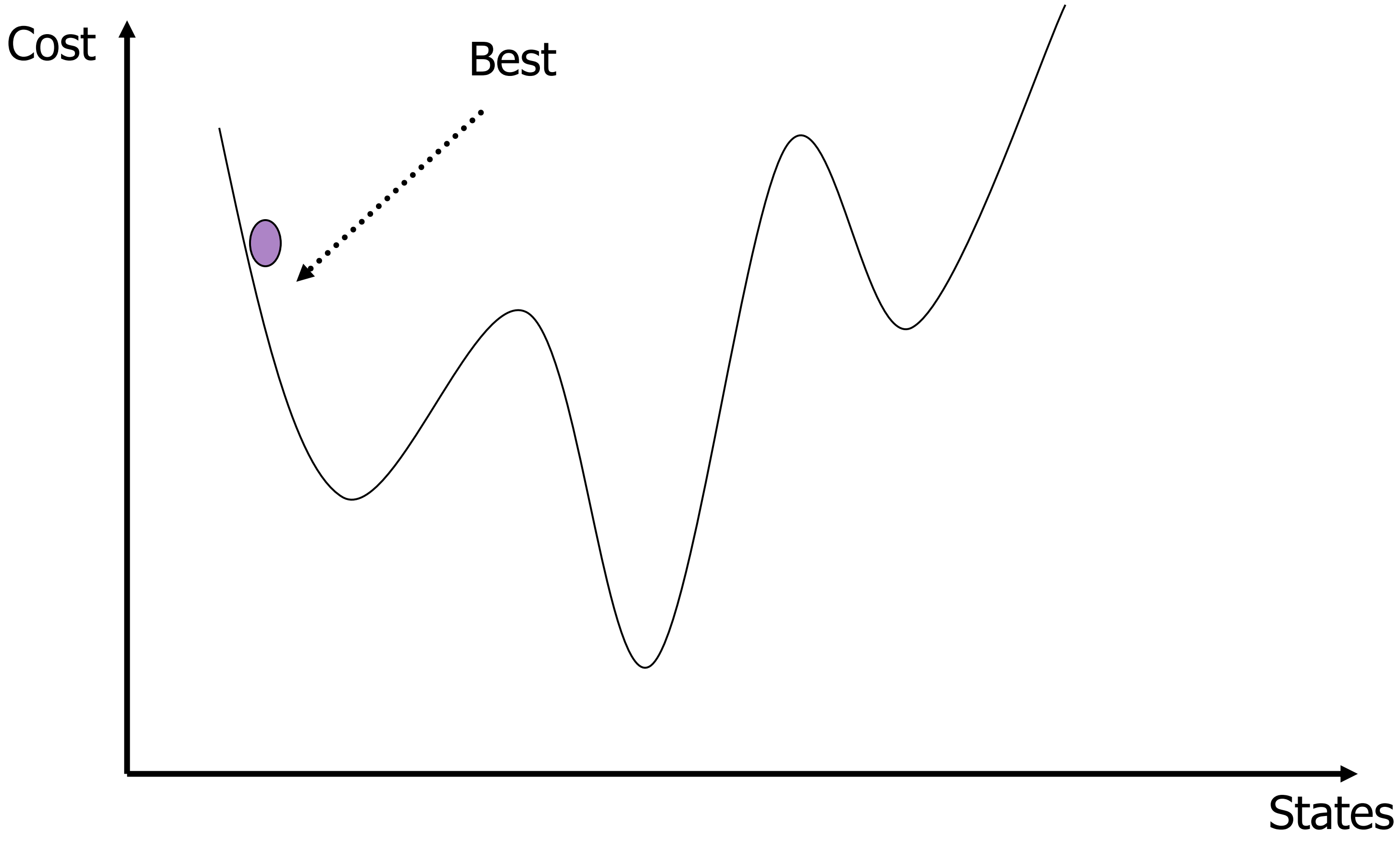

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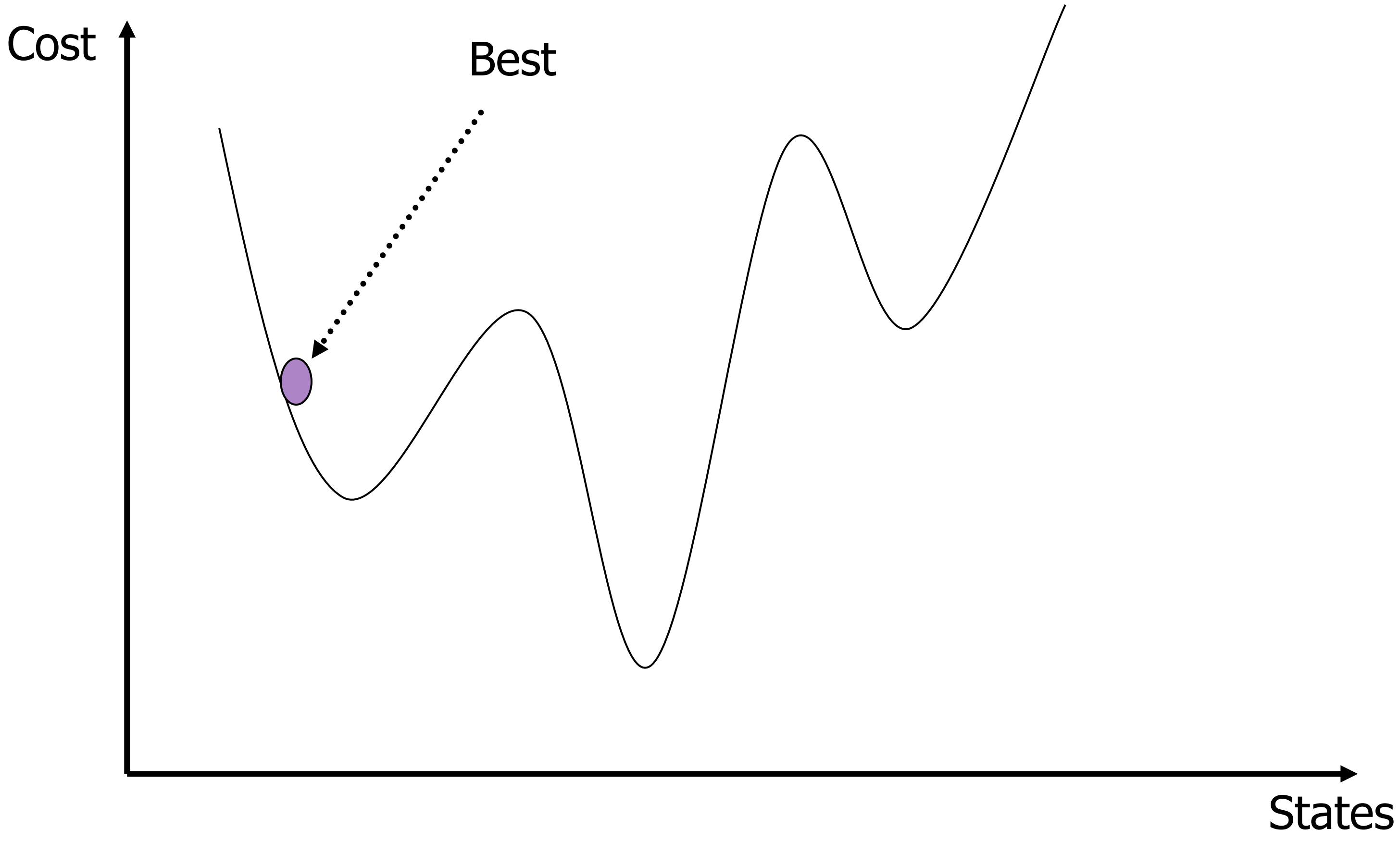

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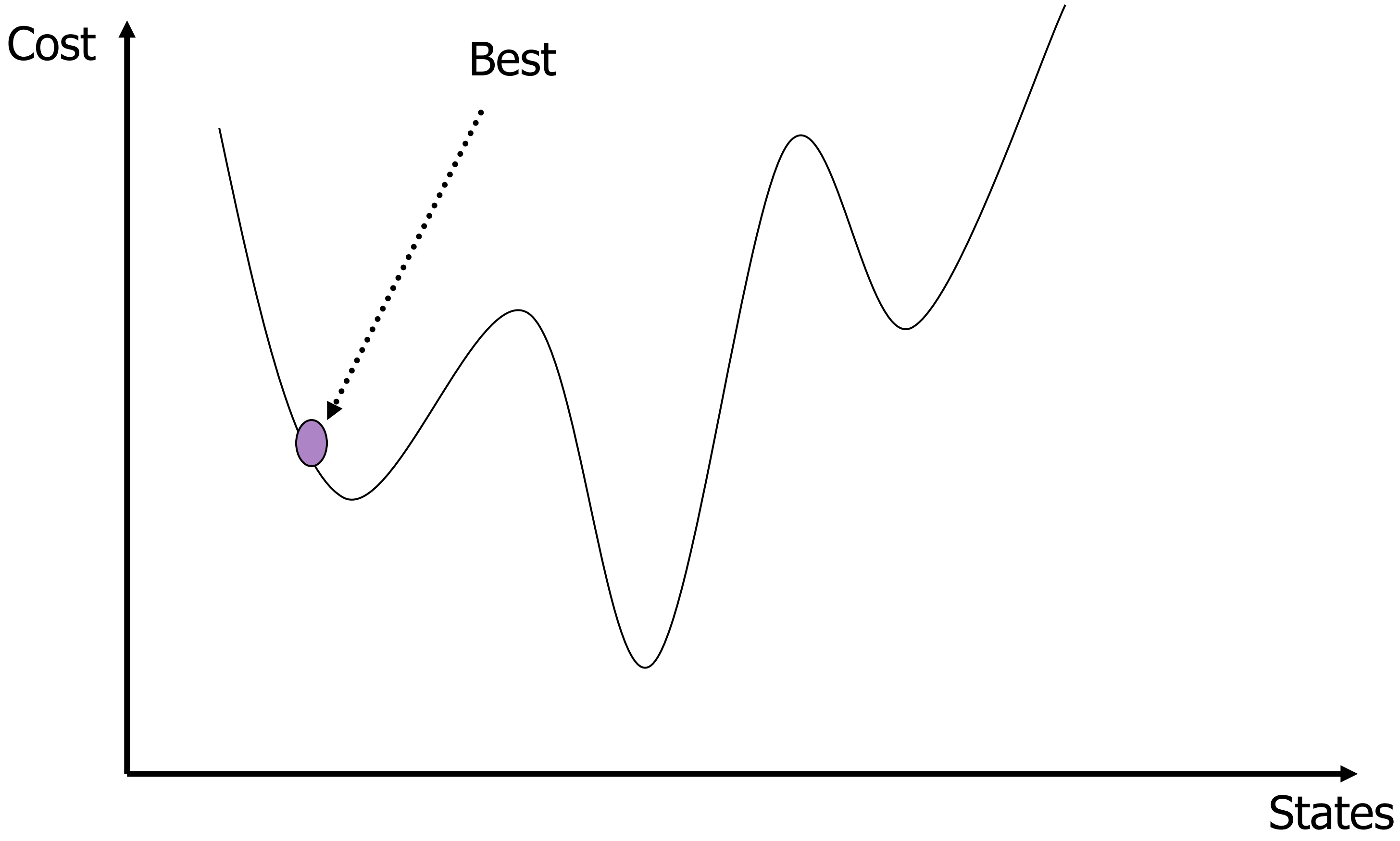

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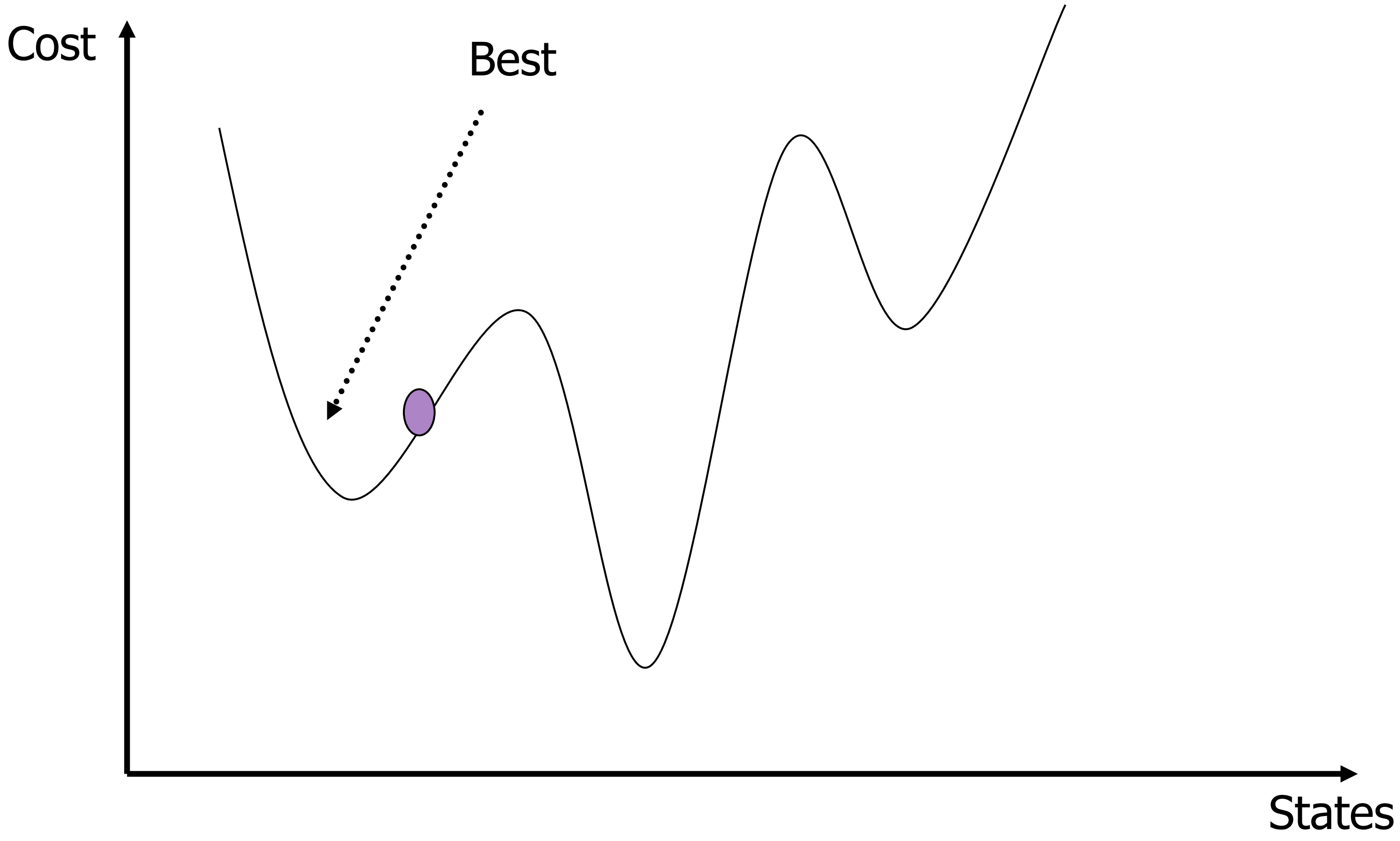

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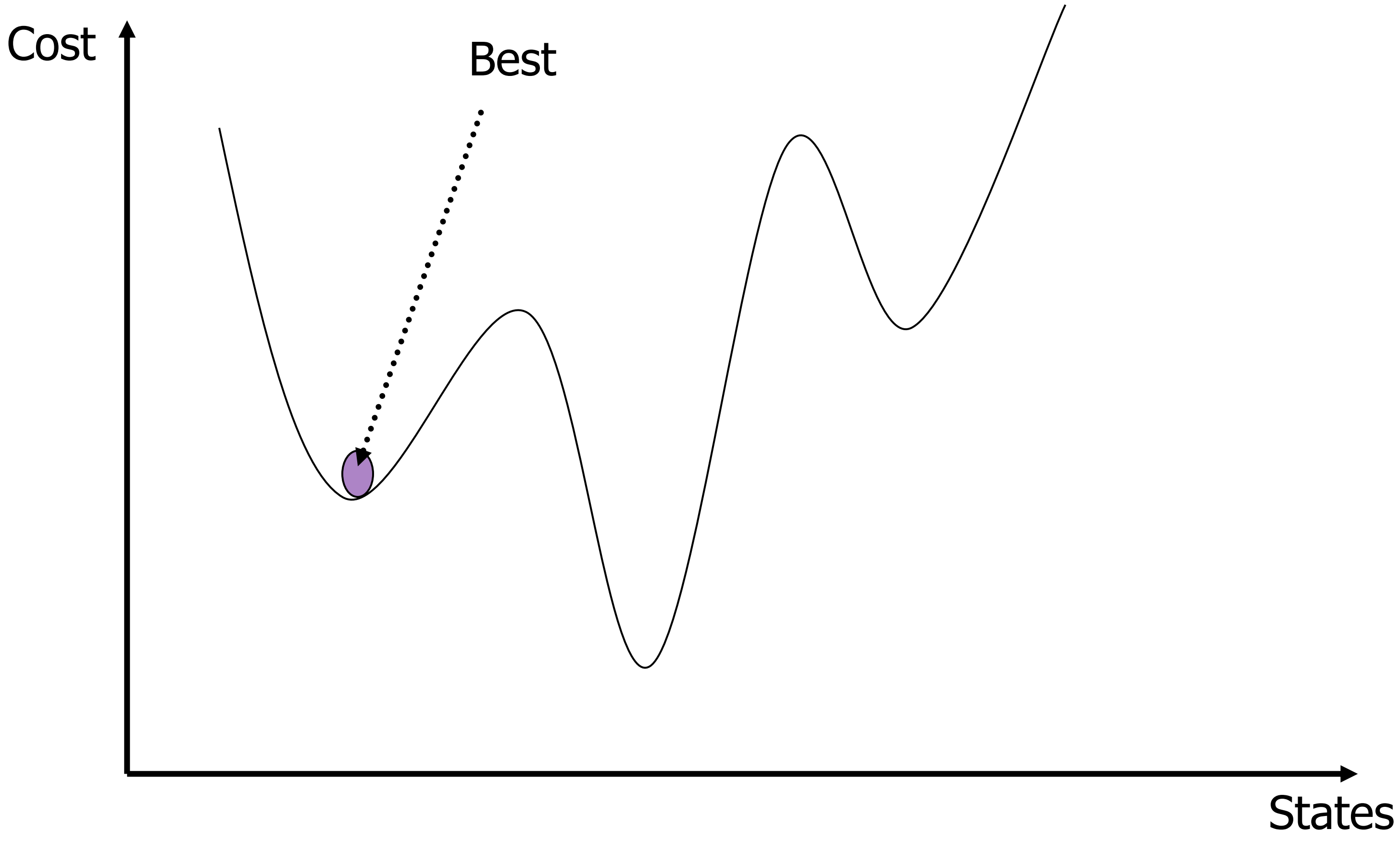

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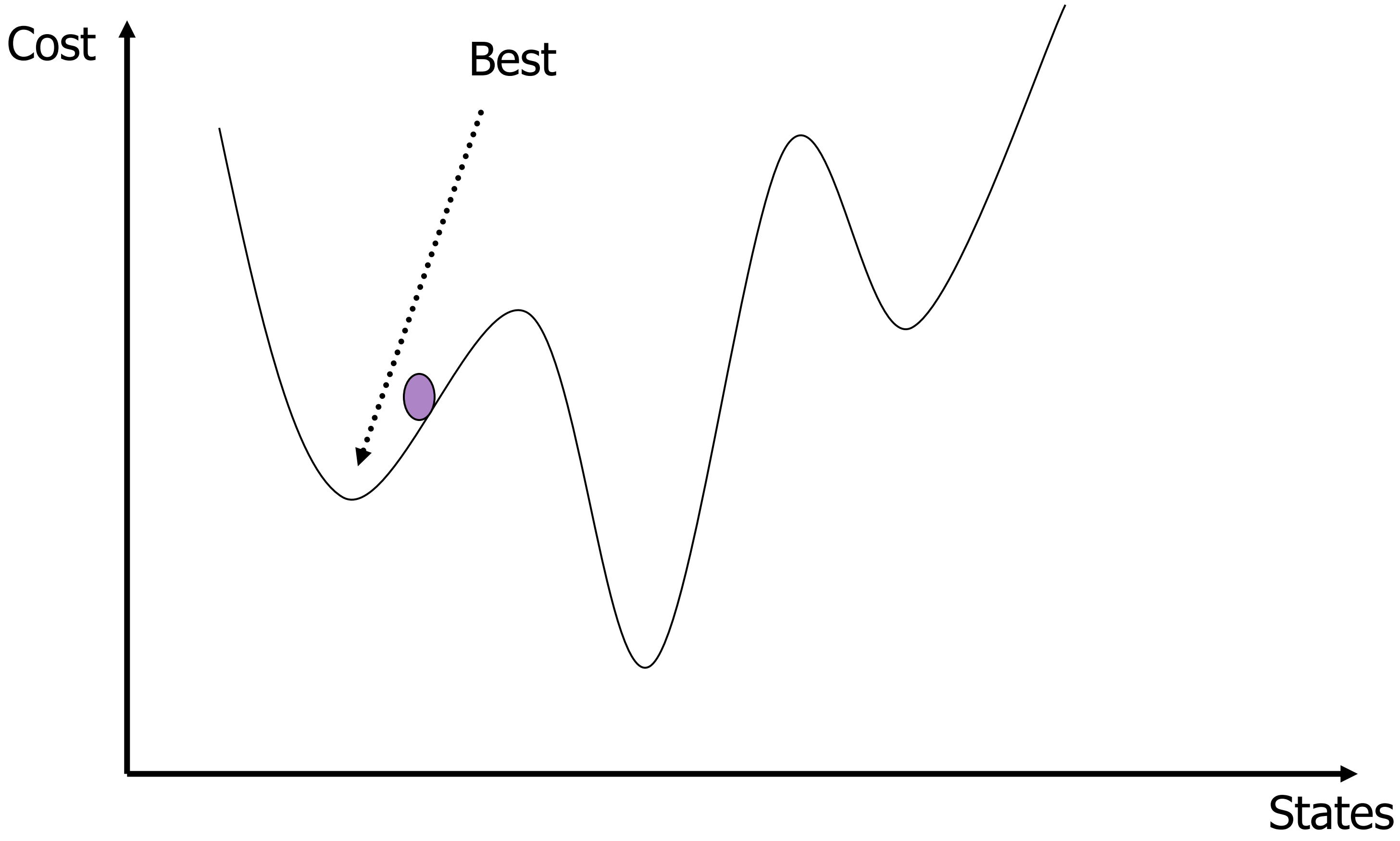

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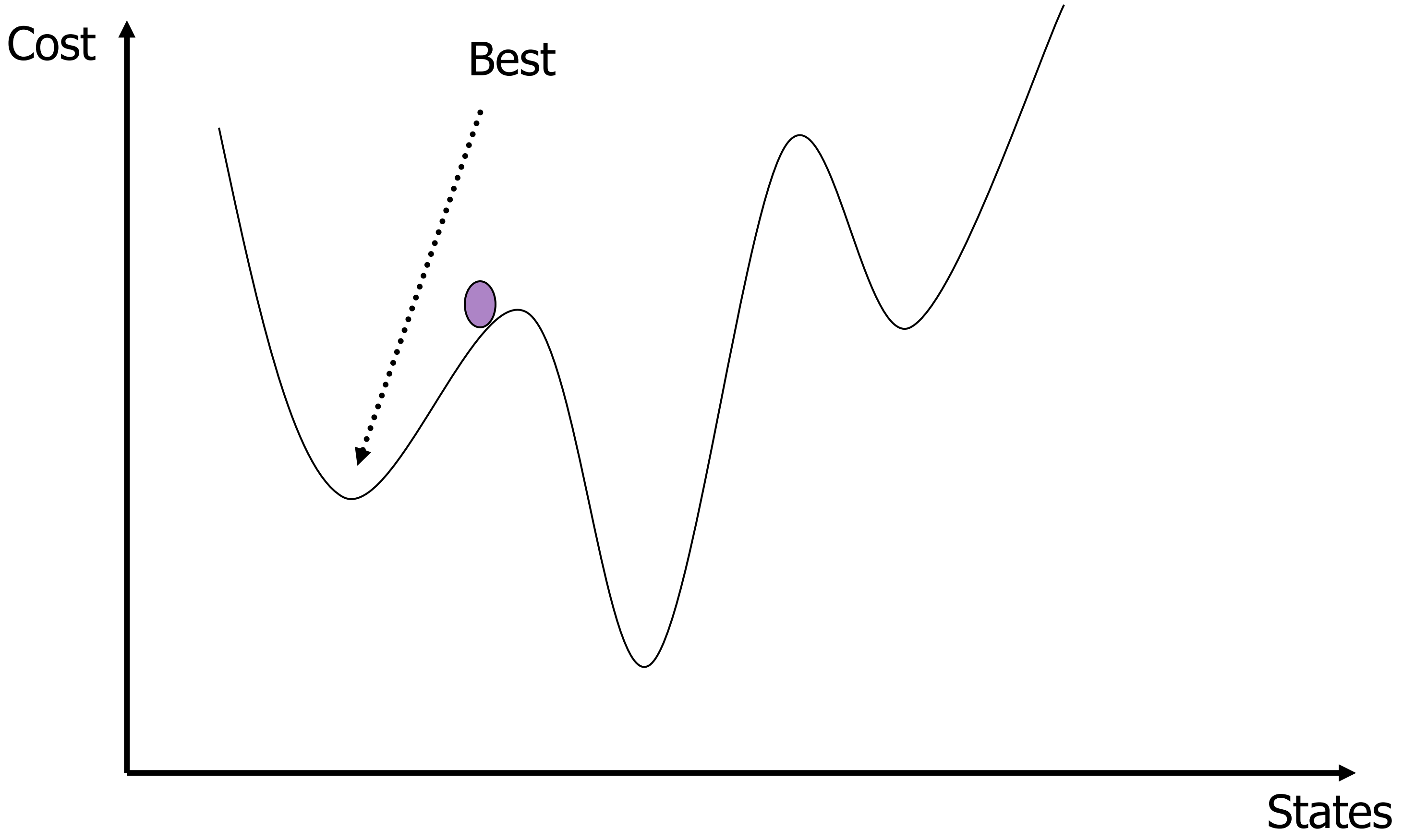

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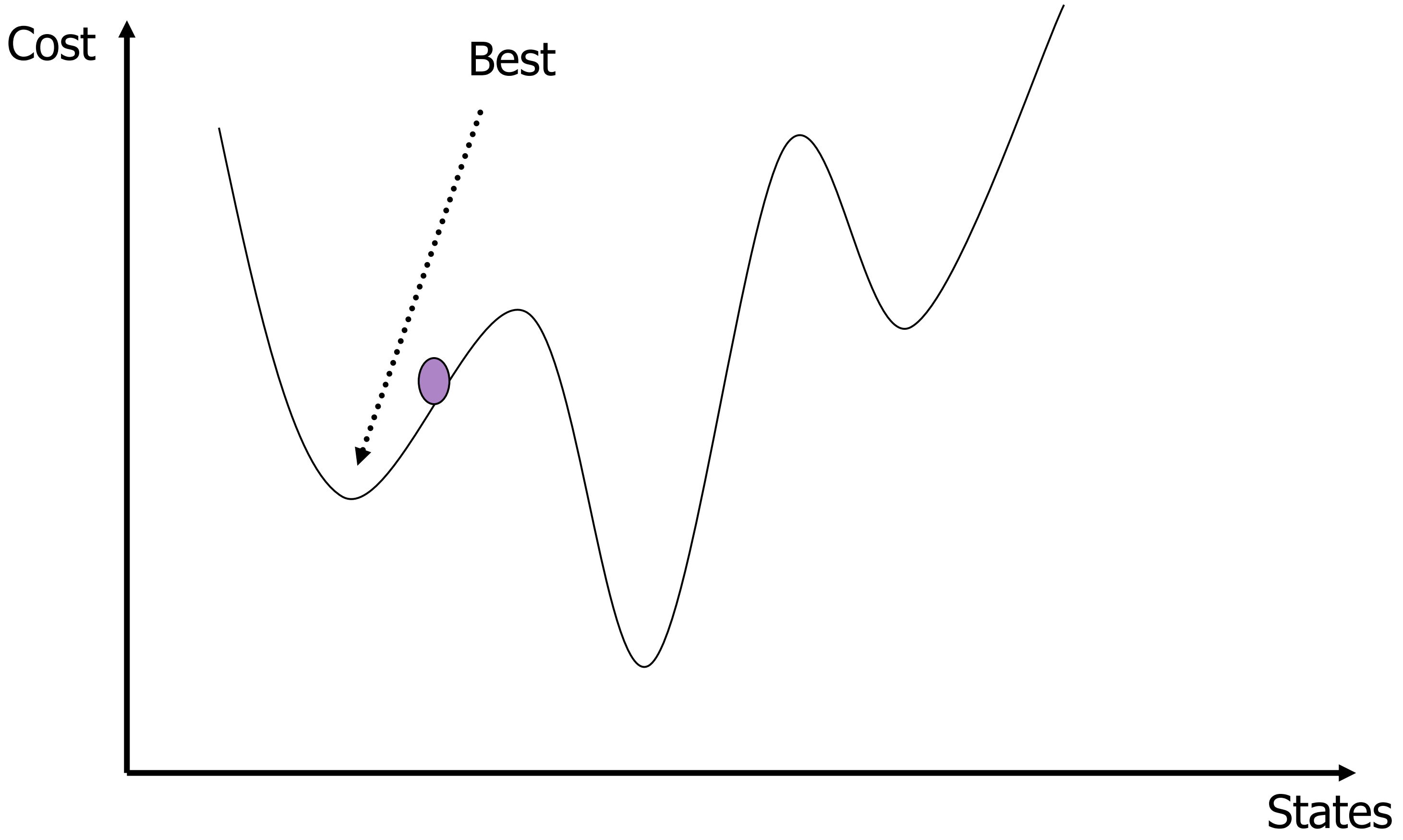

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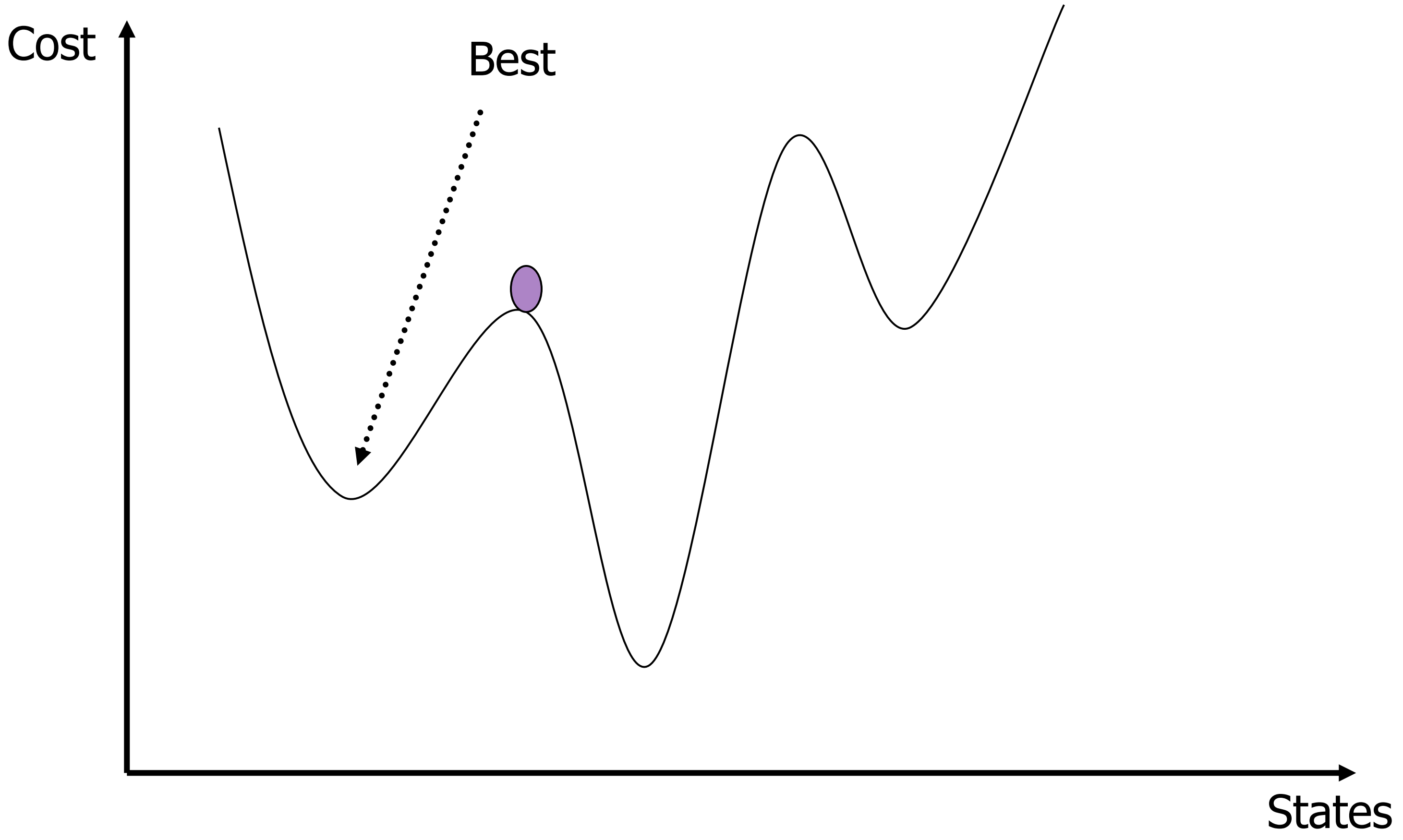

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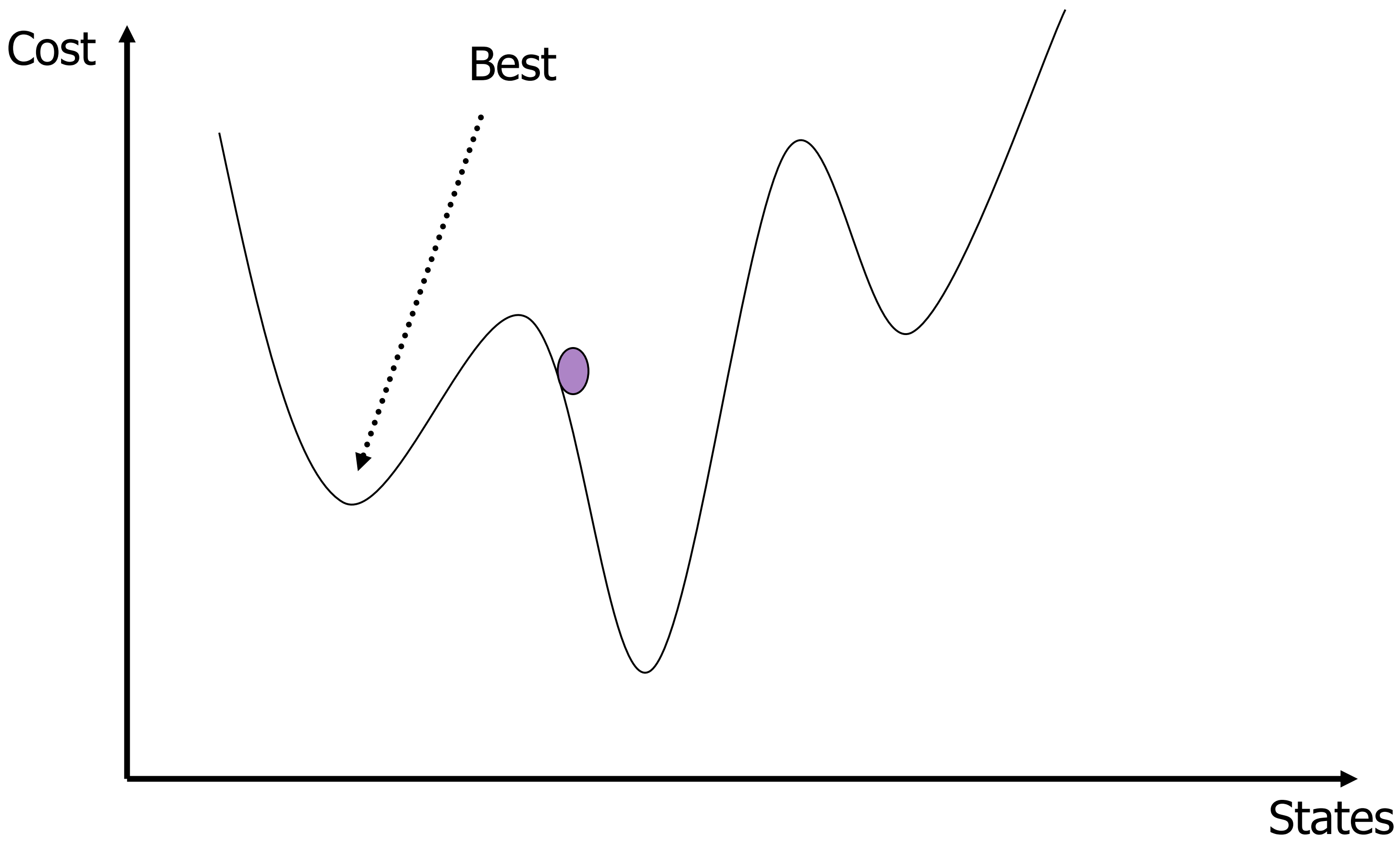

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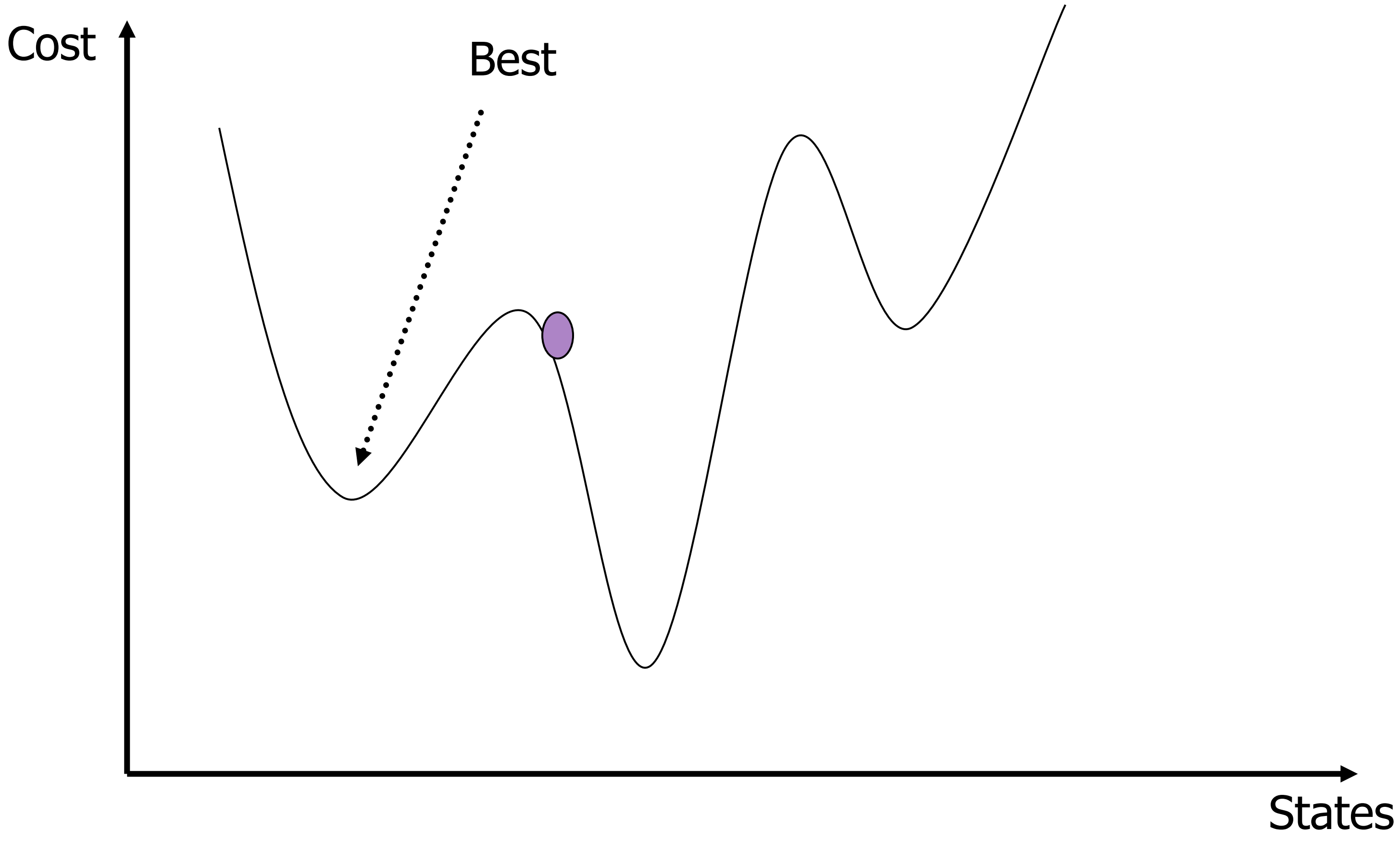

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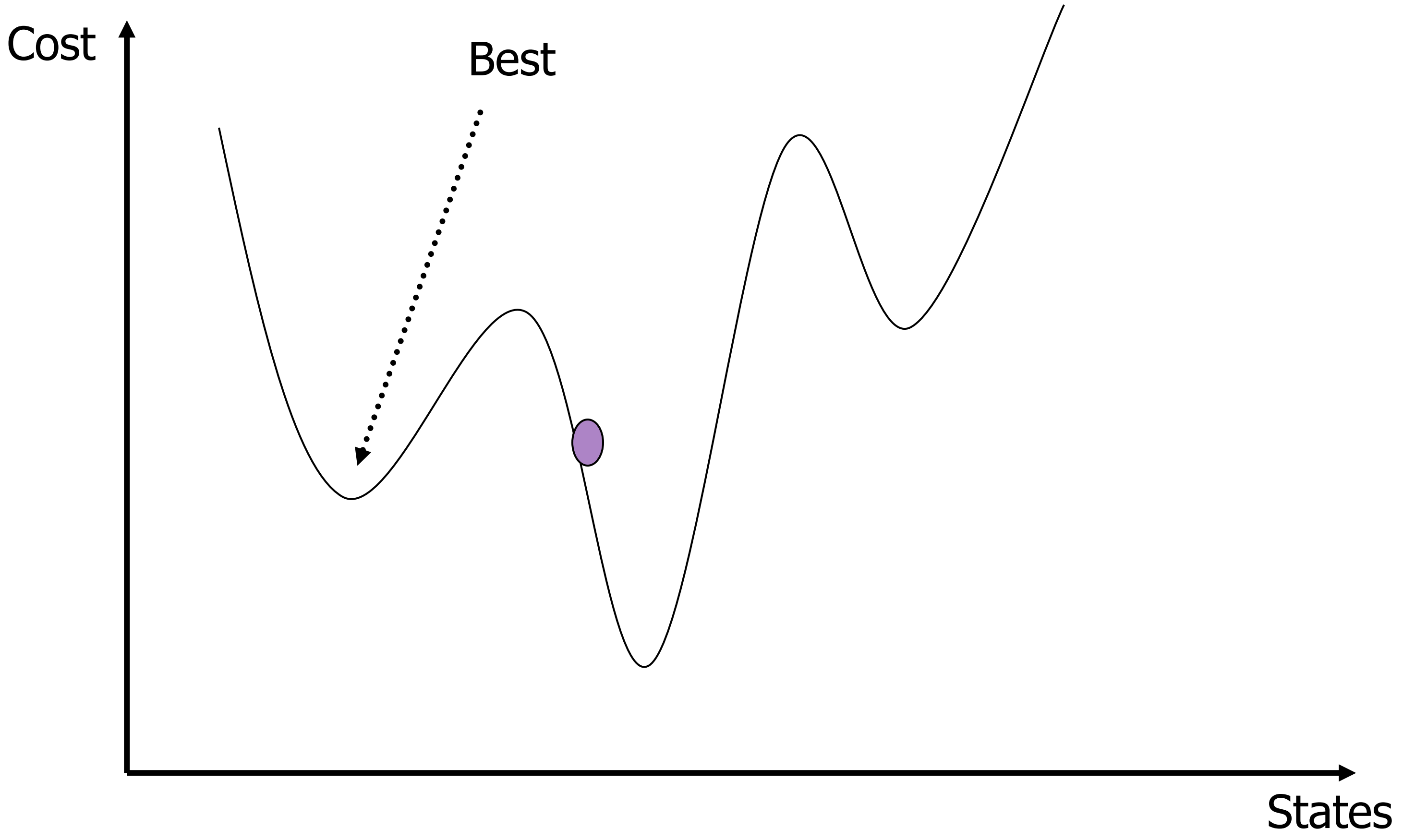

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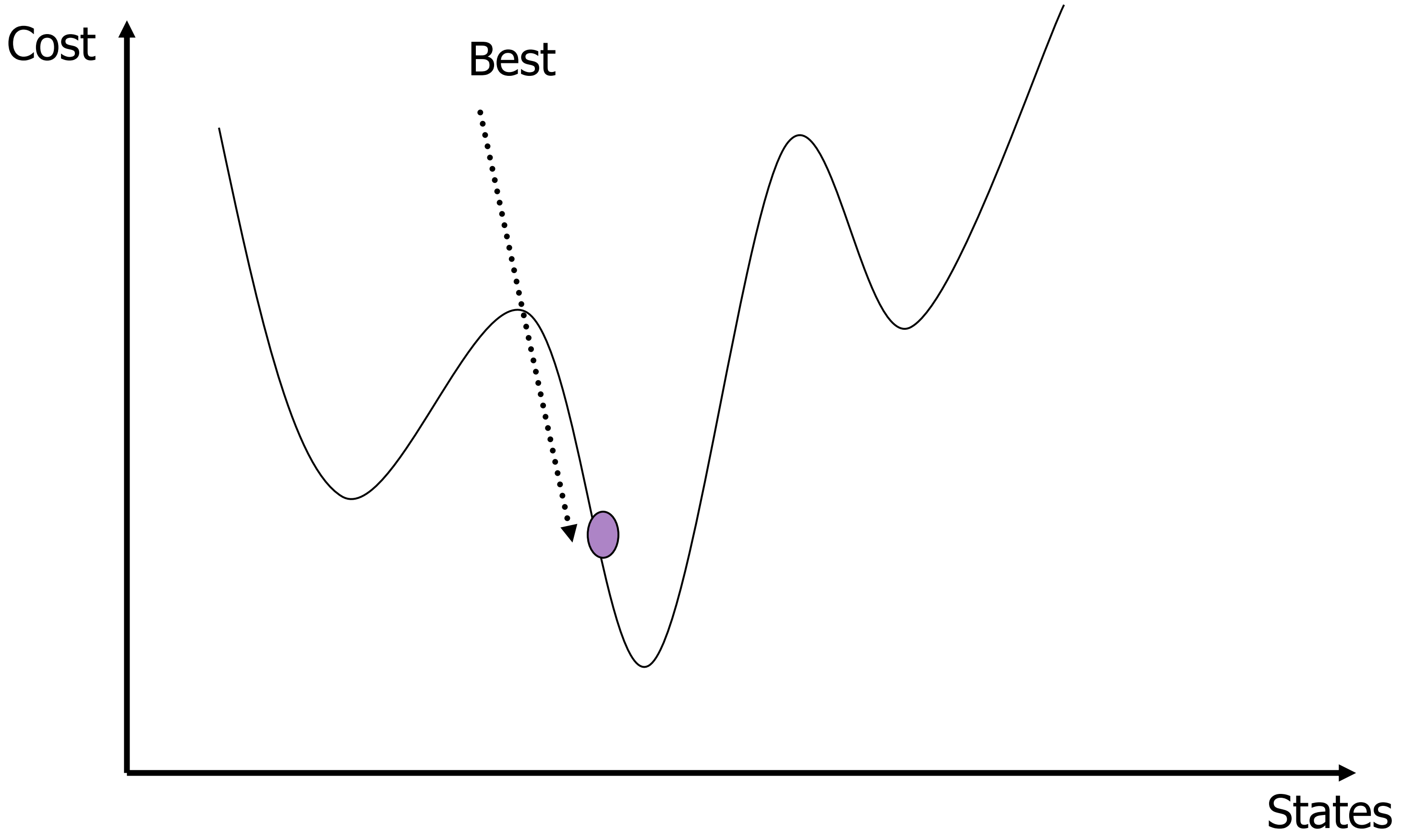

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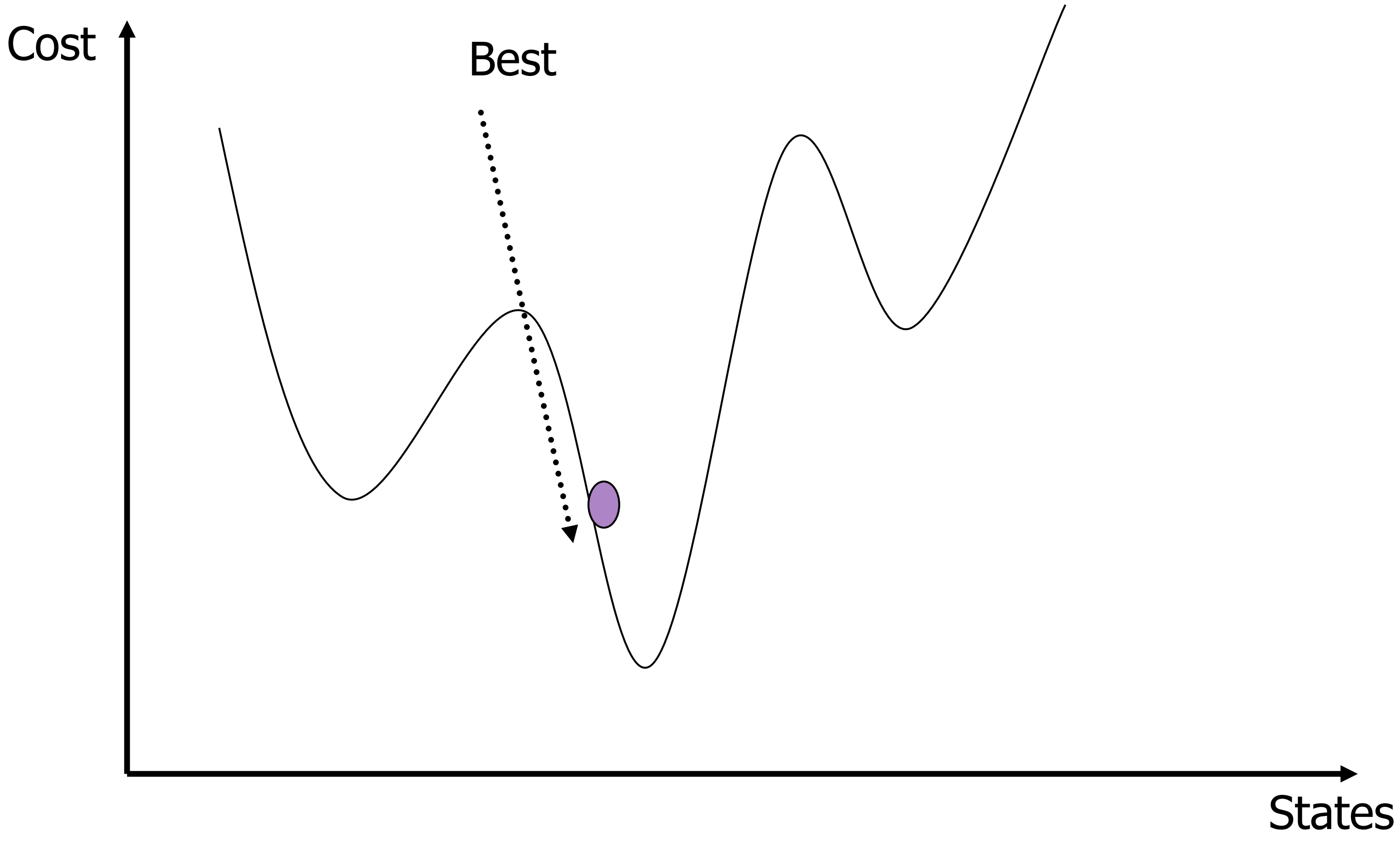

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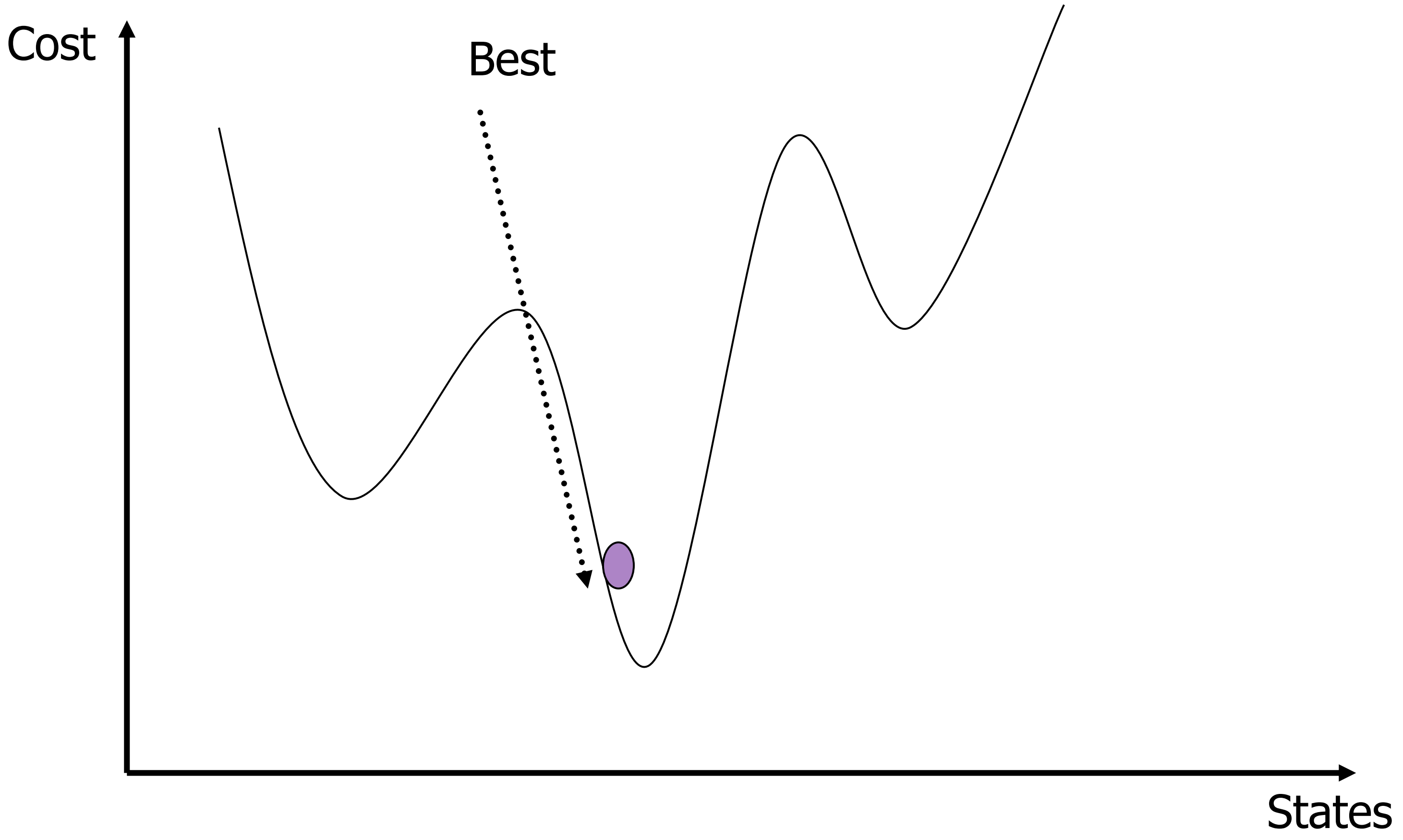

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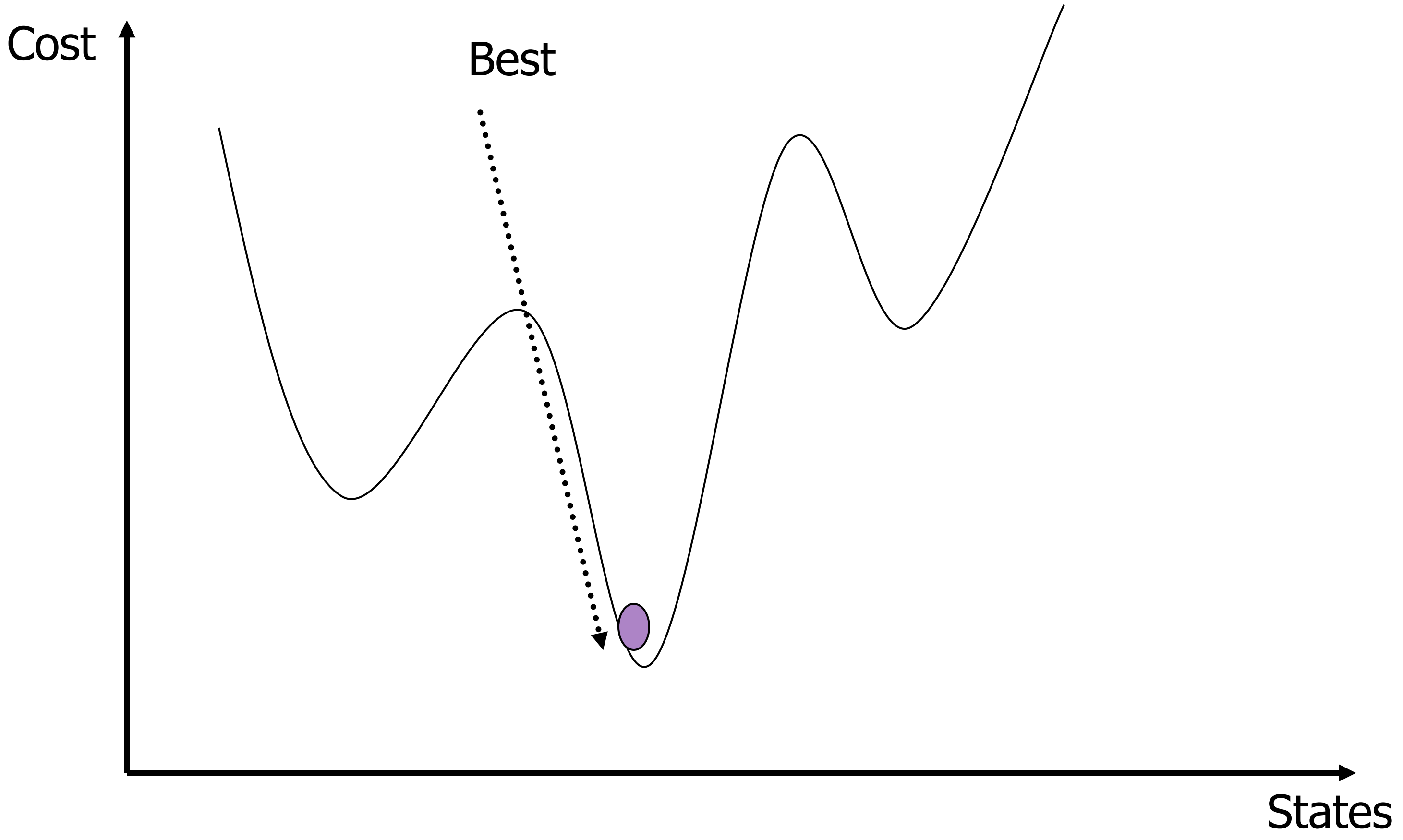

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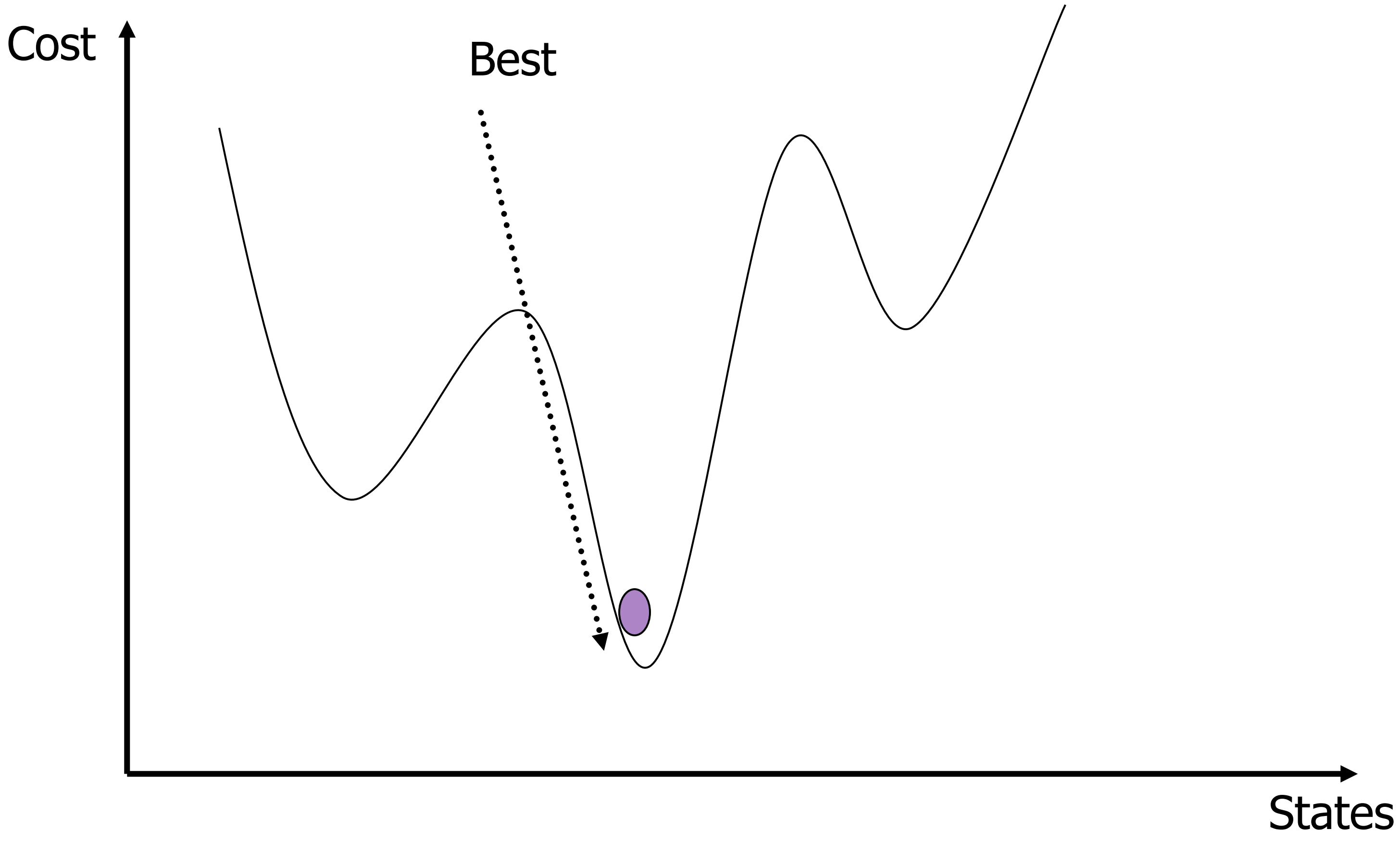

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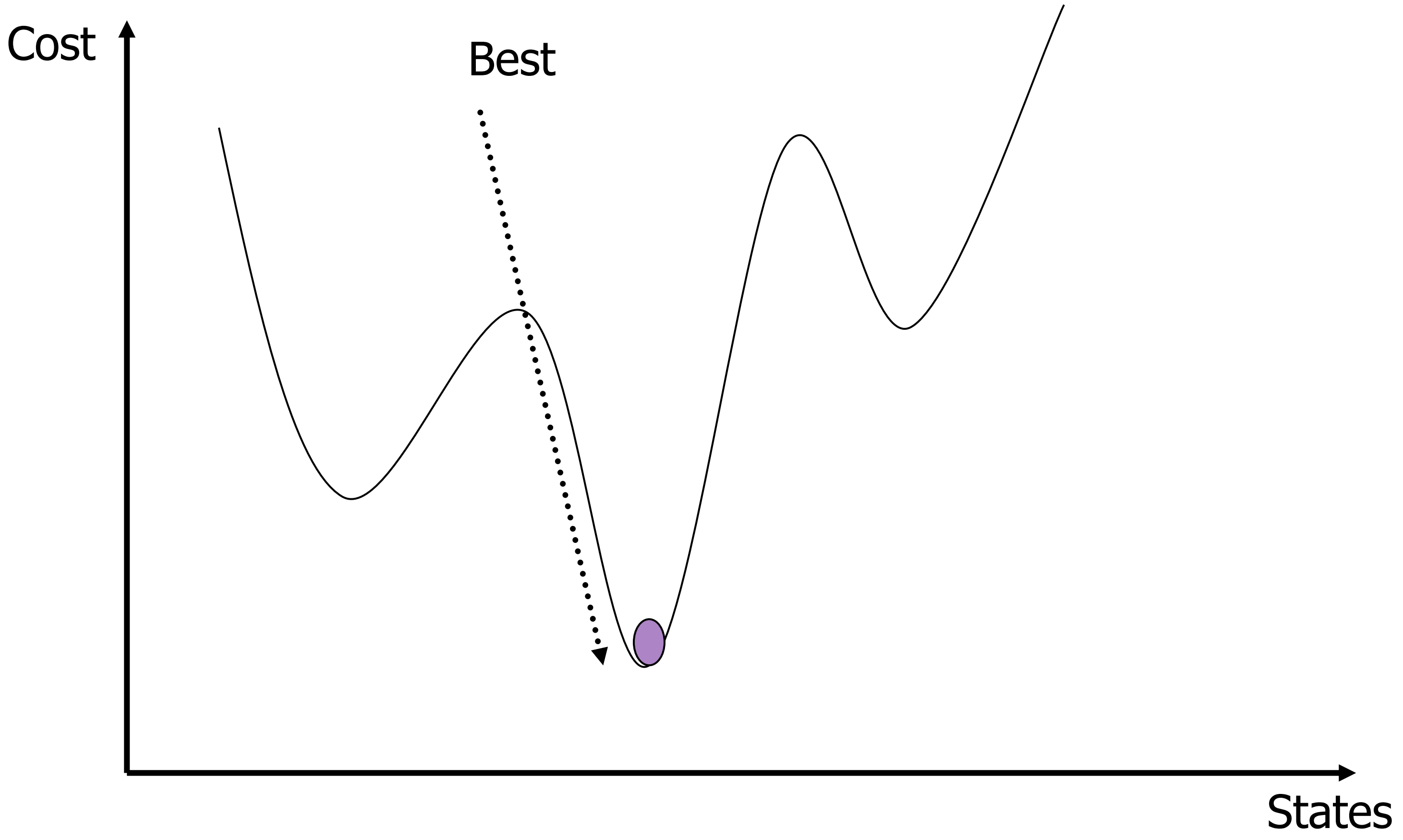

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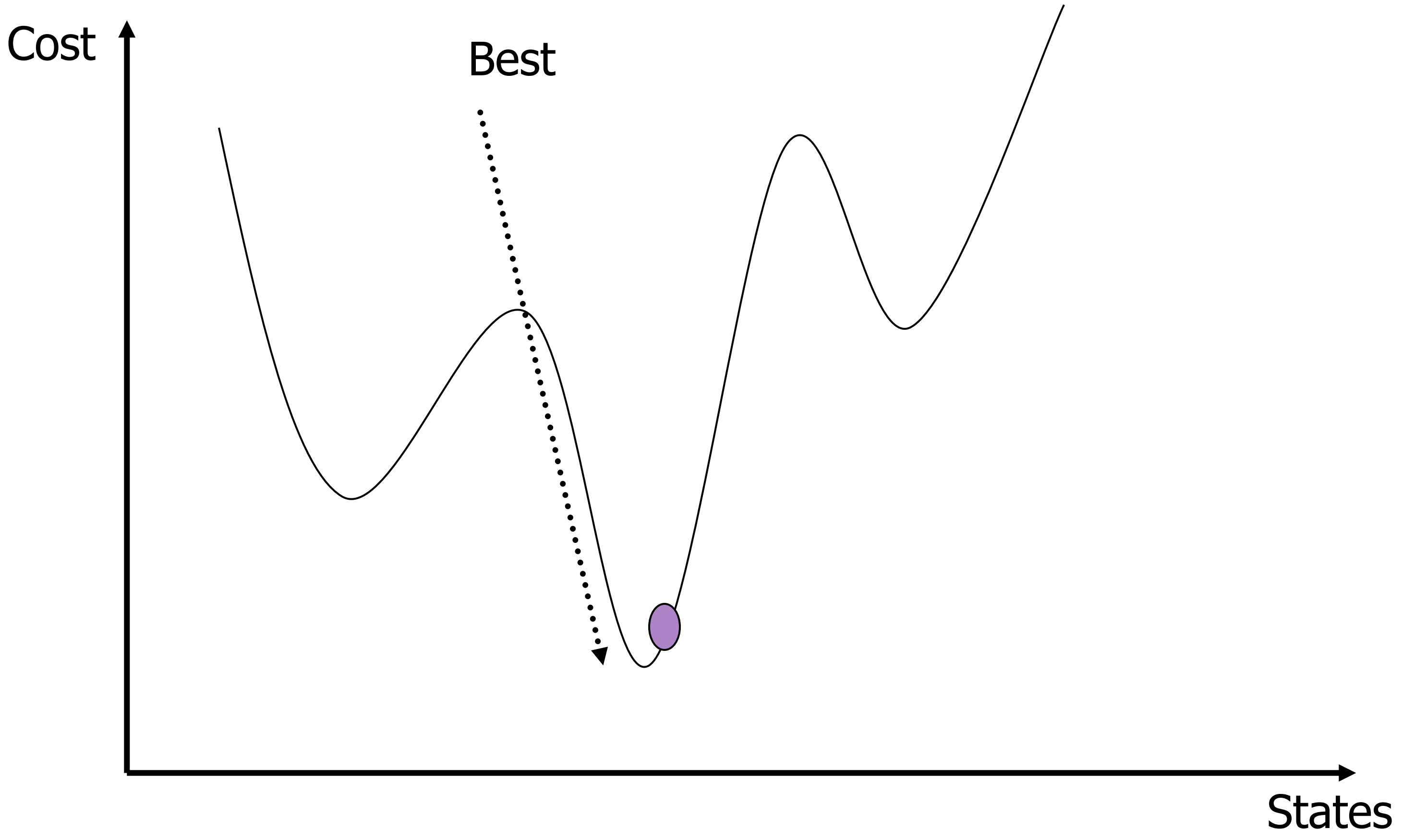

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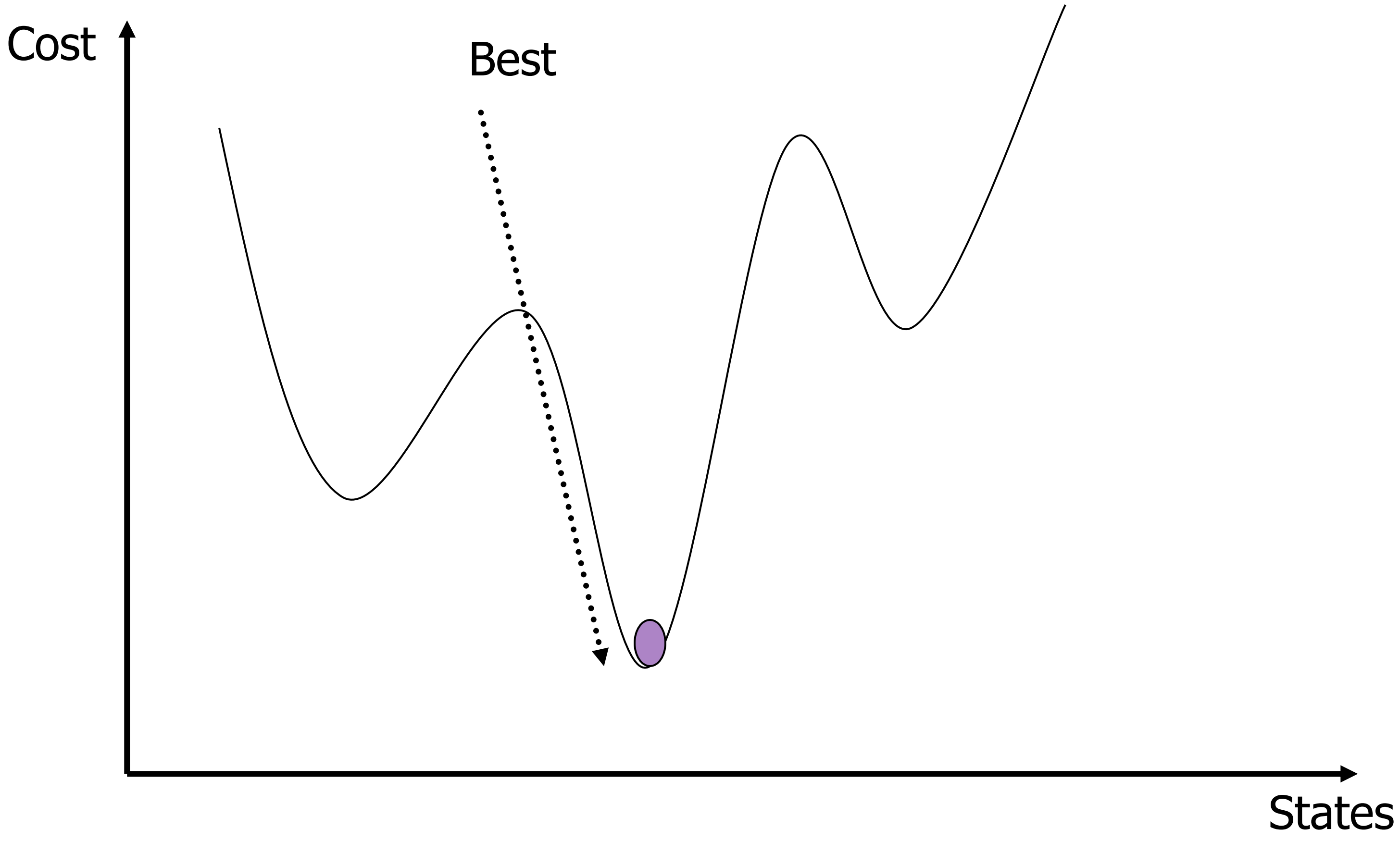

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