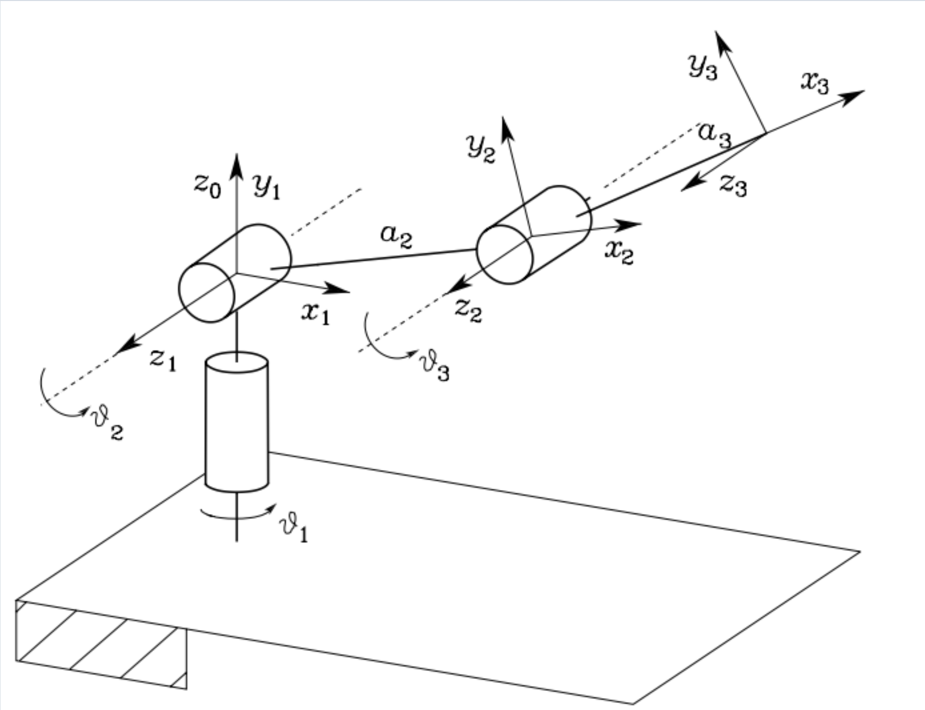

DH parameters for 4DoF Robotic Arm

Link	a_i	α_i	d_i	ϑ_i
1	0	2π	0	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

Forward Kinematics Using Denavit–Hartenberg Convention

The homogeneous transformation matrices defined in are for the single joints

$$\boldsymbol{T}_1^0(\vartheta_1) = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_2^1(\vartheta_2) = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_3^2(\vartheta_3) = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Computation of the direct kinematics function

$$\boldsymbol{T}_3^0(q) = \boldsymbol{T}_1^0\,\boldsymbol{T}_2^1\,\boldsymbol{T}_3^2 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 c_{23} & -c_1 & s_1(a_2 c_2 + a_3 c_{23}) \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $s_1 = \sin\vartheta_1$, $c_1 = \cos\vartheta_1$, $s_{23} = \sin(\vartheta_2 + \vartheta_3)$, $c_{23} = \cos(\vartheta_2 + \vartheta_3)$

Therefore, the position equation of the robot is

$x = \cos\vartheta_1\,(a_2 cos\vartheta_2 + a_3\cos(\vartheta_2 + \vartheta_3))$

$y = \sin\vartheta_1\,(a_2\cos\vartheta_2 + a_3\cos(\vartheta_2 + \vartheta_3))$

$z = a_2\sin\vartheta_2 + a_3\sin(\vartheta_2 + \vartheta_3)$