Q1:- Find all possible asymptotes & sketch the graph of the ftn $f(x) = -\dfrac{x^3+5x^2-6x}{x^2-1}$

<u>sol</u>

$$f(x) = -\frac{x[x^2+5x-6]}{(x-1)(x+1)}$$

$$f(x) = -\frac{x\,\cancel{(x-1)}(x+6)}{\cancel{(x-1)}(x+1)}$$

x-intercept
(0,0), (-6,0)

$$f(x) = \frac{-x(x+6)}{(x+1)}$$

y-intercept
(0,0)

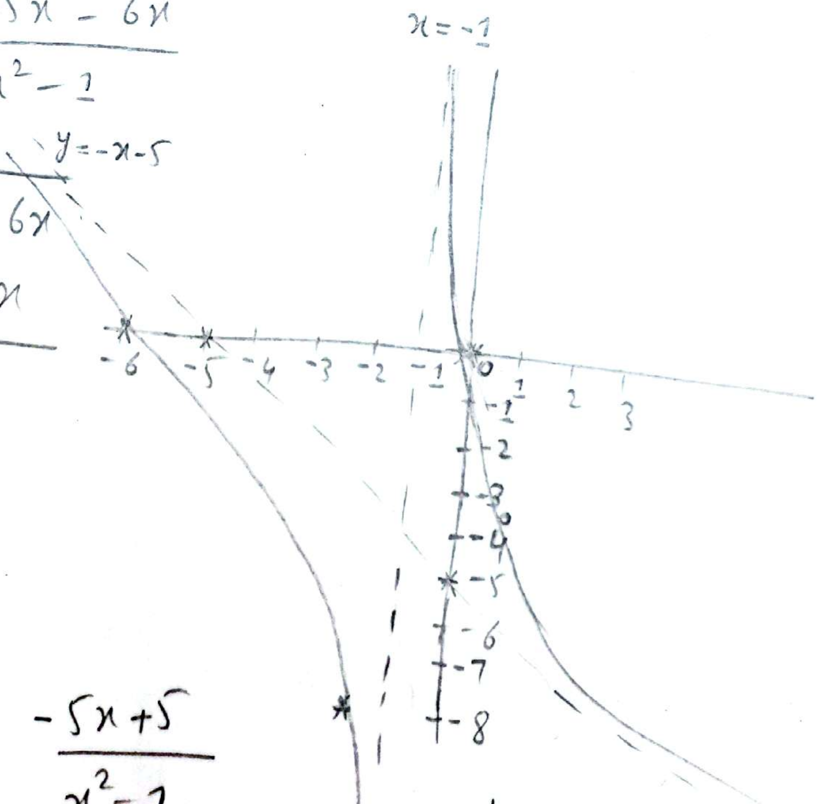
$x=-1$ is V.A

At $x=1$, $y=-7/2$

At $x=-2$, $y=-8$

$$f(x) = -\frac{x^3+5x^2-6x}{x^2-1}$$

$$\begin{array}{r l} & x+5 \\ x^2-1 \,\big) & x^3+5x^2-6x \\ & \pm x^3 \quad \mp x \\ \hline & 5x^2-5x \\ & \pm 5x^2 \quad \mp 5 \\ \hline & -5x+5 \end{array}$$



$$f(x) = -[x+5] + \frac{-5x+5}{x^2-1}$$

Oblique / Slant asymptote is $y = -x-5$

x-intercept
$x=-5$ (-5,0)
y-intercept
$y=-5$ (0,-5)

Q2:- $f(x) = \dfrac{2x-1}{x^2-1}$

when $x = 2$ $y = 1$

$x = -2$

$\dfrac{2(-2)-1}{(-2)^2-1}$

$\dfrac{-4-1}{4-1}$ $\dfrac{-5}{3}$

V.A $f(x) = \dfrac{2x-1}{(x-1)(x+1)}$

VA $x = \pm 1$

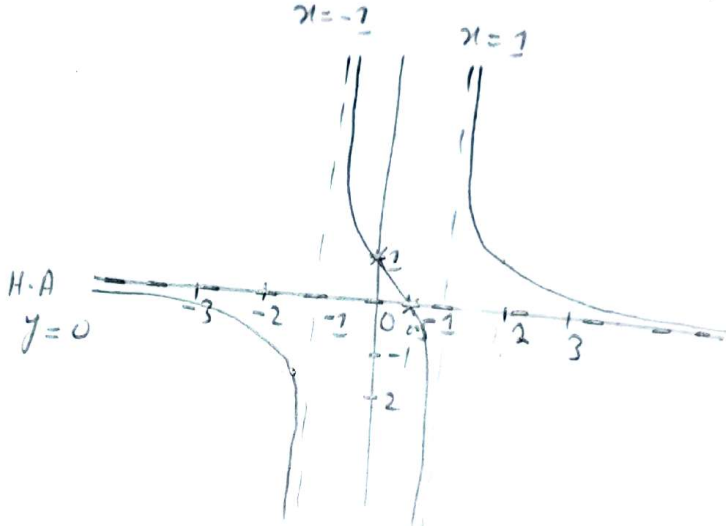
H.A Since deg (num) = 1 < deg (den) = 2 so $y = 0$ is HA

y-intercept : $x = 0$

$(0, 1)$

x-intercept : $y = 0$

$(0.5, 0)$



Q3:- Find all possible asymptotes & sketch the graph of $\dfrac{2x^2+6x-2}{x+1}$

Sol V.A $x = -1$

For S.A

$$\begin{array}{r|l} & 2x+4 \\ \hline x+1 & 2x^2+6x-2 \\ & \pm \cancel{2}x^2 + 2x \\ \hline & 4x - 2 \\ & \pm 4x \pm 4 \\ \hline & -6 \end{array}$$

x-intercept

0.3 $\left(\dfrac{-3+\sqrt{13}}{2}, 0\right)$ -3.5 $\left(\dfrac{-3+\sqrt{13}}{2}, 0\right)$

y-intercept

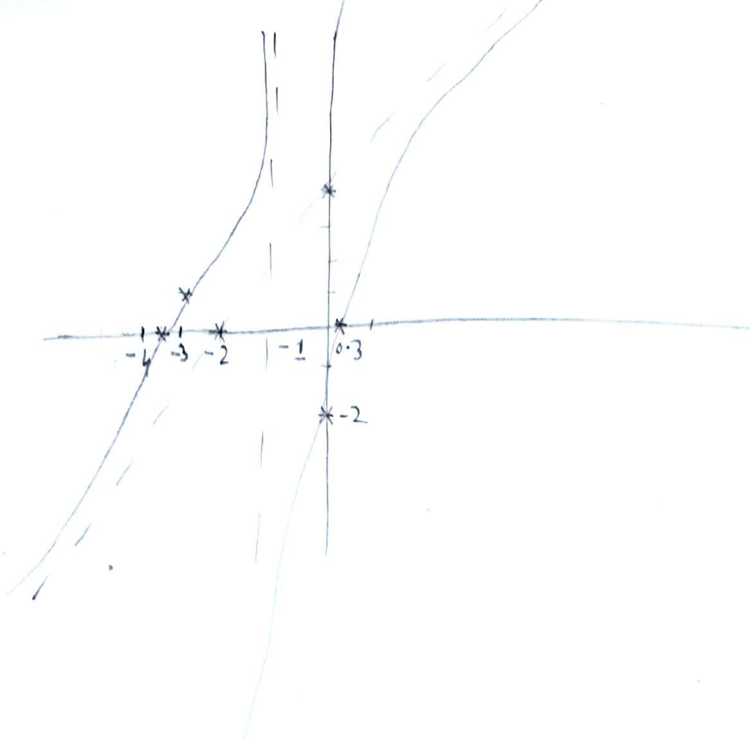
$(0, -2)$

$f(x) = 2x+4+\dfrac{(-6)}{x+1}$

Line Intercept

$x = 0$, $y = 4$

$x = -2$, $y = 0$



Q4:- Does the graph have V-T or V-C at $x = 4$

$$y = \sqrt{|4-x|}$$

Sol

$$\lim_{h\to 0^+} \frac{\sqrt{|4-(4+h)|} - 0}{h}$$

$$\lim_{h\to 0^+} \frac{\sqrt{|h|}}{h}$$

$$\lim_{h\to 0^+} \frac{\sqrt{h}}{h}$$

$$\lim_{h\to 0^+} \frac{1}{\sqrt{h}} = \infty$$

$$\lim_{h\to 0^-} \frac{\sqrt{|h|}}{h}$$

$$\lim_{h\to 0^-} \frac{\sqrt{|h|}}{-|h|}$$

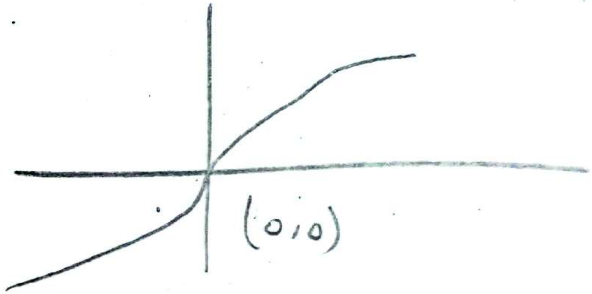
$$\lim_{h\to 0^-} -\frac{1}{\sqrt{|h|}}$$

$$\lim_{h\to 0^-} -\frac{1}{\sqrt{-h}}$$

$$-\infty$$

So graph will have V.C at $x = 4$.

Q47:- $y = \begin{cases} -\sqrt{|x|} & x \le 0 \\ \sqrt{x} & x > 0 \end{cases}$



Graph appears to have V.T at origin. Calculating the li^t of difference quotient for $x = 0$

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)-0}{h}$$

$$= \lim_{h\to 0} \frac{f(h)}{h}$$

Since the ftn changes behavior at $x = 0$ so we need to calculate one-sided li^t.

$$\lim_{h\to 0^+} \frac{f(h)}{h} = \lim_{h\to 0^+} \frac{\sqrt{h}}{h} = \lim_{h\to 0^+} \frac{1}{\sqrt{h}} = \frac{1}{\sqrt{0^+}} = \frac{1}{0^+} = +\infty$$

$$\lim_{h\to 0^-} \frac{f(h)}{h} = \lim_{h\to 0^-} \frac{-\sqrt{|h|}}{h} = \lim_{h\to 0^-} \frac{-\sqrt{|h|}}{-|h|} = \lim_{h\to 0^-} \frac{1}{\sqrt{|h|}}$$

$$\lim_{h \to 0^-} \frac{1}{\sqrt{-h}} = \infty$$

Since limit is $+\infty$, so graph has V.T at origin.

Q6:- Find the asymptotes & draw graphics of ftn

$$f(x) = \frac{2x^3 - 4x^2 + 15}{3x^2 - 12}$$

x- intercept $(-1.4, 0)$

y-intercept $(0, -5/4)$

V.A

$$f(x) = \frac{2x^3 - 4x^2 + 15}{3(x^2 - 4)}$$

$$f(x) = \frac{2x^3 - 4x^2 + 15}{3(x-2)(x+2)}$$

V.A $x = -2, 2$

S.A

$$\begin{array}{r|l} & \frac{2}{3}x - \frac{4}{3} \\ \hline 3x^2 - 12 & 2x^3 - 4x^2 + 15 \\ & \pm 2x^3 \quad \mp 8x \\ \hline & -4x^2 + 8x + 15 \\ & \mp 4x^2 \quad \pm 16 \\ \hline & 8x - 1 \end{array}$$

S.A $\frac{2}{3}x - \frac{4}{3}$

Line Intercepts

$(0, -4/3)$ 1.3

$(2, 0)$

key pts (3, 2.2)
(-3, -5)
(-1, -1)
(1, -1.4)