EE117: APPLIED PHYSICS-Part II

Serial No:______________

Final Exam

Total Time:2:30 Hour

Total Marks: 80

DATE: 19 December, 2018

Course Instructors

Ms. Ayesha Zafar

Mrs. Sabeen Sher Afgan

Dr. Muhammad Usman

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Signature of Invigilator

Solution to the Final Exam

Student Name	Roll No	Section	Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Verify at the start of the exam that you have a total of eight (6) questions printed on twelve (12) pages (single side) including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
5. Fit in all your answers in the provided space. You may use extra space on the back page if required. If you do so, clearly mark question/part number on that page to avoid confusion.
6. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
7. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of paper done in lead pencil cannot be claimed for checking/rechecking.

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Total
Total Marks	15	15	10	10	15	15	80
Marks Obtained							

Vetted By: ____________________________**Vetter Signature:** _____________________

Q1.(i) In Figure Q1-i, current is set up through a truncated right circular cone of resistivity 731 Ω·m, left radius a = 2.00 mm, right radius b = 2.30 mm, and length L = 1.94 cm. Assume that the current density is uniform across any cross section taken perpendicular to the length. What is the resistance of the cone? [10]

[Hint: Use differential resistance $dR=\rho dx/dA$ of an element, where dx and dA are the differential length and differential area of indicated circular region. Relate y and x with the slope of the cone.]

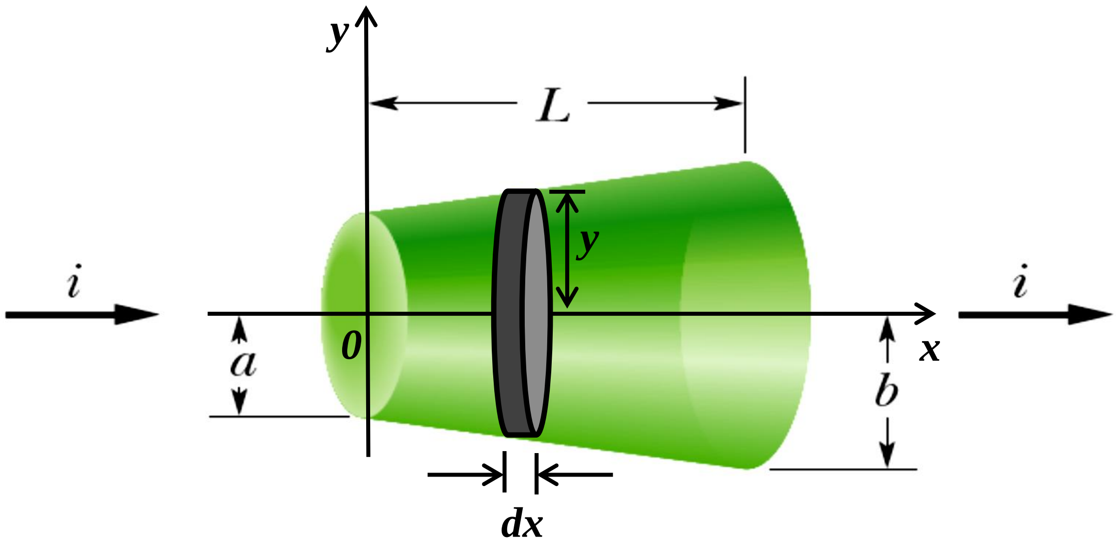


Figure Q1-i

Equation for slope side of the cone

$$y = mx + c$$

$$m = \frac{\Delta y}{\Delta x} = \frac{b-a}{L-0} = \frac{b-a}{L}$$

$$c = a$$

$$y = \frac{b-a}{L}x + a$$

Differential resistance of element of length dx and radius y is

$$dR = \rho \frac{dx}{\pi y^2}$$

$$dR = \rho \frac{dx}{\pi\left(\frac{b-a}{L}x + a\right)^2}$$

$$dR = \frac{\rho}{\pi}\left(\frac{b-a}{L}x + a\right)^{-2} dx$$

Integrate both sides

$$R = \frac{\rho}{\pi}\int_0^L \left(\frac{b-a}{L}x + a\right)^{-2} dx$$

$$R = \frac{\rho}{\pi}\left[\frac{\left(\frac{b-a}{L}x + a\right)^{-1}}{(-1)\frac{b-a}{L}}\right]_0^L$$

$$R = -\frac{\rho L}{\pi(b-a)}\left[\frac{1}{\frac{b-a}{L}x + a}\right]_0^L$$

$$R = -\frac{\rho L}{\pi(b-a)}\left[\frac{1}{\left(\frac{b-a}{L}\right)L + a} - \frac{1}{a}\right]$$

$$R = -\frac{\rho L}{\pi(b-a)}\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$R = -\frac{\rho L}{\pi(b-a)}\left[\frac{a-b}{ab}\right]$$

$$R = \frac{\rho L}{\pi(b-a)}\left[\frac{b-a}{ab}\right]$$

$$\boldsymbol{R = \frac{\rho L}{\pi a b}}$$

Substitute the given values,

$$R = \frac{(731)(1.94 \times 10^{-2})}{\pi(2.00 \times 10^{-3})(2.30 \times 10^{-3})}$$

$$\boldsymbol{R = 9.81 \times 10^{5}\ \Omega}$$

Q1. (ii) What is the current in a wire of radius R=3.40 mm if the magnitude of the current density is given by (a) $J_a=J_o r/R$ and (b) $J_b=J_o(1-r/R)$, in which r is the radial resistance and $J_o=5.50\times10^4$ A/m^2 [5]

$$i = \int_0^R \vec{j}\cdot d\vec{A}$$

(a) $J_a=J_o r/R$

$$i = \int_0^R J_a 2\pi r\,dr$$

$$i = 2\pi \int_0^R J_o \frac{r}{R} r\,dr$$

$$i = \frac{2\pi J_o}{R}\int_0^R r^2\,dr$$

$$i = \frac{2\pi J_o}{R}\left[\frac{r^3}{3}\right|_0^R$$

$$i = \frac{2\pi J_o}{R}\left[\frac{R^3}{3} - 0\right]$$

$$i = \frac{2\pi J_o R^2}{3}$$

$$i = \frac{2\pi(5.50\times10^4)(3.40\times10^{-3})^2}{3}$$

$$\boldsymbol{i = 1.33\ A}$$

(b) $J_b=J_o(1-r/R)$

$$i = \int_0^R J_b 2\pi r\,dr$$

$$i = 2\pi \int_0^R J_o\left(1-\frac{r}{R}\right) r\,dr$$

$$i = 2\pi J_o \int_0^R r\,dr - \frac{2\pi J_o}{R}\int_0^R r^2\,dr$$

$$i = 2\pi J_o\left[\frac{r^2}{2}\right|_0^R - \frac{2\pi J_o}{R}\left[\frac{r^3}{3}\right|_0^R$$

$$i = 2\pi J_o\left[\frac{R^2}{2} - 0\right] - \frac{2\pi J_o}{R}\left[\frac{R^3}{3} - 0\right]$$

$$i = \pi J_o R^2 - \frac{2\pi J_o R^2}{3} = \pi J_o R^2\left(1-\frac{2}{3}\right)$$

$$i = \frac{1}{3}\pi J_o R^2$$

$$i = \frac{\pi(5.50\times10^4)(3.40\times10^{-3})^2}{3}$$

$$\boldsymbol{i = 0.665\ A = 665\ mA}$$

Q2. Figure Q-2 shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance $r >> d$? [10]. Also find the point where the electric field is maximum? [5]

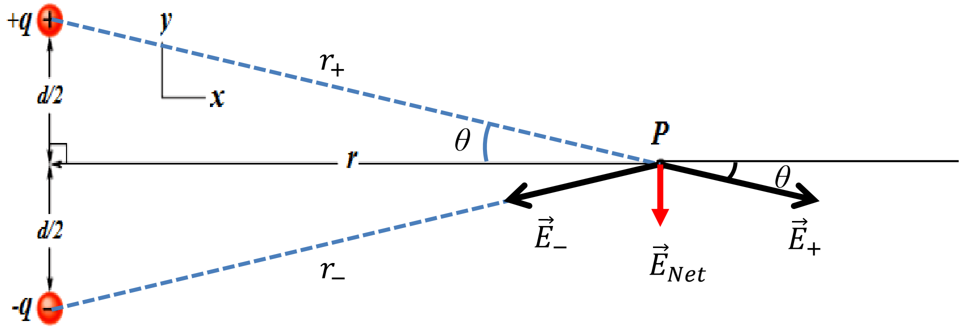


Figure Q2-i

Let $\vec{E}_+$, and $\vec{E}_-$ be the electric field due to the change $+q$ and $-q$ respectively at point P. The direction of the electric field vectors are shown in the figure.

The magnitude of the electric field vectors due to these charges are

$$|\vec{E}_+| == \frac{kq}{r_+^2}$$

$$|\vec{E}_-| == \frac{kq}{r_-^2}$$

From the figure using Pythagoras theorem,

$$r_+ = r_- = \sqrt{\left(\frac{d}{2}\right)^2 + r^2}$$

Therefore,

$$|\vec{E}_+| == \frac{kq}{\left(\frac{d}{2}\right)^2 + r^2}$$

$$|\vec{E}_-| == \frac{kq}{\left(\frac{d}{2}\right)^2 + r^2}$$

Net electric field at point P is

$$|\vec{E}_{Net}| = |\vec{E}_+| \sin\theta + |\vec{E}_-| \sin\theta$$

$$|\vec{E}_{Net}| = (|\vec{E}_+| + |\vec{E}_-|) \sin\theta$$

$$|\vec{E}_{Net}| = \left(\frac{kq}{\left(\frac{d}{2}\right)^2 + r^2} + \frac{kq}{\left(\frac{d}{2}\right)^2 + r^2}\right) \sin\theta = \left(\frac{2kq}{\left(\frac{d}{2}\right)^2 + r^2}\right) \sin\theta$$

From figure,

$$\sin\theta = \frac{d/2}{\sqrt{\left(\frac{d}{2}\right)^2 + r^2}}$$

$$|\vec{E}_{Net}| = \frac{2kq}{\left(\left(\frac{d}{2}\right)^2 + r^2\right)} \frac{d/2}{\sqrt{\left(\frac{d}{2}\right)^2 + r^2}}$$

$$\left|\vec{E}_{Net}\right| = \frac{kqd}{\left(\left(\frac{d}{2}\right)^2 + r^2\right)^{3/2}}$$

For $r \gg d$

$$\boldsymbol{\left|\vec{E}_{Net}\right| = \frac{kqd}{r^3} = \frac{1}{4\pi\varepsilon_o}\frac{qd}{r^3}}$$

(b) The direction of $\vec{E}_{Net}$, as shown in the figure ,is along –y direction

$$\boldsymbol{\vec{E}_{Net} = -\frac{1}{4\pi\varepsilon_o}\frac{qd}{r^3}\hat{j}}$$

To find the point where the $\left|\vec{E}_{Net}\right|$ is maximum, differentiate $\vec{E}_{Net}$ with respect to *r*

$$\frac{d\left|\vec{E}_{Net}\right|}{dr} = -\frac{3}{4\pi\varepsilon_o}\frac{qd}{r^4}(3r^2) = -\frac{9}{4\pi\varepsilon_o}\frac{qd}{r^2}$$

Set $\frac{d\left|\vec{E}_{Net}\right|}{dr} = 0$

$$0 = -\frac{9}{4\pi\varepsilon_o}\frac{qd}{r^2}$$

$$\boldsymbol{r = 0}$$

So at *r=0*, $\left|\vec{E}_{Net}\right|$is maximum.

Q3.(i) Three point charges lie along a circle of radius r at angles of 30°, 150°, and 270° as shown in Figure Q3-i. Find a symbolic expression for the resultant electric field at the center of the circle. [5]

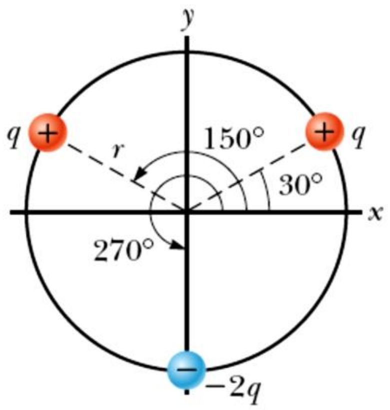
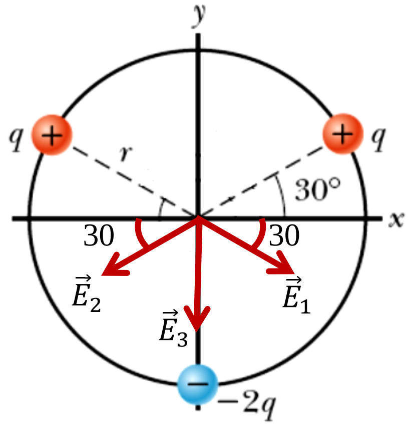


Figure Q3-i

Solution:

Let $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ be the electric field due to the change 1, 2 and 3 respectively at the center of the circle.

The direction of the electric field vectors are shown in the figure.



The magnitude of the electric field vector due to these charges are:

$$|\vec{E}_1| == \frac{kq}{r^2}$$
$$|\vec{E}_2| = \frac{kq}{r^2}$$
$$|\vec{E}_3| == \frac{2kq}{r^2}$$

The total electric field vector at the center is the vector sum of all the fileds

$$\vec{E}_{Net} = \left(|\vec{E}_1|\cos 30 - |\vec{E}_2|\cos 30\right)\hat{\imath} - \left(|\vec{E}_3| + |\vec{E}_1|\sin 30 + |\vec{E}_2|\sin 30\right)\hat{\jmath}$$

$$\vec{E}_{Net} = -\left(|\vec{E}_3| + |\vec{E}_1|\sin 30 + |\vec{E}_2|\sin 30\right)\hat{\jmath}$$

$$\vec{E}_{Net} = -\left(\frac{2kq}{r^2} + \frac{kq}{r^2}\sin 30 + \frac{kq}{r^2}\sin 30\right)\hat{\jmath}$$

$$\vec{E}_{Net} = -\left(\frac{2kq}{r^2} + \frac{kq}{r^2}(0.5) + \frac{kq}{r^2}(0.5)\right)\hat{\jmath}$$

$$\boldsymbol{\vec{E}_{Net} = -\frac{3kq}{r^2}\hat{\jmath}}$$

Q3.(ii) In Figure Q3-ii, A block securely attached to a spring oscillates vertically with a frequency of 4.00 Hz and amplitude of 7.00 cm. A tiny bead is placed on top of the oscillating block just as it reaches its lowest point. Assume that the bead's mass is so small that its effect on the motion of the block is negligible. At what displacement from the equilibrium position does the bead lose contact with the block? [5]

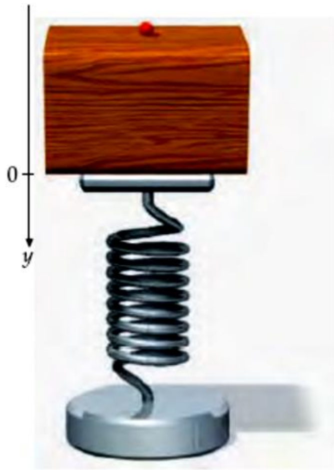


Figure Q3-ii

The forces on the bead are its weight downward and the upward normal force exerted by the block. The magnitude of this normal force changes as the acceleration changes. As the block moves upward from equilibrium, its acceleration and the acceleration of the bead are downward and increasing in magnitude. When the acceleration reaches downward, the normal force will be zero. If the block's downward acceleration becomes even slightly larger, the bead will leave the block.

Draw a sketch of the system (Figure Q3-ii). Include a coordinate axis with its origin at the equilibrium position and with down as the positive direction:

We are looking for the value of when the Figure Q3-ii acceleration is g downward.

$$a_y = -\omega^2 y$$

$$g = -\omega^2 y$$

$$g = -(2\pi f)^2 y$$

$$y = -\frac{g}{2\pi f}$$

$$y = -\frac{9.8}{2\pi(4.00)} = \boldsymbol{-0.0155\ m = -1.55\ cm}$$

Q4. (i) A delighted CS graduate throws his/her cap into the air with an initial velocity of 24.5m/s at 36.9° above the horizontal. The cap is later caught by another student. Find the total time the cap is in the air, and the total horizontal distance traveled. [4].

We choose the origin to be the initial position of the cap so $(x_o, y_o) = (0\,, 0)$.

We assume it is caught at the same height.

$v_o = 24.5\ m/s$

$\theta_o = 36.9^o$

$v_{ox} = v_o \cos\theta_o = 24.5 \cos 36.9 = 19.59\ m/s$

$v_{oy} = v_o \sin\theta_o = 24.5 \sin 36.9 = 14.71\ m/s$

The total time the cap is in the air is found by setting$y(t) = y_o$.

$$y(t) = y_o - v_{oy}t - \frac{1}{2}gt^2$$

$$y_o = y_o - v_{oy}t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = v_{oy}t$$

$$t = \frac{2v_{oy}}{g} = \frac{2(14.71)}{9.8} = \mathbf{3.00\ s}$$

$$x(t) = x_o + v_{ox}t$$

$$x(t) = 0 + (19.59)(3.00)$$

$$x(t) = \mathbf{58.8\ m}$$

2nd Method:

Time of flight

$$t = \frac{2v_o \sin\theta_o}{g} = \frac{2(24.5\,)\sin 36.9}{9.8} = \mathbf{3.00\ s}$$

Range

$$R = \frac{v_o^2 \sin 2\theta_o}{g} = \frac{(24.5)^2\ \sin 2(36.9)}{9.8} = \mathbf{58.8\ s}$$

Q4.(ii) The wave function $y(x,t) = (0.030\ m) \times sin[(2.2\ m^{-1})x - (3.5\ s^{-1})t]$ is for a harmonic wave on a string. (a) In what direction does this wave travel and what is its speed? (b) Find the wavelength, frequency, and period of this wave. (c) What is the maximum displacement of any point on the string? (d)What is the maximum speed of any point on the string? [6]

$$y(x,t) = (0.030\ m) \times sin[(2.2\ m^{-1})x - (3.5\ s^{-1})t]$$

Compare this equation with harmonic wave on a string traveling in +*x* direction

$y(x,t) = y_m sin[kx - \omega t]$

$y_m = 0.030\ m$

$k = 2.2\ m^{-1}$

$\omega = 3.5\ s^{-1}$

(a) The wave travels in +*x* direction.

Speed of the wave is

$$v = \frac{\omega}{k} = \frac{3.5\ s^{-1}}{2.2\ m^{-1}} = 1.59\ m/s = \mathbf{1.6\ m/s}$$

(b) Wavelength

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.2\ m^{-1}} = 2.85\ m = \mathbf{2.9\ m}$$

Frequency

$$f = \frac{\omega}{2\pi} = \frac{3.5\ s^{-1}}{2\pi} = 0.55\ \ s^{-1} = 0.557\ Hz = \mathbf{0.56\ Hz}$$

Period

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{1}{0.55} = 1.79\ s = \mathbf{1.8\ s}$$

(c) Maximum displacement?

$$y_m = \mathbf{0.030\ m}$$

(d) Maximum speed of any point on the string?

$$v(x,t) = \frac{\partial y(x,t)}{\partial t} = (0.030\ m)(-3.5\ s^{-1}) \times cos[(2.2\ m^{-1})x - (3.5\ s^{-1})t]$$

$$v(x,t) = \frac{\partial y(x,t)}{\partial t} = (-0.105 ms^{-1}) \times cos[(2.2\ m^{-1})x - (3.5\ s^{-1})t]$$

Comparing with

$$v(x,t) = -v_m cos[kx - \omega t]$$

$$v_m = \omega y_m = (3.5\ s^{-1})(0.030\ m) = \mathbf{0.105\ m/s}$$

Q5.(i) In Figure Q5-i, $\mathscr{E}$=12.0 V, R_1=2000 Ω, R_2=3000 Ω, and R_3=4000 Ω. What are the potential differences (a) V_A-V_B, (b) V_B-V_C.[6]

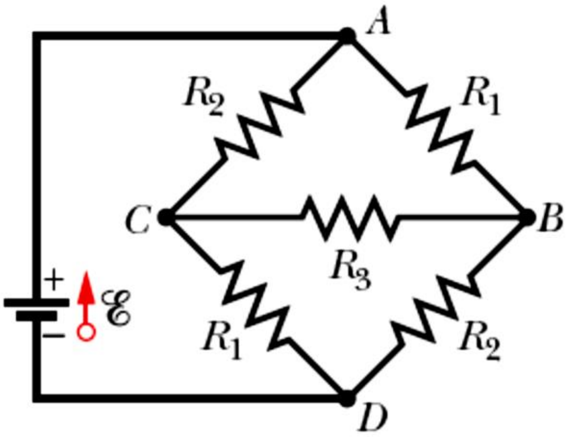


Figure Q5-i

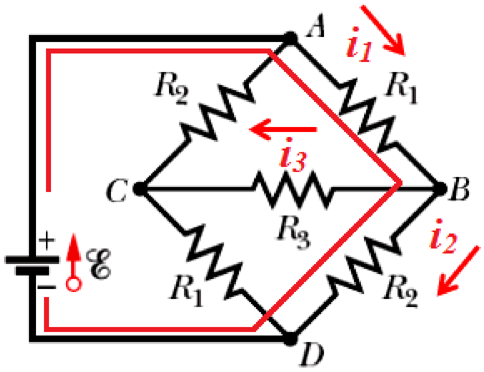
Solution:

Let i_1, i_2 and i_3 be the current through R_1, R_2 and R_3 as shown in figure of loop 1 and loop2 below.

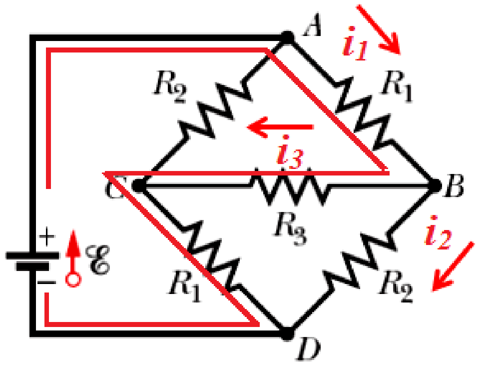
Apply KCL at junction B

$i_1 = i_2 + i_3$

$i_3 = i_1 - i_2$ (1)



Loop 1



Loop 2

(a) Consider the loop 1

Apply KVL to the loop 1 clockwise start from D.

$\mathcal{E} - i_1R_1 - i_2R_2 = 0$

$12 - 2000i_1 - 3000i_2 = 0$ (2)

Apply KVL to the loop 2 clockwise start from D.

$\mathcal{E} - i_1R_1 - i_3R_3 - i_1R_1 = 0$

$\mathcal{E} - 2i_1R_1 - i_3R_3 = 0$

Using eq 1

$\mathcal{E} - 2i_1R_1 - (i_1 - i_2)R_3 = 0$

$\mathcal{E} - (2R_1 + R_3)i_1 + i_2R_3 = 0$

$12 - (2(2000) + 4000)i_1 + 4000i_2 = 0$

$12 - 8000i_1 + 4000i_2 = 0$ (3)

Multiplying Eq. 2 by 4 on both sides

$48 - 8000i_1 - 12000i_2 = 0$ (4)

Subtract Eq. 3 from Eq. 4.

$36 - 16000i_2 = 0$

$i_2 = \frac{36}{16000} = 0.00225\,A = 2.25\,mA$

Put the value of i_2 in equation 2

$$12 - 2000i_1 - 3000(0.00225) = 0$$

$i_1 = \frac{5.25}{2000} = 0.002625\,A = 2.625\,mA$

Put the values of i_1 and i_2 in Eq. 1

$i_3 = 0.002625 - 0.00225 = 0.000375\,A = 3.75 \times 10^{-4} A$

$V_A - V_B = i_1 R_1 = (0.002625)(2000) = \mathbf{5.25\,V}$

(b) $V_B - V_C = i_3 R_3 = (0.000375)(4000) = \mathbf{1.50\,V}$

Q5. (ii) In Figure Q5-ii, a 20.0 V battery is connected across capacitors of capacitances $C_1=C_6=3.00$ μF and $C_3=C_5=2.00C_2=2.00C_4=4.00$ μF. What are (a) the equivalent capacitance C_{eq} of the capacitors and (b) the charge stored by C_{eq}? What are (c) V_1 and q_1 of capacitor? [9]

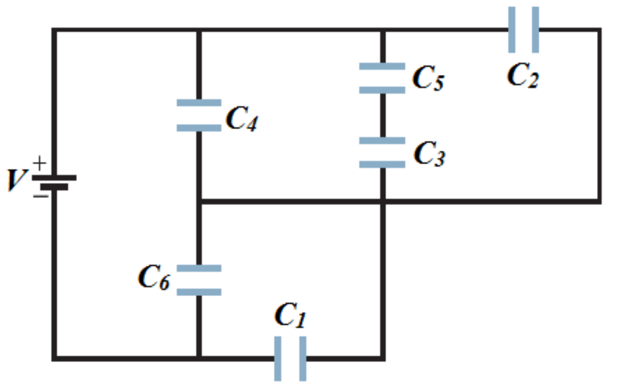


Figure Q5-ii

Given that

$C_1=C_6=3.00$ μF

$C_3=C_5=4.00$ μF

$C_2=C_4=2.00$ μF

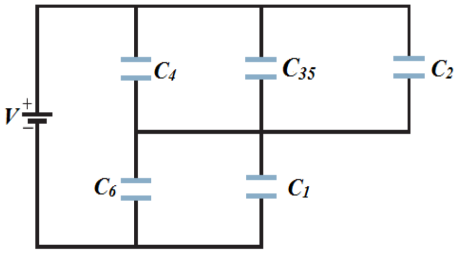


Fig-a

⇒

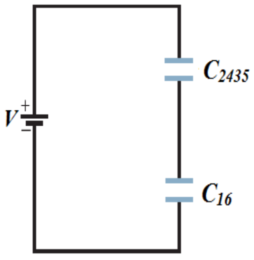


Fig-b

⇒

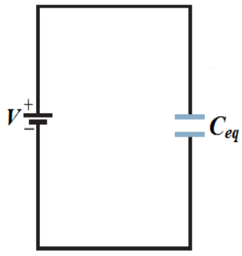


Fig-c

(a) First, the equivalent capacitance of the two capacitors in series C_3 and C_5 (Fig a)

$$C_{35} = \frac{C_3C_5}{C_3+C_5} = \frac{(4.00)(4.00)}{4.00+4.00} = 2\ \mu F$$

The equivalent capacitance of the three capacitors C_2, C_4 and C_{35} in parallel is (Fig b)

$$C_{2435} = C_2 + C_4 + C_{35} = 2.00 + 2.00 + 2.00 = 6.00\ \mu F$$

The equivalent capacitance of the two capacitors C_1, and C_6 in parallel is (Fig b)

$$C_{16} = C_1 + C_6 = 3.00 + 3.00 = 6.00\ \mu F$$

Now C_{16} and C_{2435} are in series, so C_{eq} is (Fig c)

$$C_{eq} = \frac{C_{16}C_{2435}}{C_{16}+C_{2435}} = \frac{(6.00)(6.00)}{6.00+6.00} = \mathbf{3\ \mu F}$$

(b) Let $V = 20.0$ V be the potential difference supplied by the battery. Then

$$q = C_{eq}V = (3\times10^{-6})(20.0) = 6\times10^{-5}\ C$$

(c) The potential difference across C_1 is find by using voltage divider rule (Fig b)

$$V_1 = \left(\frac{C_{16}}{C_{16}+C_{2435}}\right)V = \left(\frac{6.00}{6.00+6.00}\right)(20.0)$$

$$V_1 = \mathbf{10.0\ V}$$

Q6.(i) Determine the reference voltages provided by the network of Figure Q6-i, which uses a Blue LED to indicate that the power is on. What is the level of current through the LED and the power delivered by the supply? [7.5]

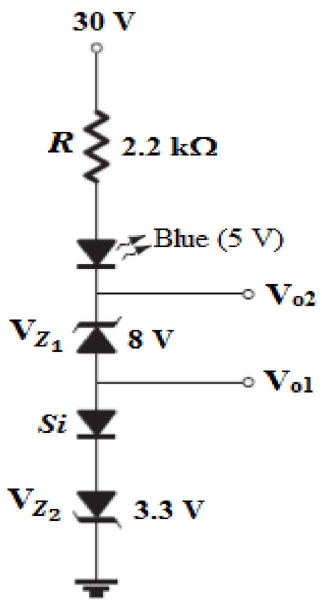


Figure Q6-i

E = 30 V

V_K(Si) = 0.7 V

V_{LED} (Blue) = 5 V

V_{Z1}= 8 V

V_{Z2}= 3.3 V

V_{o1}=?, V_{o2}=?,*and* I = ?

V_{Z2} is in forward bias to act as a normal diode with the drop of 0.7 V.

V_{o1}= V_{ZK}+V_K=0.7+0.7=***1.4 V***

Now

V_{o2}= V_{o1}+V_{Z1}=1.4+8.0=***9.4 V***

The applied 40 V is then sufficient to turn on all the elements

$$E - V_R - V_{o2} - V_{LED} = 0$$

$$I = I_{LED} = \frac{V_R}{R} = \frac{E - V_{o2} - V_{LED}}{R} = \frac{30 - 9.4 - 5}{2.2 \times 10^3} = 7.0$$

$$I = I_{LED} = 7.0 \times 10^{-3}\ mA$$

$$I = \mathbf{7.0\ mA}$$

The power delivered by the supply is simply the product of the supply voltage and current drain as follows:

$P_s = EI_s = EI$ = (30 V)(7.0 mA) = 0.212 W= **212 mW**

Q6.(ii) In the Figure Q6-ii, Determine whether the transistor is in saturation? Assume V_{CE} (Sat) = 0.2 V [7.5]

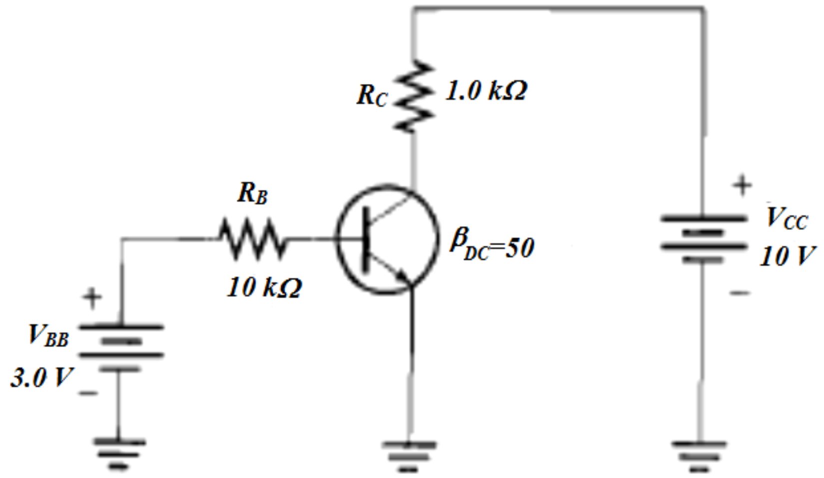


Figure Q6-ii

$$I_{C_{sat}} = \frac{V_{CC} - V_{CE_{sat}}}{R_C}$$

$$I_{C_{sat}} = \frac{10 - 0.2}{1 \times 10^3} = 9.8 \times 10^3 A = \mathbf{9.8\ mA}$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{3 - 0.7}{10 \times 10^3} = \mathbf{2.30 \times 10^{-4}\ A}$$

$$I_B = \mathbf{230\ \mu A}$$

$$I_C = \beta I_B$$

$$I_C = (50)(2.30 \times 10^{-4}) = \mathbf{0.0115\ A = 11.5\ mA}$$

Compare I_C with $I_{C_{sat}}$

As $I_C > I_{C_{sat}}$

The transistor operated in saturation region.