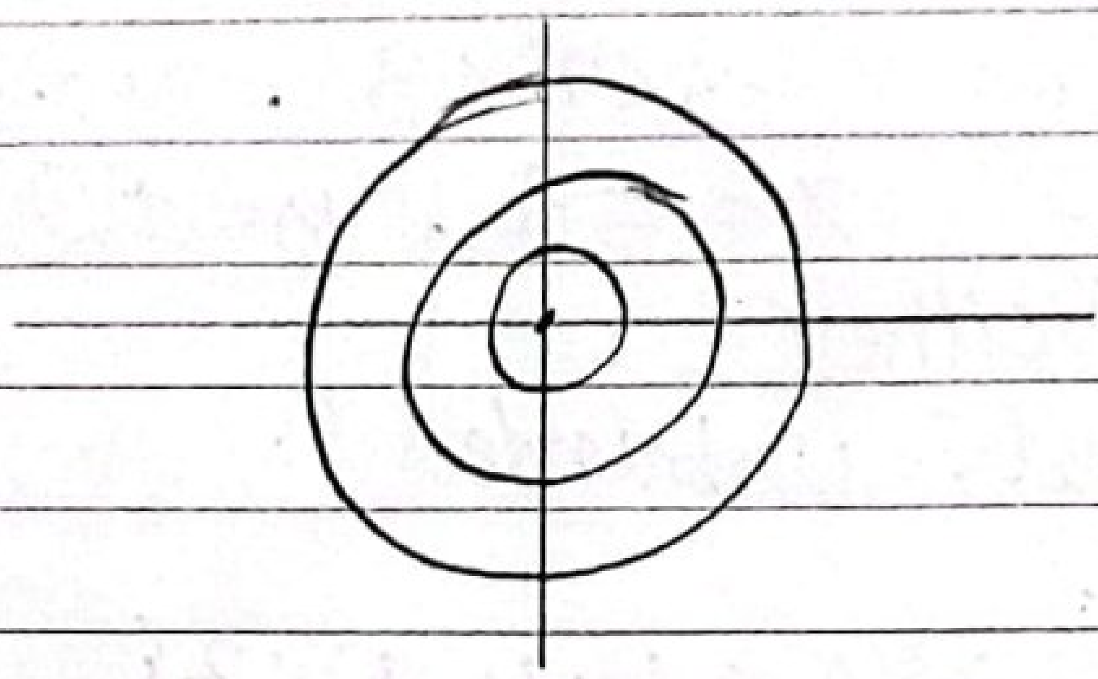
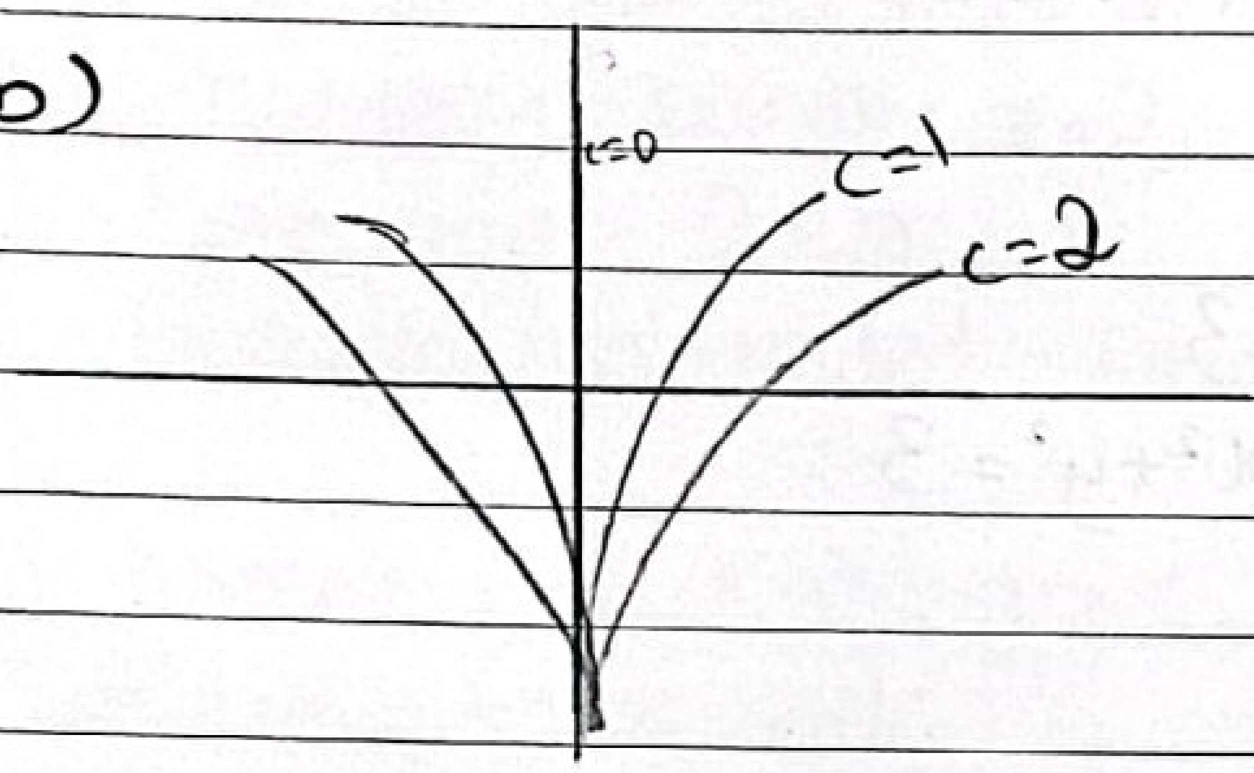
Multivariable Calculus
Assignment 1 Solution.

Question 1

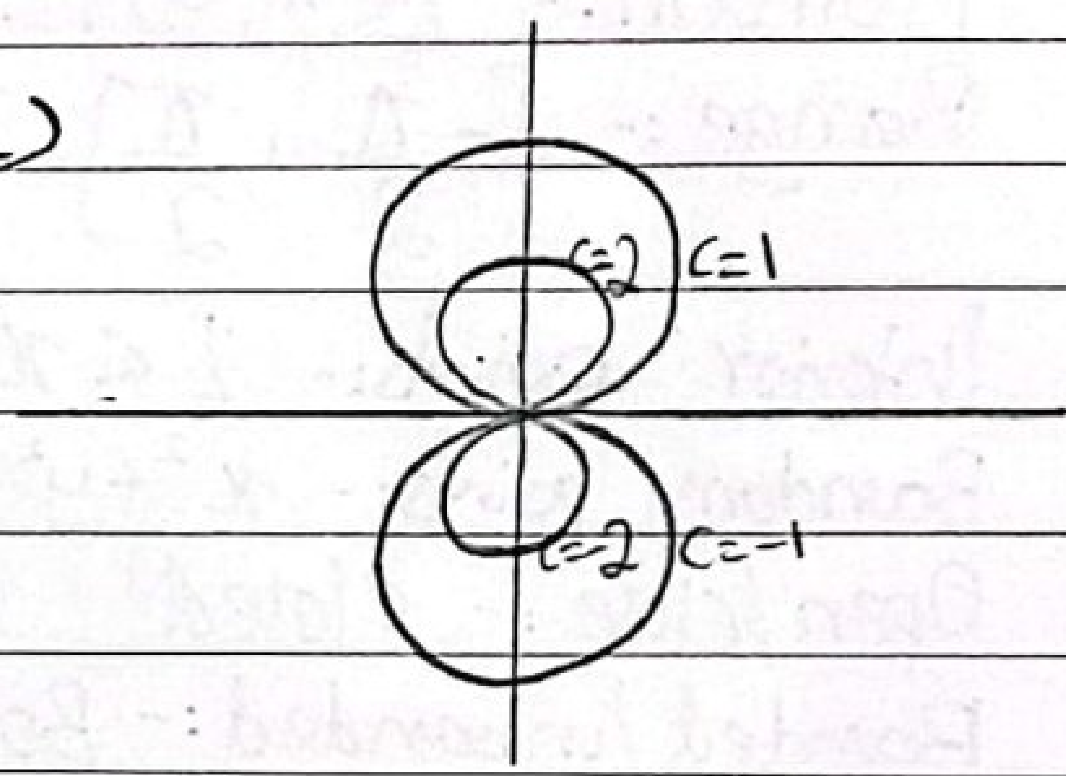
(a)



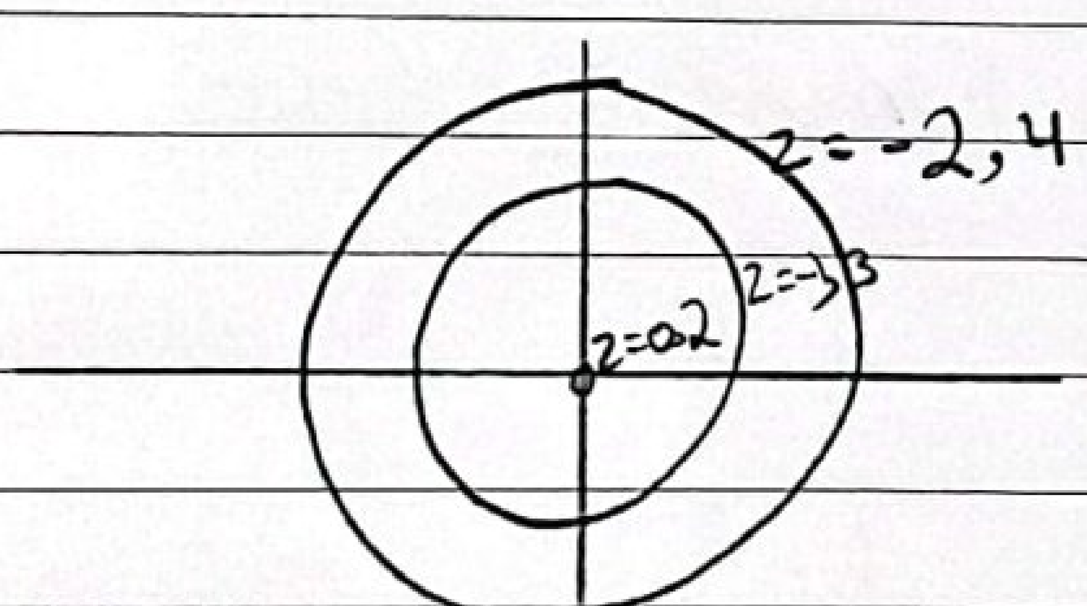
(b)



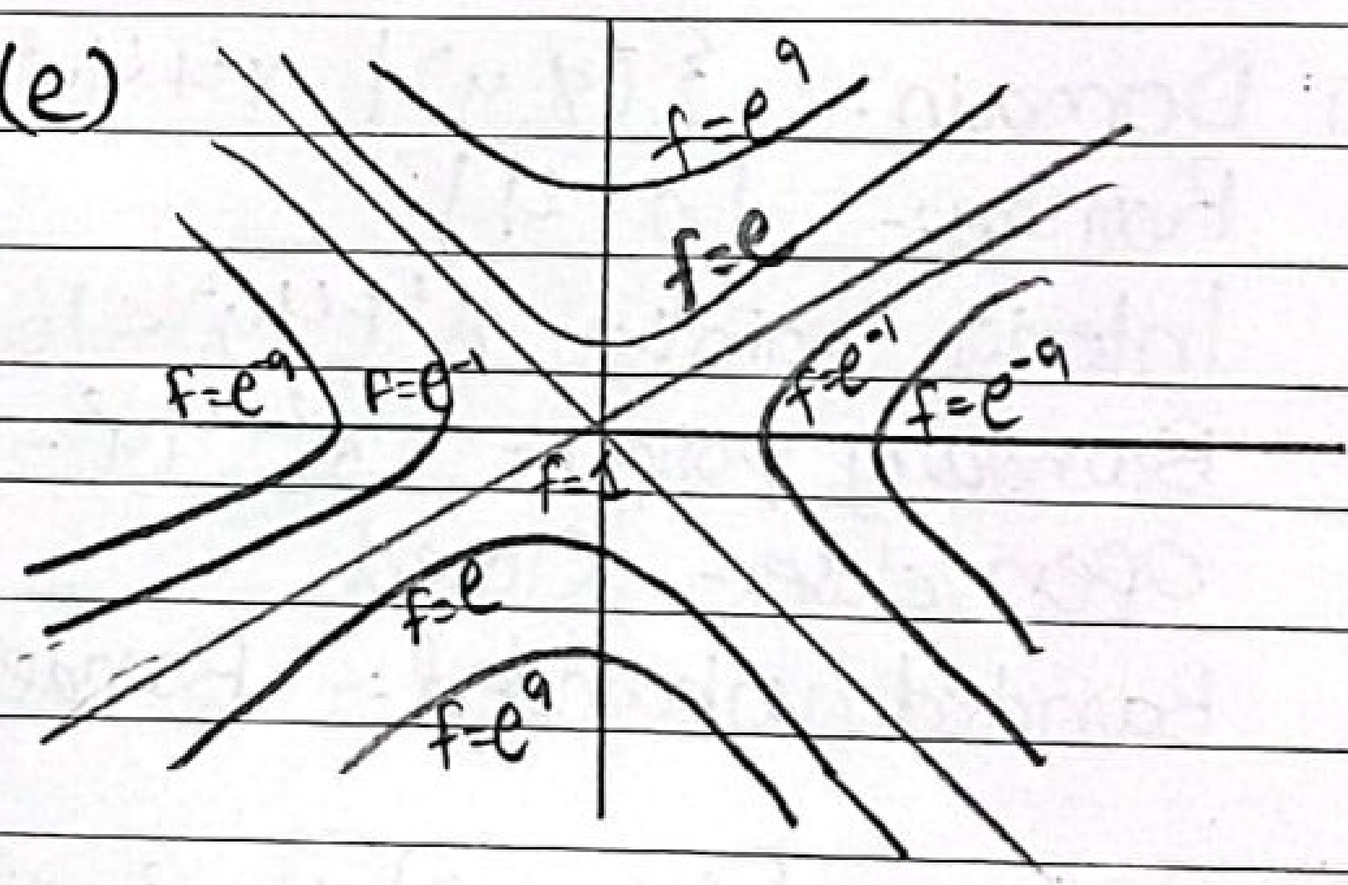
(c)



(d) $x^2+y^2=(z-1)^2-1$



(e)



Question 2

(a)	(b)	(c)
when $x=0 \rightarrow z=\|y\|$	$z=y^4$	$z=e^{-y^2}\sin(y^2)$
when $y=0 \rightarrow z=\|x\|$	$z=x^4$	$z=e^{-x^2}\sin(x^2)$
when $x=y \rightarrow z=2\|x\|$	$z=0$	$z=e^{-2x^2}\sin(2x^2)$
level curves $\rightarrow$ diamonds	hyperbolas	Circles

Question 3

(a) Domain:- $\{(x,y) \mid -3 \leq x \leq 3,\ y \geq 2,\ y \leq -2\}$
Range:- $[3, \infty)$
Interior points:- $-3 < x < 3$, $y > 2$, $y < -2$
Boundary points:- $x = \pm 3$, $y = \pm 2$
Open/close:- Neither
Bounded/unbounded:- unbounded

(b) Domain:- $\{(x,y) \mid 1 \leq (x^2+y^2) \leq 3\}$
Range:- $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Interior points:- $1 < x^2+y^2 < 3$
Boundary points:- $x^2+y^2 = 1$, $x^2+y^2 = 3$
Open/close:- Closed
Bounded/unbounded:- Bounded

(c) Domain:- $\{(x,y) \mid x^2+4y^2 \leq 16\}$
Range:- $[0, 4]$
Interior points:- $x^2+4y^2 < 16$
Boundary points:- $x^2+4y^2 = 16$
Open/close:- Closed
Bounded/unbounded:- Bounded

(d) Domain:- $\{(x,y,z) \mid x^2+y^2+z^2 < 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\}$
Range:- $\cancel{(-\infty, \infty)}$ $(-\infty, 2\ln 2)$
Interior points:- $x > 0, y > 0, z > 0, x^2+y^2+z^2 < 4$
Boundary points:- $x = 0$, $y = 0$, $z = 0$
Open/close:- ~~open~~ Neither
Bounded/unbounded:- Bounded

(e) Domain:- $\{(x,y,z) \mid 10-x-y-z \geq 0\}$
Range:- $(-\infty, \infty)$
Interior points:- $x+y+z<10$
Boundary points:- $x+y+z=10$
Open/close:- closed
Bounded/unbounded:- Unbounded

(f) Domain:- $\{(x,y) \mid x^2+y^2 \neq 1, x<2\}$
Range:- $(-\infty, \infty)$
Interior points:- $x<2$, $x^2+y^2<1$, $x^2+y^2>1$
Boundary points:- $x=2$, $x^2+y^2=1$
Open/close:- Open
Bounded/unbounded:- Unbounded

(g) Domain:- $\{(x,y) \mid y \geq x^2, x \neq \pm 1\}$
Range:- $(-\infty, \infty)$
Interior points:- $y>x^2$, $x \neq \pm 1$
Boundary points:- $y=x^2$, $x=\pm 1$
Open/close:- Neither
Bounded/unbounded:- Unbounded

Question 4

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2}-1}{x^2+y^2}$

$x = r\cos\theta$
$y = r\sin\theta$

$f(r,\theta) = \dfrac{e^{-r^2}-1}{r^2}$

$\lim\limits_{r\to 0} \dfrac{e^{-r^2}-1}{r^2} \longrightarrow \lim\limits_{u\to 0} \dfrac{e^{u}-1}{u} = 1$ (Apply L'Hopital's Rule)

$(u=-r^2)$

$$\lim_{r \to 0} \frac{e^{-r^2}-1}{r^2} = -1$$

(b) $\lim_{(x,y)\to(3,2)} \frac{x^2y}{x^2+y^2} = \frac{3^2 \times 2}{3^2+2^2} = \frac{18}{13}$

(c) $\lim_{(x,y)\to(3,2)} \frac{xy}{|xy|} = 1$

(d) $\lim_{(x,y)\to(1,0)} \frac{(x-1)^2 \ln x}{(x-1)^2+y^2}$ = indeterminate form $\left(\frac{0}{0}\right)$

Along $y=0$ ...

$$\frac{(x-1)^2 \ln x}{(x-1)^2} = \ln x = \ln(1) = 0$$

Along $x=1$

$$\frac{(1-1)^2 \ln 1}{(1-1)^2+y^2} = \frac{0}{y^2} = 0$$

limit exists & is equal to 0.

(e) $\lim_{(x,y)\to(1,0)} \frac{\sin(x)(e^y-1)}{xy} = \frac{\sin(0)(e^0-1)}{(0)(0)} = \frac{0}{0}$ (indeterminate)

Along $x=1$:

$$\frac{\sin(1)(e^y-1)}{1y} = \sin(1) . \frac{e^y-1}{y}$$

$$\lim_{y \to 0} \frac{e^y-1}{y} = 1$$

hence limit = sin(1)

(f) $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \frac{0}{0}$ (indeterminate)

let $\theta = x^2+y^2$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

(g) $\lim_{(x,y)\to(3,2)} e^{\sqrt{2x-y}} = e^{\sqrt{4}} = e^2$

(h) $\lim_{(x,y)\to(1,0)} \frac{xy^5}{x^8+y^{10}}$

Along $y=0$:

$\frac{x(0)}{x^8+0} = 0$

Along $x=1$:

$\frac{1(y^5)}{1+y^{10}}$

limit does not exist

(i) $\lim_{x\to 0} \frac{\sin 4x}{\tan x} = \frac{\frac{4\sin 4x}{4x}}{\frac{\sin x}{x\cos x}} = \frac{4 \lim_{x\to 0} \frac{\sin 4x}{4x}}{\left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \frac{1}{\cos x}\right)} = \frac{4}{1\cdot 1} = 4$

Question 5

(a) $\{(x,y) \mid x+y \neq 0\}$

(b) $\{(x,y) \mid x\neq 0, y\neq 0\}$

(c) $\{(x,y) \mid x^2+y^2 \leq 1, x \geq 0\}$

(d) $\{(x,y) \mid (x,y) \neq (0,0)\}$

Question 6

$$f(x,y,z) = xy^2z^3 + \arcsin(x\sqrt{z})$$

f_{xzy}

$$f_x = \frac{\partial}{\partial x}(xy^2z^3 + \arcsin(x\sqrt{z}))$$

$$= y^2z^3 + \frac{\sqrt{z}}{1-x^2z}$$

$$f_{xz} = 3y^2z^2 + \frac{\frac{1}{2\sqrt{z}} \cdot \sqrt{1-x^2z} - \sqrt{z} \cdot \frac{-x^2}{2\sqrt{1-x^2z}}}{1-x^2z}$$

$$= 3y^2z^2 + \frac{\frac{\sqrt{1-x^2z}}{2\sqrt{z}} + \frac{x^2z}{2\sqrt{1-x^2z}}}{1-x^2z}$$

$$f_{xzy} = \frac{\partial}{\partial y}(3y^2z^2) = 6yz^2$$

f_{yzx}

$$f_y = \frac{\partial}{\partial y}(xy^2z^3 + \arcsin(x\sqrt{z}))$$

$$= 2xyz^3$$

$$f_{yz} = \frac{\partial}{\partial z}(2xyz^3) = 6xyz^2$$

$$f_{yzx} = \frac{\partial}{\partial x}(6xyz^2) = 6yz^2$$