



National University
of Computer & Emerging Sciences
Islamabad



Department of Computer Sciences
CS1005- Discrete Structures
Assignment#3

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Semester: Spring 25

Note:

1. Please submit handwritten solutions to all of the questions given below.
 2. Scan your handwritten solution and upload to Google class room.
 3. No extension in deadline.
 4. Plagiarism would result in zero marks.
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1. Prove the following statement directly from the definitions of even and odd: For all integers a , if a is even, then $5(a + 3)$ is odd.
 2. Prove the following statement For all real numbers a and b , if $ab > 1$ then at least one of a or b is greater than 1.
 3. Prove the following statement directly from the definition of rational: For all real numbers v and s , if v and s are rational then $2v+3s$ is rational.
 4. Prove the following statement directly from the definition of divisibility: For any integers a and b , if $3 \mid a$ and $a \mid b$ then $3 \mid b$.
 5. Prove: For all integers m and n , if $m \bmod 5 = 2$ and $n \bmod 6 = 3$ then $mn \bmod 3 = 0$.
 6. All real num x , $[x]^2 = [x^2]$. Prove the statement if it is true and give a counterexample if it is false.
 7. When asked to prove that the difference of any irrational number minus any rational number is irrational, a student begins as follows: "Proof: Suppose the difference of any irrational number minus any rational number is rationalTM What is the student's mistake?

8. Prove by contradiction: $4+5\sqrt{2}$ is irrational. (You may use the fact that $\sqrt{2}$ is irrational.)
9. Prove by contradiction: If $3n + 2$ is odd, then n is odd.
10. Prove indirectly, there are infinitely many primes.
11. Prove indirectly, If $a, b \in \mathbb{Z}$ and $a \geq 2$, then a does not divide b or a does not divide $(b + 1)$.
12. If 0 = Sunday, 1 = Monday, 2 = Tuesday, ..., 6 = Saturday, then January 1 of year n occurs on the day of the week given by the following formula:

$$\left(n + \left\lfloor \frac{n-1}{4} \right\rfloor - \left\lfloor \frac{n-1}{100} \right\rfloor + \left\lfloor \frac{n-1}{400} \right\rfloor \right) \bmod 7.$$

Use the formula to January 1 of (i) 2020, (ii) 2000, (iii) year of your birth

13. Let m be a positive integer, if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then
 $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$

14. Caesar Cipher

Encrypts messages by changing each letter of the alphabet to the one three places farther along, with X wrapping around to A, Y to B, and Z to C. In other words, say each letter of the alphabet is coded by its position relative to the others—so that A=01, B=02, ..., Z=26. If the numerical version of the plaintext for a letter is denoted M and the numeric version of the ciphertext is denoted C , then

$$C = (M + 3) \bmod 26.$$

The receiver of such a message can easily decrypt it by using the formula

$$M = (C - 3) \bmod 26.$$

- a) What is the secret message produced from the message “MEET YOU IN THE PARK” using Caesar cipher?
- b) Decrypt the message encrypted using Caesar cipher
 - i) EOXH MHDQV
 - ii) HDW GLP VXP