Department of Computer sciences
Discrete Structures
Assignment#1

Discrete Mathematics

Spring 2025 (SE)

Question 1: Let p, q, and r be the propositions p : You have the flu. q : You miss the final examination. r : You pass the course. Express each of these propositions as an English sentence. **[6]**

a. p → q

b. ¬q ↔ r

c. q → ¬r

d. p ∨ q ∨ r

e. (p → ¬r) ∨ (q → ¬r)

f. (p ∧ q) ∨ (¬q ∧ r)

Question 2 : If statement forms P and Q are logically equivalent, then $P \leftrightarrow Q$ is a tautology. Conversely, if $P \leftrightarrow Q$ is a tautology, then P and Q are logically equivalent. Use $\leftrightarrow$ to convert each of the logical equivalences into a tautology. Then use a truth table to verify each tautology. **[10]**

    a) $p \rightarrow (q \lor r) \equiv (p \land \sim q) \rightarrow$ r
    b) $p \rightarrow (q \rightarrow r) \equiv (p \land q) \rightarrow$ r

Question 3: Write each of these propositions in the form "p if and only if q" in English. **[4]**
  a) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.
  b) If you read the newspaper every day, you will be informed, and conversely.
  c) It rains if it is a weekend day, and it is a weekend day if it rains
  d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him

Question 4: Use the properties to verify the logical equivalences in the following. Supply a reason for each step. **[25]**

a. $(p \wedge \sim q) \vee p \equiv p$

b. $p \wedge (\sim q \vee p) \equiv p$

c. $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

d. $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

e. $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p$

Question 5: How many assignments of truth values to p; q; r and w are there for which $((p \rightarrow q) \rightarrow r) \rightarrow w$ is true? Guess a formula in terms of the number of variables.   **[4+1]**

Question 6: "If compound X is boiling, then its temperature must be at least 150∘C." Assuming that this statement is true, which of the following must also be true?   **[5]**

a) If the temperature of compound X is less than 150∘C, then compound X is not boiling.

b) Compound X will boil only if its temperature is at least 150∘C.

c) If compound X is not boiling, then its temperature is less than 150∘C.

d) A necessary condition for compound X to boil is that its temperature be at least 150∘C.

e) A sufficient condition for compound X to boil is that its temperature be at least 150∘C.

Question 7: Write each of these statements in the form "if p, then q in English.   [8]

a. It snows whenever the wind blows from the northeast.

b. The apple trees will bloom if it stays warm for a week.

c. That the Pistons win the championship implies that they beat the Lakers.

d. It is necessary to walk 8 miles to get to the top of Long's Peak.

e. To get tenure as a professor, it is sufficient to be world-famous.

f. If you drive more than 400 miles, you will need to buy gasoline.

g. Your guarantee is good only if you bought your CD player less than 90 days ago.

h. Jan will go swimming unless the water is too cold.

Question 8: State the converse, contrapositive, and inverse of each of these conditional statements. **[3+3+3]**

    a. If it snows tonight, then I will stay at home
    b. I go to the beach whenever it is a sunny summer day.
    c.   When I stay up late, it is necessary that I sleep until noon.

Question 09   **[2+2]**

If $p \rightarrow q$ is false, can you determine the truth value of $(\sim p) \vee (p \leftrightarrow q)$? Explain your answer.

If $p \rightarrow q$ is true, can you determine the truth value of $\sim(p \rightarrow q) \wedge \sim p$? Explain your answer.

Question 10   **[3+2+2]**

a) Find simpler statement forms that are logically equivalent to

    $p \oplus p \equiv$ and $(p \oplus p) \oplus p \equiv$

c) Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? Justify your answer.
d) Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? Justify your answer.