Fast Nuces

Assignment # 1

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Question # 1

let A = (3m, 20° south of east),
B = (2m, north) and C = (5m, 70° south of west)

(a) Draw and label A, B, C with their tails at the origin. Use coridinate system with x-axis ot to the east.

(b) Write A, B, C in component form using unit vectors

(c) Find Magnitude and their direction of D = A+B+C

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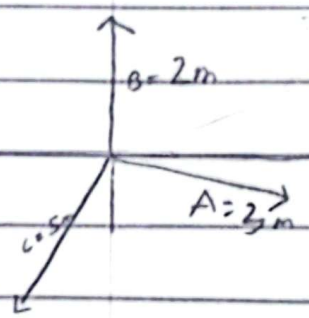
Answer # 1

(a)



(b) For A,

$A_x = A\cos\theta$, $A_y = A\sin\theta$

$A_x = 3\cos 340$ $A_y = 3\sin 340$

$A_x = 2.819\hat{i}$ $A_y = -1.026\hat{j}$

$\boxed{A = 2.819\hat{i} + (-1.026\hat{j})}$

For B,

$B_x = B\cos\theta$ $B_y = B\sin\theta$

$B_x = 2\cos 90$ $B_y = 2\sin 90$

$B_x = 0\hat{i}$ $B_y = 2\hat{j}$

$\boxed{B = 0i + 2j}$

For C,

$C_x = C\cos\theta$ $C_y = C\sin\theta$

$C_x = 5\cos 250$ $C_y = 5\sin 250$

$C_x = -1.710\hat{i}$ $C_y = -4.698\hat{j}$

$\boxed{C = -1.710\hat{i} + -4.698\hat{j}}$

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(c) For $D = A + B + C$

$$D_x = A_x + B_x + C_x$$
$$= 2.89 + 0 + (-1.71)$$
$$= \boxed{1.109}$$

$$D_y = A_y + B_y + C_y$$
$$= -1.026 + 2 + (-4.698)$$
$$= \boxed{-3.724}$$

For Magnitude:-

$$|D| = \sqrt{(D_x)^2 + (D_y)^2}$$
$$= \sqrt{(1.109)^2 + (-3.724)^2}$$
$$\boxed{|D| = 3.88}$$

For direction:-

$$\theta = \tan^{-1} \frac{D_y}{D_x}$$

$$\theta = \tan^{-1} \left(\frac{3.724}{1.109}\right) = 73.41$$

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Reference angle

$$\theta_R = 360 - \theta$$

$$= 360 - 73.41$$

$$\boxed{\theta_R = 286.59}$$

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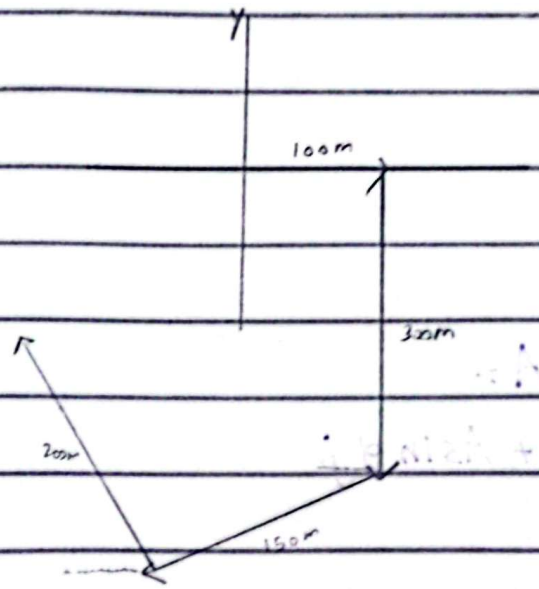
Question # 2

A person going for a walk follows the path shown in fig. The total trip consists of four straight line paths. At the end of the walk. What is the person's resultant displacesment measured from the standing point?



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Answer # 1

For Vector A,

$$A = A(\cos\theta + \sin\theta)$$

$$A = 100(\cos 0 + \sin 0) = 100\hat{i} + 0\hat{j}$$

For Vector B,

$B = B(\cos\theta + \sin\theta)$

$= 300(\cos 270 + \sin 270)$

$B = 0\hat{i} - 300\hat{j}$

For Vector C,

$C = C(\cos\theta + \sin\theta)$

$= 150(\cos 210 + \sin 210)$

$= -129.9\hat{i} - 75\hat{j}$

For vector D,

$D = D(\cos\theta + \sin\theta)$

$= 200(\cos 120 + \sin 120)$

$= -100\hat{i} + 173\hat{j}$

For Magnitude of Resultant 'R'

$R^2 = R_x^2 + R_y^2$

$R_x = A_x + B_x + C_x + D_x$

$= 100 + 0 + -129.9 - 100$

$\boxed{R_x = -129.9i}$

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$R_y = A_y + B_y + C_y + D_y$

$= 0 + 300 - 75 + 173$

$\boxed{R_y = -202j}$

$R = \sqrt{(R_x)^2 + (R_y)^2}$

$= \sqrt{(-129.9)^2 + (-202)^2}$

$\boxed{|R| = 240.16}$

For direction:-

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$= \tan^{-1}\left(\frac{202}{129.9}\right)$

$\theta = 57.26^\circ$

For reference angle

$\theta = 180 + 57.26$

$\boxed{\theta_R = 237.25^\circ}$

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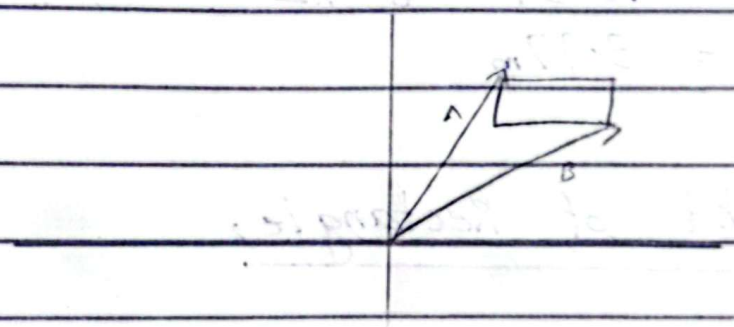
Question # 3

The rectangle shown in figure has sides parallel to the x and y-axis. The position vectors of two corners are A = 10m at 50° and B = 12m at 30°. (a) Find the perimeter of the rectangle (b) Find the magnitude and direction of the vector from origin to the upper right corner of the rectangle.



Answer # 3

For Vector A,

$$A = A_x + A_y$$
$$= A\cos\theta + A\sin\theta$$
$$= 10\cos 50 + 10\sin 50$$
$$= 6.42\hat{i} + 7.66\hat{j}$$

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For Vector B,

$B = B_x + B_y$

$= B\cos\theta + B\sin\theta$

$= 12\cos 30 + 12\sin 30$

$= 10.39\hat{i} + 6\hat{j}$

For length of rectangle

$|B_x - A_x| = 10.39 - 6.42$

$= 3.97m$

For hight of Rectangle,

$|A_y - B_y| = 17.66 - 6$

$= 1.66m$

To find Perimeter (P) :-

$P = 2 \times (\text{length} \times \text{hight})$

$= 2 \times (3.97 \times 1.66)$

$= 11.248\ m$

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For top right point (A) :-

$A = (B_x, A_y)$

$A = 10.39\hat{i} + 7.66\hat{j}$

$|A| = \sqrt{(10.39)^2 + (7.66)^2}$

$= 12.819\ m$

Direction

$\theta = \tan^{-1} \frac{A_y}{B_x}$

$= \tan^{-1} \frac{7.66}{10.39}$

$= 36.65°$

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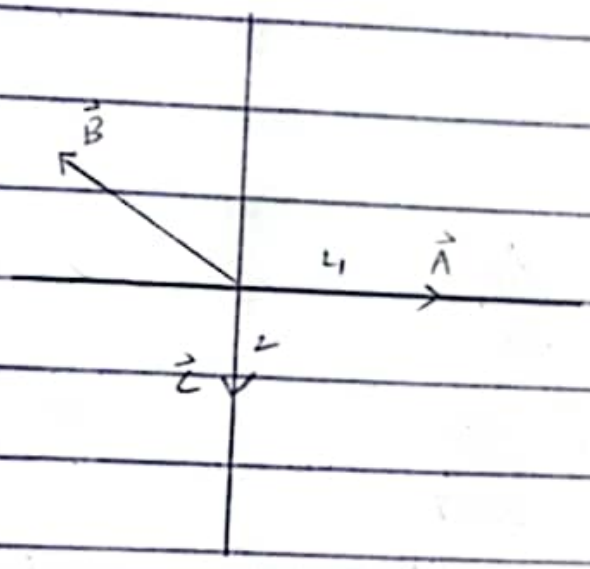
Question # 4

For the three vectors $A + B + C = 1j$. What is vector B.

(a) Write B in its components form

(b) Write magnitude and direction of vector B.



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Answer # 4

$\vec{A} = 4\hat{i} + 0j$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

$\vec{C} = 0\hat{i} - 2\hat{j}$

As,

$A + B + C = 1\hat{j}$

$4i + B_x\hat{i} + B_y\hat{j} - 2\hat{j} = 1j$

Splitting equations :-

$4\hat{i} + B_x\hat{i} = 0\hat{i}$ -- (i)

$4\hat{i} + B_x\hat{i} = 0\hat{i}$

$\boxed{B_x\hat{i} = -4\hat{i}}$

$B_y\hat{j} + -2\hat{j} = 1j$

$\boxed{B_y\hat{j} = 3\hat{j}}$

$\boxed{\vec{B} = -4\hat{i} + 3\hat{j}}$

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(b) Magnitude of B :-

$\cancel{|B| = (-4^2) + 3^2}$

$|B| = \sqrt{(-4)^2 + 3^2}$

$|B| = 5$

Direction :-

$\theta = \tan^{-1} \frac{B_y}{B_x}$

$= \tan^{-1} \left(\frac{3}{4}\right)$

$\theta = 36.8°$

Reference angle

$\theta_R = 180 - 36.8 = 143.13°$

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(a) Components of $\vec{B}$:

$B_x = B\cos\theta$

$B_x = 5\cos 143.13\,\hat{i}$

$=$

$B_y = B\sin\theta$

$B_y = 5\sin 143.13\,\hat{j}$

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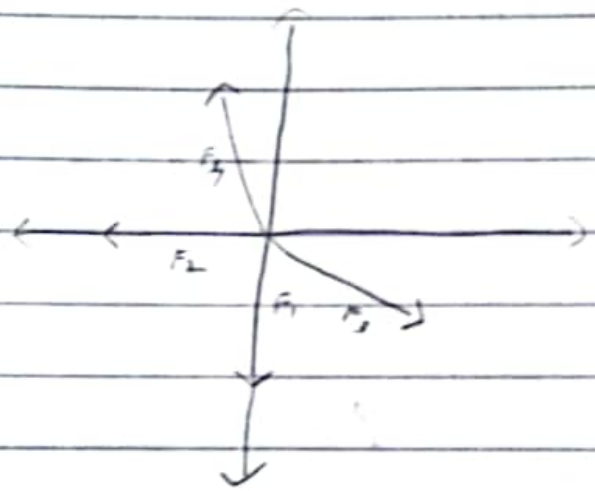
Question # 5

Four concurrent forces acts on the center of mass of a landing airplane. Calculate the resultant force and the angle it will make with x-axis



Answer # 5

F_1 = 2000 lb towards -y-axis
F_2 = 1000 lb towards -x-axis
F_3 = 500 lb towards South West
F_4 = 2000 lb towards West of North

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$F_1 = 0\hat{i} - 2000\hat{j}$

$F_2 = -1000\hat{i} + 0\hat{j}$

$F_3 = 500\cos 315 + 500\sin 315 = 353\hat{i} - 353.5\hat{j}$

$F_4 = 2000\cos 120 + 2000\sin 120 = -1000\hat{i} + 1732.0\hat{j}$

For F_x :-

$F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$= 0 + (-1000) + 353.5 - 1000$

$F_x = -1646.5\hat{i}$

For F_y :-

$F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$= -2000 + 0 - 353.5 + 1732$

$F_y = -621.5\hat{j}$

For F :-

$F = \sqrt{F_x^2 + F_y}$

$= \sqrt{(-1646)^2 + (-621.5)^2}$

$F = 1759.4$

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For δ:-

$$\delta = \tan^{-1}\left(\frac{621.5}{1646.5}\right)$$

$$\delta = 20.6^\circ$$

For Reference angle

$$\delta_R = 180 + 20.6$$

$$\boxed{\delta_R = 200.6^\circ}$$

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Question # 6

Find the gradient of the following

(a) $f(x,y,z) = x^2 y^3 z^4$

(b) $f(x,y,z) = e^x \sin(y) \ln(z)$

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Answer # 6

) $f(x,y,z) = x^2y^3z^4$

$$\nabla f = \frac{\delta f}{\delta x}\hat{x} + \frac{\delta f}{\delta y}\hat{y} + \frac{\delta f}{\delta z}\hat{z}$$

$$= 2xy^3z^4\,\hat{x} + 3y^2x^2z^4\,\hat{y} + 4z^3x^2y^3\,\hat{z}$$

(ii) $f(x,y,z) = e^x \sin y \ \ln(z)$

$$\nabla f = \frac{\delta f}{\delta x}\hat{x} + \frac{\delta f}{\delta y}\hat{y} + \frac{\delta f}{\delta z}\hat{z}$$

$$= e^x \sin y \ln z\,\hat{x} + \cos y\, e^x \ln z\,\hat{y} + \frac{1}{z} e^x \sin y\,\hat{z}$$

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Question # 7

The hight of certain hill in feet is given by

$$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

Where y is the distance in north, x is the distance in east.

(a) Where is top of hill located

(b) How high is hill

(c) How steep is the slope at point 1-mile north and 1 mile east, in what direction slope is steepest?

Answer # 7

(a) Top of hill:

Gradient of $h(x,y) = 0$

$$\frac{\delta h}{\delta x} = 0 \quad \text{--- (i)}$$

$$\frac{\delta h}{\delta y} = 0 \quad \text{--- (ii)}$$

(ii) $\frac{\delta h}{\delta x} = 0$

$\frac{\delta h}{\delta x} 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12) = 0$

$10(2y - 6x - 18) = 0$

$2y - 6x - 18 = 0 \quad \cdots (i)$

(iii) $\frac{\delta h}{\delta y} = 0$

$\frac{\delta h}{\delta y}(10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12) = 0$

$10(2x - 8y + 28) = 0$

$2x - 8y + 28 = 0 \quad \cdots (ii)$

$2x = 8y - 28$

Multiplying b/s by 3

$6x = 24y - 84 \quad \cdots (iii)$

Put value of $6x$ in eq (i)

$2y - 24y + 84 - 18 = 0$

$-22y + 66 = 0$

$\boxed{y = +3}$

For x:

Put y in eq (ii)

$2x - 8(+3) + 28 = 0$

~~$30x + 24$~~

$2x + 4 = 0$

$\boxed{x = -2}$

So top of hill is $(-2, 3)$

(b) To find highl at $(-2, 3)$
Put it in function:-

$h(-2,3) = 10(2(-2)(3) - 3(-2)^2 - 4(3)^2 - 18(-2)$
$+ 28(3) + 12$
$= \underline{720 \text{ feet}}$

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(c) To find slope at (1,1)

$$\vec{\nabla} h = \frac{\delta h}{\delta x}\hat{i} + \frac{\delta h}{\delta y}\hat{j}$$

$$= (20y - 60x - 180)\hat{i} + (20x - 80y + 280)\hat{j}$$

$= $ Put $(x,y) = (1,1)$

$$= (20 - 60 - 180) + (20 - 80 + 280)$$

$$\underline{\vec{\nabla} h = -220\hat{i} + 220\hat{j}}$$

For direction :-

$$\theta = \tan^{-1}\left(\frac{220}{220}\right)$$

$$\theta = 45^\circ$$

Reference angle

$$\theta = 180 - 45$$

$$\underline{\theta_R = 135^\circ}$$

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Question # 8

Calculate the divergence and curl of the following functions:

(a) $V = x^2\hat{i} + 3xy^2\hat{j} - 2xyz\hat{k}$

(b) $V = xy\hat{i} + 2yz\hat{j} + 3zx\hat{k}$

(c) $V = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$

Answer # 8

(a) $V = x^2\hat{i} + 3xy^2\hat{j} - 2xyz\hat{k}$

<u>divergence :-</u>

let:

$P = x^2$, $Q = 3xy^2$, $R = -2xyz$

$$\text{Div}\,V = \frac{\delta P}{\delta x} + \frac{\delta Q}{\delta y} + \frac{\delta R}{\delta z}$$

$$= 2x + 6xy - 2xy$$

$$\text{Div}\,V = 2x + 4xy$$

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Curl

$$\text{Curl } V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ P & Q & R \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ x^2 & 3xy^2 & -2xyz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\delta}{\delta y}(-2xyz) - \frac{\delta}{\delta z}(3xy^2)\right)$$
$$-\hat{j}\left(\frac{\delta}{\delta x}(-2xyz) - \frac{\delta}{\delta z}(x^2)\right)$$
$$+\hat{k}\left(\frac{\delta}{\delta x}(3xy^2) - \frac{\delta}{\delta y}(x^2)\right)$$

$$= \hat{i}(-2xz - 0)$$
$$-\hat{j}(-2yz - 0)$$
$$+\hat{k}(3y^2 - 0)$$

$$= \underline{-2xz\,\hat{i} + 2yz\,\hat{j} + 3y^2\,\hat{k}}$$

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(b) $V = xy\hat{i} + 2yz\hat{j} + 3zx\hat{k}$

Divergence :-

$P = xy$, $Q = 2yz$ $R = 3zx$

$$\text{Div } V = \frac{\delta P}{\delta x} + \frac{\delta Q}{\delta y} + \frac{\delta R}{\delta z}$$

$$\text{Div } V = y + 2z + 3x$$

Curl :-

$$\text{Curl} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ P & Q & R \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ xy & 2yz & 3zx \end{vmatrix}$$

$$= \hat{i}\left(\frac{\delta}{\delta y}(3zx) - \frac{\delta}{\delta z}(2yz)\right)$$
$$-\hat{j}\left(\frac{\delta}{\delta x}(3zx) - \frac{\delta}{\delta z}(xy)\right)$$
$$\hat{k}\left(\frac{\delta}{\delta x}(2yz) - \frac{\delta}{\delta y}(xy)\right)$$

$$= \hat{i}(0 - 2y) + -\hat{j}(3z - 0) + k(0 - x)$$

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$$= -2y\,\hat{i} - 3z\,\hat{j} - x\,\hat{k}$$

(c) $V = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$

Divergence :-

$P = y^2 \qquad Q = 2xy + z^2 \qquad R = 2yz$

$$\text{Div } V = \frac{\delta P}{\delta x} + \frac{\delta Q}{\delta y} + \frac{\delta R}{\delta z}$$

$$= 0 + 2x + 0 + 2y$$

$$= 2x + 2y$$

Curl :-

$$\text{Curl } V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ P & Q & R \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ y^2 & 2xy+z^2 & 2yz \end{vmatrix}$$

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$$= \hat{i}\left(\frac{\delta}{\delta y}(2yz) - \frac{\delta}{\delta z}(2xy + z^2)\right)$$
$$-\hat{j}\left(\frac{\delta}{\delta z}(y^2) - \frac{\delta}{\delta x}(2yz)\right)$$
$$k\left(\frac{\delta}{\delta x}(2yx + z^2) - \frac{\delta}{\delta y}(y^2)\right)$$

$$= \hat{i}(2z - 2z)$$
$$-\hat{j}(0 - 0)$$
$$k(2y - 2y)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Question # 9

A field mouse trying to escape a hawk runs east for 5m, darts south east for 3m then drops 1cm straight down a hole into its barrow. What is the magnitude of its east displacement.

Answer # 9

A = 5m at +x-axis

B = 3m at South east (315° from x-axis, anticlockwise)

For Vector A:-

$\vec{A} = 5\hat{i} + 0\hat{j}$

For Vector B:-

$\vec{B} = B\cos\theta + B\sin\theta$

$= 3\cos 315\,\hat{i} + 3\sin 315\,\hat{j}$

$= 2.12\,\hat{i} - 2.12\,\hat{j}$

For Vector C:-

$\vec{C} = 0\hat{i} - 0\hat{j}$

For Resultant R:-

$R_x = A_x + B_x + C_x$

$= 5 + 2.12 + 0$

$R_x = 7.12\ m$

$R_y = A_y + B_y + C_y$

$= \boxed{0 - 2.12 - 0 = -2.12m}$

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$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(7.12)^2 + (2.12)^2}$$

$$\boxed{R = 7.48}$$

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Question # 10

A flock of ducks is trying to migrate south for the water, but they keep being blown off course by a wind blowing from the west at 6 m/s. A wise elder duck finally realize that solution is to fly at an angle to the wind. If the duck can fly at 8 m/s relative to air, what direction should they head in order to move directly south.

Answer # 10

Speed of wind $v_w = 6 m/s$

Speed of duck $v_d = 8 m/s$

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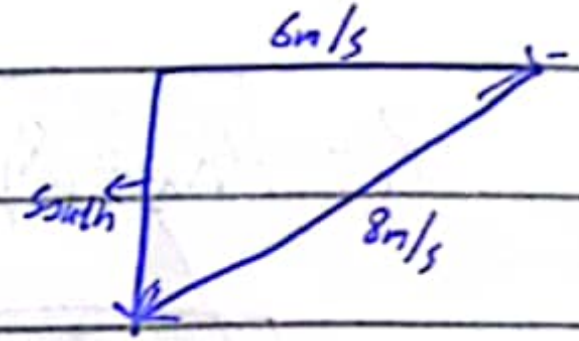
$$\sin\theta = \frac{V_w}{V_d}$$

$$\sin\theta = \frac{6}{8}$$



$$\theta = \sin^{-1}\left(\frac{6}{8}\right)$$

$$\theta = 48.6^\circ$$