

a)  $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$A_x = A \cos \theta_A = 72.4 \cos 58^\circ = 38.37$   
 $A_y = A \sin \theta_A = 72.4 \sin 58^\circ = 61.40$

$\theta_B = 216^\circ$   $B_x = -46.36$   $B_y = -33.68$   $17.8$

$C_x = 0$   $C_y = -17.8$

$R_x = -7.99$   $R_y = 9.92$

$R = 12.7$

$\theta = \tan^{-1} \left( \frac{9.92}{-7.99} \right) = -51^\circ$

$\theta = 180^\circ - 51^\circ = 129^\circ$

Q1(ii). Use concept of operators to find the directional derivative of  $\phi(x,y,z) = x^2yz + 4xz^2$  at  $(1,-2,1)$  in the direction of  $2\hat{i} - \hat{j} - 2\hat{k}$ .

$$\nabla \phi = (2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8xz)\hat{k}$$

$$(1, -2, 1)$$

$$(-4 + 4)\hat{i} + \hat{j} + (-2 + 8)\hat{k}$$

$$\nabla \phi = \hat{j} + 6\hat{k}$$

$$\hat{a} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{4 + 1 + 4}} \Rightarrow \frac{1}{3} (2\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{\nabla} \phi \cdot \hat{a} = (\hat{i} + 6\hat{k}) \cdot \frac{1}{3} (2\hat{i} - \hat{j} - 2\hat{k})$$

$$\nabla \phi \Rightarrow -13/3$$

Q.2 (i)

(a)  $\Delta y = -144 \text{ ft}$      $V_i = 0$      $a = g = 32 \text{ ft/s}^2$

$$\Delta y = V_i t - \frac{1}{2} g t^2$$

$$144 \text{ ft} = 0 - 16 t^2$$

$$t = 3 \text{ s}$$

$$V_f = V_i - g t = 0 - (32)(3) = -96 \text{ ft/s}$$

(b)  $V_i = -96$      $V_f = 0$      $\Delta y = 1.50 \text{ ft}$

$$a = \frac{V_f^2 - V_i^2}{2 \Delta y} = 3.07 \times 10^3 \text{ ft/s}^2$$

(c)  $\bar{V} = \frac{\Delta y}{\Delta t} = \frac{V_i + V_f}{2} = \frac{\Delta y}{\Delta t}$

$$\Delta t = \frac{\Delta y}{\frac{V_f + V_i}{2}} = \Delta t = 3.13 \times 10^{-2} \text{ s}$$



Q. 2 (ii)

$$a) \quad y = y_0 + x \tan \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$$

$$= 3 + 23 \tan 53^\circ - \frac{(9.8)(23)^2}{2(26.5)^2 (\cos 53^\circ)^2}$$

$$= 23.3$$

$$\boxed{\Delta y = 23.3 - 18 = 5.3 \text{ m}}$$

$$b) \quad y = y_0 + x \tan \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta_0}$$

$$= 3 + 34.5 \tan 53^\circ - \frac{9.8 (34.5)^2}{2(26.52)^2 (\cos 53^\circ)^2}$$

$$= 25.9$$

$$\boxed{\Delta y = 25.9 - 18 = 7.9 \text{ m}}$$

$$\Rightarrow \boxed{R = x = \frac{v_0^2 \sin 2\theta}{g} = 69 \text{ m}}$$

Q<sub>2</sub> (iii)

$$y = \frac{1}{2} a_y t^2 = 30 = \frac{1}{2} (0.4 \cos \theta) t^2 \quad \text{--- 1}$$

$$x_A = x_B$$

$$3t = \frac{1}{2} [(0.4) \sin \theta] t^2 \quad \text{--- 2}$$

Using eq 2.

$$t = \frac{2v}{ax}$$

Substituting in 1.

$$30 = \frac{1}{2} (0.4 \cos \theta) \left( \frac{2(3)}{0.4 \sin \theta} \right)^2$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$30 = \frac{90}{0.20} \frac{\cos \theta}{1 - \cos^2 \theta}$$

$$\cos \theta = \frac{1}{2}$$