National University of Computer and Emerging Sciences

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Q1. Two sinusoidal waves with the same amplitude of 9.00 mm and the same wavelength travel together along a string that is stretched along an x axis. Their resultant wave is shown twice in Fig.1, as valley A travels in the negative direction of the x axis by distance d = 56.0 cm in 8.0 ms. The tick marks along the axis are separated by 10 cm, and height H is 8.0 mm. Let the equation for one wave be of the form $y(x, t) = y_m \sin(kx \pm \omega t + \phi_1)$, where $\phi_1 = 0$ and you must choose the correct sign in front of ω. For the equation for the other wave, what are (a) y_m, (b) k, (c) v, (d) ϕ_2, and (e) the sign in front of ω?

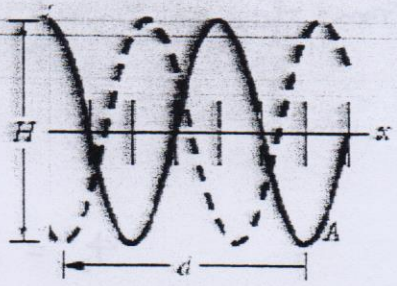


Figure 1

$y_m = 9.0\,mm = 0.009\,m$

$dx = d = 56.0\,cm = 0.56\,m$

$dt = t = 8.0\,ms = 8\times10^{-3}\,s$

Scale = 10 cm.

$H = 8.0\,mm.$

$y_1(x,t) = y_m \sin(kx \pm \omega t + \phi_1)$ with $\phi_1 = 0$.

For $y_2 = ?$

(a) $y_m = ?$

$y_m = 9.0\,mm = 0.009\,mm$

[2.0]

(b) $k = ?$

From figure $\lambda = 4 \times 10\,cm = 40\,cm = 0.4\,m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 15.70\,rad/m.$

[2.0]

(c) $v = ?$

$v = \frac{dx}{dt} = \frac{d}{t} = \frac{56.0\,cm}{8\,ms} = \frac{0.56\,m}{8\times10^{-3}\,s} = 70\,m/sec.$

Since $v = f\lambda$

$f = \frac{v}{\lambda} \Rightarrow \quad \omega = 2\pi f = 2\pi \frac{v}{\lambda} = kv$

$$\omega = (15.70)(70) = 1099 \text{ rad/sec.}$$ [2.0]

(d) $\phi_2 = ?$

The figure depicts the resultant of two travelling waves.

From figure $y'_m = \frac{H}{2} = \frac{8.0 \text{ mm}}{2} = 4.0 \text{ mm}$

Since $y'_m = 2 y_m \cos\left(\frac{\phi_2}{2}\right)$

$$\phi_2 = 2\cos^{-1}\left(\frac{y'_m}{2 y_m}\right)$$

$$\phi_2 = 2\cos^{-1}\left(\frac{4.0}{2(9.0)}\right)$$

$$\phi_2 = 2.69 \text{ rad.}$$ [2.0]

(e) Sign in front of $\omega = ?$

As said in the statement, valley travel in the -ive x direction.

$\Rightarrow$

$$y(x,t) = y_m \sin(kx + \omega t + \phi)$$

Therefore

$$y_1(x,t) = (0.009)\sin(15.7x + 1099t)$$

$$y_2(x,t) = (0.009)\sin(15.7x + 1099t + 2.69).$$ [2.0]

Q2. In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x(t) = (5.0cm)\cos\left(2t + \frac{\pi}{6}\right)$$

where x is in centimeters and t is in seconds. At $t = 0$, find (a) the position of the particle, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion. [10]

$x(t) = x_m \cos(\omega t + \phi)$

At $t = 0$.

Find.

(a) $x(0) = ?$

$x(0) = (5.0\ cm)\cos\left(2(0) + \frac{\pi}{6}\right) = (5.0\ cm)\cos\left(\frac{\pi}{6}\right) = 4.33\ cm$

[2.5]

(b) $v(0) = ?$

$v(t) = \frac{dx}{dt} = -(5.0\ cm)\, 2\sin\left(2t + \frac{\pi}{6}\right)$

$v(t) = -(10.0\ cm/s)\sin\left(2t + \frac{\pi}{6}\right)$

$v(0) = -(10.0\ cm/s)\sin\left(0 + \frac{\pi}{6}\right) = -(10.0\ cm/sec)\sin\left(\frac{\pi}{6}\right)$

$v(0) = -5.0\ cm/sec$ [2.5]

(c) $a(0) = ?$

$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -(20.0\ cm/s^2)\cos\left(2t + \frac{\pi}{6}\right)$

$a(0) = -(20.0\ cm/s^2)\cos\left(0 + \frac{\pi}{6}\right) = -20.0\ cm/s^2 \cos\left(\frac{\pi}{6}\right)$

$a(0) = -17.32\ cm/s^2$ [2.5]

(d) $T = ?$, and $x_m = ?$

$x_m = 5.0\ cm$, $\omega = 2$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi\ sec = 3.14\ sec.$ [2.5]

Q3. In Figure Q3, two blocks (m=1.8 kg and M = 10 kg) and a spring (k = 200 N/m) are arranged on a horizontal, frictionless surface. The coefficient of static friction between the two blocks is 0.40. What amplitude of simple harmonic motion of the spring–blocks system puts the smaller block on the verge of slipping over the larger block? [10]

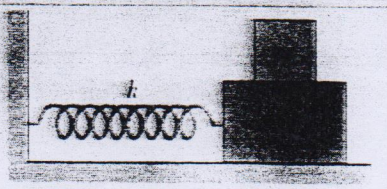
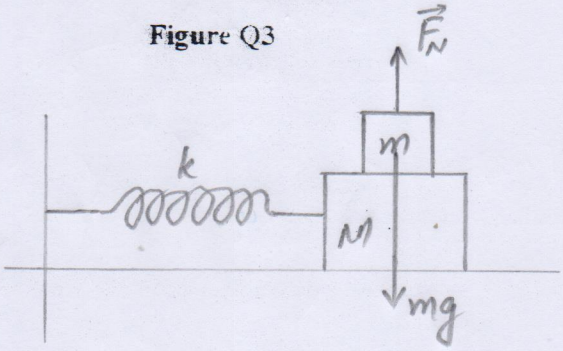


Figure Q3



$m = 1.8$ kg

$M = 10$ kg

$k = 200$ N/m

$\mu_s = 0.40$ (b/w the blocks)

$x_m = ?$

To be on the verge of slipping, the force exerted on the small block (at the point of maximum acceleration) is

$$f_{s,max} = \mu_s F_N$$

From figure $F_N = mg$

$$f_{s,max} = mg\mu_s$$ [2.5]

Since $a_m = \omega^2 x_m$

$$\omega = \sqrt{\frac{k}{m+M}}$$

$$a_m = \left(\frac{k}{m+M}\right) x_m.$$ [2.5]

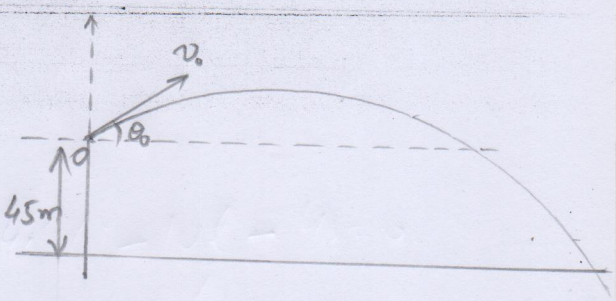
Therefore $m a_m = f_{s,max}.$

$$\cancel{m}\left(\frac{k}{m+M}\right) x_m = \cancel{m} g\mu_s$$

$$x_m = \frac{g\mu_s (m+M)}{k} = \frac{(9.8)(0.4)(1.8+10)}{200}$$

$$x_m = 0.23\,m.$$ [5.0]

Q4. A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s. The height of the building is 45.0 m. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone just before it strikes the ground? [10]



$\theta_0 = 30^\circ$

$v_0 = 20.0\ m/s$

$y = -45\ m \qquad (x_0 = 0,\ y_0 = 0)$

(a) time $t = ?$

$v_{0x} = v_0 \cos\theta = 20.0 \cos(30^\circ) = 17.3\ m/sec$

$v_{0y} = v_0 \sin\theta = 20.0 \sin(30^\circ) = 10.0\ m/sec$

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2$$

$$-45 - 0 = 10t - \frac{1}{2}(9.8)\, t^2$$

$$4.9t^2 - 10t - 45 = 0.$$

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4.9)(-45)}}{2(4.9)} = \frac{10 \pm \sqrt{982}}{9.8} = \frac{10 \pm 31.33}{9.8}$$

Choose +ive root.

$$t = \frac{41.33}{9.8} = 4.218\ sec.$$

[5]

b $v = ?$

$v_x = v_{0x} = 17.3\ m/sec$

$v_y = v_{0y} - gt = 10 - (9.8)(4.218)$

$v_y = -31.3\ m/sec$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.3)^2}$$

$v = 35.8\ m/s$

[5.0]

Q5. Figure Q5 shows a box of dirty money (mass m_1= 3.0 kg) on a frictionless plane inclined at angle θ_1=30°. The box is connected via a cord of negligible mass to a box of laundered money (mass m_2 =2.0 kg) on a frictionless plane inclined at angle θ_2=60°. The pulley is frictionless and has negligible mass. What is the tension in the cord? [10]

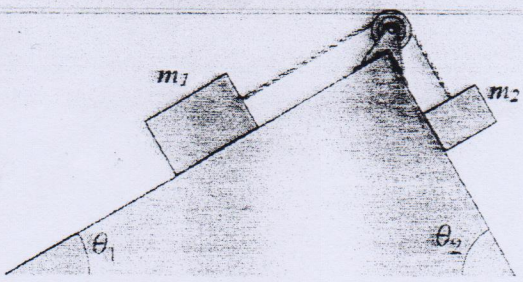


Figure Q5

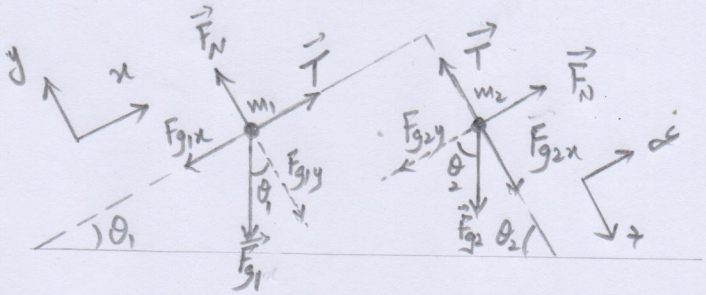
$m_1 = 3.0\,kg$

$\theta_1 = 30°$

$m_2 = 2.0\,kg$

$\theta_2 = 60°$

$T = ?$



From free body diagram. [2.0]

For box m_1; along x-direction

$$F_{net,x} = m_1 a_x$$

$$T - F_{g1x} = m_1 a$$

$$T - m_1 g \sin\theta_1 = m_1 a \rightarrow (1) \quad [2.0]$$

For box m_2; along x-direction

$$F_{net,x} = m_2 a_x$$

$$-T + F_{2gx} = m_2 a$$

$$-T + m_2 g \sin\theta_2 = m_2 a \rightarrow (2) \quad [2.0]$$

Adding Eq (1) and (2) and solve for 'a'

$$m_2 g \sin\theta_2 - m_1 g \sin\theta_1 = m_1 a + m_2 a$$

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1) g}{m_1 + m_2} \rightarrow (3)$$

$$a = \frac{[2\sin(60) - 3\sin(30)](9.8)}{3+2} = 0.44\ m/s^2 \quad [2.0]$$

From Eq (1)

$$T = m_1 a + m_1 g \sin\theta_1$$

$$T = (3)(0.44) + (3)(9.8)\sin(30°)$$

$$\boxed{T = 16.02\ N} \quad [2.0]$$

Q6. Figure Q6 shows a 4.00 kg object is attached to a vertical rod by two strings. The object rotates in a horizontal circle at constant speed 6.00 m/s. Find the tension in (a) the upper string and (b) the lower string. The upper and lower strings make angle θ with vertical. [10]

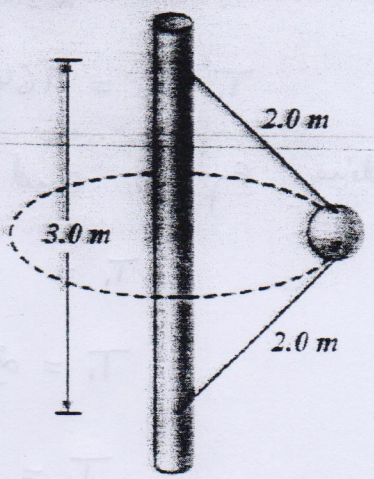


Figure Q6

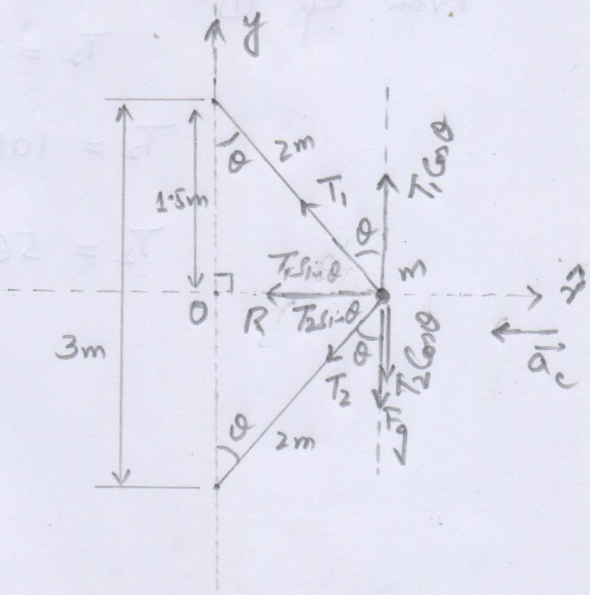
$m = 4.0\ kg$

$v = 6.0\ m/s$

(a) Tension in upper string $T_1 = ?$

(b) Tension in lower string $T_2 = ?$

Free body diagram. [1.0]



$\cos\theta = \frac{1.5}{2} = 0.75$

$\theta = \cos^{-1}(0.75) = 41.4^\circ$ [1.0]

Verticle Component of forces

$F_{net,\ verticle} = ma$

$T_1 \cos\theta - T_2 \cos\theta - F_g = m(0)$

$(T_1 - T_2)\cos\theta - mg = 0$

$T_1 - T_2 = \frac{mg}{\cos\theta} = \frac{(4.0)(9.8)}{\cos(41.4)}$

$T_1 - T_2 = 52.25 \longrightarrow (1)$ [2.5]

Radius of Circular path. $\sin\theta = \frac{R}{2}$

$R = 2\sin\theta = 2\sin(41.4) = 1.322\ m$ [1.0]

Radial Component of forces.

$-T_1 \sin\theta - T_2 \sin\theta = -m a_c$

$(T_1 + T_2)\sin\theta = m\frac{v^2}{R}$

$$T_1 + T_2 = \frac{mv^2}{R\sin\theta} = \frac{(4.0)(6.0)^2}{(1.322)\sin(41.4)}$$

$$T_1 + T_2 = 164.63 \longrightarrow ②$$ [2.5]

Adding Eq ① and ②

$$2T_1 = 52.25 + 164.63 = 216.96$$

$$T_1 = \frac{216.96}{2}$$

$$T_1 = 108.48 \text{ N.}$$ [1]

From Eq ①

$$T_2 = T_1 - 52.25$$

$$T_2 = 108.64 - 52.25$$

$$T_2 = 56.23 \text{ N.}$$ [1]