(Q 1.1) Sol. (1) [1]

[10]

$P(x,y) = R(x,y) - C(x,y)$

$= \left(-\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 300x + 240y\right)$

$ax^2 \quad a>0$

$a<0$

$- (180x + 140y + 5000)$

[2]

$P(x,y) = -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 120x + 100y - 5000$

To find max. profit

$P_x = \frac{1}{2}x - \frac{1}{4}y + 120 = 0$

$P_y = -\frac{3}{4}y - \frac{1}{4}x + 100 = 0$

[3]

$x = 208, \quad y = 64$

C.P $(208, 64)$

Test if C.P is sol. to problem

we use 2nd derivative test

$P_{xx} = \frac{-1}{2}, \quad P_{yy} = -\frac{3}{4}, \quad P_{xy} = -\frac{1}{4}$

$D(x,y) = \left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right) - \left(-\frac{1}{4}\right)^2 = \frac{3}{8} - \frac{1}{16} = \frac{5}{16}$ [4]

$D(208, 64) = 5/16 > 0$

$D(208,64) > 0, \quad P_{xx}(208,64) < 0$

$\Rightarrow$ C.P is relative maximum of P.

(Q. No 1)

(iii) (a) steepest accend $= \vec{\nabla} f = ?$ [3]

$$\left.\begin{aligned} \vec{\nabla} f &= -y^2 i - 2xy j \\ \vec{\nabla} f \Big|_{(2,4)} &= -i - 4j \end{aligned}\right\} [2]$$

unit vector $\hat{\vec{\nabla}} f = \frac{\vec{\nabla} f}{|\nabla f|} = \frac{-i}{\sqrt{17}} - \frac{4j}{\sqrt{17}}$ [1]

(iii) (b) $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$ [3]

Given $dx = 0$, $dy = 0.4$ $\Big\} \to (1)$

$df = 0 + (-4)(0.4) = -1.6 \to [2]$

(iii) (c) From above $\hat{\nabla} f = \frac{-i}{\sqrt{17}} - \frac{4j}{\sqrt{17}}$ [4]

unit vector with no change in heigh is a vector such that

$\hat{\nabla} f \cdot \vec{v} = 0$, where $\vec{v} = (a, b)$ [1]

$\left(\frac{-i}{\sqrt{17}} - \frac{4j}{\sqrt{17}}\right) \cdot (a, b) = 0$

$$\left.\begin{aligned} a &= \frac{4}{\sqrt{17}}, \text{ and } b = -\frac{1}{\sqrt{17}} \text{ or } \left(\frac{4}{\sqrt{17}}, \frac{-1}{\sqrt{17}}\right) \\ a &= -\frac{4}{\sqrt{17}} \text{ and } b = \frac{1}{\sqrt{17}} \text{ or } \left(\frac{-4}{\sqrt{17}}, \frac{1}{\sqrt{17}}\right) \end{aligned}\right\} [3]$$

Q No. 1 (2)

$$\lim_{(x,y)\to(0,0)} e^{\left(\frac{x^3}{x^2+y^2}\right)} - 1 = \left[e^{g(x,y)} - 1\right]$$

let $g(x,y) = \dfrac{x^3}{x^2+y^2}$

let $x = r\cos\theta$, $y = r\sin\theta$

$$\lim_{r\to 0} g(r) = \frac{r^3\cos^3\theta}{r^2} = \lim_{r\to 0} r\cos^3\theta = 0 \quad [2]$$

Hence $\lim_{(x,y)\to(0,0)} e^{(g(x,y))} - 1 = (e^0 - 1) = 1-1=0$ [2]

Also $f(0,0) = 0 = \lim_{(x,y)\to(0,0)} f(x,y)$ [1]

Hence prove that
$f(x,y)$ is continuous at $(0,0)$

$$\cancel{X_2 = \begin{bmatrix} 5.4143 \\ -3.1636 \end{bmatrix}}$$

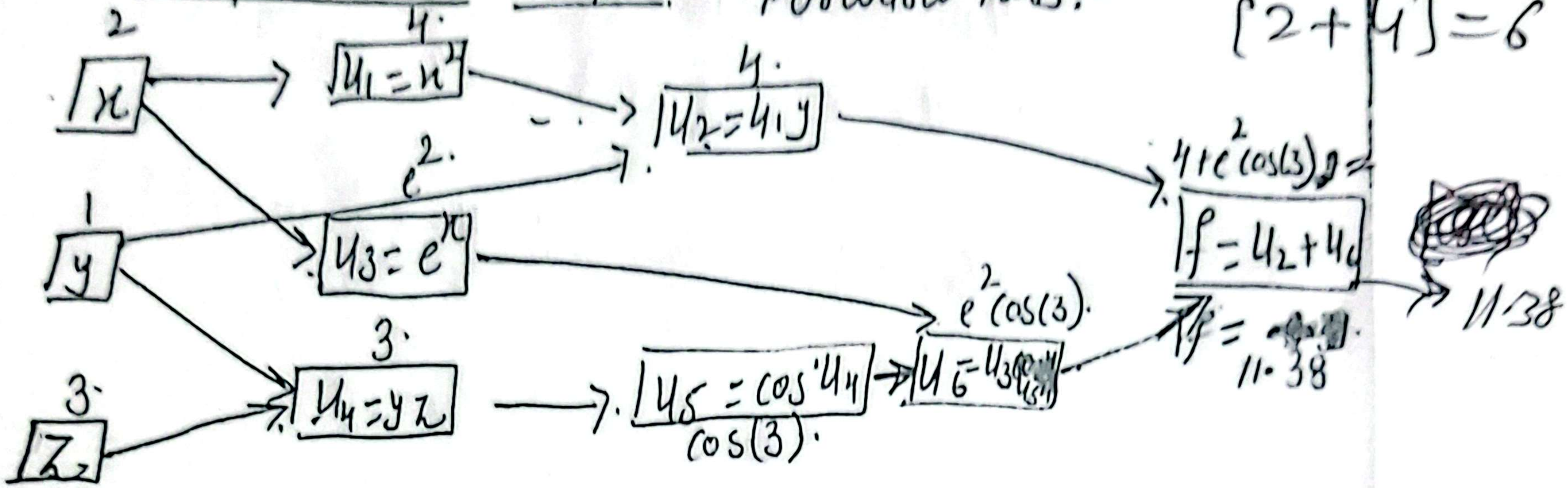
Q2).1 $f(x,y,z) = x^2 y + e^x \cos yz$; $x = 2$, $y = 1$, $z = 3$

$[15]$

$[6+9]$

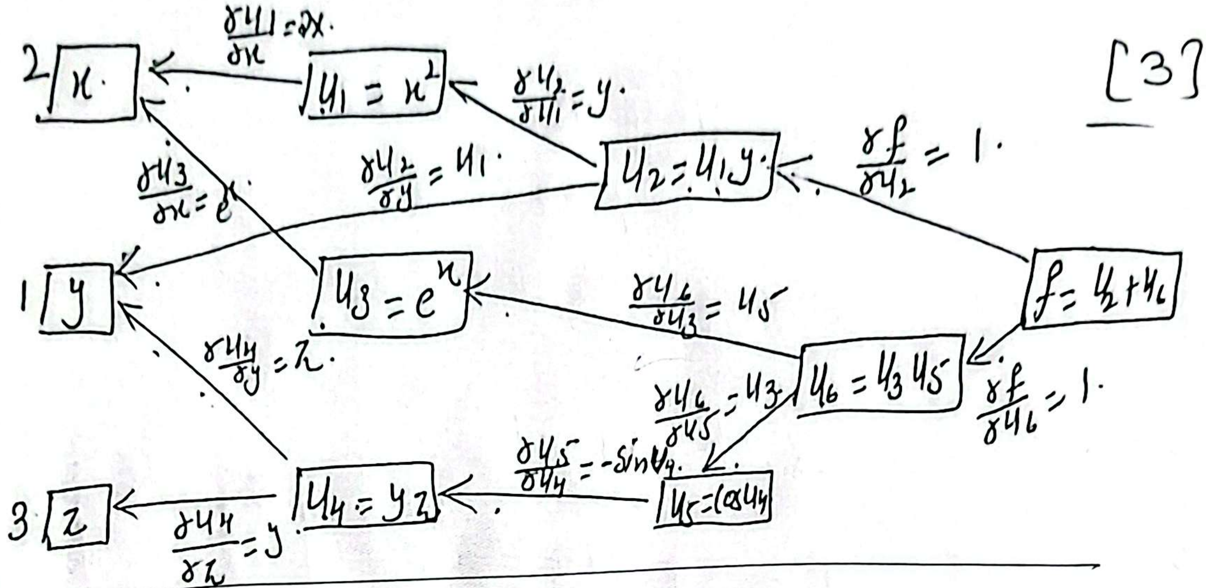
$[2+4] = 6$

Computational Graph:- Forward Pass:-



~~Forward~~ Backward Pass:-

The Backward pass is used to compute gradients using chain rule.



[3]

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u_2} \cdot \frac{\partial u_2}{\partial u_1} \frac{\partial u_1}{\partial x} + \frac{\partial f}{\partial u_6} \frac{\partial u_6}{\partial u_3} \frac{\partial u_3}{\partial x}$$ [2]

$$= 1 \times y \times 2x + 1 \times u_5 e^x = 2xy + e^x \cos(yz).$$

$$= 2(2)(1) + e^2 \cos(3)$$

$$= 4 + e^2 \cos(3) = 11.38$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u_2} \frac{\partial u_2}{\partial y} + \frac{\partial f}{\partial u_6} \frac{\partial u_6}{\partial u_5} \frac{\partial u_5}{\partial u_4} \frac{\partial u_4}{\partial y}$$ [2]

$$= 1 \times u_1 + 1 \times u_3 \times (-\sin u_4) \times z$$

$$= 4 \cancel{+} - e^2 \sin(3)\, 3 = 4 - 3e^2 \sin 3 = 2.84$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial u_6} \times \frac{\partial u_6}{\partial u_5} \times \frac{\partial u_5}{\partial u_4} \times \frac{\partial u_4}{\partial z} = 1 \times u_3 \times (-\sin u_4) \times y$$ [2]

$$= -e^2 \sin(3) \times 1$$

$$= -0.39$$

$$\cancel{8}$$

$$\cancel{= \frac{2}{\pi}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{2}{\pi} \times \frac{1}{4} = \frac{1}{2\pi}}$$

2.3 Evaluate the Integrals.

$$\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin(\pi x^2)}{x^2} \, dx \, dy$$

By Reverse order

$y = 0 \; ; \; y = 1$

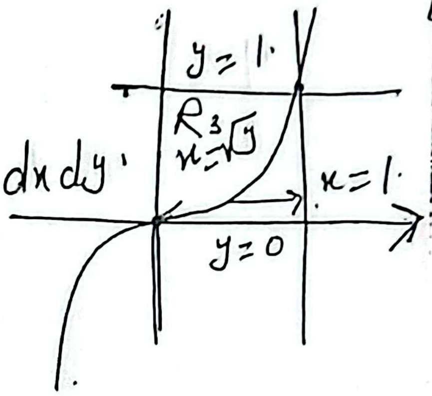
$x = 1 \; ; \; x = \sqrt[3]{y}$

$y = x^3$

[3]

$$= \int_0^1 \int_0^{x^3} \frac{2\pi \sin(\pi x^2)}{x^2} \, dy \, dx \quad [3]$$

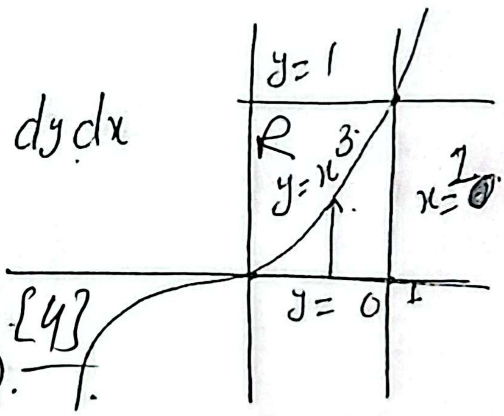
dx dy



$$= \int_0^1 \left. \frac{2\pi \sin(\pi x^2)}{x^2} y \right|_0^{x^3} dx$$

$$= \int_0^1 \frac{2\pi x^{\cancel{3}} \sin(\pi x^2)}{\cancel{x^2}} \, dx$$

dy dx



$$= \int_0^1 2\pi x \sin(\pi x^2) \, dx$$

$$- \cos(\pi x^2) \Big|_0^1 = -(-1) - (-1) \; [4]$$

$$= 1 + 1 = 2.$$