

**MT-1003: Calculus and
Analytic Geometry**
BS (SE, AI, DS, CY)

Monday, 1st January, 2024

Course Instructor

Dr. Imran Shahzad, Mr. Ahtsham ul Haq
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Serial No:

Final EXAM
Total Time: 3 Hour
Total Marks: 115

Signature of Invigilator

Student Name

Roll No

Section

Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. **ONE (01)** extra pages is attached for rough work.
3. If you need more space, write on **both sides** of the paper, and clearly mark the question and part number etc.
4. After asked to commence the exam, please verify that you have **Twelve (12)** different printed pages including this title page. There are total **Nine (09)** questions.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.

| | Q-1 | Q-2 | Q-3 | Q-4 | Q-5 | Q-6 | Q-7 | Q-8 | Q-9 | Total |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|------------|
| Total Marks | 15 | 15 | 10 | 10 | 15 | 15 | 10 | 15 | 10 | 115 |
| Obtained Marks | | | | | | | | | | |

Question 1A [07 Marks]

Show that

$$f(x) = \frac{x^2 + 17x - 38}{x^2 - 4}, \quad x \neq 2$$

is discontinuous at $x = 2$, specifying the type of discontinuity. Also show that $f(x)$ has a continuous extension to $x = 2$, and find that extension.

Question 1B [08 Marks]

Sketch the graph of function $y = -x^4 + 6x^2 - 4$ by using derivatives, identify the coordinates of local and absolute extreme points and inflection points. Analyze the symmetry of the function, if any.

Question 2A [07 Marks]

Find the area of the region in the first quadrant bounded on the left by the y -axis on the right by the curves $y = \sin x$ and $y = \cos x$.

Question 2B [08 Marks]

Evaluate the following integral:

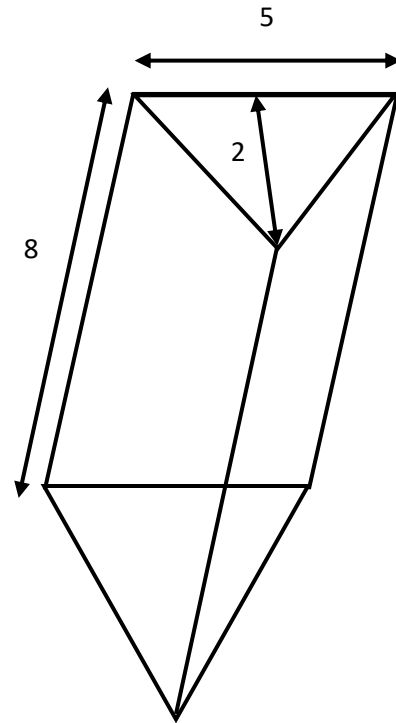
$$\int_0^1 \frac{4\pi r}{\sqrt{1-4r}} dr.$$

Question 3 [10 Marks]

Determine the convergence of the series $\sum_{n=0}^{\infty} \frac{79(-1)^n}{(13\pi)^n}$, and if convergent, calculate the sum of the series.

Question 4 [10 Marks]

A trough of water 8 meters in length and its ends are in the shape of isosceles triangles whose width is 5 meters and height is 2 meters. If water is being pumped in at constant rate of $6m^3/\text{sec}$. At what rate is the height of the water changing when water has a height of 120 cm? At what rate does the width of the water changing when water has a height of 120cm?



Question 5A [07 Marks]

Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \sin^{20} z \cos^5 z \, dz$$

Question 5B [08 Marks]

Evaluate the integral

$$\int \frac{(z + 3)^5}{(40 - 6z - z^2)^{\frac{3}{2}}} dz$$

Question 6A [3+3 Marks]

Find the limits if exists:

$$(i) \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|}$$

$$(ii) \lim_{x \rightarrow 0} \frac{8x}{3\sin x - x}$$

Question 6B [4 Marks]

Evaluate the integral

$$\int \frac{\sec x \tan x dx}{\sqrt{\sec x}}$$

Question 6C [5 Marks]

Find the formula/expression for the nth derivative of the function

$$f(x) = \ln(2x)$$

Question 7 [10 Marks]

Does the parabola $y = 2x^2 - 13x + 5$ have a tangent whose slope is -1 ? If so, find an equation for the line and the point of tangency. If not, why not?

Question 8 [4+4+4+3=15 Marks]

- (a) Evaluate the definite integral $\int_0^1 (x + 2\sqrt{1 - x^2}) dx$
- (b) Sketch the graph of $y = x$ and evaluate $\int_0^1 x dx$ by basic geometrical formulas.
- (c) Sketch the graph of $y = \sqrt{1 - x^2}$ and evaluate $\int_0^1 2\sqrt{1 - x^2} dx$ by geometric formulas.
- (d) Relate the above findings to verify that (a)=(b)+(c)

Question 9 [10 Marks]

The owner of a car rental company has determined that if they charge customers p dollars per day to rent a car, where $50 \leq p \leq 200$, the number of cars 'n' they rent per day can be modeled by the linear function $n(p) = 1000 - 5p$. If they charge 50 dollars per day or less, they will rent all their cars. If they charge 200 dollars per day or more, they will not rent any cars. Assuming the owners plan to charge customers between 50 dollars per day to 200 dollars per day to rent a car, how much should they charge to maximize their revenue?