National University of Computer & Emerging Sciences MT-1008: Multivariable Calculus		Streams: SE, DS, AI, CY
Instructors: **Muhammad Adnan, Dr. M. Usman Ashraf, Dr. Sumaira Azhar, Dr. Irfan Shah**	**Topics:** **Lagrange Multiplier, Gradient Descent Method, Computational Graphs and Automatic Differentiation**	**Launch:** **25^{th} -April- 2025** **Submission:** **5^{th}-May- 2025** **In Lecture**

Instructions:

- **Do all questions.**
- **Copying project from others will result in a straight zero.**
- **Use A4 pages or loose sheets only.**
- **Make a separate cover page with your Name, Roll Number, Course Title, and Instructor Name.**
- **Code must be typed and printed or written clearly if handwritten.**
- **Attach your handwritten solution and computer code together.**
- **Submit your work on time. Late submissions will not be accepted.**

Marks Distribution (Total: 100 Marks):

- 📝 Handwritten Solution (Theory + Manual Calculations): 60 Marks
- 💻 Code Implementation (Python/MATLAB): 25 Marks
- 🎯 Presentation & Neatness (Formatting, Tables, Graphs, Clear Steps): 15 Mark

QUESTION 1:
Choose one topic from Multivariable Calculus Course (except topic in questions 2-4) and write a comprehensive explanation of the concept.

Your answer should include:

A. A detailed theoretical overview of the topic

B. Practical applications of the concept in real-world scenarios

C. An explanation of how the concept can be implemented using a programming language (such as Python or MATLAB)

D. The specific commands or functions used to compute or visualize the concept in the chosen programming language

Note: Your answer should demonstrate both mathematical understanding and computational application.

QUESTION 2:

Using the Bordered Hessian to show that the objective function $f(w,x,y,z) = -w^2 - x^2 - y^2 - z^2$ subject to the following constraints

$$g(w,x,y,z) = 4w - 3y + z + 15 = 0$$
$$h(w,x,y,z) = -2x - y + z + 5 = 0$$

is maximized or minimized?

Your task is to:

A. Solve this optimization problem by hand, using the method of Lagrange multipliers. Clearly show all necessary steps and justifications.
B. Write a computer program in a programming language of your choice (e.g., Python or MATLAB) to verify your analytical solution.
C. Clearly mention and explain the commands or functions used in your code.

QUESTION 3:

Given the function
$f(x, y) = x^2 + y^2 + 4xy - 10x - 8y + 60$,
use the gradient descent method to find the minimum point of the function. Start with the initial guess $(x_0, y_0) = (0, 0)$ and use a fixed learning rate $\alpha = 0.05$.

A. Derive the gradient of the function $\nabla f(x,y)$.
B. Perform two iterations of the Gradient Descent algorithm manually.
C. At each iteration, calculate and record the updated values of x, y, and f(x,y).
D. Present your results in a table format and interpret whether the algorithm is converging toward the minimum.
E. *(Optional)* Discuss how the choice of the learning rate α\alphaα affects the convergence behavior (e.g., too small vs. too large).
F. Write a computer program to perform the same process and verify your results.

QUEESTION 4:

Consider the function:

$$f(x_1, x_2) = (sin(\frac{x_1}{x_2}) + \frac{x_1}{x_2} - e^{x_2})(\frac{x_1}{x_2} - e^{x_2})$$

Your task is to:

A. Draw the computation graph for the function, clearly labelling all intermediate variables.
B. Use backpropagation (reverse-mode differentiation) to compute the gradient of the function with respect to x_1 and x_2. Show all steps in detail.
C. Evaluate the function and gradients using:

- x_1 = *First digit of your roll number*
- x_2= *Any nonzero digit of your roll number.*

D. Discuss the potential applications of such gradient computations in Machine Learning and Deep Learning, particularly in the context of neural network training.