National University of Computer and Emerging Sciences

FAST School of Computing | Fall-2023 | Islamabad Campus

MT-1003: Calculus and Analytical Geometry (AI, DS, SE)

Serial No:

2nd Sessional Exam

Total Time: 1 Hour

Total Marks: 40

Tuesday, 7th November, 2023

Course Instructors

Dr. Imran Shahzad, Mr. Arif Hussain, Mr. Ahtsham Ul Haq

Signature of Invigilator

Student Name | **Roll No.** | **Course Section** | **Student Signature**

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space, write on the back side of the paper and clearly mark the question and part number etc.
4. After being asked to commence the exam, please verify that you have **Eight (08)** different printed pages including this title page. There are a total of **04** questions.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.
7. **Show full steps for scoring full credit.**
8. **Solve the questions using techniques learat in this course. Using a method other than the required method will result in deduction or zero marks.**

	Q-1	Q-2	Q-3	Q-4	Total
Marks Obtained					
Total Marks	10	10	10	10	40

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Question 1 [10 Marks]

Sketch the graph of $f(x) = \frac{x^2+1}{x}$ by finding:

(a) All the critical points.
(b) Intervals for increasing and decreasing.
(c) Intervals for concave up and concave down.
(d) Relative extrema (if exists)
(e) $x - intercept$ and $y - intercept$ (if exists)

Question 2 [10 Marks]

A manufacturer wants to create an optimal cylindrical container for storing a liquid substance. The objective is to maximize the volume of the container while minimizing the material used in its construction. The cylindrical container is to be open-topped and has a semi-circular cross-section at both ends. The radius of each semi-circular end is r cm, and the length of the container is L cm. The metal sheet used for construction is 600π cm^2.

a. Derive the constraint equation from the total surface area of the cylindrical container.
b. Express length L in terms of radius r.
c. Write the equation of volume V in terms of r and L.
d. Write the equation of volume V in terms of r alone.
e. Solve for the maximum value of V with respect to r to determine the optimal value of r that yields the maximum volume V.
f. Verify that the value of r found in the optimization step gives the maximum value of V.

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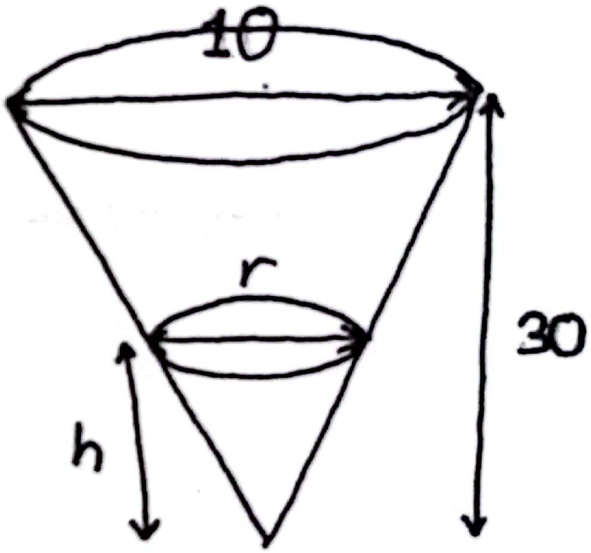
Question 3 [10 Marks]

Water is poured into a conical container at the rate of $10\ cm^3/sec$. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm; see below. How fast is the water level rising when the water is 4 cm deep (at its deepest point)?



Question 4A [05 Marks]

Find the linear approximation to $g(z) = 4\sqrt{z}$ at $z = 2$. Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximated values to the exact values.

Question 4B [05 Marks]

Find $\lim_{x \to 0} (1 + \sin x)^{1/x}$.