



Department of Computer sciences
Discrete Structures

Assignment # 2
Spring 2025 (SE)

Question 1: [10 Marks]

Fill in the following table considering x and y belong to Real numbers.

	Statement	Truth Value T/F	Sample value of x & y corresponding to Truth Value
a)	$\forall x \forall y (x^2 < y + 1)$		
b)	$\exists x \forall y (x^2 < y + 1)$		
c)	$\exists y \forall x (x^2 < y + 1)$		
d)	$\exists x \exists y (x^2 < y + 1)$		
e)	$\forall y \exists x (x^2 < y + 1)$		
f)	$\forall x \exists y (x^2 + y^2 = 9)$		
g)	$\forall x \forall y ((x < y) \rightarrow (x^2 < y^2))$		
h)	$\forall x \exists y ((x < y) \rightarrow (x^2 < y^2))$		
i)	$\exists x \forall y ((x < y) \rightarrow (x^2 < y^2))$		
j)	$\exists x \exists y ((x < y) \rightarrow (x^2 < y^2))$		

Question 2: [10 Marks]

Translate these system specifications into English where the predicate $S(x, y)$ is “ x is in state y ” and where the domain for x and y consists of all systems and all possible states, respectively.

- a) $\exists x S(x, \text{open})$
- b) $\forall x (S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$
- c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$
- d) $\exists x \neg S(x, \text{available})$
- e) $\forall x \neg S(x, \text{working})$

Question 3: [8 Marks]

Translate these specifications into English where

$F(p)$ is “Printer p is out of service,”

$B(p)$ is “Printer p is busy,”

$L(j)$ is “Print job j is lost,” and

$Q(j)$ is “Print job j is queued.”

- a) $\exists p (F(p) \wedge B(p)) \rightarrow \exists j L(j)$
- b) $\forall p B(p) \rightarrow \exists j Q(j)$
- c) $\exists j (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$
- d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

Question 4: [8 Marks]

You are given a computer program and some information draw a reasonable conclusion that finds the mistake in the program:

- a) There is a syntax error in the first five lines of the program or there is an undeclared variable.
- b) If there is a syntax error in the first five lines, then there is a missing semicolon, or a variable name is misspelled.
- c) There is not a missing semicolon.
- d) There is not a misspelled variable name.

Question 5: [10 Marks]

Rewrite each of the following statements in the form “ $\forall \dots$, if \dots then \dots ”

- a. All Java programs have at least 5 lines.
- b. Any valid argument with true premises has a true conclusion.
- c. Any student with a GPA of better than 3.5 must study a lot.
- d. Any polygon with three sides is a triangle.
- e. All real numbers are positive when squared.

Question 6: [10 Marks]

Use logical equivalences and the rules of inference to determine whether the following arguments are valid. The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

- I. Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.
- II. Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.
- III. If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.
- IV. If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.
- V. If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.
- VI. If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

Is it possible for the detective to deduce the identity of the murderer from these facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death)

Question 7: [10 Marks]

Happy-birthday card logic: "WHY WORRY???"

There are only two things to worry about, either you're healthy or you're sick. If you're healthy, there's nothing to worry about, and if you're sick, there are two things to worry about, either you'll get well or you won't. If you get well there is nothing to worry about, but if you don't, you'll have two things to worry about, either you'll go to heaven or to hell. If you go to heaven, you have nothing to worry about, and if you go to hell you'll be so busy shaking hands with all of us that you'll have no time to worry.

What conclusion can you draw from this.

Question 8 :[10 Marks]

Use truth tables to determine whether the argument form is valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

- a) $p \wedge w \rightarrow \sim r \vee \sim s$
- b) $q \rightarrow p$
- c) $\sim q \rightarrow p$
- d) $w \vee \sim p$
- e) $w \rightarrow s$
- f) $\therefore \sim r$

Question 9: [10 Marks]

There are only three basic constructs in Prolog: facts, rules, and queries. A collection of facts and rules is called a knowledge base (or a database). If we have given multiple facts like

male(haris).

female(yasmin).

parent_of(haris,yasmin).

Make the rules about

- i. father_of(X,Y):-
- ii. mother_of(X,Y):-
- iii. grandfather_of(X,Y):-
- iv. grandmother_of(X,Y):-
- v. cousin_of(X,Y):-

Question 10:[20 Marks]

- a) Suppose you know of an argument only that it is valid and has a true conclusion. What, if anything, can you tell about its premises? (Defend your answer, including examples.)
- b) Suppose you know of an argument only that it is valid and has a false conclusion. What, if anything, can you tell about its premises? (Defend your answer.)

- c) Suppose you know that an argument is sound. What can you determine about its conclusion? (Defend your answer.)
- d) Suppose you know of an argument only that it has all true premises and a true conclusion. Can you tell from that whether it is valid or invalid? (Defend your answer.)
- e) Suppose you know of an argument only that it has all true premises and a true conclusion. Can you tell from that whether it is valid or invalid? (Defend your answer.)
- f) Suppose you know that a set of sentences is consistent. Can you tell from that whether every set member is actually true? (Defend your answer.)
- g) Suppose you know every member of a set of sentences is false. Can you tell from that whether the set is inconsistent? (Defend your answer.)

Question 11 : [10 Marks]

Translate the following statement into English.

a) $\exists x \forall y \forall z ((F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow \neg F(y, z))$

$F(x, y)$: x and y are friends.

Domain of x, y and z : all students

b) $\forall x (C(x) \exists y (C(y) \wedge F(x, y)))$

$C(x)$: x has a computer.

$F(x, y)$: x and y are friends.

Domain of x, y and z : all students