National University
of Computer & Emerging Sciences
Islamabad

MT1008 – Multivariable Calculus

Assignment # 2 Software Engineering

Total Marks: 110

Deadline: 26-02-2025
Submission: In Lecture

Question 1- Optimize: $f(x, y, z) = x^3 + 3xz + y - y^2 - 3z^2$ **[10 marks]**

Question 2- A rectangular box with open top is to be formed from a rectangular piece of cardboard which is 3×8 inches. What size square should be cut from each corner to form a box with maximum volume? **[10 marks]**

Question 3- Suppose the temperature at (x, y, z) is given by $T = xy + sin(yz)$. In what direction should you go from the point $(1,1,1)$ to decrease the temperature as quickly as possible? What is the rate of change of temperature in this direction? **[10 marks]**

Question 4-

A drone is flying over a mountainous region, and its altitude above sea level (in meters) at any point (x, y) is given by the function:

$$h(x, y) = 1500 - \frac{x^2}{3} - \frac{y^2}{2}$$

where x and y are distances in kilometers.

Answer the following:

1. The drone is currently at the location (3,2). Find the gradient vector $\nabla h(x, y)$ at the point.
2. Find the equation of the tangent plane to the altitude surface at (3,2).

3. The drone starts moving in the direction of the vector $v = (-4,5)$. What is the rate of change of altitude?
4. In which direction should the drone fly to ascend most rapidly? What is the maximum rate of altitude increase?
5. The drone wants to maintain constant altitude while flying. Find a vector direction in which the drone should travel to stay at the same height. **[20 marks]**

Question 5- A boundary stripe 3 in. wide is painted around a rectangle whose dimensions are 100 ft. by 200 ft. Use differentials to approximate the number of square feet of paint in the stripe.

[10 marks]

Question 6-

a. Find the critical points of,
$f(x,y) = \sqrt{4y^2 - 9x^2 + 24y + 36x + 36}$
b. Find all the local maxima, local minima, and saddle points of,
$f(x,y) = e^{x^2-y} + x^2y - y^3$ **[20 marks]**

Question 7- A rectangular box with no top is to be constructed to have a volume $V = 12ft^3$.

The cost per square foot of the material to be used is \$4 for the bottom, \$3 for two of the opposite sides, and \$2 for the remaining pair of opposite sides. Find the dimensions of the box that will minimize the cost. **[10 marks]**

Question 8-

Find the maximum and minimum values of f on the range R.

$$f(x,y) = x^2 - 3xy - y^2 + 2y - 6x$$

$$R = (x,y): |x| \leq 3, |y| \leq 2$$ **[10 marks]**

Question 9-

Suppose an xyz −coordinate system is located in space that the temperature T at the point (x,y,z) is given by the formula $T = \frac{100}{x^2+y^2+z^2}$

a) Find the rate of change of *T* with respect to distance at the point *P* (1,3,-2) in the direction of the vector $a = i - j + k$.
b) In what direction from P does *T* increase most rapidly? What is the maximum rate of change of *T* at *P*? **[10 marks]**