National University
of Computer & Emerging Sciences
Islamabad

MT1008 – Multivariable Calculus

Assignment # 1 SE

Total Marks: 150
Deadline: 10th February 2025

Question 1 - Sketch the level curves of the following for 3 different values of c (constant). **[25 marks]**

a. $f(x,y) = x^2 + 2y^2$
b. $f(x,y) = xe^{-y}$
c. $f(x,y) = \frac{2y}{x^2+y^2}$
d. $f(x,y) = x^2 + y^2 - z^2 + 2z = 0 \ \ (z = constant)$
e. $f(x,y) = e^{-x^2+4y^2}$

Question 2 - Determine the equations and shapes of the cross-sections when $x = 0, y = 0, x = y$, and describe the level curves of the following. **[15 marks]**

a. $f(x,y) = |x| + |y|$
b. $f(x,y) = (x^2 - y^2)^2$
c. $f(x,y) = e^{-(x^2+y^2)} sin(x^2 + y^2)$.

Question 3 - State the domain, range, interior points, boundary points, open/close, bounded/unbounded for each of the following functions. **[35 marks]**

a. $f(x,y) = \sqrt{9 - x^2} + \sqrt{y^2 - 4}$
b. $f(x,y) = arcsin(x^2 + y^2 - 2)$
c. $f(x,y) = \sqrt{16 - x^2 - 4y^2}$
d. $f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + ln(4 - x^2 - y^2 - z^2)$
e. $f(x,y,z) = x^3y^2z\sqrt{10 - x - y - z}$
f. $f(x,y) = \frac{ln(2-x)}{1-x^2-y^2}$
g. $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$

Question 4 - Determine whether each limit exists. If it does, find the limit and prove that it is the limit; if it is not, explain how you know. **[45 marks]**

a. $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$
b. $\lim_{(x,y)\to(3,2)} \frac{x^2y}{x^2+y^2}$

c. $\lim_{(x,y)\to(3,2)} \frac{xy}{|xy|}$

d. $\lim_{(x,y)\to(1,0)} \frac{(x-1)^2 \ln x}{(x-1)^2+y^2}$

e. $\lim_{(x,y)\to(1,0)} \frac{\sin(x)(e^y-1)}{xy}$

f. $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

g. $\lim_{(x,y)\to(3,2)} e^{\sqrt{2x-y}}$

h. $\lim_{(x,y)\to(1,0)} \frac{xy^5}{x^8+y^{10}}$

i. $\lim_{(x,y)\to(1,0)} \frac{\sin(4x)}{\tan(x)}$

Question 5 - Determine the set of points at which the function is continuous. **[20 marks]**

a. $f(x,y) = \tan^{-1}((x+y)^{-2})$

b. $f(x,y) = \frac{e^x+e^y}{e^{xy}-1}$

c. $f(x,y) = \sqrt{x} + \sqrt{1-x^2-y^2}$

d. $f(x,y) = \begin{cases} \frac{xy}{x^2+xy+y^2} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$

Question 6 - If $f(x,y,z) = xy^2z^3 + arcsin(x\sqrt{z})$, find f_{xzy} and f_{yzx} **[10 marks]**