Question Number 1:

Determine if each statement below is True or False, and explain your reasoning

1: **If a conditional statement "P Implies Q" is true, it means that Q implies P is also true?**
False: The Truth of P implies Q does not guarantee the Truth of Q implies P

2: **The Contrapositive of the condition statement P implies Q is logically equivalent to the negation of the original statement.**
False: The contrapositive of the condition statement is logically equivalent to the original statement.

3: The converse and Inverse of a conditional statements are logically equivalent to each other.
True: Converse and Inverse are logically equivalent each other

4: **(¬P v Q) ∧ ¬ (¬P v Q) is contradiction**
Yes, its equals to contradiction

5: Use the logical equivalence p ∨ q → r ≡ (p → r) ∧ (q → r), to rewrite the following statement.
If x > 2 or x < −2, then x 2 > 4.

$$\text{If } x>2, \text{ then } x^2>4 \text{ and if } x<-2, \text{ then } x^2>4.$$

6: The contrapositive of the statement "If an animal is a cat, then it has whiskers" is "if an animal does not have whiskers, it is a cat.

No, it is not correct, the correct contrapositive is "if an animal does not have whiskers, it is not a cat.

7: Write the negation of the statement "if today is New Year's Evening, then tomorrow is January.

Today is New Year's Eve and tomorrow is not January

8: In Propositional logic, a double negation is equivalent to a positive. There is one fairly common English usage in which a "double positive" is equivalent to a negative. What is it?
No: There is no as such case in which "double positive is equal to negation".

9: ¬(P∧Q) ⟺(¬P∨¬Q) represents?
"The negation of both event P and event Q is true if and only if either event P is not true or event Q is not true (or both).

10: prove that (p ∨ ~q) ∧ (~p ∨ ~q) is logically equivalent to **¬q**

(~q ∨ p) ∧ (~q ∨ ~p)
~q ∨ (p ∧ ~p)
~q ∨ c
~q

Q2: Simplify the following expression:

$\neg a \vee \neg b \vee (a \wedge b \wedge \neg c)$

Ans:

$= [\neg a \vee \neg b] \vee (a \wedge b \wedge \neg c)$

$=([\neg a \vee \neg b] \vee a) \wedge ([\neg a \vee \neg b] \vee b) \wedge ([\neg a \vee \neg b] \vee \neg c)$

$=([\neg a \vee a] \vee \neg b) \wedge ([\neg b \vee b] \vee \neg a) \wedge (\neg a \vee \neg b \vee \neg c)$

$=(\boldsymbol{T} \vee \neg b) \wedge (\boldsymbol{T} \vee \neg a) \wedge (\neg a \vee \neg b \vee \neg c)$

$=(\boldsymbol{T}) \wedge (\boldsymbol{T}) \wedge (\neg a \vee \neg b \vee \neg c)$

$= \neg a \vee \neg b \vee \neg c$

Q # 3 : Compute $(fog)(x)$ and $(gof)(x)$ for $f(x) = x^2 + 3, g(x) = \sqrt{5 + x^2}$.

Solution: $(fog)(x) = f[g(x)] = f\left[\sqrt{5+x^2}\right] = (\sqrt{5+x^2})^2 + 3 = 8 + x^2$

$(gof)(x) = g[f(x)] = g[x^2+3] = \sqrt{5+(x^2+3)^2} = \sqrt{x^4+6x^2+14}$

Q # 4: Are these system specifications consistent? "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

Date

m : The System is in Multiuser state
n : The System is operating normally
K : The kernel is functioning
i : The System is in interrupt mode

$m \leftrightarrow n,\ n \rightarrow K,\ \sim K \vee i,\ \sim m \rightarrow i, \sim i$

1. $m \leftrightarrow n$
2. $n \rightarrow K$
3. $\sim K \vee i$
4. $\sim m \rightarrow i$
5. $\boxed{\sim i}$

Premises

6. $m \rightarrow n$ — 1, $\leftrightarrow$ Rule
7. $m \rightarrow K$ — 2, 6, Transitivity
8. $\sim K$ — 3, 4, $\vee$ - Elimination
9. $\sim m$ — 7, 8, Modus Tollens
10. $\boxed{i}$ — 4, 9, Modus Ponens

The given premises are inconsistent as there is a contradiction in values of i.