



National University
of Computer & Emerging Sciences
Islamabad



MT1008 – Multivariable Calculus

Assignment # 3

Total Marks: 110

Deadline: 19-03-2025

Submission: In Lecture

Question 1: By Using Lagrange Multipliers Method.

[30 Marks]

- A. A cargo container (in the shape of a rectangular solid) must have a volume of 480 cubic feet. The bottom will cost \$5 per square foot to construct, and the sides and the top will cost \$3 per square foot to construct. Find the dimensions of the container of this size that has minimum cost.
- B. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108 inches. Find the dimensions of the package with largest volume that can be mailed.
- C. A grain silo is to be built by attaching a hemispherical roof and a flat floor onto a circular cylinder. Show that for a total surface area S (Constant), the volume of the silo is maximized when the radius and height of the cylinder are equal.

Question 2: Make a computation graph and calculate the gradient by using automatic differentiation

[20 marks]

- A. $f(x, y, z) = x^2y + \sin(yz) + e^{xz}$ with $(x, y, z) = (1, 2, 3)$
- B. $f(x, y, z) = e^{xy} \sin(z) + \ln(x^2 + y^2) + yz^3$ with $(x, y, z) = (4, 5, 6)$

Question 3: Perform the Steepest Descent 3 Iterations with initial guess $[0 \ 0]$.

[20 marks]

- A. $F(X) = \frac{1}{2}X^T H X + c^T X + 13$, where $H = \begin{bmatrix} 10 & -9 \\ -9 & 10 \end{bmatrix}$, $c = \begin{bmatrix} 4 \\ -15 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- B. $f(x, y) = 5x^2 + 5y^2 - xy - 11x + 11y + 11$

Question 4:

Solve these double integrals.

[40 Marks]

- A. Find the volume of the region bounded above by the elliptical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R: 0 \leq x \leq 2, 0 \leq y \leq 2$.
- B. Find the volume of the given solid under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$
- C. Sketch the region of integration, reverse the order of integration, and evaluate the integral.
- $\int_0^2 \int_{\frac{x^2}{2}}^{\frac{2}{x^2}} \sqrt{y} \cos(y) dy dx$
- D. Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral over the region R .
- $\iint x dA$
 R : Sector of a circle in the first quadrant bounded by $y = \sqrt{25 - x^2}$, $3x - 4y = 0$, $y = 0$