Rick's Rocks sells decorative landscape rocks in bulk quantities. For quantities up to and including 500 lb, Rick charges \$2.50 per pound. For quantities above 500 lb, he charges \$2 per pound. He realizes that his customers are taking advantage of him: for example, they pay less for 550 lb of rocks than they would for 500 lb of rocks. For Rick, this means lost revenue, so he decides to add a quantity discount surcharge for quantities above 500 lb. If k represents this surcharge, the price function becomes:

$$p(x) = \begin{cases} 2.50x, & \text{for } 0 < x \leq 500, \\ 2x + k, & \text{for } x > 500. \end{cases}$$

Find k such that the function is continuous at $x = 500$.

$$P(x) = \begin{cases} 2.50x & x \leq 500 \\ 2x + K & x > 500 \end{cases}$$

For fn to be continuous at $x = 500$

$$LHL = RHL = P(500) \longrightarrow (i)$$

$$\underline{LHL} \quad \lim_{x \to 500^-} (2.50x) = 1250 \longrightarrow (a)$$

$$\underline{RHL} \quad \lim_{x \to 500^+} (2x + k) = 1000 + K \longrightarrow (b)$$

$$P(500) = 2.50 \times 500 = 1250 \longrightarrow (c)$$

Substituting (a), (b) & (c) in (i) we get

$$1250 = 1000 + K$$

$$\boxed{K = 250}$$

Value of K should be 250 inorder to get fn continuous at $x = 500$.

. The Candy Factory sells candy by the pound, charging \$1.50 per pound for quantities up to and including 20 pounds. Above 20 pounds, the Candy Factory charges \$1.25 per pound for the entire quantity, plus a quantity surcharge k. If x represents the number of pounds, the price function is

$$p(x) = \begin{cases} 1.50x, & \text{for } x \leq 20, \\ 1.25x + k, & \text{for } x > 20. \end{cases}$$

a) Find k such that the price function p is continuous at $x = 20$.

$$P(x) = \begin{cases} 1.50\,x & x \le 20 \\ 1.25x + K & x > 20 \end{cases}$$

For ftn to be continuous at $x = 20$

$$LHL = RHL = P(20) \longrightarrow (i)$$

<u>LHL</u> $\lim_{x \to 20^-} (1.50x) = 30 \longrightarrow (a)$

<u>RHL</u> $\lim_{x \to 20^+} (1.25x + K) = 25 + K \longrightarrow (b)$

$P(20) = 1.50 \times 20 = 30 \longrightarrow (c)$

Substituting (a), (b) & (c) in (i)

$$30 = 25 + K$$

$$\boxed{K = 5}$$

So K should be 5 inorder to get price ftn continuous at $x = 20$.

(b) $\{x \mid 0 < x \leq 3\}$ or $(0, 3]$

(c) Ftn is continuous on intervals $(0,1]$, $(1,2]$ & $(2,3]$

(d) At $x = 1$ & 2 ftn has jump discontinuity.

Q3:- In December 2023, Florida Power & Light had the following monthly rate schedule for electric usage in single family residences:

Monthly customer charge	\$ 7.87
Fuel charge	
≤ 1000 KWH	0.02173 per KWH
> 1000 KWH	0.03173 for each KWH in excess of 1000

(a) Find a ftn C that models the monthly cost of using x KWH of electricity.

(b) What is domain of C?

(c) Determine intervals on which C is continuous.

(d) At numbers where C is not continuous (if any), what type of discontinuity does C have.

Sol (a) When $x \leq 1000$

$$C(x) = 7.87 + 0.02173x$$

when $x > 1000$

The cost for first 1000 KWH is $7.87 + (0.02173 \times 1000)$, & the the cost for the remaining $x - 1000$ KWh is charged at 0.03173 per KWh. So total cost is

$$C(x) = 7.87 + (0.02173 \times 1000) + 0.03173\,(x - 1000)$$

$$C(x) = 7.87 + 21.73 + 0.03173\,(x - 1000)$$

$$C(x) = 29.6 + 0.03173x - 31.73$$

$$C(x) = -2.13 + 0.03173x$$

$$C(x) = \begin{cases} 7.87 + 0.02173x & x \leq 1000 \\ -2.13 + 0.03173x & x > 1000 \end{cases}$$

(b) $C = [0, \infty)$ or $\{x \mid x \geq 0\}$

(c) C is Continuous on its domain

(d) No.

Q3:- A pizza delivery service charges a fee based on distance. The delivery fee is calculated as follows

* \$10 for deliveries within a 5 mile radius
* \$2 for each additional mile beyond the initial 5 miles.

If 'd' represent the distance in miles from the pizza shop to the delivery location & let $f(d)$ represent the delivery fee.

Write an expression for the delivery fee ftn based on given pricing structure. Then, determine whether the limit of $f(d)$ as $d \to 5$ miles exists or not?

sol

$$f(d) = \begin{cases} \$10 & 0 < d \leq 5 \\ \$10 + \$2(d-5) & d > 5 \end{cases}$$

LHL

$$\lim_{d \to 5^-} f(d) = \$10$$

RHL

$$\lim_{d \to 5^+} f(d) = 10 + 2(5-5) = \$10$$

LHL = RHL so limit exists & is \$10.

Q5:- A shipping Company charges customers based on the weight of their packages. The pricing structure is as follows:

* \$10 for packages weighing upto 5 lb
* \$2 for each additional lb beyond the initial 5 lb.

If 'w' represent the weight in lb of package, & C(w) represent the shipping cost. Write a fn for C(w) & find limit of C(w) as $w \longrightarrow 5$.

Sol

$$C(w) = \begin{cases} \$10 & 0 < w \leq 5 \\ \$10 + \$2(w-5) & w > 5 \end{cases}$$

LHL

$$\lim_{w \to 5^-} C(w) = \$10$$

RHL

$$\lim_{w \to 5^+} C(w) = \$10$$

As LHL = RHL so limit is \$10 when $w \to 5$.