$$M = \frac{1}{2}\cdot\frac{(x-1)^3}{3}\Big|_0^1 - \frac{1}{2}\int_0^1 (x^3 - 2x^2 + x)\,dx + \frac{1}{3}\cdot\frac{(x-1)^4}{4}\Big|_0^1$$

$$= \frac{1}{6}\left(0 - (-1)^3\right) - \frac{1}{2}\left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right)_0^1 + \frac{1}{12}\left(0 - (-1)^4\right)$$

$$= \frac{1}{6}(1) - \frac{1}{2}\left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2}\right) + \frac{1}{12}(1)$$

$$= \frac{1}{6} - \frac{1}{8} + \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$$

$$M = \frac{17}{24}$$

[3]

Q4)

2.4 Given $\int_C (-y + xz + 2z)\, ds$

$$C : r(t) = \cos t\, i + \sin t\, j + t\, k \quad 0 \le t \le 2\pi$$

$$V = r'(t) = -\sin t\, i + \cos t\, j + k \quad (2)$$

$$|V| = |r'(t)| = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}$$

$$\int_C f(x,y,z)\, ds = \int_0^{2\pi} (-\sin t + t\cos t + 2t)\sqrt{2}\, dt \quad (1)$$

$$= \left[\cos t + t\sin t + \cos t + t^2\right]_0^{2\pi} \sqrt{2} \quad (2)$$

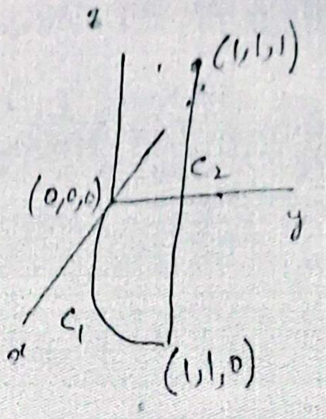
$$= \left[1 + 0 + 1 + 4\pi^2 - 1 - 1\right]\sqrt{2}$$

$$= 4\sqrt{2}\,\pi^2$$

Q # 2.5

[F(6.1-0.4)]

$$\int_C \vec{F}\cdot d\vec{s} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{s}$$



For C_1

$$\left.\begin{aligned} C_1 = r_1(t) &= ti + t^2 j + 0k \\ \vec{V} = r_1'(t) &= (1i + 2tj + 0k) \end{aligned}\right\} \quad 0 \le t \le 1 \quad (03)$$

$$\vec{F} = xi + \sqrt{y}\, j + zk$$

$$\int_{C_1} \vec{F}\cdot d\vec{s} = \int_0^1 \left(\vec{F}\cdot\frac{dr}{dt}\right)dt$$

$$= \int_0^1 (t + 2t^2)\,dt = \left.\frac{t^2}{2} + \frac{2}{3}t^3\right]_0^1 \quad (2)$$

$$\int_{C_1} \vec{F}\cdot d\vec{s} = \frac{1}{2} + \frac{2}{3} = \frac{7}{6} \quad (1)$$

For C_2

$$\left.\begin{aligned} r_2(t) &= 1i + 1j + tk \\ \frac{dr_2}{dt} = r_2'(t) &= 0i + 0j + k \end{aligned}\right\} (2) \quad 0 \le t \le 1$$

$$\boxed{\int_{C_2} \vec{F}\cdot d\vec{s} = \int_0^1 t\,dt = \left.\frac{t^2}{2}\right]_0^1 = 1/2} \rightarrow (1)$$

$$\int_C \vec{F}\cdot d\vec{s} = \frac{7}{6} + \frac{1}{2} = \frac{7+3}{6} = \frac{10}{6} = \boxed{\frac{5}{3}} \rightarrow (1)$$

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Q#2.6

Solution. First we compute

a

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ 3x^2y^2z + 5y^3 & 2x^3yz + 15xy^2 - 7z & x^3y^2 - 7y + 4z^3 \end{vmatrix}$$

[5]

$$= \vec{i}\,(2x^3y - 7 - (2x^3y - 7)) - \vec{j}(3x^2y^2 - 3x^2y^2) + \vec{k}\,(6x^2yz + 15y^2 - (6x^2yz + 15y^2))$$

$$= (0, 0, 0).$$

b

Moreover, $\vec{F}$ is defined (and smooth) on all of $\mathbb{R}^3$, hence it is conservative. Let us find a potential function $f(x, y, z)$ for $\vec{F}$. We want

[12]

$\vec{F} = \nabla f$

$$\left.\begin{aligned} M &= f_x = F_1 = 3x^2y^2z + 5y^3 \\ N &= f_y = F_2 = 2x^3yz + 15xy^2 - 7z \\ P &= f_z = F_3 = x^3y^2 - 7y + 4z^3. \end{aligned}\right\} [2]$$

Using the first equation, we obtain

$$f = \int F_1 \, \mathrm{dx} \quad [1]$$

$$= \int 3x^2y^2z + 5y^3 \, \mathrm{dx}$$

$$\boxed{f(x,y,z) = x^3y^2z + 5xy^3 + g(y, z)} \quad [2]$$

whose derivative with respect to y is

$$2x^3yz + 15xy^2 + g_y(y, z).$$

Using the second equation, we equate this with F_2:

$$2x^3yz + 15xy^2 + g_y(y, z) = 2x^3yz + 15xy^2 - 7z$$

$$\Rightarrow g_y(y, z) = -7z$$

$$\Rightarrow g(y, z) = \int -7z \, dy$$

$$g(y,z) = -7yz + h(z). \longrightarrow [2]$$

$g(y,z) = -7yz + h(z)$. $\longrightarrow$ [2]

Plugging this back into the expression for f, we obtain

$$\boxed{f = x^3y^2z + 5xy^3 - 7yz + h(z)} \longrightarrow [1]$$

whose derivative with respect to z is

$$x^3y^2 - 7y + h'(z).$$

Using the third equation, we equate this with F_3:

$$x^3y^2 - 7y + h'(z) = x^3y^2 - 7y + 4z^3$$

$$\Rightarrow h'(z) = 4z^3$$

$$\Rightarrow h(z) = \int 4z^3 dz$$

$$\boxed{h(z) = z^4 + c.} \longrightarrow [2]$$

$$\boxed{f(x,y,z) = x^3y^2z + 5xy^3 - 7yz + z^4} \longrightarrow [2]$$

[3]

$$= \int_C \vec{F} \cdot d\vec{r} = \int_A^B d(f) = f(B) - f(A) = \Big[x^3y^2z + 5xy^3 - 7yz + z^4 \Big]_{(0,0,0)}^{(\pi^2,0,\pi)}$$

$$= \pi^4 - 0 = \pi^4$$

(Q.7)

$$F = \ln(x^2+y^2)i - \left(\frac{2z}{x}\tan^{-1}\frac{y}{x}\right)j + z\sqrt{x^2+y^2}\,k$$

D: The thick walled cylinder $1 \leq x^2+y^2 \leq 2$,

$$-1 \leq z \leq 2$$

Solution:-

$$\frac{\partial}{\partial x}\ln(x^2+y^2) = \frac{1}{(x^2+y^2)}(2x)$$

$$\frac{\partial}{\partial y}\left(-\frac{2z}{x}\tan^{-1}y/x\right) = -\frac{2z}{x} \times \frac{1}{1+\frac{y^2}{x^2}}(1/x)$$

$$= \frac{-2z}{\cancel{x}} \times \frac{\cancel{x}}{x^2+y^2} = \frac{-2z}{x^2+y^2}$$

$$\frac{\partial}{\partial z}\left(z\sqrt{x^2+y^2}\right) = \sqrt{x^2+y^2}$$

[4]

$$\boxed{\nabla \cdot F = \left(\frac{2x}{x^2+y^2} - \frac{2z}{x^2+y^2} + \sqrt{x^2+y^2}\right)}$$

(1) $\text{Flux} = \iiint_D \left(\frac{2x}{x^2+y^2} - \frac{2z}{x^2+y^2} + \sqrt{x^2+y^2} \right) dz\,dy\,dx$

(2) $= \int_0^{2\pi}\int_1^{\sqrt{2}}\int_{-1}^{2} \left(\frac{2r\cos\theta}{r^2} - \frac{2z}{r^2} + r \right) dz\; r\,dr\,d\theta$

$\Rightarrow \int_0^{2\pi}\int_1^{\sqrt{2}} \left(\frac{2\cos\theta}{r} z - \frac{z^2}{r^2} + rz \right)\Big|_{-1}^{2} r\,dr\,d\theta$

(3) $= \int_0^{2\pi}\int_1^{\sqrt{2}} \left(\frac{2\cos\theta}{r}(2) - \frac{4}{r^2} + 2r \right) - \left(\frac{-2\cos\theta}{r} - \frac{1}{r^2} - r \right) r\,dr\,d\theta$

$\Rightarrow \int_0^{2\pi}\int_1^{\sqrt{2}} \left(6\cos\theta - \frac{3}{r} + 3r^2 \right) dr\,d\theta$

(3) $= \int_0^{2\pi} \left[6\cos\theta\, r - 3\ln r + r^3 \Big|_1^{\sqrt{2}} \right] d\theta$

(4) $= 2\pi\left(-\frac{3}{2}\ln 2 + 2\sqrt{2} - 1 \right)$

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