CSE218 Homework 1

You are working for 'DOWN THE TOILET COMPANY' that makes floats for ABC commodes. The ball has a specific gravity of 0.55 and has a radius of 6 cm. You are asked to find the depth to which the ball will get submerged when floating in water (see Figure 1).

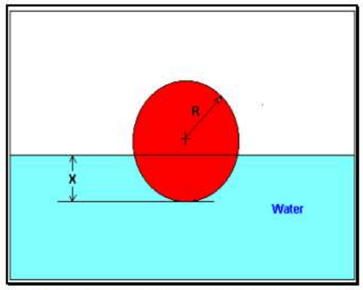

Figure 1 Depth to which the ball is submerged in water

According to Newton's third law of motion, every action has an equal and opposite reaction. In this case, the weight of the ball W_B is balanced by the buoyancy force B_F (Figure 2).

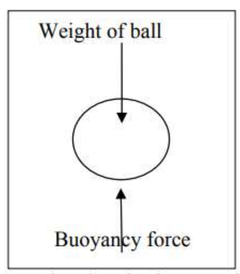

Figure 2 Free Body Diagram showing the forces acting on the ball immersed in water

Weight of ball is given by

W_B = (volume of ball)*(density of ball)*(acceleration due to gravity).

$W_B = \left(\frac{4}{3}\pi R^3\right)(\rho_b)(g)$, where R = radius of the ball (m), ρ = density of the ball (kg/m^3), and g = acceleration due to gravity (m/s^2).

According to Archimedes principle, the buoyancy force of the liquid is given by

B_F = weight of water displaced = (volume of water displaced)*(density of water)*(acceleration due to gravity).

$B_F = \pi x^2 \left(R - \frac{x}{3}\right)\rho_w g$, where x = depth to which ball is submerged and ρ_g = density of water.

At equilibrium, we get

$$W_B = B_F$$

$$\left(\frac{4}{3}\pi R^3\right)(\rho_b)(g) = \pi x^2 \left(R - \frac{x}{3}\right)\rho_w g \text{ --------------------------(1)}$$

Note that specific gravity of the ball $\gamma_b = \frac{\rho_b}{\rho_w}$.

Transform equation (1) into a suitable form of a non-linear equation in the form $f(x) = 0$. Apply bisection method to find the value of x.

Write a python program that does the following (20):

- Draw a graph of the function $f(x)$ using python matplotlib library. From this function, visually find the approximate location of the root. (4)

- Uses bisection method to estimate the value of *x* for ε_s=0.5%. The bisection method should be implemented as a function having the following parameters: lower bound of the bracket, upper bound of the bracket, expected relative approximation error, and max iteration. The function should return the approximate value of the root. Use the graph plotted in the first step to choose the initial values that bracket the root. (12)

- Modify the above method (as a second function/program) to output a table showing the absolute relative approx. error after each iteration of the bisection method for up to 20 iterations. (4)