Contents in the book

- Preliminaries
  - > Polar, cylindrical and spherical coordinates
  - > Parameterized curves
  - > Parameterized surfaces in R^3
  - > Multivariable calculus
  - > Directional derivatives
  - > Summation Convention
- Tensors
  - > Motivation behind tensors: stating that different coordinate systems makes solving problems simpler. Quantities in nature stays the same in any coordinates and hence tensors are those quantities that remain unaffected by a coordinate transformation.
  - > Cartesian and Polar coordinate system
  - > Forward and Backward transforms (Derivative form of component transformation matrices - Jacobian)
  - > Vector component tranformation rule (stating that derivative symbol and basis vector notation is the same and why and also explaining why derivative notation is better)
  - > Vector basis transformation rule
  - > Covector basis and components(Introduce covectors as 1-forms, tell that we'll discuss more about in Chapter #)
    - > Integration with differential forms
    - > Gradient (vector field) and 'd' operators (covector field) w/ gradient explanation and examples
  - > Covector component transformation rule
  - > Covector basis tranformation rule
  - > Formal definition of a tensor (A tensor is a multilinear map $T: V^* \times \ldots \times V^* \times V \times \ldots \times V \rightarrow K \in \mathbb{R}$) (Eigenchris almost last lecture in tensor algebra series and Frederic Schuller tensor introduction)
  - > Tensor Product spaces (EC)
  - > Tensor Addition, Subtraction and Multiplication (Tensor Algebra)
  - > Transformation rules of general tensors
- Metric Tensor
  - > Using Metric Tensor to find lengths and angles (and symmetry of metric tensor)
  - > Transformation rules of metric tensors
  - > Finding the metric tensor of different coordinate systems
  - > Measuring arc length in curved spaces
  - > Index raising and Index lowering (also say that gradient is the "dual" of differential form and we can also change one to another by metric and inverse metric tensor)
  - > Finding Divergence(Components of gradient of f is just components of inverse metric tensor as found from the previous part), Curl and Laplacian in any orthogonal coordinate system (Then difference between Contravariant, Covariant and "Ordinary" Vectors and

Gradients)

> Tensor densities

- Derivatives of Tensors

  > Covariant derivative (Flat Space) → Christoffel Symbols (Provide Examples)

  > Covariant derivative component definition → Christoffel Symbols are not tensors

  > Geodesics (Professor Lia Vas' course note part 3) (w/ examples of different geodesics on different surfaces)

  > Parallel Transport

  > Covariant derivative (Extrinsic Curvature) (Start with "There are two ways of looking at curved spaces" also end with covariant derivative helps us find parallel tranported vector fields!) (concept of tangent plane is needed in the derivation? Maybe Sculler's Lectures can help)

    > Now approach in the opposite direction, given an intitial vector and and a path, change the vector such that the vector becomes parallel transported

  > Covariant derivative (Intrinsic Curvature) (Don't forget provide the motivation!)

    > Derivation of Christoffel symbols using metric tensors (and symmetry of Christoffel symbol)

    > Deriving geodesic equations (again) by parallel transporting a vector along itself

  > Abstract covariant derivative (Motivation: Used for taking covariant derivative of general tensor fields)

- Curvature

  > Flow curve

  > Lie Bracket (coordinate lines are flow curves along basis vectors)

  > Torsion Tensor (separation vector between parallel transported basis vectors)

  > Riemann curvature tensor definition

    > Motivation: Attempt 1 – A space is flat if the geodesics are straight therefore connection coefficients are 0 i.e, when the metric tensor is an identity matrix but metric tensor is not an identity for polar coordinates, Attempt 2 – Flat if connection coefficients are 0 if we can change the basis vectors to cartesian using Jacobian matrix but a sphere has connection coefficients at the equator (Local inertial frame) therfore we can always change coordinates to make connection coefficients = 0. So we use a tensor called Riemann curvature tensor.

    > Holonomy (Derivation) (Maybe try to get a better derivation that explains lie bracket in covariant derivative)

    > Geodesic deviation

    > Derivation of Components of Riemann Curvature Tensor

    > Symmetries of Riemann Curvature Tensor

    > Examples of calculating Riemann Curvature Tensor

  > Ricci Tensor ("Tracks volume change in curved space")

    > Sectional curvature (orthonormal basis only)

      > Ricci Curvature ("Ricci Curvature is the average of all sectional curvature" w/ examples of + and – Ricci curvature and finding Ricci curvature of a sphere)

    > Volume element derivative (Any Basis)

        > Volume element/form (gives the "volume" formed by two vectors) (maybe introduce n-forms after introducing vectors?)

> Ricci Scalar ("Compares the volume of a ball in curved space vs the volume of a ball in flat space")

> Properties of Ricci Tensor and Ricci Scalar

    > Einstein Tensor