Opinion Dynamics and Price Formation: Simulating the GameStop Corp. Short Squeeze

FTEC FYP Group I
Tianmeng, Tie 115104270
Meiqi, Zeng 1155107891
Supervisor: Professor WAI, Hoi To

Section I: Introduction

Summary of Results from First Semester

During last semester we had conducted an extensive study on complex network models and discrete opinion dynamics models in order to understand the effects of different parameter settings on the overall dynamics of the model. Specifically, in terms of complex networks, we analyzed the Erdős–Rényi-Gilbert model, Watts-Strogatz model and the preferential attachment model. Combined with these network models, we implemented both the voter model and the label propagation variant and explored their convergence properties. One question that was left unanswered how can we utilize these models in the field of finance, and consequently this is the focus of our research this semester.

Background for GameStop Corp. Short Squeeze

As we pondered upon the potential applications for opinion dynamics model in finance, an extraordinary event occurred in the U.S. stock market and it captivated our interests. The said event is the infamous GameStop Corp. short squeeze that began on Jan. 11 [1]. Before we briefly introduce the event, we need to explain the context first.

The number of retail investors (or colloquially referred as armchair traders) in the stock market has surged over the past year thanks to various factors such as technology and the pandemic [2] [3]. Thanks to commission-free online trading platforms such as Robinhood and E-Trade, stock trading has become more accessible and affordable than ever to retail investors. In addition, due to the pandemic and the many lockdowns that it caused, consumer spending was down and many people were eager to find ways to spend their stimulus cheques. These people found themselves getting involved with sports gambling as well as the stock market. This surge can also be seen through the emergence of social media groups that are dedicated to stock trading, the most famous one being r/wallstreetbets from the social platform Reddit. This particular group is depicted by the public as the culprit for the GameStop Corp. short squeeze and its user base had grown significantly over the past year. Even before the short squeeze, it had accumulated more than 1.8 million users. To conclude, just before the GameStop Corp. short squeeze, the stock market had just experienced a surge in the number of retail investors with limited experience in trading. Furthermore, these retail investors are likely to be frequent browsers of internet forums such as r/wallstreetbets.

What had happened during the GameStop Corp. (GME hereafter) short squeeze will be illustrated using the following timeline:

- Dec. 8, 2020: GME reported an operating loss of $63 million in the third quarter [4]. Stock price plummeted nearly 20% the next day Following this news, a few hedge funds (Citron Research, Melvin Capital, Point72 Asset Management and many more) had taken a short position on GME [1].
- Jan. 11, 2021: Ryan Cohen taking over GME: Ryan Cohen, whose RC Ventures had taken a 13% stake in GME and effectively gained three seats on the directors board [5]. Cohen intends to transform GME and this news sparked some chatter on r/wallstreetbets [6].
- Jan. 13, 2021: Many contribute this spike in price to the influence of a frequent r/wallstreetbets user as well as YouTuber Keith Gill. Gill was extremely bullish on GME and publicly voiced his opinion.
- Jan. 19, 2021: On this day, Citron Research, which is run by famous short-seller Andrew Left publicly criticized those who are buying GME on Twitter and soon announced that GME price will soon fall back to $20. This had only fueled buyers and price continued to increase [7].
- Jan. 26, 2021: Elon Musk tweeted a term frequently used on Reddit: “Gamestonk!!” as well as a link to the r/wallstreetbets forum [8].
- Jan. 27, 2021: Citron Capital and Melvin Capital announced that they were closing their short positions and had suffered significant loss. Andrew Left uploaded a video message saying that he respects the people on r/wallstreetbets and Reddit.
- Jan. 28, 2021: Robinhood and TD Ameritrade, the two most popular platforms used by retail traders, unexpectedly restricted most transactions involving GME [9].

- Jan. 29, 2021: SEC intervened and re-enabled GME transactions on trading platforms [10].
- Feb. 4, 2021: Treasury Secretary Janet Yellen announced that she will be meeting with regulators to discuss the GME saga [11].

The above timeline roughly sums up the short squeeze event that we are interested in. During the event, the lowest price was $19.01 and the highest price was $483.00. Such volatility is without a doubt unusual and perhaps even harmful to the stability of the stock market. It also served as a warning sign for regulators: armed with accessible, commission-free trading platforms and social media forums, retail investors have become a force to be reckoned with.

Problem Statement

After witnessing the GME short squeeze event unfold, we became inspired to simulate the GME trading activities among investors during that event period (Jan. 11 to Feb. 5) with the intention of studying the opinion dynamics of investors and the price dynamics. Furthermore, we would like to explore the price dynamics of a particular stock when exposed to a large inflow of irrational investors (as was the case of GME when an army of Reddit investors flocked to buy shares). Specifically, we would like to gain insights into questions such as "what percentage of total investors need to be irrational so that price can only move in the expected direction of the irrational investors". The GME short squeeze had proven that a new type of risk consequential to large stock trading social forums and prevalence of trading platforms had emerged in the market, and we believe that understanding the aforementioned dynamics will be of paramount importance to regulators as well as institutional investors going forward in the future.

Goals

Our goal for this study is to investigate the effects on the price dynamics of a stock when it is exposed to a large influx of irrational traders with homogenous belief. Specifically, we would like to design a number of simulations to mimic the GME short squeeze event and run experiments with different parameter settings to potentially create a model that provides insights as to the kind of effects unified retail investors can cause on the stock market and potential patterns institutional investors can identify as early signals of an incoming wave of unexpected trading frenzy initialized by social media groups.

Section II: Literature Review

Application of Opinion Dynamics Model in Financial Market

Opinion dynamics model's application in the financial market has been quite limited for some obvious reasons. These reasons include the difficulty of generating a social network of investors, difficulty in obtaining the opinion/sentiment of individual investors and the inherent complexity of the financial market itself. Consequently, existing research of applying opinion dynamics model to the financial market has omitted the use of real data and instead opting to build theoretical models that can demonstrate statistical characteristics that are observed in financial markets.

One of the opinion dynamics models studied by Sabatelli and Richmond [12] as well as Sznajd-Weron and Weron [13] is the Sznajd model. Under this model, market participants are assumed to be trend followers who place orders according to the opinions of the local opinion leader, and if such leader is absent, then these participants will act randomly. In addition to these trend followers, there also exist rational traders who place orders according to the demand and supply in the current market. Under this model, both papers were able to observe the main statistical characteristics found in financial markets such as fat-tails in the probability distribution of volumes and long memory dependence between random noise changes and volatility shocks.

Another opinion dynamics model used by Krause and Bornholdt [14], Zubillaga et al. [15], Vilela et al. [16], and Wang et al. [17] is the voter model which we studied extensively in the previous semester. The purposes of those research are all along the lines of studying trends, bubbles, and crashes of the financial market. Another commonality among these research is that agents typically have three states representing the actions of buying, selling, and holding.

GameStop Short Squeeze Analysis

Though It has only been 3 months since the GME Short Squeeze, there is some research conducted on the event already.

Umar et al. studied the effect of investors' sentiment represented by Twitter publication count, news publication count excluding Twitter, put-call ratio, and short-sale volume on the GME stock prices and returns during the short squeeze [18]. They apply the wavelet coherency framework to test the co-movement between GME returns and the aforementioned indicators. It is found that, firstly, there is a strong positive relationship between the GME returns and the number of Twitter/non-Twitter publications, indicating the retail investors' sentiment had indeed driven the stock prices up. In addition, the put-call ratio also demonstrates a positive co-movement with GME returns, which is also the key driver of the stock price. Note that the increase in put-call ratio is caused by the large increase in put option volume. Lastly, the authors discovered that increase in short sales volume is another important driver in the soaring of stock price as it eventually leads to market inefficiency since stock prices deviate from the fundamental value.

In another research conducted by Vasileiou et al. [19], a different approach was adopted to test whether Efficient Market Hypothesis (EMH) is violated or not during the short squeeze. Subsequently. Granger causality test and GARCH family model were conducted. From the testing result, the authors conclude that GME stock is not in line with the EMH. Moreover, while increases in trading volume lead to the sudden rise of GME stock prices, an increase in Google Searches can lead to a decline in stock price as GME drew wide attention.

Lyócsa et al. expands the researched stock from only GME to AMC Entertainment Holdings, Blackberry, and Nokia, which are all subject to a decentralized short squeeze, and test whether the price fluctuation can be explained by the discussion on r/wallstreetbets [20]. They defined a variable *relative Reddit intensity* $YOLO_1$ as the ratio between the level of interest in a given stock on the r/wallstreetbets and the level of interest in short squeeze indicated by Google Searches. If the discussions on r/wallstreetbets exert an impact on the stock price, then price variation should be associated with $YOLO_1$. The result shows that for all four stocks, when the related discussions on r/wallstreetbets intensified, the price volatility increased.

Section III: Opinion Dynamics and Price Formation Simulations

Overview

Recall that our goal is to investigate the effects on the price dynamics of a stock when the market is exposed to a large influx of irrational traders with homogeneous belief regarding the movement of the price. Specifically, we would like to simulate the GME short squeeze event that occurred from Jan. 11 to Feb. 5, 2021. We begin by building simpler models and then expanding them to include more realistic dynamics, and in the end, we have designed three models which we extensively tested.

The first model strictly follows an existing framework proposed by Marco D'Errico [21] which updates the opinion of agents using a bounded confidence model. We explore this model in detail and discuss some of the questionable assumptions made.

In the second model, we modify the first model to include more dynamics and relax some assumptions which we thought to be unreasonable. Specifically, trading dynamics are added to the model and each agent updates its shares based on opinion dynamics during each period.

The final simulation model combines the second model with another model which is aimed to represent the dynamics of irrational agents (analogous to retail investors in the GME event). The idea of this model is that rational investors are represented by the second simulation model and during the simulation process, irrational investors are sequentially added to the existing network of rational agents. Once irrational agents are added to the network, they begin to influence price by creating demand. This process is supposed to mimic the GME event and we extensively analyze the results of our simulation and compare them with real data from the GME event.

Simulation Model 1

Introduction

In this section, we strictly followed the approach proposed by Marco D'Errico [21] to simulate the price formation using opinion dynamics models. In the original paper, the author attempts to simulate the 2010 flash crash in stock price, which is believed to be caused by the opinion shifting due to erroneous reporting of a bombing event by a reputable news outlet. In the model, the author establishes a connection between the stock price and investors' opinions and observes the dynamics between the two. Two specific models are used. The first mode is the Present Discounted Value (PDV) asset pricing model proposed by Brock and Hommes [22], which can be used to connect the opinions of agents and the price of stock. The second model is the Bounded Confidence Model proposed by Deffuant et al. [23] and Hegselmann and Krause [24], which is a continuous opinion dynamics model in which an agent's opinion will only be affected by another agent if the difference between two agents' opinions is smaller than a certain deterministic threshold of confidence.

1) *Present Discounted Value (PDV) Model*

The PDV asset pricing model with heterogeneous, adaptive beliefs is as follows:

Assume in the network, all agents trade only one risk free asset and one risky asset stock. We have wealth dynamics:

$$\mathbf{W}(t+1) = R\mathbf{W}(t) + [p(t+1) + y(t+1) - Rp(t)]z(t) \qquad (1)$$

where R>1 denotes the gross return of risk free asset, $p(t)$ denotes the price per share of stock at time t, $y(t)$ denotes the stochastic dividend paid by the stock, and $z(t)$ denotes the number of shares of the stock purchased at t. Let E_{it}, V_{it} denote the beliefs of agent i about the expectation and variance. We assume that each agent shares the same belief about the variance of excess returns and this variance is a constant, i.e., $V_{it} \equiv \sigma^2$ for all agents. Next, we assume all agents are myopic mean variance maximizer, therefore we can solve each agent demand for stock shares

$$\max_{z}\{E_{it}\mathbf{W}(t+1)-\frac{a}{2}V_{it}(\mathbf{W}(t+1))\} \tag{2}$$

i.e.,

$$z_i(t)=\frac{E_{it}[p(t+1)+y(t+1)-Rp(t)]}{a\sigma^2} \tag{3}$$

where a denotes the risk aversion parameter. We assume a is a constant and is equal for all agents, i.e., each agent has the same risk preference. Now let $z_s(t)$ denote the supply of shares per agent. According to the equilibrium of demand and supply, we have

$$\sum\frac{E_{it}[p(t+1)+y(t+1)-Rp(t)]}{a\sigma^2}=nz_s(t)$$

If all agents are of the same type, we can derive the following pricing equation

$$Rp(t)=\frac{1}{n}\sum_{i=1}^{N}E_{it}[p(t+1)+y(t+1)]-a\sigma^2 z_s(t) \tag{4}$$

Without loss of generality, we specialize to the case of zero supply of outside shares, i.e., $z_s(t)=0$ [25]. Furthermore, we take $E_t[p(t+1)+y(t+1)]$ as agent opinion on the future stock price and denote it as $x_i(t)$. The final relation between stock price $p(t)$ and agent opinion can be represented by

$$Rp(t)=\frac{1}{n}\sum_{i=1}^{N}x_i(t) \tag{5}$$

Note that in Brock and Hommes's model, more parameters that can affect agent's opinion on stock price are introduced in Equation 4. However, in this simulation, we simplify it to one opinion $x_i(t)$.

2) *Opinion Evolution with Bounded Confidence Model*

The opinion evolution process can be represented by

$$\mathbf{x}(t)=\mathbf{A}(t)\mathbf{x}(t-1) \tag{6}$$

where $\mathbf{A}(t)$ is a confidence matrix and can be considered as an adjacency matrix representation of the network. If a_{ij} is 0, it means that agent i and agent j does not have edge in between; else it means from agent i to agent j there is an edge with weight a_{ij}. Therefore, this model differs from models we simulated last semester as it is an adaptive network and has an evolving topology. Next, we have an update matrix $\mathbf{C}(t)$ for evolution of $\mathbf{A}(t)$

$$a_{ij}(t) = \alpha_i c_{ij}(t) + (1 - \alpha_i) a_{ij}(t-1) \tag{7}$$

where α_i is an update propensity parameter and

$$[c_{ij}(t) := \begin{cases} \frac{1}{\#I(i,\mathbf{x}(t-1),p(t-1))} & \text{if } j \in I(i, \mathbf{x}(t-1), p(t-1)) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

In the bounded confidence model, an agent's opinion will only be affected by another agent if the difference between two agents' opinions is smaller than a certain deterministic threshold of confidence. In this case, agent i's opinion is affected by agent j if and only if $\frac{|x_i(t-1)-x_j(t-1)|}{x_i(t-1)} < \epsilon_i$. Therefore, we have

$$I(i, \mathbf{x}(t-1), p(t-1)) = \{j \text{ s.t. } \frac{|x_i(t-1) - x_j(t-1)|}{x_i(t-1)} < \epsilon_i\} \tag{9}$$

With the PDV model and bounded confidence model, we now have the opinion evolution through time and can get the price accordingly. Next, we introduce our simulation, including the initialization of network and different parameters and our algorithm combining two models.

Simulation

Initialization: Before running the simulation the following parameters need to be initialized:

- n, number of agents in the network
- r, risk-free rate. Note that $R = r + 1$
- $x_i(t = 0)$, agent i's opinion at the beginning
- $\mathbf{A}(t = 0)$, confidence matrix at the beginning
- α_i, update propensity parameter for agent i
- ϵ_i, the threshold in bounded confidence model for agent i

The network structure of the simulation is determined by parameter n and the confidence matrix $\mathbf{A}$. We set n to be 100 and $\mathbf{A}$ as an identity matrix which indicates that initially each agent only values his/her own opinion. However, as simulation runs, $\mathbf{A}$ evolves which means the topology of the network is dynamic.

The risk-free rate r is needed for Equation 5 and we set it has the overnight LIBOR USD rate quoted on Jan. 12, 2021, which is when the GME price hike was about to begin. The exact rate was 0.08588%[1].

For opinion matrix $\mathbf{x}$, we initialize $\mathbf{x}(t = 0)$ randomly following the normal distribution $N(19.95, 3)$ where 19.95[2] is the adjusted closing price of GME on Jan. 12, and the standard deviation is set to be 3 arbitrarily in order to reflect the opinion variance among agents.

For update propensity parameter α_i, we initialize it with uniform distribution $U(0, 1), U(0, 0.6), U(0, 0.2)$. From Equation 7, we learn that update propensity parameter α_i determines how much a_{ij} is affected by new network topologies and its new neighbors resulted from the opinion evolution. Intuitively, with higher α_i agent i puts more weight on the opinions of its new neighbors when updating its opinion, which leads to a faster change of opinions. The use of uniform distribution is to capture different degrees of stubbornness of agents.

For ϵ_i, we initialize it with uniform distribution $U(0, 0.2), U(0, 0.1), U(0, 0.05)$. If agent i's opinion is 19.95, $U(0, 0.2)$ gives a interval of $(15.96, 23.94)$ as neighbor. Similarly, $U(0, 0.1), U(0, 0.05)$ gives $(17.955, 21.945), (18.9525, 20.9475)$, respectively. Adjust the upper bound of ϵ_i is easier to understand as we want to limit the number of nodes that can affect agent i. Again, we use uniform distribution is to capture different degree of stubbornness of agents.

For each round t, starting from $t = 0$, do:

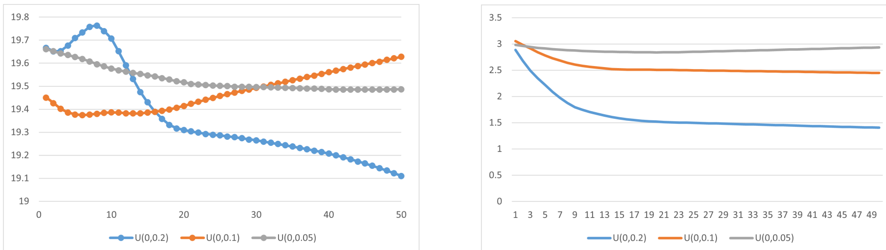
1. Calculate price $p(t)$ following Equation 5 with opinion $x(t)$
2. Compute $\mathbf{C}(t + 1)$ based on current opinion $x(t)$ according to Equation 8
3. Generate $\mathbf{A}(t + 1)$ following Equation 7 with $\mathbf{C}(t + 1)$ and $\mathbf{A}(t)$
4. Update agents' opinion $x(t + 1) = \mathbf{A}(t + 1)x(t)$

Analysis

Combining the different settings of α_i and ϵ_i, we have in total 9 different initializations. We run each setting 10 times and observe the evolution of price and the standard deviation of 100 agents' opinions during the evolution. Below are the figures of the movement of stock price and the standard deviation of agents' opinions in the network in one run.

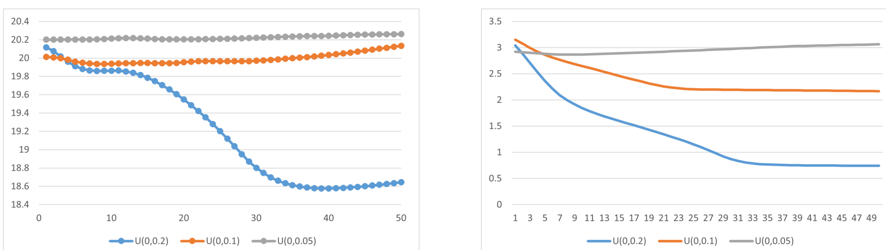
First, we consider the effects of different ϵ_i settings. A higher ϵ_i value directly translates to more neighbors for each agent as there are likely more agents in the network who can satisfy equation 9. In addition, higher ϵ_i will likely cause the agent to deviate from its original opinion as its opinion is determined by the

[1]This rate is cited from https://fred.stlouisfed.org/series/USDONTD156N
[2]The price is cited from https://finance.yahoo.com/quote/GME/history?p=GME



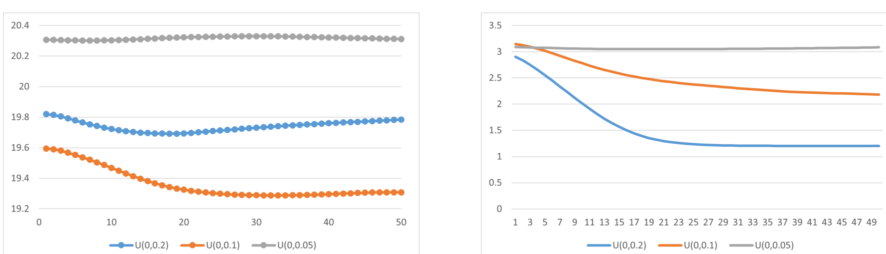
(a) Price Evolution (b) Standard Deviation

Figure 1: $\alpha_i = U(0,1)$. Labels in figure represent different settings of ϵ_i



(a) Price Evolution (b) Standard Deviation

Figure 2: $\alpha = U(0,0.6)$



(a) Price Evolution (b) Standard Deviation

Figure 3: $\alpha = U(0,0.2)$

weighted mean of its neighbors. Furthermore, in terms of the standard deviation of agents' opinions during the simulation process, we can observe that higher ϵ_i leads to a lower standard deviation. This is also expected as a more connected network of agents can lead to greater convergence of opinion. The opposite is true for lower ϵ_i value: higher standard deviation is observed. Lastly, in terms of the price dynamics, we observe that a higher ϵ_i leads to greater change in price during the initial period but settles down over time. This is likely due to the fact that with higher ϵ_i value, the network topology changes significantly in the beginning, and consequently, each agent's opinion may be influenced by other very different opinions. As the simulation continues, the topology of the network settles down and hence we observe a more stable price dynamic.

Next, we consider the effects of different settings of the update propensity parameter α_i. From the graph, we can see that a smaller upper bound of α_i directly translates to smaller changes in stock price in each round. The reason is that a smaller α_i means each agent puts less weight on the opinions of its neighbors when updating its opinion. Intuitively, it also affects the standard deviation as the decline of standard deviation is slower with smaller α_i because of the same reason above. The convergence of opinions becomes slower when α_i is small.

In general, the price evolution in this simulation is relatively smooth, which means the average of all opinions does not experience sharp changes. Therefore, without external factors, such as breaking news that greatly affects agents' opinions, the price will not fluctuate much in this simulation.

Simulation Model 2

Introduction

Simulation Model 1 had some flaws in its mechanism which we believe to be unreasonable, and thus Model 2 aims to get rid of these and create another model that can perhaps simulate the dynamics between investors' opinions and price better. In Model 1, the equilibrium condition in Equation 2 implies that the number of shares held by each agent during each round is constant at z_s, which is the limited outside supply per agent (and nz_s denotes the overall outside supply). This is unreasonable as the portfolios of agents will likely change during the simulation. Consequently, we introduce dynamics to the portfolio of agents in Model 2. Furthermore, in Model 1, price is solely determined by the opinions of agents in the network (since we adopt the special case of zero supply of outside shares). We modify the price formation formula to include the dynamics of both opinions and portfolio in Model 2.

Recall in Equation 3, z_i is the optimal demand for shares at time t for agent i. In other words, z_i represents the change in number of shares held by each agent for time t. Here for clarity, from now on we use $\Delta z_i(t)$ to denote $z_i(t)$ in above equation. Further recall that $x_i(t)$ denotes $E_t[p(t+1)+y(t+1)]$ in Model 1 and we have the following equation:

$$\Delta z_i(t) = \frac{x_i(t) - Rp(t)}{a\sigma^2} \tag{10}$$

The value of $\Delta z_i(t)$ can convey to trading action of agent i during each round:

1. $\Delta z_i(t) > 0$: Agent i would proceed to buy $\Delta z_i(t)$ shares given that:
   - The agent has not reached his/her wealth constraint (in case where there is one)
2. $\Delta z_i(t) < 0$: Agent i proceeds to sell $\Delta z_i(t)$ shares, given that:
   - The agent cannot short sell the stock
3. $\Delta z_i(t) == 0$: No action is taken

Once $\Delta z_i(t)$ is determined using equation 10, combined with trading action, we can generate an order for each agent once we generate the order price with the following method:

1. Buy
$$y_i(t) = U((1 - \beta_i)(1 + r)p(t - 1), x_i(t)) \tag{11}$$
2. Sell
$$y_i(t) = U(x_i(t), (1 + \beta_i)(1 + r)p(t - 1)) \tag{12}$$

where β_i can be considered as a risk preference of agent i when agent i places order. For instance, a higher β_i means that the lower bound of the purchasing order price will be lower (and vice versa), and the upper bound of the selling order price will be higher.

After obtaining the full information of orders placed by agents, we can generate the stock price as the weighted mean of all order price as shown by the following equation:

$$p(t) = \frac{\sum \Delta z_i(t) y_i(t)}{\sum \Delta z_i(t)} \tag{13}$$

As for opinion dynamics, we can follow the bounded confidence model described in Model 1 to update the opinion $x(t)$.

In this model we have introduced several assumptions which are summarized in the following:

1. Short selling is not allowed, i.e. agents cannot hold a negative amount of stock
   - In order to limit the complexity of the model, we prevent agents from short selling shares
2. There is no limitation on the number of outstanding shares in the network

- As the number of agents in our network is quite small, we hope to simulate only a portion of the entire shareholder network while retaining its characteristics.

3. All selling and buying order will be fulfilled
   - The model executes all orders placed by agents regardless of price. We believe this is reasonable as under this model order prices will never be ridiculously high or low.

Simulation

Initialization Before running the simulation the following parameters need to be initialized:

- n, number of agents in the network
- r, risk free rate. Note that $R = r + 1$
- a, risk averse parameter
- $p(0)$, initial stock price
- σ^2, corresponding variance of stock
- $z(0)$, current holding of shares of agents
- β_i, risk preference parameter of agent i on placing order
- $x_i(t = 0)$, agent i's opinion at the beginning
- $\mathbf{A}(t = 0)$, confidence matrix at the beginning
- α_i, update propensity parameter for agent i
- ϵ_i, the threshold in bounded confidence model for agent i

To be consistent with previous simulation, we use a network consisting of 100 agents in this simulation and run 50 updates. For equation 10, we need to initialize the initial stock price $p(t = 0)$, variance σ^2, risk free rate r, and risk aversion parameter a. Note that variance $\sigma^2 = V_{it}(p(t + 1) + y(t + 1) - Rp(t))$ here. We initialize the following variable using GME historical data: $p(t = 0)$ is set to be 19.95, which again is the adjusted close price on 12 Jan. 2021. For variance σ^2, we use 1 year historical data of GME starting from 30 Oct. 2020 to 12 Jan. 2021, that is 1.152757[3]. And lastly for risk-free rate r, again we use the overnight LIBOR USD rate 0.08588% quoted on 12 Jan. 2021. For each agent, the risk aversion parameter a is initialized as 1.

We adopt the same opinion update strategy used in previous equilibrium simulation. We initialize $\mathbf{x}(0)$ with normal distribution $N(19.95, 3)$, $\mathbf{A}(t = 0)$ as an identity matrix, α_i with uniform distribution $U(0, 0.8)$, and ϵ_i with uniform distribution $U(0, 0.1)$, which is considered as a reasonable setting based on the result from Simulation Model 1. $z_i(t = 0)$ is initialized as a random integer from $(100, 500)$ so that each agent initially hold 100 to 500 shares of stock. In our simulation, we initialize β_i using the following uniform distribution $U(0, 0.2), U(0, 0.1), U(0, 0.05)$ to capture the effect of β_i on price evolution.

[3] The σ^2 is cited from https://finance.yahoo.com/quote/GME/history?p=GME

For each round t, starting from $t = 0$, do:

1. Calculate the change in demand for stock shares $\Delta z_i(t)$ using Equation 10, determine the action taken by each agent
2. Generate an order price $y_i(t)$ for each agent that places an order
3. Combine $\Delta z_i(t)$ and $y_i(t)$ to generate $p(t+1)$ following Equation 13
4. Settle down the order by updating $z_i(t+1)$ from $z_i(t)$ and $\Delta z_i(t)$

Analysis

We run each setting of β_i 10 times to check for extreme cases and arbitrarily choose one to be represented by the graphs below:

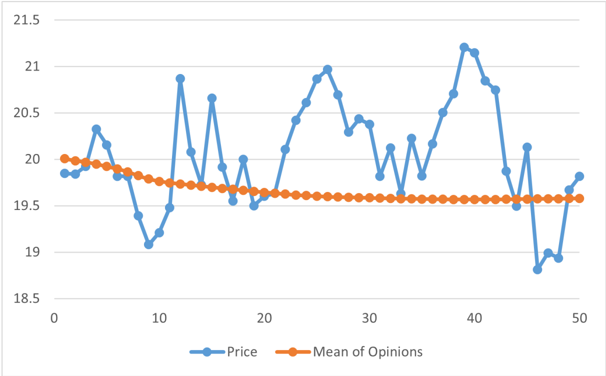


Figure 4: Price Evolution When $\beta_i = U(0, 0.2)$

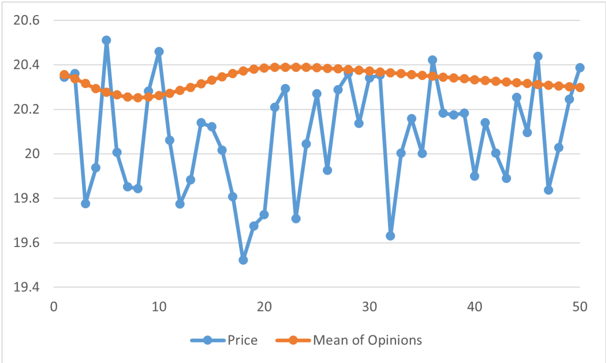


Figure 5: Price Evolution When $\beta_i = U(0, 0.1)$

Looking at the price pattern in Figure 4, Figure 5, and Figure 6, we immediately notice that it fluctuates more than Model 1. In fact, the orange line representing the mean of opinions would be the price evolution pattern for Model 1. This

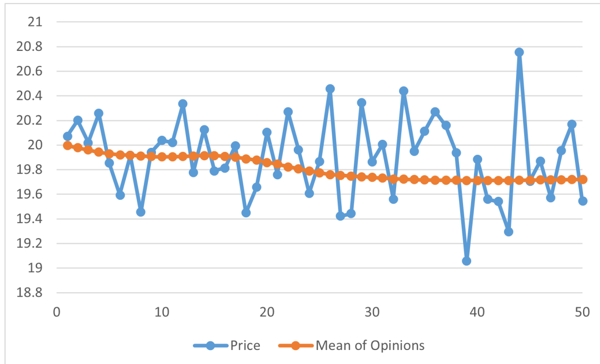


Figure 6: Price Evolution When $\beta_i = U(0, 0.05)$

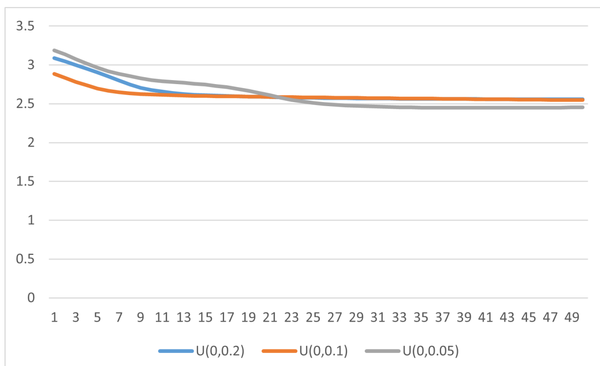


Figure 7: Standard Deviation of Different Settings of β_i

pattern is perhaps a more realistic representation of the actual price dynamics of stocks.

For different settings of β_i, we expect that with a higher upper bound of β_i the change in price in each round will likely be larger as the range of prices in which an agent chooses as the order price y_i is larger. Nevertheless, judging from Figure 7 the differences in terms of standard deviation of prices for different settings of β_i is indistinguishable. However, after taking the average from 10 simulation runs, the following results are observed:

	U(0,0.2)	U(0,0.1)	U(0,0.05)
Standard Deviation	0.702148658	0.553659136	0.361238393

Table 1: Mean of Standard Deviation of Different Setting of β_i

From Table 1, we can notice that a higher β_i value does indeed induce a larger standard deviation for price.

Simulation Model 3: Combined Network Simulation:

Introduction

In this model, we try to simulate the sudden rise of GME stock price which began on 11 Jan. 2021 by combining two different networks together. The first one is a network of rational investors who already hold GME stocks at the beginning of the simulation and continues to trade as simulation goes and update their opinions by communicating with other rational investors. This model is basically the Model 2 we presented previously. The second network consists of only stubborn and irrational investors who all hold a homogenous opinion. These investors represent the wave of retail investors coming from the social media forum r/wallstreetbets. In this case, these irrational investors have a high expected price for GME and will only place buy orders. Because the rational network (simulated using Model 2) has already been explained, we will describe the dynamics of the irrational network in the following:

Irrational Network As mentioned before, irrational agents all hold a single homogenous opinion with regards to the direction of the price, and in our case, they all hold the belief that price will only go up. This sentiment is reflected during the GME short squeeze event by the famous slogan "stonks only go up". Therefore, to simulate this, we remove the opinion dynamics from the irrational network as these investors are essential stubborn agents. Irrational agents, therefore, obtain $\Delta z_{i,i}$, which is the demand for shares at time t under the following equation:

$\Delta z_{i,i} = U(0, \Delta z_{max})$ where Δz_{max} is the maximum number of shares that could be bought by an irrational agent at each time period. Once $\Delta z_{i,i}$ is determined, the irrational agent then determines the order price using the following equation:

$$y_{i,i}(t) = U((1+r)p(t-1), (1+\gamma)(1+r)p(t-1)) \tag{14}$$

where γ serves as risk preference parameter similar to β_i found in the rational agent order price Equation 11 and Equation 12.

After the irrational network has been established, we combine the two networks by sequentially introducing irrational agents into the existing rational network. Therefore, as the simulation runs, the total number of agents in the network increases with time.

We further introduce the three variants of this simulation model. The first variant is to add irrational agents without restricting the total number of shares they can purchase; the second variant is to add irrational agents with limit on the maximum number of shares they can purchase, denoted by z_{max}; the third variant is to add rational agents to the existing rational network as opposed to adding irrational agents. The last variant can be viewed as a control to the first two variants.

In the first variant, at $t = 0$, there is a network consisting of n rational agents and each agent holds opinion $x_i(0)$ on the stock price and initializes other parameters in the same way as in Model 2. At each time period, each agent can trade $\Delta z_{r,i}(t)$ number of shares at the proposed order price $y_{r,i}(t)$. At the same time, irrational agents are added sequentially and these irrational agents purchases $\Delta z_{i,i}(t)$ shares at each time period following uniform distribution $U(0, \Delta z_{max})$. Irrational agents place order at price $y_{i,i}(t)$ which follows Equation 14. Combining the orders of all agents, the stock price is updated as the weighted mean of all order price (similar to equation 13) and the new price is used to update rational agents' opinions.

In the second variant, we add limitation z_{max} on the shares of stock an irrational agent can buy. Therefore, when an irrational agent i's' current portfolio $z_{i,i}(t)$ reaches z_{max}, the agent cannot purchase stock and $\Delta z_{i,i}(t)$ is set to 0.

In the third variant control simulation, we add the same number of rational agents in each round as in the first and second variant. The newly added rational agents are initialized in the same way as we initialize the rational agents in original network except that their initial holding $z_i(t)$ is set to be 0. Note that the newly added agents do not have connection with existing agents or agents added in the same round, i.e. $a_{ij} = 0$ for all $j \neq i$ and $a_{ii} = 1$. They only start to build edges with other agents during the opinion updating with the change of $\mathbf{A}(t)$. Then, we can update the network the same way as in Model 2 as it only involves network.

Simulation

Initialization Before running the simulation the following parameters need to be initialized:

- n, number of agents in the network
- r, risk-free rate
- a, risk averse parameter
- $p(0)$, initial stock price
- σ^2, corresponding variance of stock
- $z_r(0)$, current holding of shares of rational agents
- β_i, risk preference parameter of rational agent i on placing order
- $x_i(t = 0)$, rational agent i's opinion at the beginning
- $\mathbf{A}(t = 0)$, confidence matrix of rational agent at the beginning
- α_i, update propensity parameter for rational agent i
- ϵ_i, the threshold in bounded confidence model for rational agent i
- z_{max}, the maximum shares an irrational agent can hold in variant 2
- Δz_{max}, the maximum shares an irrational agent can buy in variant 1 and variant 2
- γ, risk preference parameter for irrational agent in variant 1 and variant 2
- S, a sequence of number of agents added in each round in all variants

In this simulation, we introduce new parameters z_{max}. Δz_{max}, γ, and S and we

	Case γ	Case z_{max}	Case Δz_{max}
n	100	100	100
r	30	30	30
a	1	1	1
$p(0)$	19.95	19.95	19.95
σ^2	1.152757	1.152757	1.152757
$z(0)$	$U(100, 500)$	$U(100, 500)$	$U(100, 500)$
β_i	$U(0, 0.1)$	$U(0, 0.1)$	$U(0, 0.1)$
$x_i(0)$	$N(19.95, 3)$	$N(19.95, 3)$	$N(19.95, 3)$
$\mathbf{A}(0)$	Identity matrix	Identity matrix	Identity matrix
α_i	$U(0, 0.8)$	$U(0, 0.8)$	$U(0, 0.8)$
ϵ_i	$U(0, 0.1)$	$U(0, 0.1)$	$U(0, 0.1)$
z_{max}	500	**200, 500, 800**	500
Δz_{max}	90	90	**60, 90, 120**
γ	**0.1, 0.2, 0.3**	0.2	0.2
S	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]

Table 2: Initialization of γ, z_{max}, and Δz_{max}

want to study how these parameters affect the price dynamics. First, we test z_{max}. Δz_{max}, and γ, and the initialization of parameters is shown in Table 2. Other than the parameter in question, the settings of all other parameters are the same as settings in Model 2. In addition, we set the base value of S to be the sequence that is proportional to the volume of the corresponding trading day of GME stock from Jan. 11 to Feb. 5. Note that after all irrational agents are added to the network, trading is continued for 12 days for both rational and irrational agents. Moreover, we designed 8 cases to study the effects of number of agents introduced to the network on each round as shown in Table 3. The details and reasons for setting will be explained together in Analysis.

In the first variant, for each round t, starting from $t = 0$, do:

1. Calculate the change in demand for stock shares $\Delta z_{r,i}(t)$ for rational agent

	n	S	Rational/Irrational After adding
Case 1	100	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	0.70
Case 2	100	[2, 0, 28, 18, 8, 14, 6, 10, 38, 34, 34, 18, 10, 10, 6, 14, 8, 12, 16]	0.35
Case 3	100	[1, 0, 14, 8, 4, 7, 2, 5, 19, 17, 17, 8, 5, 4, 3, 7, 3, 5, 7]	0.74
Case 4	100	[0, 0, 0, 0, 0, 0, 47, 49, 47, 0, 0, 0, 0, 0, 0, 0, 0, 0,]	0.70
Case 5	100	[7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8]	0.70
Case 6	200	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	1.40
Case 7	500	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	3.50
Case 8	800	[1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]	5.59

Table 3: Initialization of n and S

following Equation 10
2. Generate an order price $y_{r,i}(t)$ for each rational agent that places an order
3. Introduce $S(t)$ irrational agents to the network
4. Generate the number of shares $\Delta z_{i,i}(t)$ purchased by each irrational agent
5. Generate an order price $y_{i,i}(t)$ for each irrational agent
6. Combine $\Delta z_{r,i}(t)$, $\Delta z_{i,i}(t)$, $y_{r,i}(t)$ and $y_{i,i}(t)$ to generate $p(t+1)$
7. Settle down the order and update $z_{r,i}(t+1)$ and $z_{i,i}(t+1)$ from $\Delta z_{r,i}(t)$, $\Delta z_{i,i}(t)$, $z_{r,i}(t)$, $z_{i,i}(t)$

The update process of the second variant is the same as the first variant. However, in control simulation, we introduce $S(t)$ initialized rational agents to the network and update correspondingly.

Analysis

In the following section, we will go through the simulation results obtained from the aforementioned proposed test cases. The effects of each of the four parameters (γ, z_{max}, Δz_{max} and S) are studied and individually presented. Note that the the base case follows the setting $\gamma = 0.2, z_{max} = 500, \Delta z_{max} = 90, and S = [1, 0, 14, 9, 4, 7, 3, 5, 19, 17, 17, 9, 5, 5, 3, 7, 4, 6, 8]$ and the full results of base case are listed in Figure 8, 9, 10, and 11. Note that we add the full results for all testing cases in Appendix.

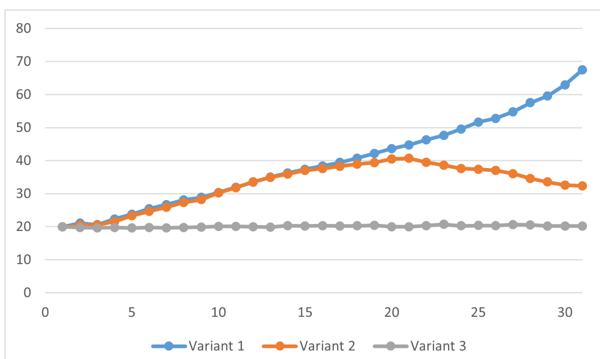
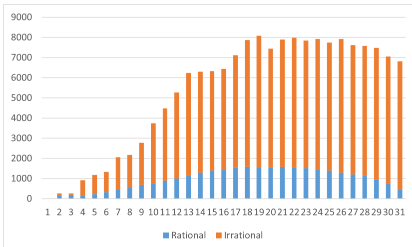


Figure 8: Price Evolution of Base Case for 3 Variants

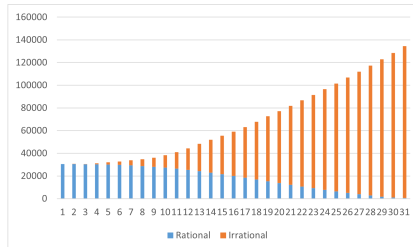
1) *Effects of* γ

Recall the γ is the risk preference parameter for Equation 14, and consequently changes in γ directly influence the order price of irrational agents and consequently the price of shares. The following observations are noted:

- A larger γ leads to higher prices at all time periods throughout the simulation. This would also mean that the peak price achieved would be higher as γ increases. This is expected as

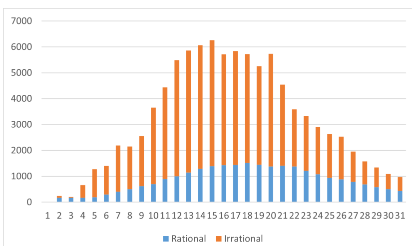


(a) Trading Volume

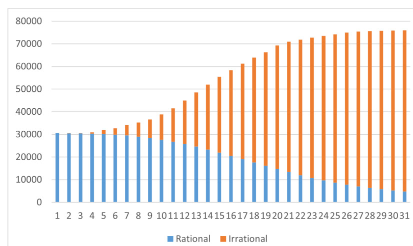


(b) Total Volume

Figure 9: Volume of Base Case for Variant 1

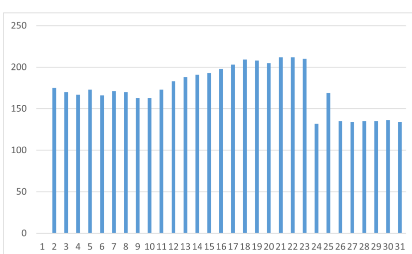


(a) Trading Volume

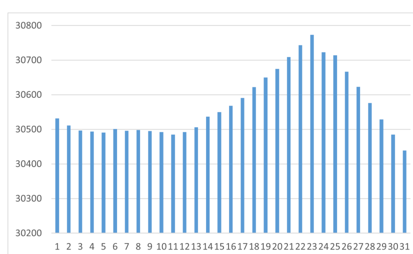


(b) Total Volume

Figure 10: Volume of Base Case for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 11: Volume of Base Case for Variant 3

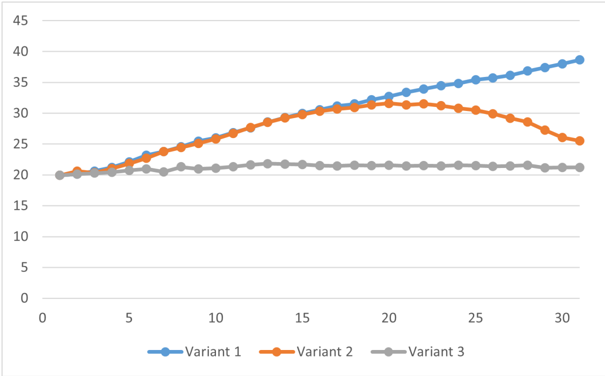


Figure 12: Price Evolution When $\gamma = 0.1$ for 3 Variants

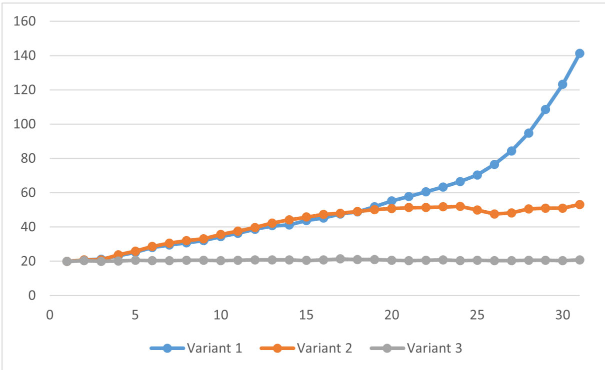


Figure 13: Price Evolution When $\gamma = 0.3$ for 3 Variants

higher γ leads to higher order price from irrational orders which would then increase the weighted average or order prices leading to an increase in the updated price.

- Higher prices caused by larger γ would motivate rational agents to sell more stock as simulation proceeds due to the Equation 10. More specifically, the difference between rational agents' expected prices and current price widens.

2) *Effects of* z_{max}

Recall that z_{max} refers to the maximum number of shares that could be held by an irrational agent for the entire duration of the simulation process. The base case setting of z_{max} is 500 and we test two additional cases where $z_{max} = 200, 800$ to analyze its effect on the overall dynamics. The following observations are found during our simulation. Note that z_{max} is only applicable for variant 2 model:

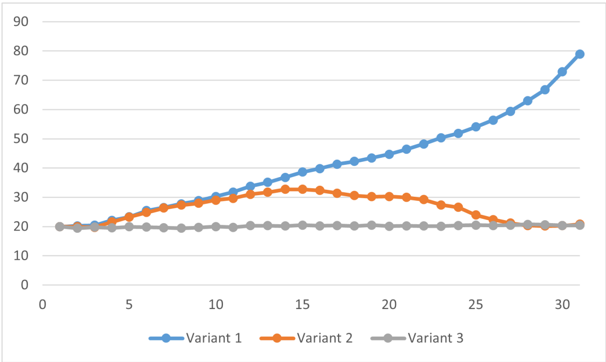


Figure 14: Price Evolution When $z_{max} = 200$ for 3 Variants

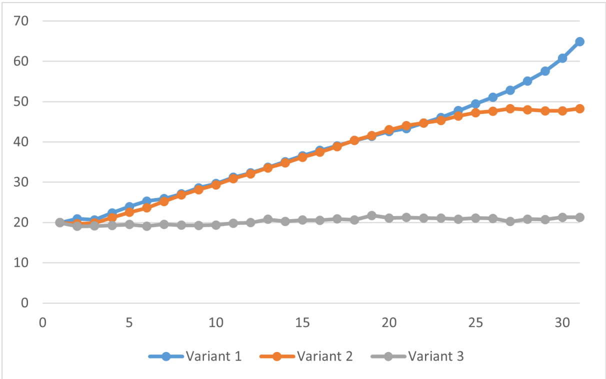
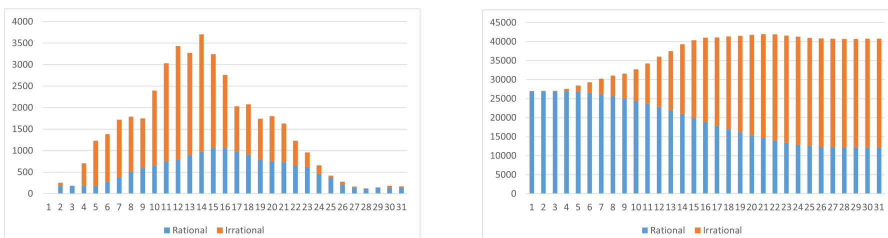


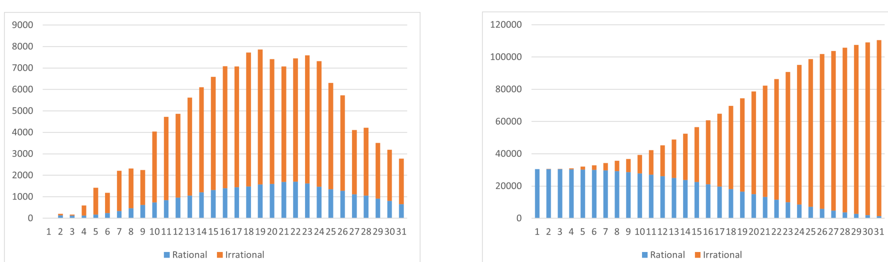
Figure 15: Price Evolution When $z_{max} = 800$ for 3 Variants

- In terms of price pattern, the base case where $z_{max} = 500$ exhibits expected behavior that loosely follows the real price pattern of GME during the corresponding period. Looking at Figure 8, we notice that the price peaks at $t = 20$ which is just after all the irrational agents have been added to the network. After that price begins to fall slowly. As we decrease z_{max}, a similar pattern can be observed in Figure 14, however, the price peak is lower and is reached much earlier at $t = 15$. This is due to the fact that irrational agents who entered earlier in the simulation process exhaust their lowered purchase power earlier and thus the momentum to drive up the price is diminished. On the contrary, if z_{max} increases, not only do we observe a higher peak price, we also observe a strong price momentum and the price hike continues to even at the end of the simulation process in Figure 15.



(a) Trading Volume (b) Total Volume

Figure 16: Volume When $z_{max} = 200$ for Variant 2



(a) Trading Volume (b) Total Volume

Figure 17: Volume When $z_{max} = 800$ for Variant 2

- A similar story is presented in trading volume during the simulation. Compared the base case, as shown in Figure 16, the lower $z_{max} = 200$ setting leads to a greater concentration of irrational agent trading activity during the first half of the sim-

z_{max} Max Total Volume	Model Variant	Max Price	Min Price	Max Trading Volume by Rational	Max Trading Volume by Irrational	Max trading Volume
200	32.74	19.74	1068	2726	3706	41976
500	40.71	19.95	1517	4867	6259	75927
800	48.27	19.70	1707	6293	7865	110550

Table 4: Summary of Variant 2 with different settings of z_{max}

ulation process, although the trading volume has decreased by almost 40%. In contrast, for the higher $z_{max} = 800$ setting as shown in Figure 17, irrational traders' trading activities are more concentrated during the later phase of the simulation and the volume is higher than the base setting.

- The results of setting different z_{max} values are largely in line with our expectation: if irrational agents have deeper pockets, they are expected to conduct more trading and their ability to drive up stock prices will be amplified.

3) *Effects of* Δz_{max}

Recall that Δz_{max} represents the maximum amount of shares that could be traded during each round for irrational agents. The base setting of Δz_{max} is 90 and the other two settings which will be tested are 60 and 120.

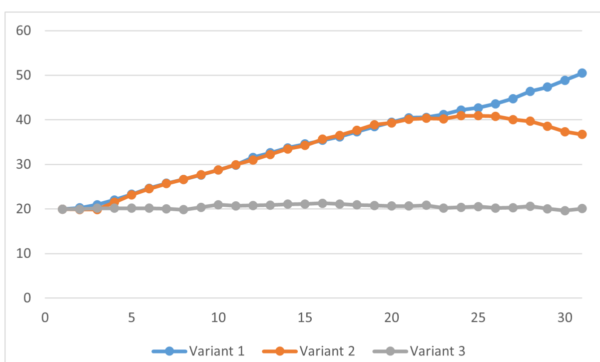
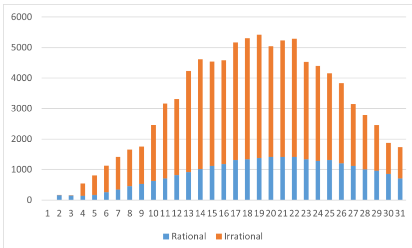
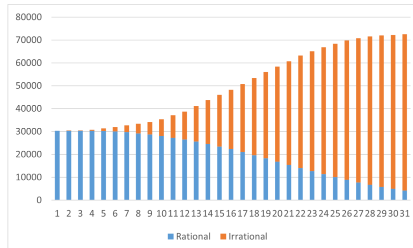


Figure 18: Price Evolution When $\Delta z_{max} = 60$ for 3 Variants



(a) Trading Volume



(b) Total Volume

Figure 19: Volume When $\Delta z_{max} = 60$ for Variant 2

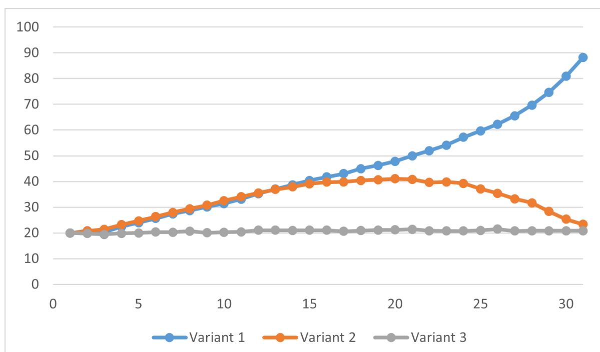
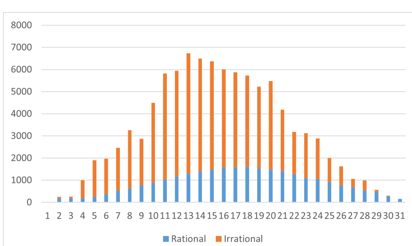
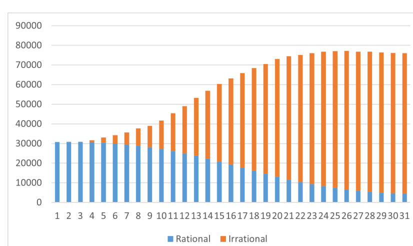


Figure 20: Price Evolution When $\Delta z_{max} = 120$ for 3 Variants



(a) Trading Volume



(b) Total Volume

Figure 21: Volume When $\Delta z_{max} = 120$ for Variant 2

Δz_{max}	Model Variant	Max Price	Min Price	Max Trading Volume by Rational	Max Trading Volume by Irrational	Max trading Volume	Max Total Volume
60	1	50.50	19.95	1461	4640	5873	89628
	2	40.94	19.87	1419	4047	5421	72546
90	1	67.47	19.95	1586	6624	8079	134431
	2	40.71	19.95	1517	4867	6259	75927
120	1	88.26	19.95	1873	9115	10719	177512
	2	41.09	19.95	1599	5446	6739	77126

Table 5: Summary of Variant 1, 2 with different settings of Δz_{max}

- It is apparent that a higher Δz_{max} would lead to greater trading volume, especially for irrational agents. However, a slight increase in max trading volume can also be observed in rational traders. In addition, the overall distribution of trading volume of rational agent, in fact, mirrors that of the irrational agent.
- Similarly, higher Δz_{max} leads to a higher total volume of shares in the combined network.
- Furthermore, as shown by Figure 19 and Figure 21, higher Δz_{max} means that z_{max} is reached sooner for irrational agents. This is rather self-explainable as irrational agents are buying more shares each round with higher Δz_{max}.
- In terms of price pattern, it can be observed from Figure 18 and Figure 20 that a higher Δz_{max} causes the peak price to be reached earlier in the simulation process. Nevertheless, the peak price itself seems to be unaffected by Δz_{max}.

4) *Effects of S*

Recall that in this combined model, irrational agents are added to the existing rational network sequentially. In the base setting the sequence corresponds to the volume GME shares traded on each day, as we are using volume traded as a proxy to the number of retail investors entering the market. In addition to this case, other sequences are explored and their effects are analyzed:

- Case 1:
  - Base case as mentioned before.

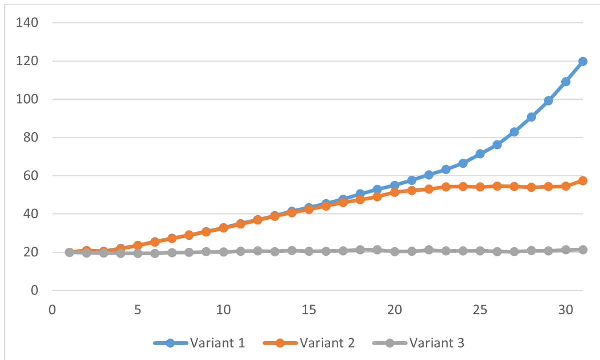


Figure 22: Price Evolution in S Case 2 for 3 Variants

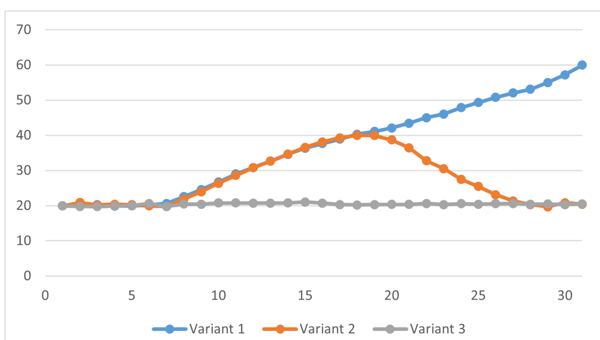


Figure 23: Price Evolution in S Case 4 for 3 Variants

- Case 2:
  - Base case multiplied by a factor of 2. In this case, we explore the effect of double the number of irrational agents added to the sequence.
  - Using this sequence, the prices have indeed increased throughout the simulation process as shown in Figure 22. In addition to higher prices, the price momentum has also sustained for longer. By the end of simulation, prices have not dropped but rather increased a little.
  - As explained in the γ section, these higher prices would lead to greater sell orders from rational agents.
- Case 3:
  - In case three, instead of using the volume of GME shares traded during the corresponding day, we use an adjusted volume. The total traded volumes from Jan. 11 onwards are subtracted by the 100 day average trading volume prior to Jan. 11.
  - Since the case 3 sequence is quite similar to the base case, there are no novel observations.
- Case 4:
  - In case four, our focus is to simulate a wave effect where the majority of the irrational agents enter the networking in a short period. In our case, all of the irrational agents enter the network in three periods from $t = 6$ until $t = 9$. The total number of irrational agents entered is kept the same as the base case.
  - In terms of price pattern, while peak price has not changed significantly, it is apparent that the price hike is much less sustained as shown in Figure 23.
  - As expected, trading activities remain relatively dormant until the sudden arrival of irrational traders.
- Case 5:
  - In case five, we attempt to simulate the opposite of the wave effect. Here irrational agents are evenly added to the network, and similarly, the total number of irrational agents added is kept constant.
  - The results resemble that of the base case but with smoother increases in terms of trading volumes and prices.

5) *Effects of n*

In general, increasing n has the same effect of decreasing the number of agents added in each round. We use the base case configuration and run the simulation with three different values of n (200, 500, and 800) and compare the results.

- The effect of n is mostly demonstrated by the decrease in prices. As n increase, and price levels throughout the simulation decreases. At the highest n setting, the peak price has fallen to just over $25 compared to around $40 for the base setting.

Section IV: Discussions and Future Extensions

Similar to existing literature regarding the application of opinion dynamics model in financial market which mostly focuses on analyzing trends, bubbles, and crashes, our research has also been focusing on a specific event where an exceptional price hike has taken place. However, in contrast to previous studies, the event in question for our research has a much greater social aspect to it as the event is directly triggered by a social media group and the group's influence in the financial market has inflicted lasting pain on large hedge funds. This event has largely transformed the public perception of influence of retail investors and motivated both regulators and institutional investors to pay greater attention to their activities, especially collective activities.

Our simulations have shown that our models can largely mimic the GME short squeeze event, however, there are some assumptions to our models which can limit their usabilities

- Order matching mechanism: In our models, orders are matched regardless of whether there exists an opposite order. Nevertheless, agents within our model do not set order prices that are too far off from the current price which makes the mechanism more acceptable.
- Total number of shares: In a way this is related to the previous assumption. Because we do not set a limit on the total number of shares that circulate the network, our model cannot strictly represent the entire market for GME stock. However, since we are simulating on a small number of agents, we believe the lack of a constraint on total number of shares is acceptable.
- Fixed strategy: In our models, both rational and irrational agents all follow fixed strategies. However, it is highly likely in real life that some rational agents would adopt irrational behaviors. It is equally likely that some irrational agents would adopt rational behaviors or even some other behaviors that fall somewhat in the middle.

Lastly, we would like to propose several future extensions to our study:

- Add opinion dynamics to irrational agents: The current irrational agents are thought to be stubborn and do not adopt or influence anyone else. Adding opinion dynamics among irrational agents to mimic the interactions found on Reddit forums (mutual idea reinforcement, encouragement, etc.) could potentially generate interesting results.
- Scale up the networks: Instead of simulating hundreds of agents, potentially

scale up the network to be hundreds of thousands of agents. One immediate benefit of this is that the order mechanism can be designed to mimic that of the real world and the total number of shares constraint could be added.

- Dynamic-strategy: Allow agents to switch strategies under different market conditions. Potentially use real data and supervised learning to teach agents what actions to take under different situations.

Summary and Contributions:

In this study, we explored a model to simulate opinion dynamics and price formation of risky assets in the financial market and expanded that model to simulate the recent GME short squeeze event with the intention of discovering the effects of various parameters on the price of the stock. Our results may provide useful insights to both market regulators and institutional investors to better prepare for a market that is transforming consequential to accessible online trading platforms and a generation of retail investors who are extremely active on various social media platforms. We hope this study could pave way for further research in this area.

In the months leading up to this report, we continued to read and discuss papers related to the applications of opinion dynamics model in finance. Meanwhile, Meiqi expanded upon our simulations last semester to include directed graphs and adaptive networks. Once the problem statement has been finalized, we begin planning to read specific papers to help us design our simulation. Tianmeng is mainly responsible for designing mechanisms of our simulation model and Meiqi is tasked with implementing the simulation models. Once the basic framework of the simulation is built, we both worked on improving the simulations and come up with new ideas to make the model more realistic. Once we had finalized the simulations, Tianmeng designed several test cases and Meiqi is responsible for running the simulations and conduct initial analysis. The detailed analysis later on as well as this report is done by both members together. Finally we would like to thank Professor WAI, Hoi To for his invaluable patience, time and guidance during our weekend meetings which helped us tremendously throughout this final year project.

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Appendix: Results from Combined Network Simulation

Below we list all results generated from Combined Network Simulation by different settings of γ, z_{max}, Δz_{max}, and S.

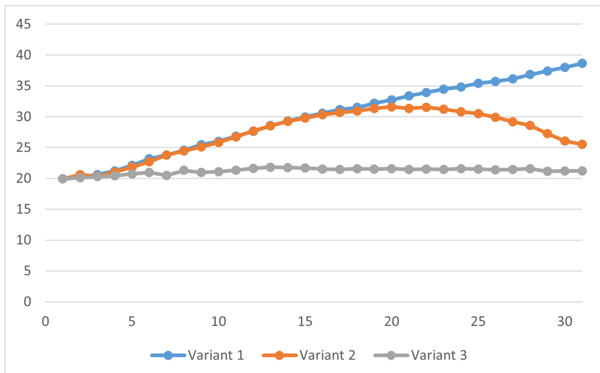
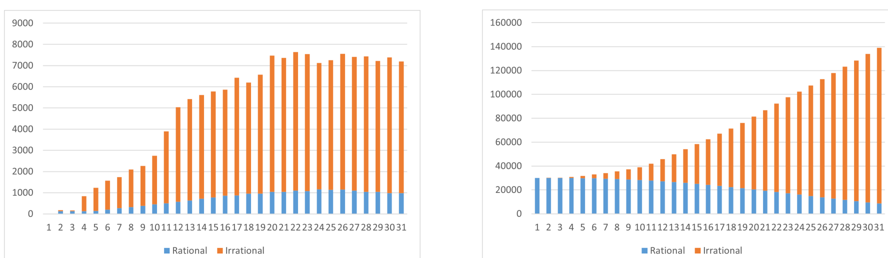


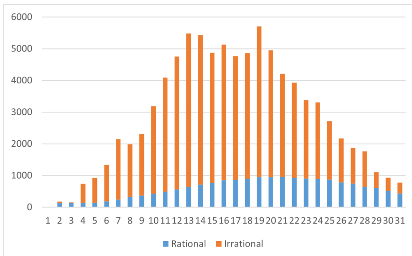
Figure 24: Price Evolution When $\gamma = 0.1$ for 3 Variants



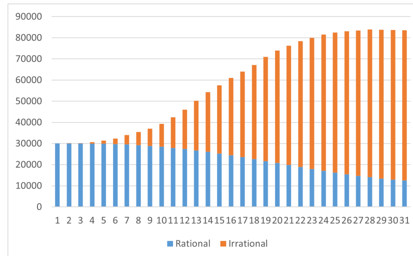
(a) Trading Volume

(b) Total Volume

Figure 25: Volume When $\gamma = 0.1$ for Variant 1

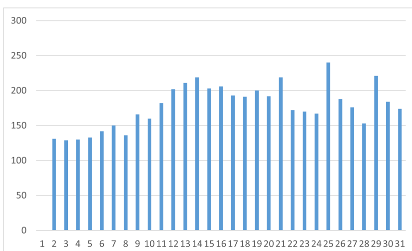


(a) Trading Volume

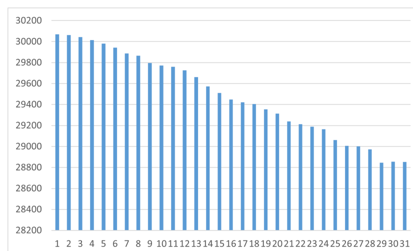


(b) Total Volume

Figure 26: Volume When $\gamma = 0.1$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 27: Volume When $\gamma = 0.1$ for Variant 3

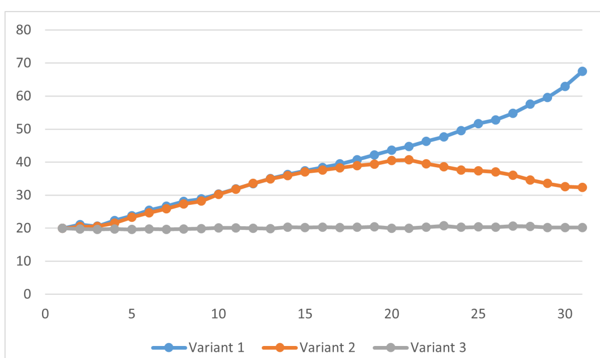
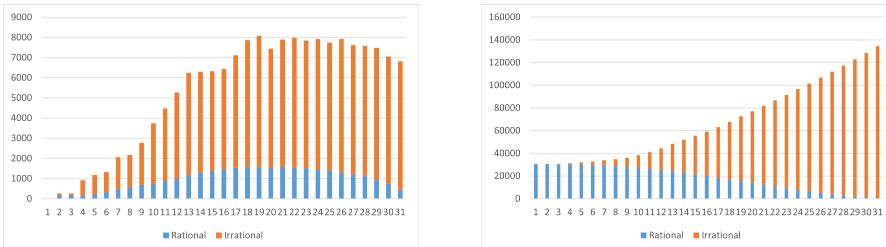
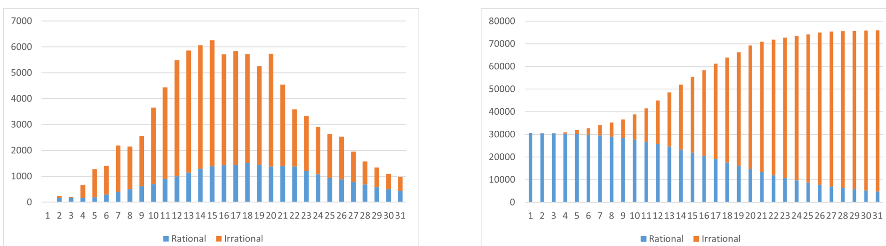


Figure 28: Price Evolution When $\gamma = 0.2$ for 3 Variants



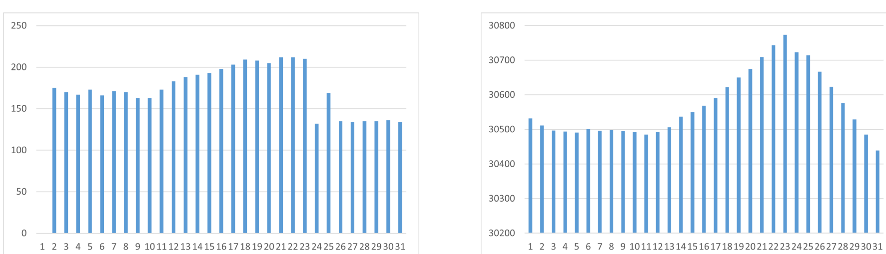
(a) Trading Volume (b) Total Volume

Figure 29: Volume When $\gamma = 0.2$ for Variant 1



(a) Trading Volume (b) Total Volume

Figure 30: Volume When $\gamma = 0.2$ for Variant 2



(a) Trading Volume (b) Total Volume

Figure 31: Volume When $\gamma = 0.2$ for Variant 3

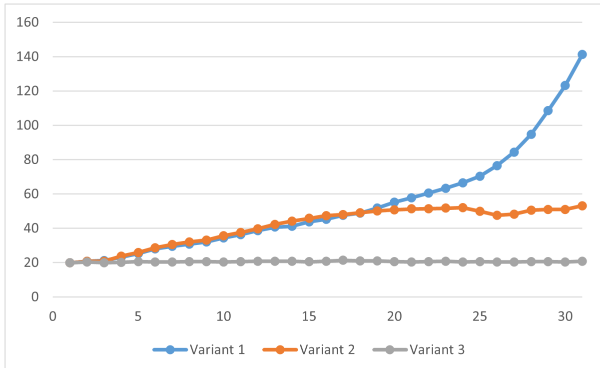
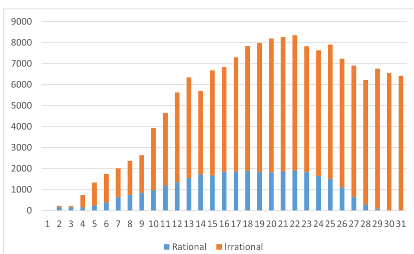
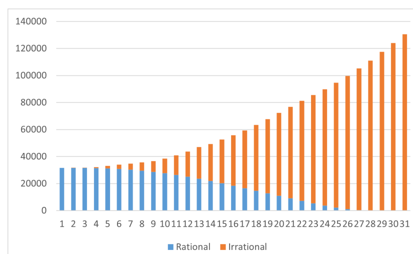


Figure 32: Price Evolution When $\gamma = 0.3$ for 3 Variants

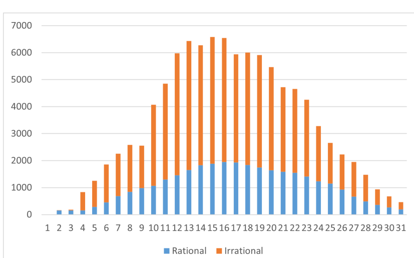


(a) Trading Volume

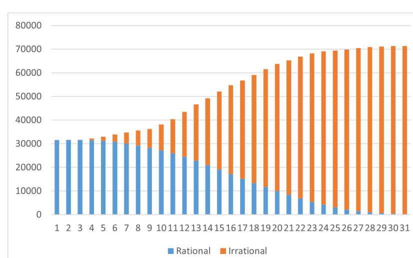


(b) Total Volume

Figure 33: Volume When $\gamma = 0.3$ for Variant 1

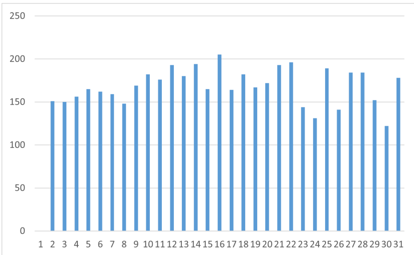


(a) Trading Volume

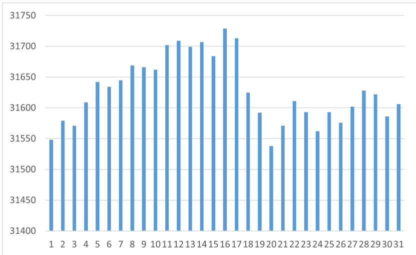


(b) Total Volume

Figure 34: Volume When $\gamma = 0.3$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 35: Volume When $\gamma = 0.3$ for Variant 3

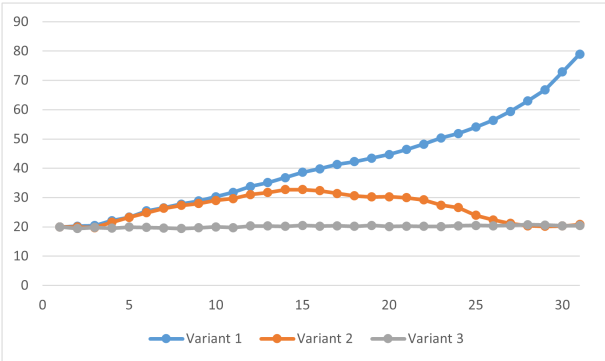
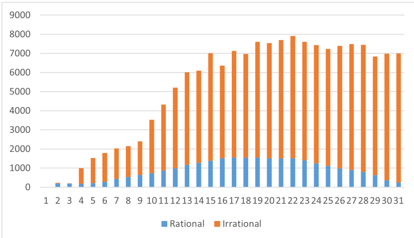
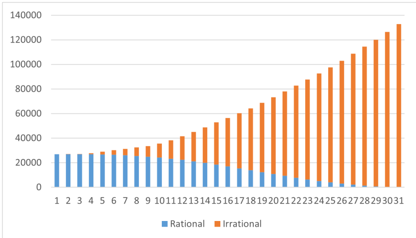


Figure 36: Price Evolution When $z_{max} = 200$ for 3 Variants

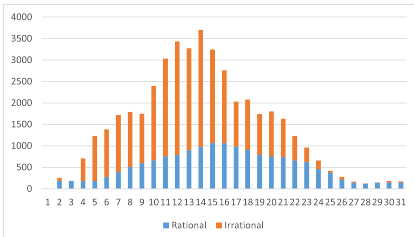


(a) Trading Volume

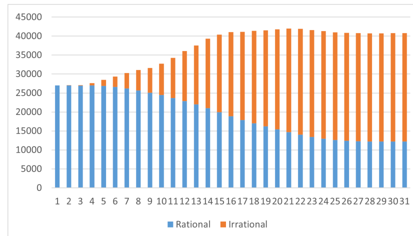


(b) Total Volume

Figure 37: Volume When $z_{max} = 200$ for Variant 1

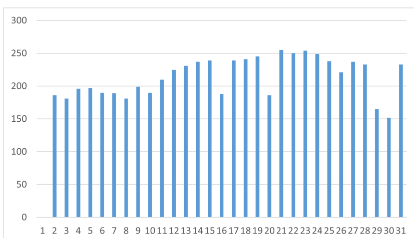


(a) Trading Volume

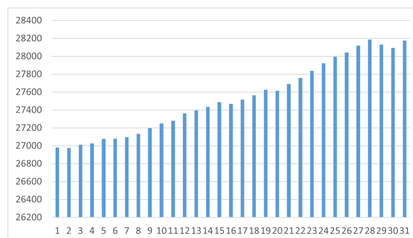


(b) Total Volume

Figure 38: Volume When $z_{max} = 200$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 39: Volume When $z_{max} = 200$ for Variant 3

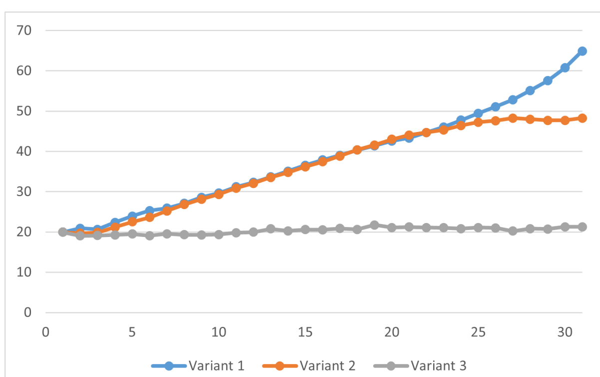
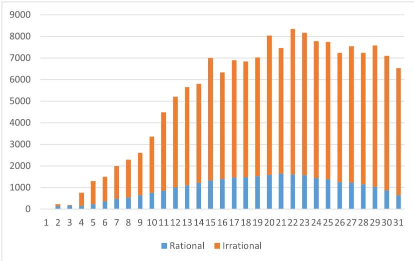
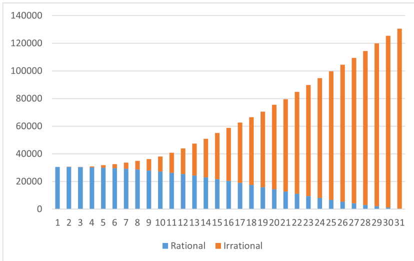


Figure 40: Price Evolution When $z_{max} = 800$ for 3 Variants

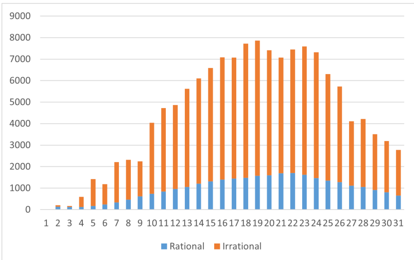


(a) Trading Volume

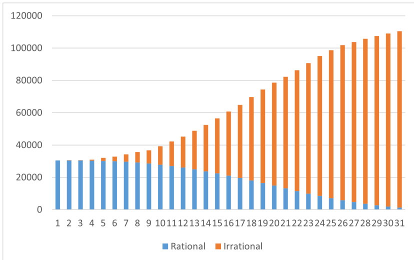


(b) Total Volume

Figure 41: Volume When $z_{max} = 800$ for Variant 1

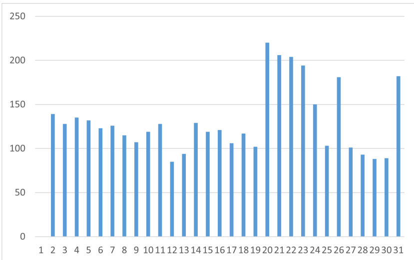


(a) Trading Volume

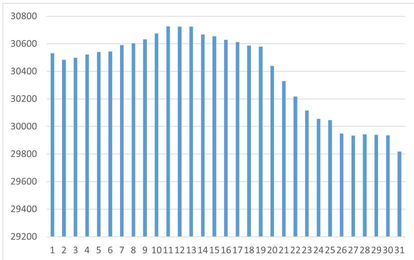


(b) Total Volume

Figure 42: Volume When $z_{max} = 800$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 43: Volume When $z_{max} = 800$ for Variant 3

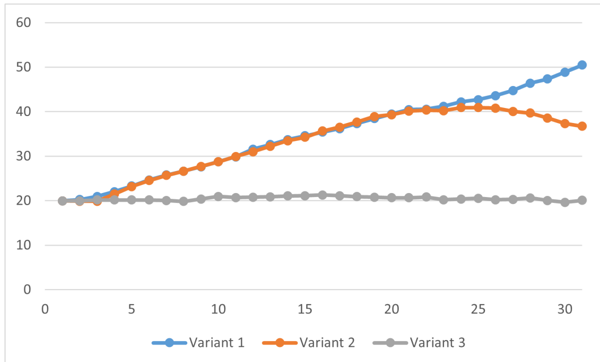
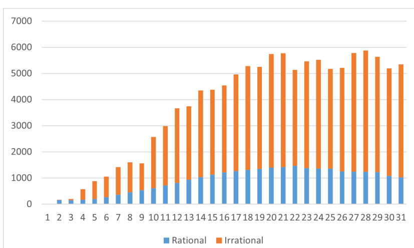
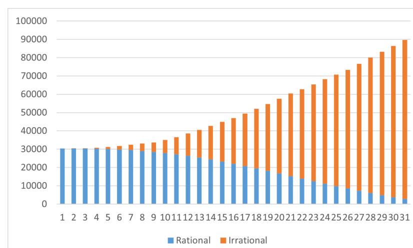


Figure 44: Price Evolution When $\Delta z_{max} = 60$ for 3 Variants

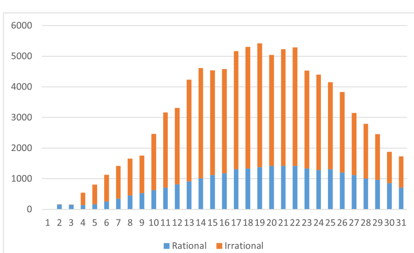


(a) Trading Volume

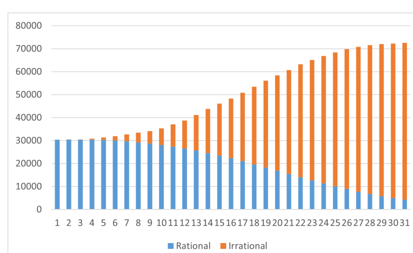


(b) Total Volume

Figure 45: Volume When $\Delta z_{max} = 60$ for Variant 1

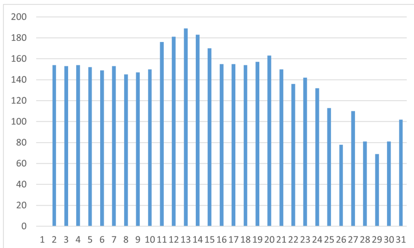


(a) Trading Volume

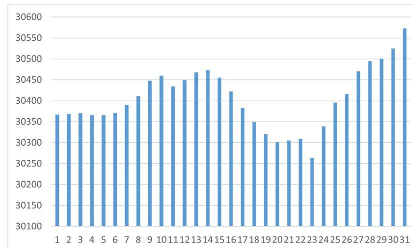


(b) Total Volume

Figure 46: Volume When $\Delta z_{max} = 60$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 47: Volume When $\Delta z_{max} = 60$ for Variant 3

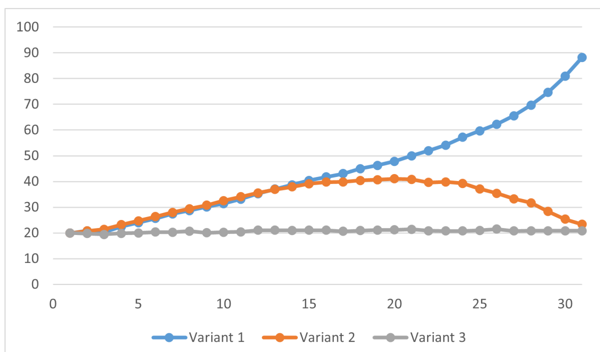
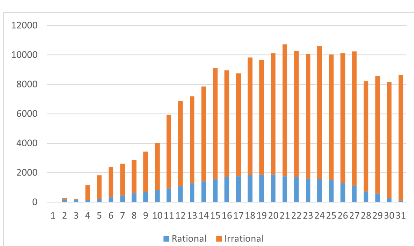
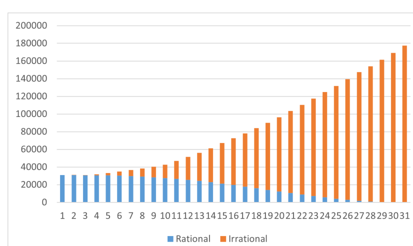


Figure 48: Price Evolution When $\Delta z_{max} = 120$ for 3 Variants

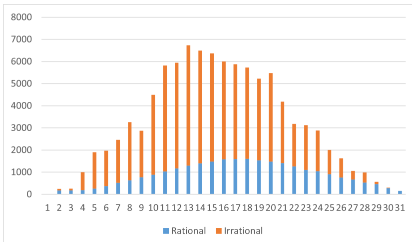


(a) Trading Volume

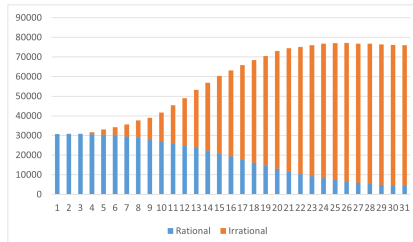


(b) Total Volume

Figure 49: Volume When $\Delta z_{max} = 120$ for Variant 1

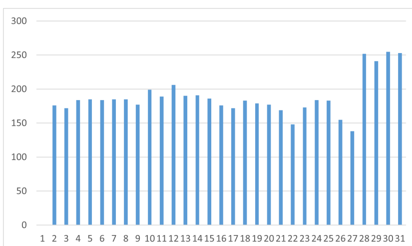


(a) Trading Volume

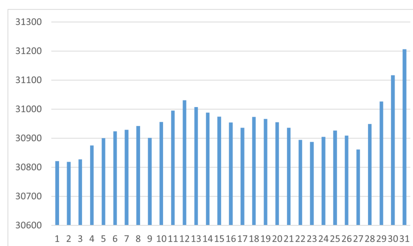


(b) Total Volume

Figure 50: Volume When $\Delta z_{max} = 120$ for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 51: Volume When $\Delta z_{max} = 120$ for Variant 3

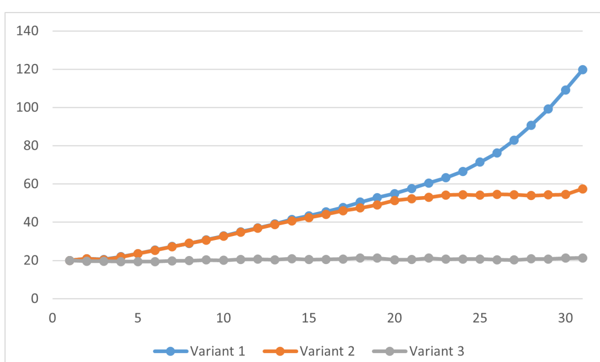
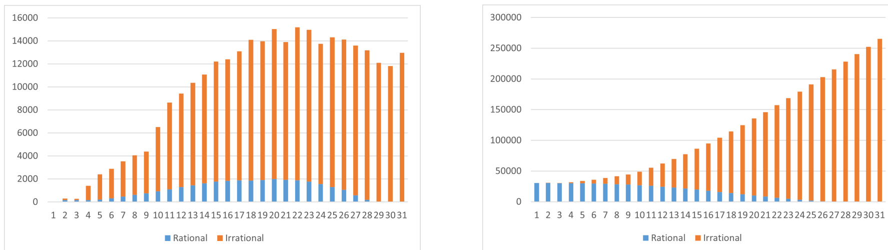
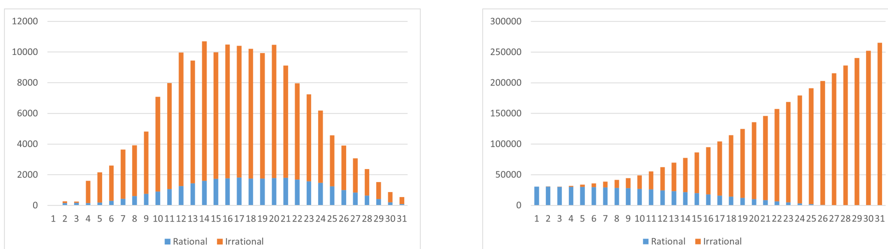


Figure 52: Price Evolution in S Case 2 for 3 Variants



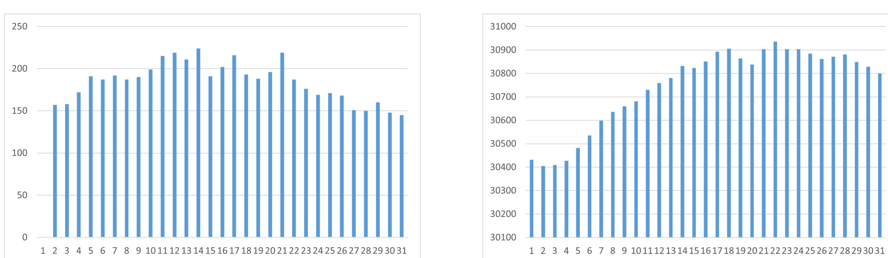
(a) Trading Volume (b) Total Volume

Figure 53: Volume in S Case 2 for Variant 1



(a) Trading Volume (b) Total Volume

Figure 54: Volume in S Case 2 for Variant 2



(a) Trading Volume (b) Total Volume

Figure 55: Volume in S Case 2 for Variant 3

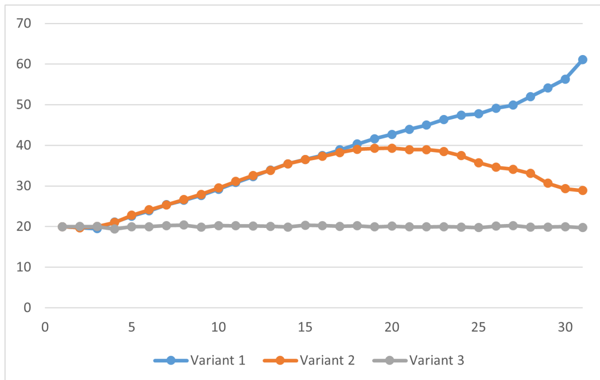
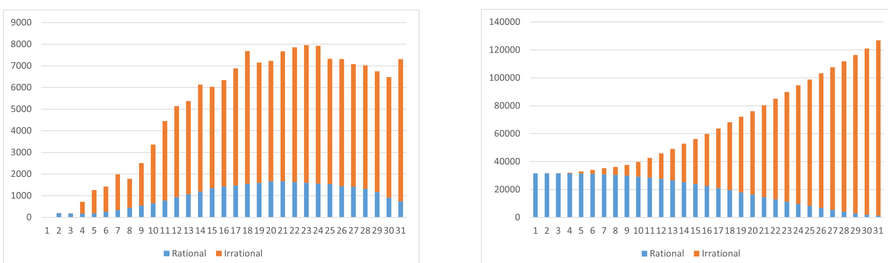


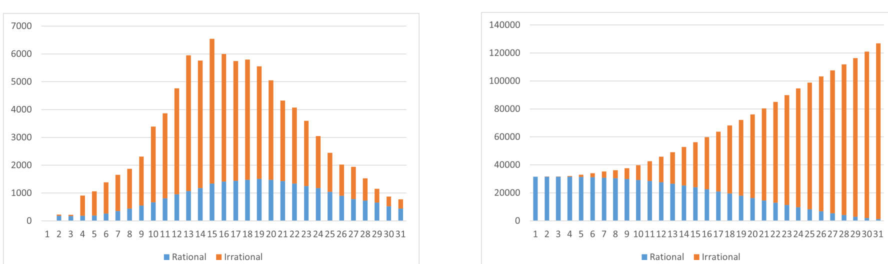
Figure 56: Price Evolution in S Case 3 for 3 Variants



(a) Trading Volume

(b) Total Volume

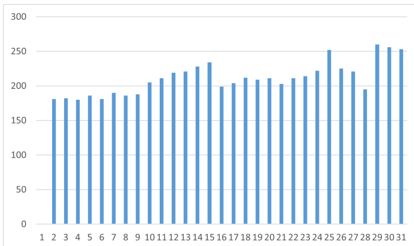
Figure 57: Volume in S Case 3 for Variant 1



(a) Trading Volume

(b) Total Volume

Figure 58: Volume in S Case 3 for Variant 2



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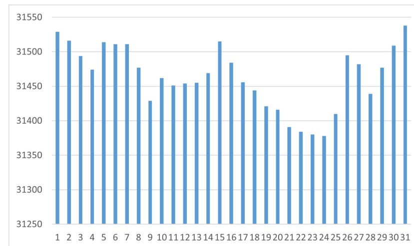


(b) Total Volume

Figure 59: Volume in S Case 3 for Variant 3

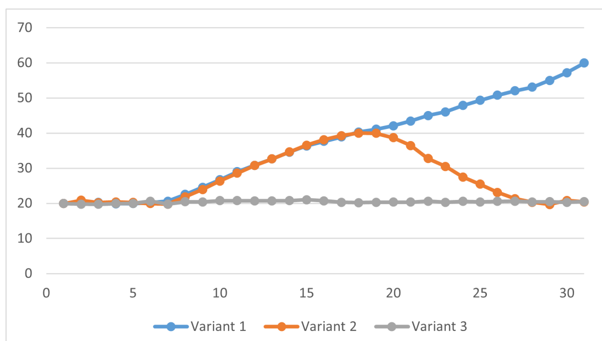
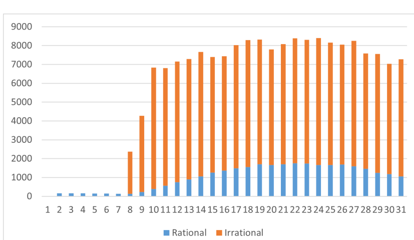
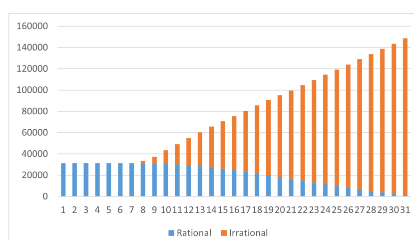


Figure 60: Price Evolution in S Case 4 for 3 Variants

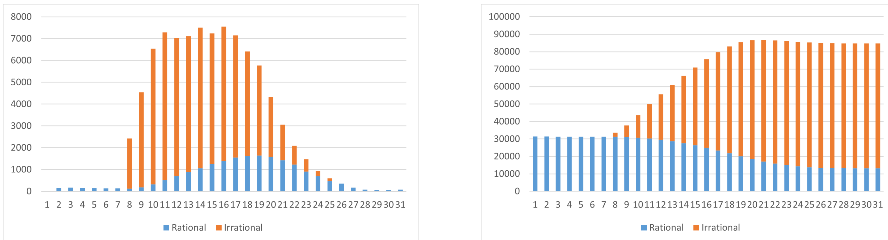


(a) Trading Volume



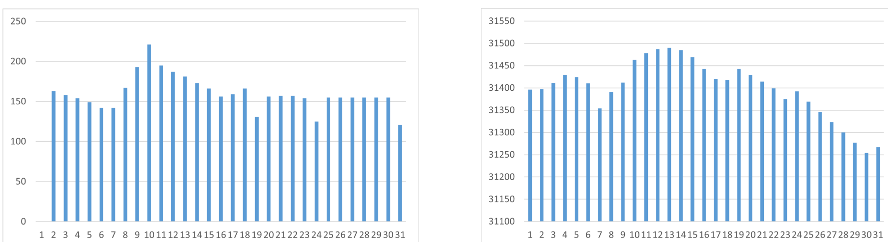
(b) Total Volume

Figure 61: Volume in S Case 4 for Variant 1



(a) Trading Volume (b) Total Volume

Figure 62: Volume in S Case 4 for Variant 2



(a) Trading Volume (b) Total Volume

Figure 63: Volume in S Case 4 for Variant 3

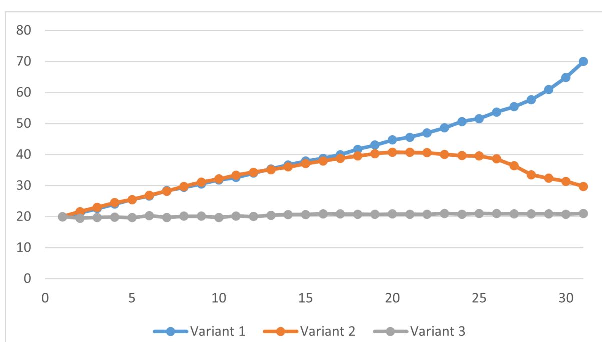
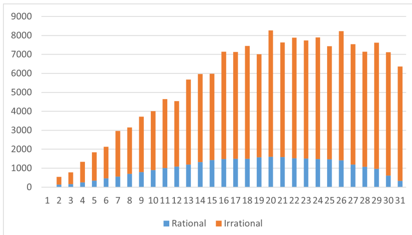
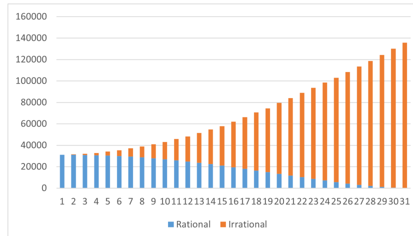


Figure 64: Price Evolution in S Case 5 for 3 Variants

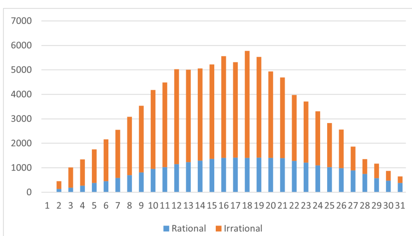


(a) Trading Volume

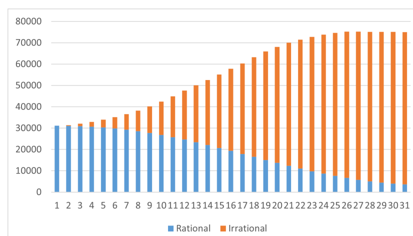


(b) Total Volume

Figure 65: Volume in S Case 5 for Variant 1

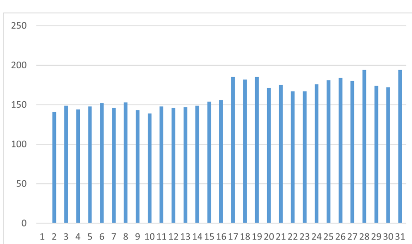


(a) Trading Volume

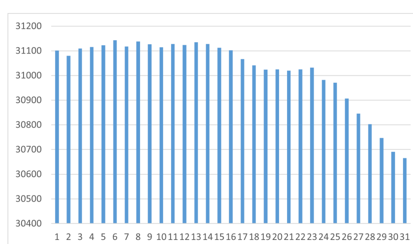


(b) Total Volume

Figure 66: Volume in S Case 5 for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 67: Volume in S Case 5 for Variant 3

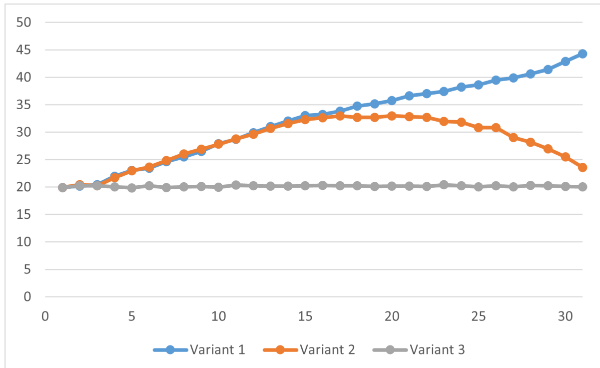
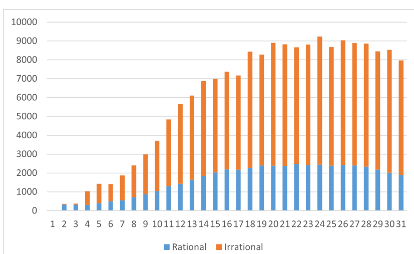
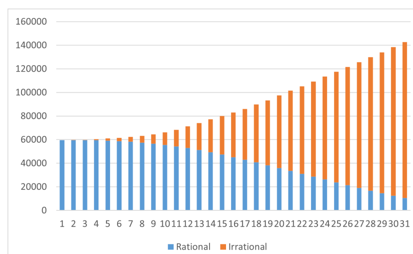


Figure 68: Price Evolution in S Case 6 for 3 Variants

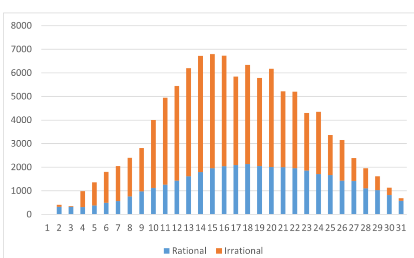


(a) Trading Volume

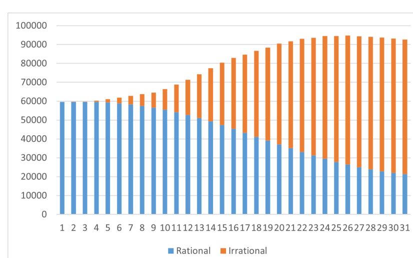


(b) Total Volume

Figure 69: Volume in S Case 6 for Variant 1

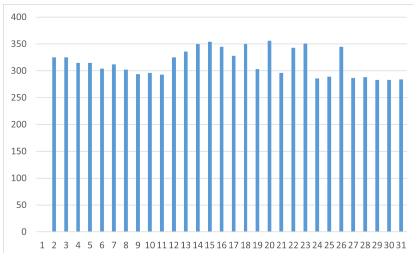


(a) Trading Volume

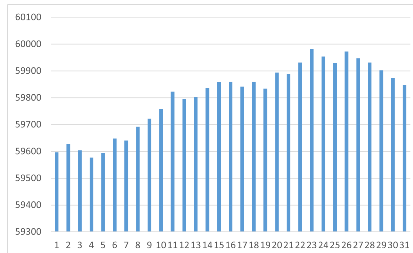


(b) Total Volume

Figure 70: Volume in S Case 6 for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 71: Volume in S Case 6 for Variant 3

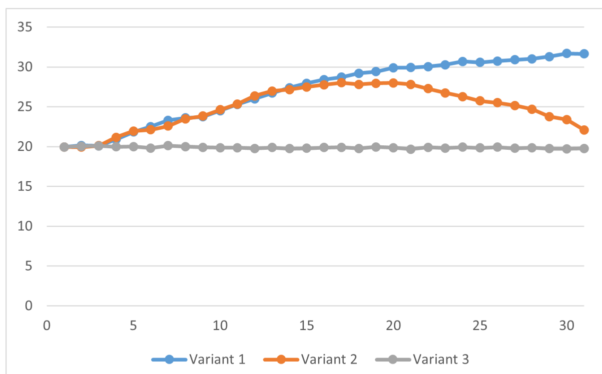
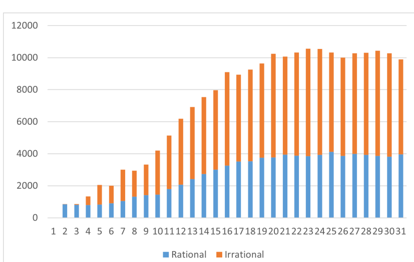
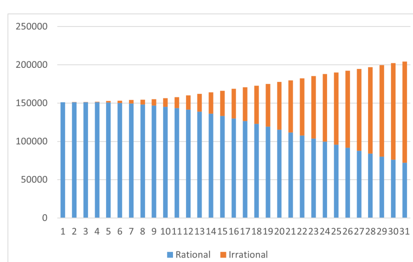


Figure 72: Price Evolution in S Case 7 for 3 Variants

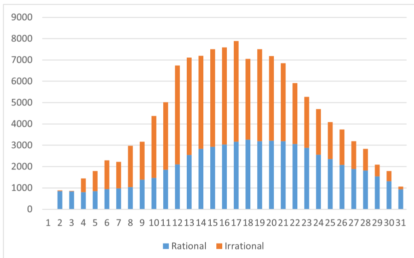


(a) Trading Volume

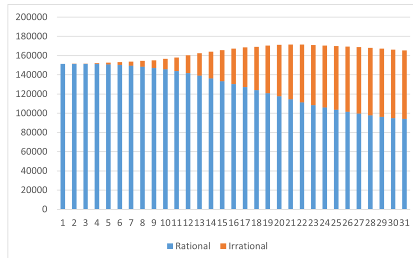


(b) Total Volume

Figure 73: Volume in S Case 7 for Variant 1

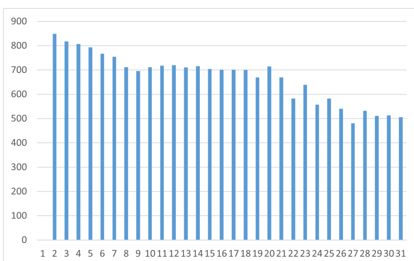


(a) Trading Volume

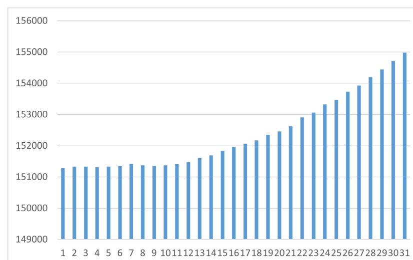


(b) Total Volume

Figure 74: Volume in S Case 7 for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 75: Volume in S Case 7 for Variant 3

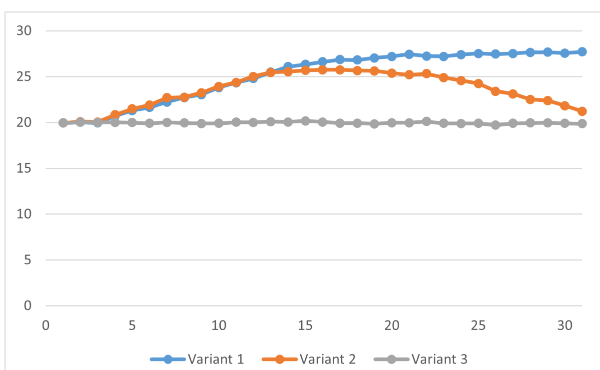
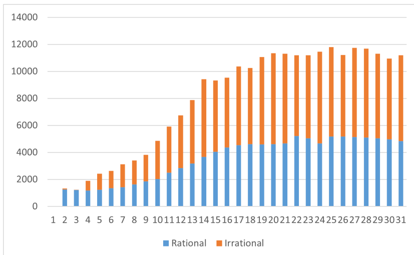
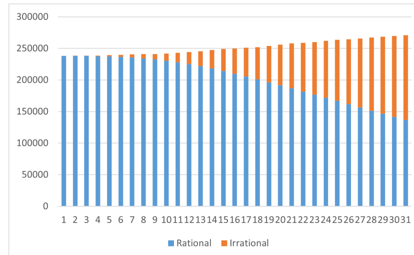


Figure 76: Price Evolution in S Case 8 for 3 Variants

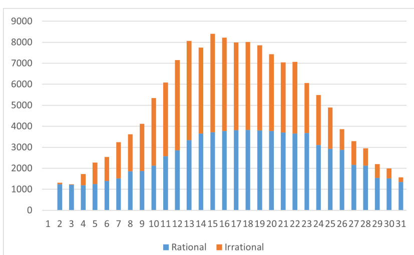


(a) Trading Volume

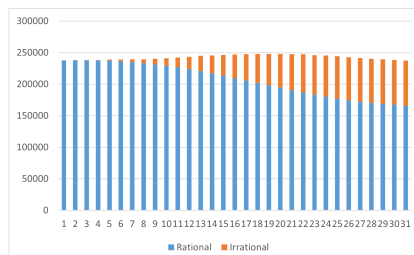


(b) Total Volume

Figure 77: Volume in S Case 8 for Variant 1

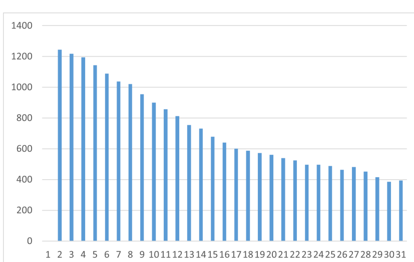


(a) Trading Volume

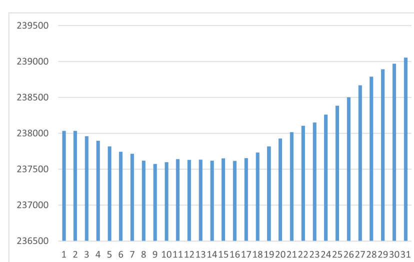


(b) Total Volume

Figure 78: Volume in S Case 8 for Variant 2



(a) Trading Volume



(b) Total Volume

Figure 79: Volume in S Case 8 for Variant 3