

Parametri Iniziali

Input: $x = [x_1, x_2, x_3, x_4] = [10, 30, 30, 50]$

Hidden layer weights:
 $w_1=0.1, w_2=0.2, w_3=0.1, w_4=0.3$ (per z_1)
 $w_5=0.2, w_6=0.1, w_7=0.2, w_8=0.2$ (per z_2)

Hidden layer biases:
 $b_1=0.1, b_2=0.2$

Output layer weights:
 $w_9=0.3, w_{10}=0.2$

Output bias:
 $b_3=0.1$

Target:
 $y_{target} = 1$

Learning rate:
 $\eta = 0.01$

1 Forward Pass

$$z_1 = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3 + w_4 \cdot x_4 + b_1 = 25.1$$
$$z_2 = w_5 \cdot x_1 + w_6 \cdot x_2 + w_7 \cdot x_3 + w_8 \cdot x_4 + b_2 = 21.2$$

$$a_1 = \text{ReLU}(z_1) = 25.1$$
$$a_2 = \text{ReLU}(z_2) = 21.2$$

Output:
$$\text{output} = w_9 \cdot a_1 + w_{10} \cdot a_2 + b_3 = 11.87$$

2 Calcolo Errore

$$E = \frac{1}{2} \cdot (\text{output} - y_{target})^2$$
$$E = \frac{1}{2} \cdot (11.87 - 1)^2 \approx 59.285$$

3 Retropropagation - Delta Output

$$\delta_{\text{output}} = \text{output} - y_{target} = 11.87 - 1 = 10.87$$

4 Aggiornamento Pesi Output

$$w_{9_new} = w_9 - \eta \cdot \delta_{\text{output}} \cdot a_1 \approx -2.43$$
$$w_{10_new} = w_{10} - \eta \cdot \delta_{\text{output}} \cdot a_2 \approx -2.10$$
$$b_{3_new} = b_3 - \eta \cdot \delta_{\text{output}} \approx -0.0087$$

5 Delta Layer Nascosto

$$\delta_1 = \delta_{\text{output}} \cdot w_9 \cdot \text{ReLU}'(z_1) = 10.87 \cdot 0.3 \cdot 1 \approx 3.261$$
$$\delta_2 = \delta_{\text{output}} \cdot w_{10} \cdot \text{ReLU}'(z_2) = 10.87 \cdot 0.2 \cdot 1 \approx 2.174$$

6 Aggiornamento Pesi Layer Nascosto

Per z_1 :
$$w_{1_new} = 0.1 - 0.01 \cdot 3.261 \cdot 10 \approx -0.2261$$
$$w_{2_new} = 0.2 - 0.01 \cdot 3.261 \cdot 30 \approx -0.7783$$
$$w_{3_new} = 0.1 - 0.01 \cdot 3.261 \cdot 30 \approx -0.8783$$
$$w_{4_new} = 0.3 - 0.01 \cdot 3.261 \cdot 50 \approx -1.3305$$
$$b_{1_new} = 0.1 - 0.01 \cdot 3.261 \approx 0.0674$$

Per z_2 :
$$w_{5_new} = 0.2 - 0.01 \cdot 2.174 \cdot 10 \approx -0.0174$$
$$w_{6_new} = 0.1 - 0.01 \cdot 2.174 \cdot 30 \approx -0.5522$$
$$w_{7_new} = 0.2 - 0.01 \cdot 2.174 \cdot 30 \approx -0.4522$$
$$w_{8_new} = 0.2 - 0.01 \cdot 2.174 \cdot 50 \approx -0.887$$
$$b_{2_new} = 0.2 - 0.01 \cdot 2.174 \approx 0.1783$$

7 Riepilogo Pesi Aggiornati

Peso	Vecchio	Nuovo
w1	0.1	-0.2261
w2	0.2	-0.7783
w3	0.1	-0.8783
w4	0.3	-1.3305
w5	0.2	-0.0174
w6	0.1	-0.5522
w7	0.2	-0.4522
w8	0.2	-0.887
w9	0.3	-2.43
w10	0.2	-2.10
b1	0.1	0.0674
b2	0.2	0.1783
b3	0.1	-0.0087