Implied Volatility and Stock Market Jumps, an Exploratory Analysis

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Abstract

We use high-frequency financial data on SPX options and on the S&P500 index to analyze the relationship between jumps and implied volatility in the options market. Specifically, we estimate jump regressions of the changes in the Black-Scholes implied volatility (BSIV) of SPX options when the S&P500 index jumps. To do so, we use jump identification techniques and the jump regression idea recently developed in [Li et al., 2017]. The estimated regressions indicate a negative correlation between jumps in the market index and changes in the BSIV. Furthermore, this relationship is stable over different moneyness categories, and is stronger for out of the money options. These empirical results contradict the assumptions of well known option pricing models.

1 Introduction

Stock market indices exhibit jumps in their dynamics. These jumps are important risk factors when determining the price of options on the market index. This importance is confirmed by the implied volatility smile observed in the data, and by the expensiveness of out of the money options. Therefore, it is relevant to understand the impact of market index jumps on the price of options.

To understand the impact of jumps in the options market, we use recent developments in high-frequency financial econometrics theory to estimate the relationship between the Black-Scholes implied volatility (BSIV) of options and jumps in the market index. More specifically, through the use of high-frequency financial data, we identify jumps in the market index, and estimate jump regressions of the BSIV on the jumps in the market index. These jump regressions illuminate the relationship of market index jumps and option implied volatilities, and allow for the comparison between what is empirically observed and the implications of option pricing models.

The use of the Black-Scholes implied volatility of options instead of their actual prices is due to the market practice of quoting and trading options based on the implied volatility. [Carr and Wu, 2016] best explain this common practice:

> (...) institutional investors manage their volatility views and exchange their quotes not through option prices, but through the option implied volatility computed from the Black-Merton-Scholes (BMS) model. This common practice does not mean that investors agree with the assumptions made by Black and Scholes (1973) and Merton (1973); rather, they use the BMS model as a transformation to enhance quote stability and to highlight the information in the option contract.

The BSIV is directly related to how much volatility investors are expecting for the underlying asset of the price. And, under the put-call parity, the BSIV is the same for both puts and calls. As so, the BSIV is a convenient way to characterize option prices.

The put-call parity also implies that we can focus the analysis solely on put options, since they contain the same implied volatility information of call options. Additionally, put options are more frequently traded than call options, leading to more accurate mid-quote prices.

The objective of this work is to analyze the BSIV of options at jump times. It is important to analyze what other models imply about the behavior of the BSIV when the underlying asset jumps. Specifically, given an option pricing model, we can compute option prices, and from these prices we obtain the Black-Scholes implied volatility. We can then analyze what changes in the BSIV when the underlying asset price jumps.

An effective way of implementing this analysis is to compute the Black-Scholes implied volatility curve for a given model, and analyzing what changes in the curve after a jump in the underlying price. The usual BSIV curve shows a skew when compared to the curve for the Black-Scholes model. This skew happens in models where the distribution of the underlying price at expiration has heavier tails compared to the normal distribution in the Black-Scholes model, leading to relatively higher prices for out of the money options.

Consider the option pricing model in [Merton, 1976]. In Merton's model, the underlying asset's price dynamics are given by:

$$\underbrace{\frac{dS_t}{S_t}}_{\text{change in price}} = \mu dt + \sigma dB_t + \underbrace{(y_t - 1)dN_t}_{\text{change due to jump}}$$

This model is a natural extension of the model in [Black and Scholes, 1973]. Merton's model includes the usual diffusive component from the Black-Scholes model, but also introduces a component for jumps. Notice that the model has a constant diffusive volatility, and that jumps only affect the underlying asset's price, but not the diffusive volatility.

The jump component changes the distribution of the underlying price at expiration, leading to heavier tails and a skew in the BSIV. An illustrative simulation of the BSIV curve under Merton's model is depicted in Figure 1.

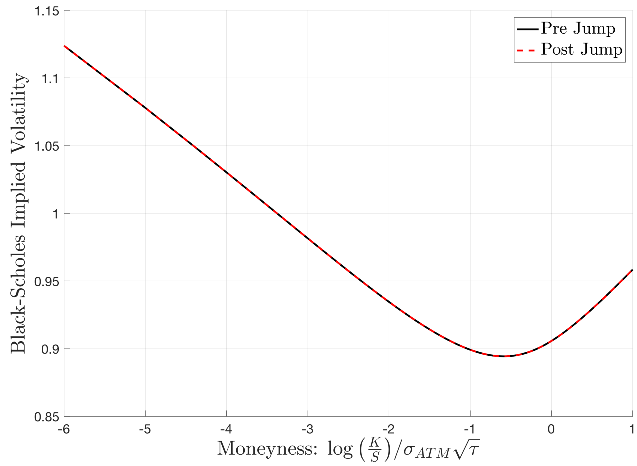


Figure 1: Simulated Black-Scholes Implied Volatility Curve under Merton's Model; Before a Jump in the Underlying (black) and After a Jump in the Underlying (red, dashed).

The black curve is the simulated BSIV under Merton's model, for different moneyness levels (moneyness is discussed in Section 2.2).

Consider a put option with a negative moneyness m, say $m = -1$. Under the simulation parameters used in Figure 1, this option has an implied volatility of about 0.9. If there is a positive jump in the underlying price, then this put option becomes more out of the money, that is, m becomes more negative. This leads to an increase in the implied volatility of this option, or a move along the BSIV curve.

Now, instead of analyzing the change in BSIV by considering a single specific option, let's focus on a given moneyness. That is, given a moneyness of $m = -1$, for example, what would happen to the BSIV at that moneyness level when there is a positive jump in the underlying price? To answer this question we first compute the BSIV for the option with moneyness -1, and then we recompute the BSIV for the same moneyness after the jump in the underlying price. It is important to notice that when there is a jump in the underlying price S, all put options have a mechanical shift in their moneyness. And when we recompute the BSIV for the same moneyness, we are not necessarily using the same option as before, since that option's moneyness is not equal to -1 anymore.

Under Merton's model, a jump in the underlying price does not cause a change in the BSIV for a given moneyness. That is, given a moneyness m, the BSIV after the jump is exactly the same as the BSIV before the jump. This happens because the jump only impacts the underlying price, but not the diffusive volatility. This way, as S increases, for example, the moneyness of every option decreases. And because the BSIV ultimately only depends on the strike-to-underlying ratio, which is fixed when we fix the moneyness, we get the same BSIV.

The red dashed curve in Figure 1 is the simulated BSIV curve after a positive jump in the underlying price. As expected, the curve did not change because of the jump, implying that for each moneyness we observe the same implied volatility from the put options.

The implication from Merton's model is that jumps in the underlying asset should not affect the BSIV of options. However, when we move on to the data, specifically options on the S&P500 index, we observe that jumps in the underlying asset price are negatively correlated with changes in the BSIV. Not only the correlation is negative, but it is also persistent across different moneyness categories, and its effect is stronger for out of the money options. This indicates that there is a disconnect between option pricing models and the empirical evidence regarding jumps.

A possible explanation for the negative correlation between the BSIV and jumps is the leverage effect. The leverage effect was first identified by [Black, 1976] and [Christie, 1982], and refers to the empirical findings that asset prices and their volatilities are negatively correlated. If we consider the leverage effect, then a positive jump in the asset price would be accompanied by a decrease in the diffusive volatility. Under Merton's model, a jump in S does not affect the diffusive volatility, but if it did, then the decrease in the volatility would make all put options less valuable. That is, for any given moneyness, the put options at that moneyness are less valuable. This would lead to a downward shift in the BSIV curve, and would reconcile the option pricing model with the negative correlation observed in the options data.

The remaining of the paper is divided as follows. Section 2 discusses the theory used in this work. Specifically, subsection 2.1 reviews the jump separation theory, which identifies the jump times for asset prices. Subsection 2.2 defines the moneyness measure used to categorize the options data. And subsection 2.3 briefly reviews the jump regression framework and applies it to the options context. Section 3 discusses the data sources for the SPX options and the S&P500 index. Subsection 3.1 identifies the jump times and filters the data. Subsection 3.2 analyzes the options data at the identified jump times. Lastly, section 4 presents the estimated jump regressions and discusses the results.

2 Theory

The theory section is divided into 3 parts: jump detection, moneyness, and jump regression. The first part discusses the theory that allows the separation of diffusive returns from jump returns, and the identification of the trading intervals in which a jump occurred. The second part discusses the moneyness measure used to aggregate different options into categories, that are then used in the jump regressions. The last part discusses the theory of jump regressions, and how it is used in the context of options.

2.1 Jump Detection

The jump detection theory is due to the works of [Mancini, 2001], [Mancini, 2009], [Jacod and Protter, 2012] and [Li et al., 2017]. The theory takes the efficient price process of a stock as a stochastic process of the form:

$$dX_t = \sqrt{c_t} dW_t + J_t \tag{1}$$

Where X_t is the logarithm of the stock price, c_t is the stochastic volatility process, W_t is a Brownian motion, J_t is a compound poisson process, and $t \in [0, T]$. Equation (1) is known as a jump diffusion process, and is a commonly used model for stock prices.

It is assumed that we observe this price process at discrete times, with equally spaced sampling, and for a fixed number of days T. At each day we observe n returns, or $n+1$ prices, which corresponds to sampling the price process every $\Delta_n \equiv \frac{1}{n}$ time interval. The discretely sampled prices are denoted by $X_{t,i}$, for $i = 0, \ldots, n$ and $t = 1, \ldots, T$. And the returns are given by $r_{t,i} \equiv X_{t,i} - X_{t,i-1}$, for $i = 1, \cdots, n$ and t as above.

Given the sampled returns, one can ask whether a return was due to diffusive or jump moves. That is, whether $r_{t,i}$ is mostly driven by the diffusive component, $\sqrt{c_t} dW_t$, or by the jump component, J_t. By identifying what returns are due to the jump component, we identify the dates and times in which a random jump occurred in the price process.

To classify whether a return is due to the jump component, the following threshold is used:

$$\text{diffusive return: } r^c_{t,i} \equiv r_{t,i} 1_{\{|r_{t,i}| \leq \alpha_n \Delta_n^{0.49} \sqrt{\text{tod}_i BV_t}\}} \tag{2}$$

$$\text{jump return: } r^d_{t,i} \equiv r_{t,i} 1_{\{|r_{t,i}| > \alpha_n \Delta_n^{0.49} \sqrt{\text{tod}_i BV_t}\}} \tag{3}$$

If the absolute value of the return is smaller than the threshold, it is classified as a diffusive return, if it surpasses the threshold, then it is deemed a jump return.

The term BV_t is the bipower variance, which is an unbiased estimator of the integrated variance $\int_0^1 c_s ds$ for a given day t. The term tod_i is the time-of-day factor, that estimates the diurnal pattern of the volatility, and is normalized to have mean one.

The threshold can be split in two terms: $\Delta_n^{0.50} \sqrt{\text{tod}_i \text{BV}_t}$ and $\alpha_n \Delta_n^{-0.01}$. The first term estimates the local volatility of the price process: BV_t estimates the total variance for the day, tod_i redistributes this variance over the day, and $\Delta_n^{0.50}$ adjusts the variance for the discrete sampling interval. The second term selects a big number α_n of the estimated local volatility that was inflated by a factor $\Delta_n^{-0.01}$. Thus, the threshold captures how much we expect the return to fluctuate in a given time interval. If it exceeds this expectation, we have an indication that the return does not originate from the diffusive part of the process.

The jump separation by thresholding assumes that there is an infill asymptotic scheme, in which the sampling frequency increases, or $\Delta_n \to 0$, so that more and more prices are sampled at each time interval. It also requires choosing $\alpha_n \to 0$ appropriately as $\Delta_n \to 0$, in order to correctly disentangle the diffusive returns from the jump returns.

Under the infill asymptotics, [Li et al., 2017] prove that the threshold in (2) and (3) correctly classifies the returns as diffusive or jump returns. More specifically, denote the set of true jump times by $\mathcal{T} \equiv \{t \in [0, nT] : \Delta J_t \neq 0\}$. This set collects the times at which the jump component of the price process is nonzero. One of the assumptions about the jump process is that it has finite activity, implying that the set $\mathcal{T}$ is finite, so we can denote its P elements by $t_p, p = 1, \ldots, P$. This set is unknown, but we can estimate it using the thresholding of returns.

Another implication of the number of jumps being finite, is that for Δ_n small enough, we will observe at most one jump at each discrete interval of size Δ_n. That means that for each jump time t_p, there is a unique index i_p such that $t_p \in ((i_p - 1)\Delta_n, i_p\Delta_n)$. We can collect these indices i_p in a set $\mathcal{I} = \{i_p\}_{p=1,\ldots,P}$. This is the set of indices for intervals in which a jump occurred.

We can also define the set of indices for intervals in which the return exceeded the jump threshold:

$$\begin{aligned}\mathcal{I}_n \equiv \{j = (t-1)n + i\Delta_n : \exists (t, i) \in [1, \ldots, T] \times \{1, \ldots, n\} \\ \text{with } |r_{t,i}| > \alpha_n \Delta_n^{0.49} \sqrt{\text{tod}_i \text{BV}_t}\}\end{aligned}$$

This set contains the jump intervals that were identified by the jump detector. And the set $\mathcal{I}_n \cap \mathcal{I}$ contains the jump intervals that were identified via the threshold, and in which a jump actually occurred. Ideally, we want $\mathcal{I}_n$ to be identical to $\mathcal{I}$. That might not be the case in finite samples, but [Li et al., 2017] prove that $\mathcal{I}_n \to \mathcal{I}$ as $\Delta_n \to 0$. Therefore, as we move to higher and higher frequency data, the detected jumps approximate more and more the true jumps.

Using the theory above, we can identify the market jumps by using data on the S&P500 index. Next, we discuss how to categorize options on the market index by moneyness.

2.2 Moneyness

The price of an option contract depends on its strike price K, on its time to maturity τ, on the underlying price S, and on the dividend yield of the underlying and the risk-free interest rate of the economy. All these parameters characterize the option contract, and so, analyzing the BSIV of an option ultimately depends on all of the mentioned parameters.

In order to analyze the BSIV of options and simplify the problem, we choose to categorize the options through a number that measures how far an option is from the money. This measure is known as moneyness:

$$m \equiv \frac{\overbrace{\log\left(\frac{K}{S}\right)}^{\text{strike-to-underlying}}}{\underbrace{\sigma_{ATM}}_{\substack{\text{BSIV for at-the-money}\\ \text{option}}} \times \sqrt{\underbrace{\tau}_{\text{tenor}}}} \tag{4}$$

The denominator is the usual strike-to-underlying ratio in logarithm. It gives a measure of how far from the money an option is. In the case of a put option, a negative ratio means that the option is out of the money, since $K < S$. However, the strike-to-underlying ratio by itself does not consider the time to maturity of the option, or the current volatility of the underlying asset. These values are taken into account in the numerator of the moneyness. Indeed, the numerator considers the time to maturity, and the BSIV of the option as an approximation of the volatility of the underlying asset.

The moneyness defined in (4) is interpreted as the number of standard deviations that the underlying asset price has to move in order for the option to be at the money. For example, if a put option has moneyness equal to -3, it means that the underlying price has to decrease by 3 standard deviations for the option to be at the money.

We can categorize options into groups of options with similar moneyness, and then use those options to estimate the BSIV for that category. And by fixing a moneyness category and analyzing the options within that category, we remove the impact of the jump in the underlying in the moneyness of options. For example, a positive jump in the underlying would automatically decrease the moneyness of all put options. If we analyzed the same options before and after the jumps, then we would be capturing two effects: the reduction in the moneyness, and the change in the underlying's volatility due to the leverage effect. However, by fixing a moneyness category and analyzing the options within that category, we eliminate the mechanical change in the moneyness when the underlying jumps, since the options within that moneyness category also change. Thus, by computing the BSIV of this new set of options, and comparing to the BSIV of the old set of options, we capture only the leverage effect.

Now that we can categorize options by moneyness, we can use the identified jump times to estimate the BSIV before and after the jumps. The difference between these values is analyzed in a regression with the size of the jumps in the underlying. This topic is discussed in next subsection.

2.3 Jump Regression

The jump regression discussed here is based on [Li et al., 2017]. In their work, the authors study the dependence of a univariate process Y on the jumps of another univariate process Z. They assume that $X \equiv (Z, Y)^T$ follows an Itô semimartingale, like the jump diffusion in Section 2.1. Under this and other assumptions, they arrive at the following expression:

$$\Delta Y_{t_p} = \beta^J_{t_p} \Delta Z_{t_p} \text{ for all jump times } t_p \text{ of Z}$$

Where $\beta^J_{t_p}$ is the jump beta of Y for moves in Z, and $\Delta Y_t \equiv Y_t - Y_{t-}$. This expression defines the jump beta as a stochastic process that varies across each jump time. And without any other restriction, is a definition without empirical content. However, under the assumption that the jump beta is a constant, $\beta^J_{t_p} = \beta$ for all t_p, we obtain a linear relationship between the processes Y and Z:

$$\Delta Y_{t_p} = \beta \Delta Z_{t_p} \text{ for all jump times } t_p \text{ of Z}$$

This expression asserts that the dependence of Y on the jumps in Z is linear and exact. However, it can fail to hold, for example, if the true relationship is nonlinear, or if the slope varies with time. Nonetheless, it provides a base scenario to analyze the jump dependence.

In the context of options, Z is the underlying asset, or the S&P500 index, and Y is the Black-Scholes Implied Volatility (BSIV). And under the linear jump beta assumption, the following linear model is implied:

$$\Delta \log BSIV_t = \beta \Delta \log S_t + \Delta \epsilon_t, \quad \Delta \log S_t \Delta \epsilon_t = 0 \text{ and } t \in [0, T]$$

Where S_t represents the $S\&P500$ index. The jump beta in the model can be estimated by least squares:

$$\hat{\beta} = \frac{\sum_{p=1}^{P_n} \Delta^n_{t_p} \log BSIV \Delta^n_{t_p} \log S}{\sum_{p=1}^{P_n} \Delta^n_{t_p} \log S^2}$$

Where $\Delta^n_i X \equiv X_{i\Delta_n} - X_{(i-1)\Delta_n}$.

We now have all the necessary tools to analyze the impact of jumps in the S&P500 on the BSIV of SPX options. We can identify the jump intervals of S&P500, categorize the SPX options by moneyness, compute the average BSIV for each category, and estimate the jump regressions.

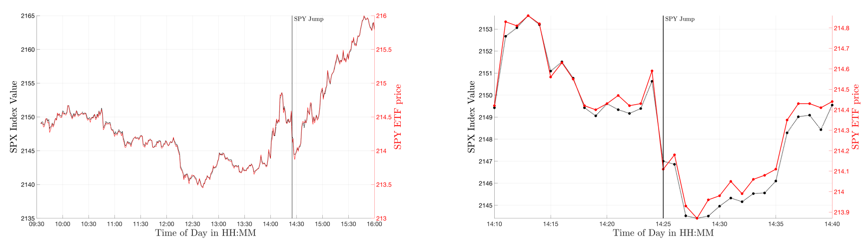
In the next section, we discuss the stock and options data used in this project, present the data sources and summary statistics, detect, filter and analyze the stock market jumps, and evaluate the options data at the stock market jump times.

3 Data

There are two main components in the jump regressions: the change in the BSIV of an option, and the change in the price of the underlying asset at a jump time. In this work, these two components are: SPX options, and the SPY exchange traded fund (ETF).

SPX options are European options in which the underlying asset is the S&P 500 index. The S&P 500 index is based on the 500 largest companies listed in the U.S., and spans many different industry groups. It is a theoretical index that represents the U.S. stock market, but it is not directly traded in any exchange. However, there are funds that mimic the behavior of the S&P 500 index and that are traded in exchanges. One of these funds is the SPDR S&P 500 ETF, traded under the ticker symbol SPY.

The SPY ETF was the first ETF listed in the U.S. and is one of the largest ETFs in the world. It closely tracks the S&P 500 returns, but is traded at price levels close to the S&P 500 index divided by 10. SPY also tracks the index at jump times, as illustrated by the figure below:



a) Entire Trading Day b) Zoomed in at the Jump Time

Figure 2: S&P 500 Index (black) being tracked by the SPY ETF (red) on a jump date (09/21/2016)

The Chicago Board Options Exchange (CBOE) is the only exchange where SPX options are traded, and is the source of our dataset on these options. The dataset contains 1-minute quote data on all SPX options from 2012 to 2016. In addition to the basic contract information for each option, like expiration date, strike price and option type, the dataset also records the best bid and best ask prices for each option, and the number of contracts being offered at those prices. The regular trading hours for SPX options are from 9:30am to 4:15pm EST, totaling 405 1-minute quotes per day for each option.

The dataset also records trade information. Whenever trades occur within a 1-minute interval, the prices and volumes of all trades within that minute are recorded. Specifically, the dataset records the open, close, high and low prices for each 1-minute interval where a trade occurred, and the total volume of contracts negotiated in that minute. The figure below illustrates the information available for each option:

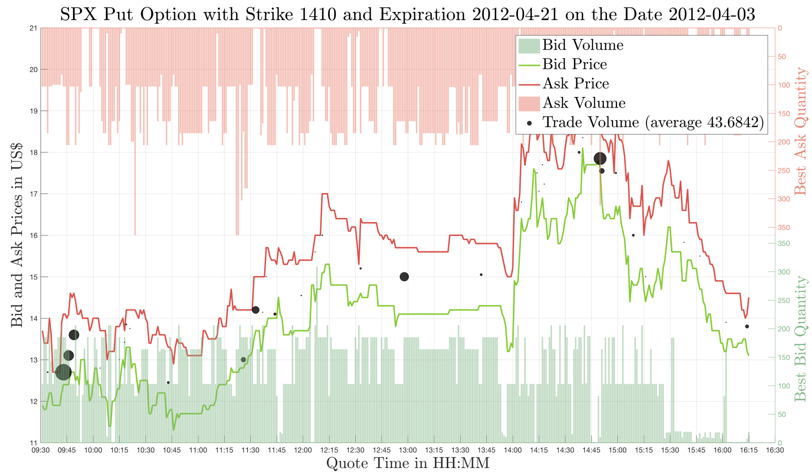


Figure 3: Quote Data for a Close to the Money Put Option

The red and green lines represent the ask and bid prices for that option throughout the day. The red and green bars represent the number of contracts being offered at the best ask and best bid prices. The black circles represent the average price of trades that occurred in a 1-minute interval, and the width of the circle represents the number of contracts traded within that interval.

The figure above illustrates that there is great interest in trading SPX options. We see that there is a big number of contracts being offered at the best bid and ask prices, and that these prices are constantly changing throughout the day. However, notice that out of the 405 1-minute intervals in a day, there are only 40 intervals in which trades occurred. This contrasts with what we observe in the stock market with liquid assets, where trades occur at much smaller intervals. This low number of trades in the options market could indicate that the quoted prices are not truly reflecting the true price of these option contracts. However, the bid and ask prices are binding, meaning that if someone is willing to pay the ask price, then the seller must oblige and sell the option contract. Therefore, even though the number of trades for SPX options is relatively small, the prices we observe in the quote data are still representative of the true market prices for SPX options. So, we can define the price options by their mid-quote prices.

The dataset on SPY was obtained from TickData.com. It contains 1-minute quote data from 2012 to 2016. The market hours for the SPY ETF are from 9:30am to 4:00pm EST. However, due to the high volatility at the opening of the market, we removed the first 5 minutes of data, leading to 386 1-minute SPY quotes per day.

In addition to data on SPX options and on the SPY, we also need data on the risk-free interest rates, and dividend yields on the stocks that constitute the

S&P 500 index. The interest rates and the dividend yields are used to compute the BSIV on SPX options. The data on risk-free interest rates was obtained from the Federal Reserve Bank of St. Louis website, and is given by the 3-Month Treasury Bill (Secondary Market) series. The data on dividend yields was obtained from Dividend.com.

3.1 Jump Detection

The jump detection theory allows us to separate jump returns from continuous returns for the SPY, identifying its jump times i_p. Using an $\alpha = 7$ in the threshold parameter, we classify returns as jump returns if they are larger than 7 local standard deviations. We obtained 51 total jumps in SPY during 2012 and 2016, averaging 10.2 jumps per year. For each jump detected, we recovered the jump date and the interval at which the jump happened. We then filtered the jumps according to whether a jump occurred at the beginning or end of the day, if the detected jump was too small, and if the more than one jump occurred at the same day.

The reason for filtering jumps that occur at the beginning or end of the day is that at those times it is hard to estimate the local volatility. This difficulty could lead to falsely identifying jumps and contaminating the regressions. The reason for filtering small jumps is that at calm periods, when the volatility is small, it is easier for a return to be identified as a jump return, since the cut threshold is very small. This could also lead to falsely identifying jumps. In the 2012-2016 sample, these two filters did not remove any of the 51 identified jumps.

The filter for dates in which more than one jump return occurred did remove 15 jump returns out of the 51 previously identified. This filter seeks to remove jumps that did not occur randomly, but because of pre-scheduled announcements, like FOMC meetings. These jump returns that arrive close to pre-scheduled announcements do not correspond to a Poisson process, in which jumps arrive at random times, and so are not considered by the jump regression framework in use. For example, two of the identified jump returns happened on 09/13/2012, one at 12:32pm and one at 12:39pm. However, on the same day, there was an FOMC announcement at 12:30pm. That is, in the 1-minute between 12:30pm and 12:31pm, no jump return was detected, but in the next 1-minute interval, a jump return happened, and then again a few minutes later. These jump returns are directly connected to the announcement by the FOMC, and, therefore, are filtered out of the sample. The table below summarizes the jumps that were filtered out:

Jump Returns		News Announcement	
Date	Times	Type	Time
2012/09/13	12:32, 12:39	FOMC Meeting	12:30
2013/09/18	14:01, 14:04	FOMC Meeting	14:00
2013/12/18	14:01, 14:02, 14:03	FOMC Meeting	14:00

2014/12/17	14:01, 14:41	FOMC Meeting	14:00
2015/12/16	14:01, 14:04	FOMC Meeting	14:00
2016/03/16	14:01, 14:02	FOMC Meeting	14:00
2016/12/20	14:49, 14:50	-	-

Table 1: Filtered Jump Dates due to Announcements

The first 13 jump returns in the table happened within minutes of the FOMC meetings. This indicates that those jumps are not random in nature, but are reactions to the FOMC announcements. And the subsequent jumps are usually corrections of the first jumps, often moving in the opposite direction of the first jump return.

The last 2 jump returns occurred in adjacent intervals, indicating that they are most likely related to some news arrival. However, none of the announcements that we checked for occurred on 2016/12/20. Nevertheless, these 2 jumps were removed from the analysis. The full list of announcements that were checked for is available in the appendix.

There are 36 jumps left after applying the filters mentioned above. The figure below plots the values of these jump returns:

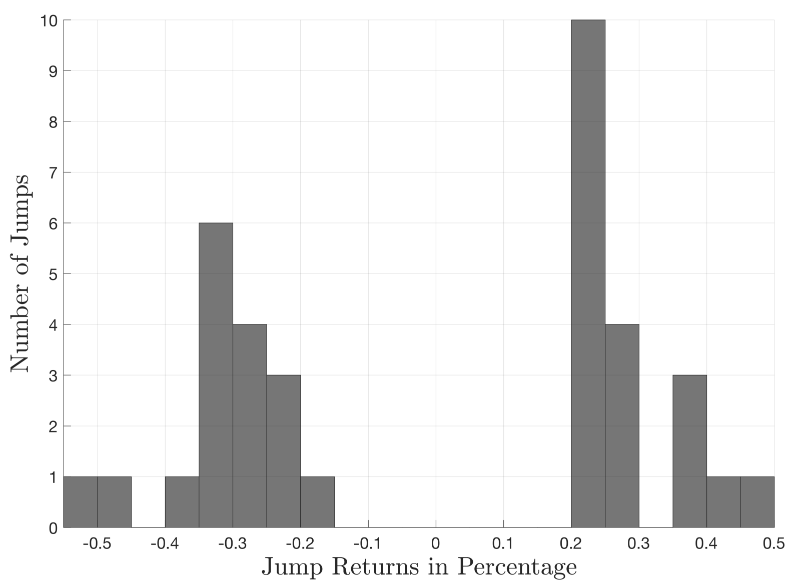


Figure 4: Histogram of the 36 Jump Returns in 2012-2016

The distribution of the jumps is approximately symmetric, with 19 positive jumps and 17 negative jumps. The biggest jump is 47 basis points (bp), and the smallest jump is -52 bp. However, most of the jumps are around ± 30 bp. Now we can analyze the options data at the identified jump dates.

3.2 Options Data at the 36 Jump Dates

The SPX options data available at the jump dates stores information on a large number of options. This large number is due to various strike prices and tenors, and because for each strike and tenor there are two option types: call and put. In this study, we only consider put options, due to their higher liquidity and the equivalence with call options (put-call parity).

In order to estimate the change in BSIV at jump times, we first aggregate this large number options into seven moneyness categories: $i \leq m_i < (i+1)$ for $i = 0, -1, \cdots -6$. A put option with moneyness in the category m_0 is an option that is slightly in the money. As the moneyness of an option moves closer and closer to the m_{-6} category, it gets more and more out of the money.

We compute the moneyness for each option at the times before and after the jump, so that they may change from one moneyness category to another after the jump. In order to compute the moneyness, we must first compute σ_{ATM}, which is the BSIV for the at the money option. To do so, we find the options that are out of the money, but as close to the money as possible, one for each tenor, and compute their BSIV. This BSIV is used to compute the moneyness of each option with the same tenor.

As with jumps, the options data also requires filtering. We remove: options with moneyness outside the moneyness categories of interest, options where the best bid and best ask quantities are too low, and options that have undetermined BSIV values.

The problem with considering very out of the money and very in the money options is that their prices are misleading due to the very low or nonexistent amount of quotes. In some cases, the bid or the ask quantities are zero, so that the quotes are one-sided. When there are only people willing to sell or only people willing to buy, then the quoted prices are unreliable, and are removed from the analysis. This is also a problem for some of the option that are in the moneyness categories we are interested in, and so are also removed from the analysis. Some of the options also violate arbitrage conditions. When it happens, their BSIV cannot be computed, as it diverges to infinity. Those options are also removed from the analysis.

The figure below summarizes the total number of put option that are available at each jump date, and how many were removed from the analysis by the aforementioned filters.

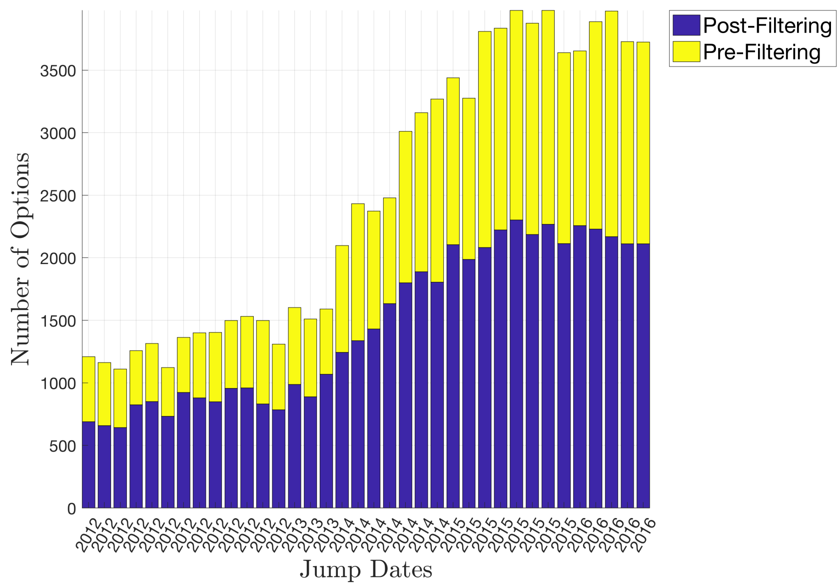


Figure 5: Number of Put Options and Put Options Filtered Out at the 36 Jump Dates

Notice in the figure the sudden increase in the total number of options at the beginning of 2014. At that date, CBOE announced the expansion of the number of weekly SPX options. An average of 40% of the total number of options were removed from the analysis. Around 15% of the options had moneyness above the category m_0, while 24% of the options had moneyness below the category m_{-6}. Also, approximately 10% of the options had low bid or ask quantities being quoted. And about 9% had undetermined BSIV.

The remaining options are used to estimate the BSIV for each moneyness category, at each jump date, before and after the jump occurs. The table below presents the average number of options and the BSIV per moneyness category:

	Moneyness Category						
	m_0	m_{-1}	m_{-2}	m_{-3}	m_{-4}	m_{-5}	m_{-6}
Number of Options	307	362	343	327	307	259	132
BSIV	0.14	0.18	0.22	0.26	0.29	0.32	0.35

Table 2: Average Number of Options and Black-Scholes Implied Volatility per Moneyness Category

Notice that as we move out of the money, less and less options are analyzed on average. This is natural, since there are less out of the money options being traded. Also notice that the BSIV increases as we move out of the money, indicating a volatility skew.

Now that we have filtered both options and stocks, and computed the changes in BSIV and the jump returns, we can move on to estimating the jump regressions, one for each moneyness category.

4 Results

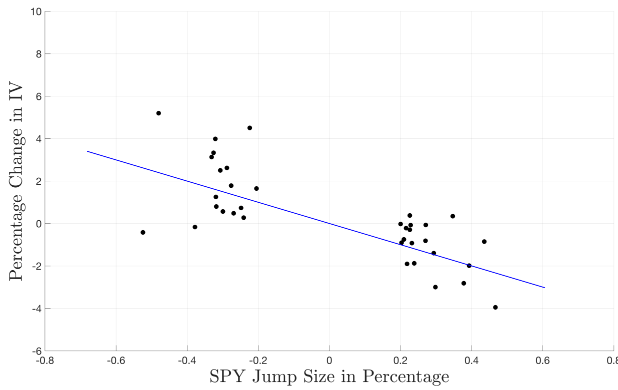
We begin with the jump regressions for each moneyness category. Figure 6 presents the changes in BSIV and the jump sizes, both in percentage, and the fitted regression lines. Table 3 summarizes the jump regression estimates for each moneyness category.

Moneyness	Jump Beta Estimate	OLS Standard Deviation	R^2
m_0	-4.9942	0.76763	0.5596
m_{-1}	-4.2642	0.43787	0.73045
m_{-2}	-4.0793	0.39622	0.75239
m_{-3}	-4.3935	0.41685	0.76825
m_{-4}	-5.0982	0.52423	0.75222
m_{-5}	-4.3011	0.77706	0.48252
m_{-6}	-7.063	0.98544	0.60686

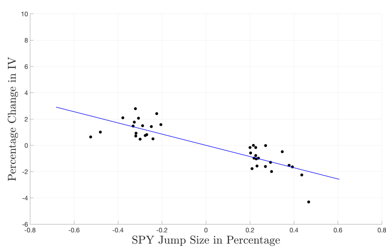
Table 3: Jump Regression Estimates

The jump regressions estimate the elasticity of the average BSIV for a moneyness category with respect to the jump size of the underlying. The estimated elasticity in all regressions is negative, so that positive jumps in the underlying leads to a decrease in the average BSIV. The estimated elasticity is surprisingly stable across different moneyness category, being roughly -4.5 for the categories $m_0, \dots, m_{-5}$, and somewhat more negative for category m_{-6}. The R^2 of the regressions are also relatively high, averaging 66% across all regressions, where the smallest R^2 is 48%, and the biggest is 76%. Also, the ordinary least squares standard (OLS) deviations are small, indicating that the estimates are relatively sharp. It is important to notice that the OLS standard deviations might not be the theoretically correct standard deviation estimator for the jump regression estimator. However, they serve as an indication of the accuracy of the estimates.

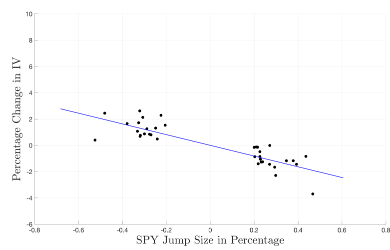
The first regression in Figure 6 is for put options that are slightly in the money (m_0). A visual inspection reveals that the relationship between jumps and change in the BSIV is negative. Furthermore, this relationship gets tighter as we move from slightly in the money options to out of the money options. In fact, the subsequent plots in Figure 6 demonstrate that the linear relationship becomes stronger for the categories m_{-1}, m_{-2}, m_{-3} and m_{-4}. However, for very



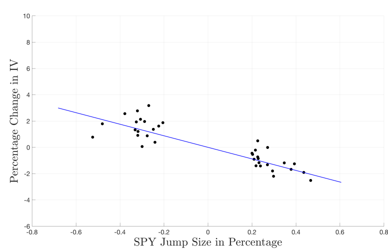
(a) m_0



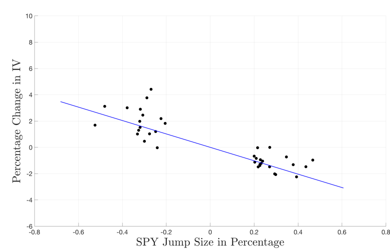
(b) m_{-1}



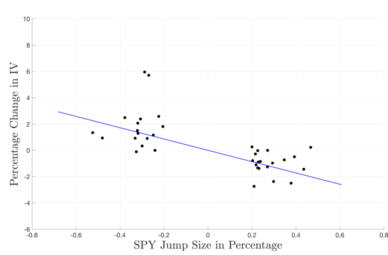
(c) m_{-2}



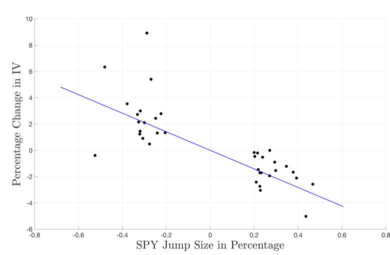
(d) m_{-3}



(e) m_{-4}



(f) m_{-5}



(g) m_{-6}

Figure 6: Jump Regressions for Each Moneyness Category

out of the money options, as in categories m_{-5} and m_{-6}, the linear relationship starts to break down.

The linearity throughout the regressions, as well as the closeness of the jump regression estimates, suggests that at jump times, the pricing of options can be well approximated by a linear model. That is, if we expect a negative jump of 50 bp in SPY, then we should also expect an increase in the BSIV of approximately $-4.5 \times (-50) = 225$ basis points. The contrary is valid for a positive jump. And given the change in the BSIV, we can recover the change in the option prices if necessary. As an example, we consider the average BSIV of options on a given day, for each moneyness category, and compute what would happen to the BSIV according to the jump regressions:

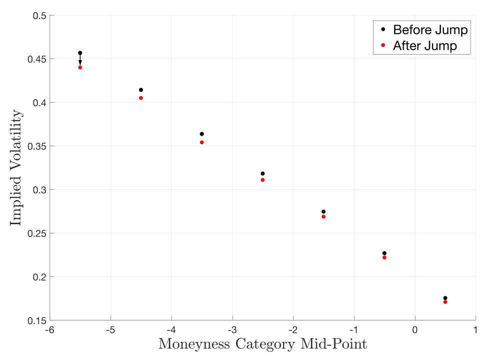


Figure 7: Change in BSIV under the Results of the Jump Regressions

The black dots represent the BSIV before the jump at a given day. Notice that there is a volatility skew. At this day a 52 bp positive jump occurred in the SPY. The jump regressions imply that there should be a decrease of approximately 232 bp in the average BSIV at each moneyness category. The red dots represent the new BSIV after the jump. Notice that the absolute change in BSIV is smaller for options closer to the money, since the starting BSIV is smaller.

We can also compare these results to existing options pricing models. According to [Merton, 1976], a jump in the underlying asset has no impact on the average BSIV of options within a fixed moneyness category. Thus, we should not see a negative relation between the change in the BSIV and the jump size. However, in the data we see that jumps are negatively correlated with the changes in BSIV. Therefore, the jump regressions present evidence against Merton's options pricing model.

Another important options pricing model in the literature was proposed by [Andersen et al., 2015]. In their three-factor double exponential model, the equity index dynamics are given by:

$$\frac{dX_t}{X_t} = (r_t - \delta_t)dt + \sqrt{V_{1,t}}dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}dW^{\mathbb{Q}}_{2,t} + \eta\sqrt{U_t}dW^{\mathbb{Q}}_{3,t} + \int_{\mathbb{R}^2}(e^x - 1)\tilde{\mu}^{\mathbb{Q}}(dt, dx, dy) \tag{5}$$

$$dV_{1,t} = \kappa_1(\bar{v}_1 - V_{1,t})dt + \sigma_1\sqrt{V_{1,t}}dB^{\mathbb{Q}}_{1,t} + \mu_1\int_{\mathbb{R}^2} x^2 1_{\{x<0\}}\mu(dt, dx, dy) \tag{6}$$

$$dV_{2,t} = \kappa_2(\bar{v}_2 - V_{2,t})dt + \sigma_2\sqrt{V_{2,t}}dB^{\mathbb{Q}}_{2,t}$$

$$dU_t = -\kappa_u U_t dt + \mu_u\int_{\mathbb{R}^2}[(1-\rho_u)x^2 1_{\{x<0\}} + \rho_u y^2]\mu(dt, dx, dy) \tag{7}$$

The three volatility factors are given by V_1, V_2 and U. They impact both the diffusive volatility and the jump intensities. The jumps are modeled with two separate components: x and y. The component x represents jumps that affect the price, the volatility V_1, and U. The component y represents jumps that only affect U. Equation (5) reveals that the price jumps are exponentially distributed, while equations (6) and (7) indicate that the jump affects the volatility. Notice that because of the indicator $1_{\{x<0\}}$, only negative jumps bear an impact on the volatility. According to this model, positive jumps only impact the prices, but not the volatility. However, that is not what we see in the data. In fact, as indicated by the jump regressions in Figure 6, positive jumps do impact the volatility. Furthermore, the impact is negative and stable over different moneyness categories. This is evidence that the three-factor double exponential model is misspecified, it should also account for positive jumps.

5 Conclusion

This project analyzed the impact of jumps in SPY on the change in the average BSIV of SPX options. Based on the jump regression theory by [Li et al., 2017], we identified the jump returns in SPY, and estimated jump regressions with the change in BSIV, controlling for the moneyness of options. Seven moneyness categories were used to aggregate the SPX options, leading to seven jump regressions.

The jump regressions resulted in the following: there is a negative correlation between jump sizes and the change in the average BSIV, the estimates are relatively stable across different moneyness categories, the estimated elasticity is approximately -4.5 for most moneyness categories, and the relationship is tighter for out of the money options, but not for extremly out of the money options.

The empirical evidence from the jump regressions was contrasted to the theoretical implications of the options pricing models in [Merton, 1976] and in [Andersen et al., 2015]. The former stated that jumps in the underlying have no effect in the BSIV, while the latter stated that only positive jumps did not have an impact. Both models were contradicted by the empirical evidence: jumps have an impact on the BSIV.

This work analyzed the impact of underlying jumps in the BSIV of SPX options. It presented evidence of a linear relationship between both factors, and contrasted data to theoretical options pricing models. However, this work faults at giving a theory of why we should expect a linear relationship in the jump regressions, present theoretically valid confidence intervals for the jump regression estimates and provide robustness checks.

References

[Andersen et al., 2015] Andersen, T. G., Fusari, N., and Todorov, V. (2015). The risk premia embedded in index options. *Journal of Financial Economics*, 117(3):558 – 584.

[Black, 1976] Black, F. (1976). Stuedies of stock price volatility changes.

[Black and Scholes, 1973] Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654.

[Carr and Wu, 2016] Carr, P. and Wu, L. (2016). Analyzing volatility risk and risk premium in option contracts: A new theory. *Journal of Financial Economics*, 120(1):1–20.

[Christie, 1982] Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of financial Economics*, 10(4):407–432.

[Jacod and Protter, 2012] Jacod, J. and Protter, P. (2012). Discretization of processes. stochastic modelling and applied probability 67.

[Li et al., 2017] Li, J., Todorov, V., and Tauchen, G. (2017). Jump regressions. *Econometrica*, 85(1):173–195.

[Mancini, 2001] Mancini, C. (2001). Disentangling the jumps of the diffusion in a geometric jumping brownian motion. *Giornale dell'Istituto Italiano degli Attuari*, 64(19-47):44.

[Mancini, 2009] Mancini, C. (2009). Non-parametric threshold estimation for models with stochastic diffusion coefficient and jumps. *Scandinavian Journal of Statistics*, 36(2):270–296.

[Merton, 1976] Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of financial economics*, 3(1-2):125–144.

6 Appendix

The appendix discusses the list of news announcements used to check whether adjacent jumps were random in nature or due to news announcements.

6.1 News Announcements

The news announcements dates and types were obtained from Bloomberg. It includes announcements on financial indicators, real estate, cyclical indicators, and other categories. See Table 4 below for detailed descriptions.

Category	Announcement
Financial Indicator	FOMC Rate Decision Upper Bound
Industrial Service	Market US Services PMI
Personal Household Sector	Consumer Credit
Prices	ISM Prices Paid
Labor Market	JOLTS Job Openings
Government Finance And Debt	Monthly Budget Statement
Purchasing Manager index	ISM New York
LaborMarket	Nonfarm Payroll
Retail and Wholesale	Wards Total Vehicle Sales
	Wholesale Inventories MoM
Cyclical Indicators	Chicago Purchasing Manager
	Consumer Confidence Index
	Dallas Fed Manf Activity
	ISM Manufacturing
	ISM Milwaukee
	ISM NonManf Composite
	ISM NonManufacturing
	Kansas City Fed Manf Activity
	Leading Index
	Market US Composite PMI
	Market US Manufacturing PMI
	Philadelphia Fed Business Outlook
	Richmond Fed Manufact Index
	Univ. of Michigan Sentiment
Housing and Real Estate	Existing Home Sales
	Existing Home Sales MoM
	FHFA House Price Index MoM
	House Price Purchase Index QoQ
	NAHB Housing Market Index
	New Home Sales
	New Home Sales MoM
	OneFamily Home Resales
	Pending Home Sales MoM
	Pending Home Sales NSA YoY

Table 4: News Announcements by Categories

These news announcements were used to check whether the jumps in dates with more than one jump could have been due to news announcement, and not random in nature.

Of the 15 jumps that were removed from the analysis, 13 of them occurred in the minutes after FOMC announcements. However, two of the jumps that occurred in adjacent intervals did not match the date and time of any of the announcements listed above. I have two hypotheses about what could be driving these two jumps: some other important news announcement not listed above, or a market bounce-back.

It is possible that some other news announcement affected the market and caused the jump. The list of news announcements above is not comprehensive, and there are other financial indicators that could be important into explaining jumps. In this case, the two jumps would not be truly random in nature, but would be due to some news announcement, and removing them from the analysis would be the correct thing to do. Another possibility is that a bounce-back occurred. For example, if a liquidity trader has to sell a portfolio, then it is likely that the price would increase momentarily, since only a limited number of stocks can be traded at the best bid price. However, because nothing fundamentally changed about the company (or the market), the price would go back to its previous level, creating a bounce-back. Since there are many automated traders in the market, it is plausible that a bounce-back could occur in the span of 2 minutes. This jump is random in nature, and excluding it from the analysis would be a mistake.