Volume, Volatility, and Market Index Options

Guilherme Salomé

April 30, 2019

Abstract

We provide new empirical evidence of disagreement in the S&P 500 options market. Our findings are consistent with the implications of a model where investors observe public information but agree to disagree about its interpretation. We argue that there are two natural measures of disagreement for the options market: moneyness and tenor. These two measures speak to the distribution of the market index at different quantiles and at different times. We estimate a volume-volatility elasticity equal to unit for options that are at-the-money and near expiration, which is consistent with the case of no disagreement among investors. Our elasticity estimates decrease with increases in the absolute value of moneyness, implying that new information leads to more disagreement about tail events of the market index. We also find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index.

Keywords: SPX options, market index, high-frequency data, disagreement, volume-volatility elasticity, public information

1 Introduction

This paper studies disagreement between investors in the S&P 500 options market. The strength of the relationship between a financial asset's trade volume and its volatility reveals investors' disagreement about new information. This study uses high-frequency data on market index options to estimate the volume-volatility elasticity. Our results are consistent with the implications of a theoretical model where investors are subject to new information and may disagree about its interpretation, leading to trade based on such disagreement. The model implies that the volume-volatility elasticity is equal to unity in the case of no disagreement among investors, and that the elasticity should decrease as disagreement increases. We find volume-volatility elasticity estimates below unit, consistent with the case of disagreement among investors. We also find that the volume-volatility elasticity decreases with the measures of disagreement considered in the study. These disagreement measures also speak to the distribution of the market index, and the elasticity estimates reveal greater disagreement regarding the tails of the distribution, but a lack of disagreement regarding the center of the distribution.

The relationship between trade volume and volatility has been previously analyzed by several works. A survey of the earlier empirical evidence is available in Karpoff (1987).

The earlier research finds a positive contemporaneous correlation between trade volume and variance (see Table 1 of Karpoff (1987)). A statistical explanation for such correlation comes from Tauchen and Pitts (1983), where the authors argue that changes in price and in volume are both driven by a process that models the rate of information arrival, which leads to the positive correlation found in many empirical works. The explanation based on this common driving process is known as the mixture of distribution hypothesis (MDH). While the MDH provides a statistical explanation for the empirical findings, it is deficient in providing the economic channel between new information and the relation of volume and volatility.

Another strand of the literature on volume and volatility considers heterogeneous-agent models in which agents disagree about new information. This disagreement between agents generates an additional trading motive, which leads to changes in prices and trade volume. Hong and Stein (2007) argue that this class of disagreement models is compelling since they model the joint behavior of prices and volume, while providing the underlying mechanisms for the connection of new information to volatility and volume.

The work of Kandel and Pearson (1995) supplies one such disagreement model. In the Kandel-Pearson model, investors trading a risky asset observe a public piece of news and use different economic models to interpret this information. The difference in models leads to disagreement about interpreting the new information. The disagreement between investors generates an additional motive for trading. The trading generated by disagreement is not directly dependent on changes in the asset price. The Kandel-Pearson model implies that it is possible to have high trade volume even when there is no price change.

In Bollerslev, Li, and Xue (2018) the authors extend the findings of Kandel and Pearson (1995) and get an expression directly connecting disagreement between investors to the elasticity between volume and volatility. The expression that connects disagreement to the volume-volatility elasticity has two main implications. The first is that the volume-volatility elasticity is equal to unity only when there is no disagreement between traders. The second is that the elasticity decreases when the disagreement between traders increases.

Bollerslev, Li, and Xue (2018) validate the two implications from the volume-volatility elasticity for the market index. The authors take into account proxies of investor disagreement and estimate a volume-volatility elasticity equal to unity and that decreases when the disagreement proxies increase. The present work analyzes these two implications in the context of the S&P 500 options market (SPX options). A benefit of using options data instead of stock data is the availability of two direct measures of disagreement: moneyness and tenor. We argue that moneyness and tenor speak to the distribution of the market index at different quantiles and at different times. This allows us to uncover disagreement between investors about different parts of the distribution.

We present empirical evidence consistent with the implications from the Kandel-Pearson model and Bollerslev, Li, and Xue (2018). We estimate a volume-volatility elasticity equal to unity for options that are at-the-money and near expiration, which is consistent with the case of no disagreement among investors. We also find that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, implying that new information leads to more disagreement regarding tail events for the market index. We find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index. Last, we estimate the volume-volatility elasticity in a nonparametric setting and obtain results that support our initial findings.

The article is organized as follows. Section 2 discusses the Kandel-Pearson model and the extension proposed by Bollerslev, Li, and Xue (2018), including the volume-volatility elasticity and its interpretation regarding disagreement about new information. Section 3 argues why moneyness and tenor are measures of disagreement and how to interpret these measures with respect to the volume-volatility elasticity. Section 4 discusses how to measure the volatility of options and argues for the use of a measure of residual volatility. Section 5 describes the high-frequency options data used in the empirical section and discusses how to avoid microstructure noise effects. Section 6 discusses the parametric specification used to estimate the volume-volatility elasticity. Section 7 presents the main empirical findings using SPX options data. This section also discusses findings from a nonparametric estimation procedure that confirms the main findings from the parametric setting. Section 8 concludes the paper. Last, the Appendix offers details on cleaning procedures and additional figures.

2 How to Interpret the Relationship Between Trade Volume and Volatility

We study the relationship between trade volume and volatility in the options market. The economic interpretation of this volume-volatility relationship originates in the model of Kandel and Pearson (1995) where investors disagree about the interpretation of new information. Bollerslev, Li, and Xue (2018) extend the findings from Kandel and Pearson (1995) and provide an interpretation for the volume-volatility elasticity. This section discusses the Kandel-Pearson model and the interpretation of the volume-volatility elasticity proposed by Bollerslev, Li, and Xue (2018). If the reader is familiar with both works, this section can be skipped.

2.1 Trade Motivated by Disagreement About New Information

In Kandel and Pearson (1995) the authors explore the relationship between an asset's trade volume and its return. More specifically, the authors seek to understand why there are numerous trades in the market even when there is no price change in the asset. The authors argue that this happens due to a difference in information among traders.

In the setting of Kandel and Pearson (1995), there is a competitive market with a risk-free and a risky asset. The risk-free asset has a zero return rate, but the risky asset has a random future payoff of X. There are two types of traders trading in this market, type 1 and type 2, and the proportion of type 1 traders is given by $\alpha \in [0, 1]$. Each trader has prior beliefs about the random payoff of the risky asset. Specifically, agent of type $i \in \{1, 2\}$ believes $X \overset{d}{\sim} \mathcal{N}\left(X_i, Z_i^{-1}\right)$ and $X_1 > X_2$ is assumed without loss of generality. Given these beliefs the agents price the risky asset and a competitive market equilibrium is achieved.

The traders then observe a public signal $L = X + \varepsilon$ about the risky asset payoff, but each agent interprets the signal differently. The difference in interpretation stems from each agent's assumption about the noise term ε. The agent of type $i \in \{1, 2\}$ believes that $\varepsilon \overset{d}{\sim} \mathcal{N}\left(\mu_i, b_i^{-1}\right)$, so that the difference in interpretations originates from different values of μ_i and b_i for each agent type. Different agents know that they have different interpretations of the signal, but they agree to disagree about the signal's interpretation. The agents update their beliefs about the risky asset's future payoff, and a new competitive market equilibrium is achieved.

After the new equilibrium is achieved it is possible to compute the trade volume of the risky asset resulting from the arrival of the public signal:

$$\text{Volume} = |\beta_0 + \beta_1 \underbrace{(P_{\text{after signal}} - P_{\text{before signal}})}_{\Delta P}| \tag{1}$$

$$\beta_0 = \frac{\alpha(1-\alpha)}{\lambda(\alpha b_1 + (1-\alpha)b_2)} b_1 b_2 (\mu_2 - \mu_1)$$

$$\beta_1 = \frac{\alpha(1-\alpha)}{\lambda(\alpha b_1 + (1-\alpha)b_2)} (Z_2 b_1 - Z_1 b_2)$$

where $\lambda > 0$ is the common risk aversion of the traders. Equation 1 above relates the trade volume of the risky asset to two terms: the return of the risky asset and a term β_0 that captures disagreement. We can interpret this equation in terms of the effect of the asset return on its trade volume. The return depends on the public signal and the disagreement generated by it, and there are two cases to be analyzed. First, the public signal leads to a change in the price of the risky asset. In this case we have $\Delta P \neq 0$ and the trade volume depends on how big the asset return is. Second, the public signal does not lead to a change in the price of the risky asset. In this case we have $\Delta P = 0$, but the trade volume is not zero and depends on the term β_0, which captures the disagreement among the traders. Notice that the term β_0 is nonzero as long as the traders disagree about the mean of ε, that is $\mu_1 \neq \mu_2$.

The Kandel-Pearson model generates an additional motive of trade other than a price change. Even if there is no price change, it is still possible to have trades due to disagreement among traders. Furthermore, as the level of disagreement (measured by $\mu_2 - \mu_1$) increases, the relationship between trade volume and price changes weakens, and most of the trade is motivated by disagreement.

2.2 Measuring Disagreement by Volume-Volatility Elasticity

In Bollerslev, Li, and Xue (2018) the authors propose a weaker version of the volume equation from Kandel and Pearson (1995) (Equation 1). The authors argue that while the volume equation summarizes the relationship between trade volume and price changes in response to the release of new information, it also establishes an exact relationship between random quantities, which is a condition too strong to impose. A more realistic condition, according to the authors, is that Equation 1 should hold on average, resulting in a moment condition.

The setting in Bollerslev, Li, and Xue (2018) simplifies the model from Kandel and Pearson (1995) by assuming both types of traders have the same precision regarding the new information: $h \equiv b_1 = b_2$. This simplification implies that β_0 and β_1 can be written as:

$$\beta_0 = r\alpha(1-\alpha)h(\mu_2 - \mu_1)$$
$$\beta_1 = r\alpha(1-\alpha)(Z_2 - Z_1)$$

where $r \equiv 1/\lambda$ is the degree of risk tolerance of traders. Now the disagreement regarding the public signal is captured by β_0 via $\mu_2 - \mu_1$.

Under the additional assumption that $\Delta P \overset{d}{\sim} \mathcal{N}(0, \sigma)$, the authors derive the expression for the expected trade volume in function of the volatility σ:

$$\mathbb{E}[\text{Volume}] = \frac{2}{\pi} |\beta_1| \sigma \mathrm{e}^{\frac{-\beta_0^2}{2\beta_1^2\sigma^2}} + |\beta_0| \left(2\Phi\left(\frac{|\beta_0|}{|\beta_1|\sigma}\right) - 1\right)$$

where Φ is the cumulative density function of a standard normal distribution. While it is possible to interpret the expectation above in terms of disagreement, a clearer expression is obtained by computing the volume-volatility elasticity:

$$\text{Volume-Volatility Elasticity} = \frac{1}{1 + \psi\left(\frac{|\beta_0|}{|\beta_1|\sigma}\right)} = \frac{1}{1 + \psi\left(\frac{h}{\sigma} \cdot \frac{|\mu_2 - \mu_1|}{|Z_2 - Z_1|}\right)} \tag{2}$$

where ψ is an increasing function with $\psi(0) = 0$ and $\lim_{x\to\infty} \psi(x) = \infty$.

The expression for the elasticity has two important characteristics. First, the elasticity is no greater than one and is equal to one only if $\mu_2 = \mu_1$. That is, the volume-volatility elasticity is maximized only when there is no disagreement among traders regarding the public signal. Second, the elasticity is a decreasing function of the level of disagreement. Thus, a higher level of disagreement (larger $|\mu_2 - \mu_1|$) implies in a smaller elasticity.

In Bollerslev, Li, and Xue (2018) the implications of disagreement on the volume-volatility elasticity are taken to the market index data. The authors study the behavior of the elasticity around macroeconomic news announcements. Initially, the authors estimate a volume-volatility elasticity below unity, which is in line with the predictions of the Kandel and Pearson (1995) model. The authors then explicitly control for disagreement using proxies for disagreement based on the survey of professional forecasters and economic uncertainty. When disagreement is accounted for, the volume-volatility elasticity estimates are close to unity, and the disagreement proxies have a negative impact on the elasticity. This work also focuses on understanding disagreement via the volume-volatility elasticity but centers the analysis on the options market. The next section discusses the advantages of doing so.

3 Measures of Disagreement for Options

The present work studies disagreement among investors in the S&P 500 index options market. A motivation for this study is the existence of two natural measures of disagreement for the options market: moneyness and tenor. We begin by defining these two measures and then how to interpret them from the point of view of disagreement.

3.1 Moneyness and Tenor

We define the moneyness of an option as:

$$\text{Moneyness} = \frac{\ln\left(\frac{K}{S}\right)}{\sigma\sqrt{\tau}}$$

where K is the strike price of the option, S is the current underlying price (in this case the S&P 500 index), σ is the volatility of the underlying asset and τ is the time to expiration of the option.

The numerator in the expression above is the logarithm of the strike-to-underlying ratio, which is negative for out-of-the-money (OTM) puts, and positive for in-the-money (ITM) puts. The numerator represents by how much the underlying asset would have to move so that the option is exactly at-the-money (ATM). Observe that the numerator is measured as a rate of geometric return. The denominator standardizes the logarithm of the strike-to-underlying ratio, so that moneyness is measured in units of standard deviation.

We interpret moneyness as the number of standard deviations the underlying price needs to move for the option to be ATM. For example, a put option with moneyness equal to -3 would need the underlying price to decrease by three times its standard deviation for the option to be ATM.

As stated by Bollen and Whaley (2004):

> (...) an option's "moneyness" is intended to reflect its likelihood of being in the money at expiration.

Following this understanding, a put option that has a very negative moneyness (deep OTM) is unlikely to expire in the money. On the other hand, a put option with highly positive moneyness (deep ITM) is likely to expire in the money.

The tenor of an option is the amount of time until the expiration date of an option. With options on the S&P 500 index (SPX options) special care needs to be taken due to differences in the types of expiration. There are two types of SPX options: standard and weekly. The standard SPX options have expiration dates set for the 3rd Friday of every month, and the actual expiration time is at the market open (9:30 a.m. Eastern Time). The weekly SPX options, however, have expiration dates at multiple days every week, but their actual expiration time is at the close of the trading day (4:00 p.m. ET)[1]. In this case, the tenor of these options is computed at the minute level and then the units are converted to days.

3.2 Interpretation of Moneyness and Tenor as Measures of Disagreement

The moneyness and tenor of an SPX option speak to the distribution of market returns. To discuss the interpretation of these two measures, we need to understand the pricing of options.

The price of an option can be computed as the risk-neutral expectation of its payoff. The existence of a risk-neutral measure is assured under no-arbitrage conditions (see Section 6K of Duffie (2001)). Let $\mathbb{Q}$ denote the risk-neutral distribution, then the price of an SPX put option is given by:

$$P_t(K,T) = \mathrm{e}^{-(r-q)T}\mathbb{E}_t^{\mathbb{Q}}[\max\{K - S_T, 0\}]$$

where we denote the price of the market index at time t by S_t, K as the strike price of the put option and T its time to expiration, and r and q denote the corresponding interest rate and dividend yield.

The equation above shows that the price of a put option is given by the risk-neutral expectation of its payoff. This expectation depends on the distribution of the market index at expiration time and also on the strike price of the option. If the strike price

[1]For details see CBOE (2014).

K is very low, then the price of the option is derived mostly from the left-tail of the distribution of the market index, S_T. As the strike price increases, the dependence of the price on the left-tail of the distribution of S_T decreases. At the same time, the price of the option depends increasingly more on the center of the distribution of S_T. If the strike price is very high, then the price of the option is more dependent on the right tail of the distribution of S_T.

The distance between the strike price and the market index is captured by the option's moneyness. Therefore, moneyness measures how much the price of an option depends on the different parts of the distribution of the market index. From this perspective, depending on the moneyness of options we analyze, new information regarding the market index would lead to disagreement regarding different quantiles of the distribution of S_T. For this reason, we interpret moneyness as a measure of disagreement about different parts of the distribution of the market index. In addition, due to the difficulty of analyzing tail events, we expect new information to generate more disagreement regarding the tails of the distribution of the market index. Therefore, there should be a negative relationship between the absolute value of moneyness and the volume-volatility elasticity.

A similar explanation applies to the tenor of an option. The option pricing equation shows that the expectation depends on the distribution of S_T, which depends on the time to expiration T. Therefore, the pricing of the option at different tenors depends on how the distribution of S_T changes for different values of T. From this perspective, the tenor captures disagreement regarding the distribution of the market index at different times. Since events far in the future are subject to more uncertainty than events about to happen[2], the value of options with longer times to expiration are subject to more uncertainty than options with short tenors. This is especially true for the case of the market index, where the uncertainty regarding the future price increases with time. Thus, we expect a negative relationship between tenor and the volume-volatility elasticity.

The moneyness and tenor are readily available for any option. These two measures are directly related to disagreement and are used in the analysis of the volume-volatility elasticity. While options have these direct measures of disagreement, the volatility of an option is not as straightforward to measure. This is discussed in the next section.

4 Measure of Volatility for Options

An option is an asset with a payoff that depends on the value of another asset, the underlying. Even though there is this dependence, we can think of an option as an asset on its own. In this case, the volatility of an option is just the volatility of this option's price. If we let $r_{i,t}$ denote the geometric return of the price of an option at a time interval i of a day t, then the volatility of this option can be estimated via the realized volatility estimator:

$$RV_t^{\text{option}} = \sum_i \left(r_{i,t}^{\text{option}}\right)^2$$

[2]Forecasting the expectation of a stochastic process may or may not be more difficult as the forecast horizon increases. Indeed, for a moving average process the mean-squared error of the forecast converges to the unconditional variance of the series as the forecasting horizon increases. However, for a unit-root process the mean-squared error of the forecast diverges as the forecasting horizon increases (see page 440 of Hamilton (1995)). Since the market index is a unit-root process, its forecasting error increases with the forecasting horizon.

where the summation is across all time intervals in a day. The volatility estimate above is a measure of daily variance in the prices of an option.

We know, however, that an option is fundamentally linked to its underlying. Indeed, the value of an option is partially derived from the current price and volatility of the underlying. In the Black-Scholes world, this connection is made explicit by the Black-Scholes Delta:

$$\Delta_t^{Black-Scholes} \equiv \frac{\partial \mathrm{P}_t}{\partial S_t} = \Phi\left(\frac{\ln\left(\frac{S_t}{K}\right) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

where P_t is the price of a put option at time t with some strike price K and expiration time T, and S_t is the underlying price at time t. In this case, a price change in the underlying translates into a price change in the option. This connection implies that the volatility of an option can be divided into two terms. The first term is the price volatility of an option due to changes in the underlying price. In our case, the underlying is the market index, so we denote this first term as the impact of the market volatility. The second term is the price volatility of an option due to other changes, such as changes in the underlying volatility. This second term is denoted as the residual volatility. The separation of an option's volatility yields the following equation:

$$RV_t^{option} = RV_t^{\text{impact of market volatility}} + RV_t^{\text{residual volatility}}$$

The separation of the volatility is relevant because different economic agents are interested in different parts of the total volatility. According to Hull (2003), traders in financial institutions maintain delta-neutral portfolios, by at least daily re-balancing their portfolios, but cannot maintain neutral portfolios with relation to the other Greeks due to the difficulty of finding financial instruments to hedge with at competitive prices. This suggests that portfolios of financial institutions are hedged against changes in the market index (via delta-hedging), but are still exposed to the residual volatility. This point of view is supported by S. Ni, Pan, and Poteshman (2008), who argue that market makers can delta-hedge directional moves but cannot avoid volatility risk. Since the position of market makers is market-neutral, it is possible that the option volatility relevant to these agents is not the total volatility (RV_t^{option}), but actually the residual volatility ($RV_t^{\text{residual volatility}}$). On the other hand, the total option volatility would be relevant to speculators that trade options for directional moves, since the position of these agents would not be delta-neutral.

This work assumes that the sophisticated agents trading in the SPX options market are either maintaining a delta-neutral position or are speculators that trade on volatility information (as in S. Ni, Pan, and Poteshman (2008)). This is partially supported by the fact that financial institutions (sophisticated agents) maintain delta-neutral portfolios (Hull (2003)), that there is a high demand for SPX puts due to the hedging demand of institutional investors (Bollen and Whaley (2004)), and that unsophisticated traders account only for 3% of the total volume of trades in the SPX options market (Lemmon and S. X. Ni (2008)).

The discussion above motivates mapping the moves in an option's price into moves due to changes in the option's underlying and moves due to other (residual) changes:

$$r_{i,t}^{\text{option}} = \beta r_{i,t}^{\text{market}} + \varepsilon_i$$

To recover the impact of the market volatility on the option volatility, we estimate the equation above separately for each option and for each day in the sample. We then use the residuals to recover the impact of the market volatility and compute the residual volatility:

$$RV_t^{\text{impact of market volatility}} \equiv \sum_{i=1}^{n} \left(\hat{\beta} r_{i,t}^{\text{market}}\right)^2$$
$$RV_t^{\text{residual volatility}} \equiv \sum_{i=1}^{n} \hat{\varepsilon}_i^2$$

We use the residual volatility as the measure of volatility for an option. It represents how much variation in an option's price is due to factors not directly hedged by sophisticated agents.

The next section discusses the data used in the empirical analysis.

5 Data

This work uses data on S&P 500 options (SPX options) made available by the Chicago Board Options Exchange (CBOE). SPX options are European style and are cash-settled with a multiplier of $100 on the market index. The multiplier implies that if the S&P 500 is at the 2,867.24 level, then the notional value of one option is $286,724.00. The data set contains bid and ask quotes and trade volume at the 1-minute frequency for each option (varying strike and time to expiration), from 2007 to 2016, totaling 2458 days in the sample after cleaning procedures[3]. To provide some sense of the scale of the analysis, observe that there were 3,479,352,243 records in the raw data, and a number of techniques had to be employed to clean and condense the set.

The focus of the analysis is on put options (call options are excluded), since these options are heavily traded and provide a natural hedge against market downturns (Bollen and Whaley (2004)).

In the Kandel-Pearson model investors disagree about a public signal. In Bollerslev, Li, and Xue (2018), the public signals considered are macroeconomic announcements, and the authors compute the elasticity from data around the announcement times. In this work, however, we consider public signals as being the information disseminated throughout a trading day. Therefore, we analyze the volume and volatility at the daily frequency.

The bid and ask quotes of each option are used to compute mid-prices. We estimate an option's residual variance from its high-frequency intra-day mid-prices. However, it is known that high-frequency prices can be affected by microstructure noise, which lead to high biases in variance estimates. To mitigate the effect of microstructure noise, we sparsely sample the mid-prices to the 5-minute frequency. Sampling sparsely is a standard practice in the literature and, in this case, is supported by a volatility signature plot for SPX options.

The volatility signature plot is displayed in Figure 1. The figure shows the average realized volatility for different sampling frequencies, with the options being categorized by their normalized moneyness. The volatility signature can indicate whether microstructure noise affects the return observations. If there is no microstructure noise, then the volatility

[3]See Section 9.1 in the Appendix for details.

estimate should converge to a value as the sampling frequency increases. However, if microstructure noise is present, then the volatility estimate would diverge as the sampling frequency increases.

The figure indicates that the realized volatility estimator is stable for sampling frequencies as high as 5-minutes. For sampling frequencies above 5-minutes, we notice a sharp increase in the volatility estimates for out-of-the-money options, indicating the presence of microstructure noise at higher sampling frequencies. To avoid the effect of microstructure noise, we sparsely sample the options prices to the 5-minute frequency.

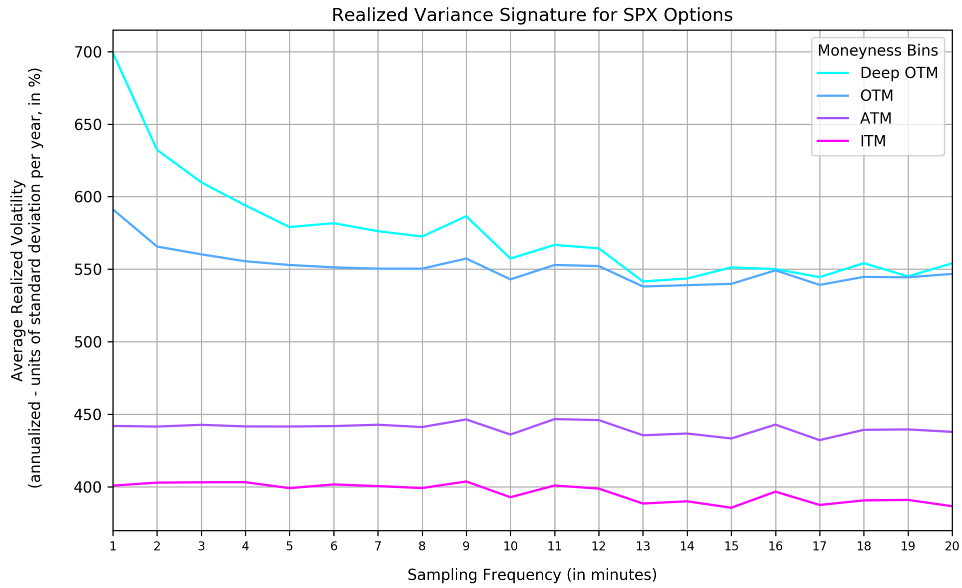

Figure 1: Volatility Signature for SPX Options. The figure shows the average realized volatility (annualized in percentage) for options of different moneyness based on different sampling frequencies. The realized volatility is estimated as the sum of squared returns over each day for each option, where the frequency of returns varies from 1-minute up to 20-minutes. The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

To compute the residual volatility of each option, we use data at the 5-minutes frequency. The residual volatility is estimated separately for each option at each day in the sample. The moneyness of each option is computed daily at the end of the day.

The trade volume is computed daily for each option and is given by its total volume of trades at that day. However, as discussed in Tauchen and Pitts (1983), the presence of a trend in the volume data could mislead the results of the volume-volatility analysis. We verify that indeed a trend exists in the data and mitigate its effects by detrending the volume with a moving average based on the volume on the previous 252 days.

The data set on SPY was obtained from TickData[4], and it contains the last closing price of every 5-minute interval for all days from 2007 to 2016.

This study also considers two measures of sentiment. We want to explore the impact of sentiment on the disagreement between investors. The idea is that at times of op-

[4]See `https://www.tickdata.com`

timism investors disagree less about new information, while at times of pessimism new information leads to greater disagreement.

The first sentiment measure is from Baker and Wurgler (2006) and Baker and Wurgler (2007) [5]. Baker and Wurgler construct a sentiment measure based on the principal component of various market proxies for sentiment. Specifically, they consider the dividend premium of stocks, the closed-end fund discount, the first-day return and trade volume on IPOs and the equity issues over total new issues[6]. The authors find evidence that their sentiment measure explains stock returns on the cross-section (increase in expected return on speculative and difficult-to-arbitrage stocks when sentiment increases, but the opposite for safe and easy-to-arbitrage stocks), is correlated with an equal-weighted market index and predict future average returns (lower average return for speculative stocks when past sentiment was high). The sentiment index is available at the monthly frequency from 2007 to September of 2015.

The second sentiment measure is the FEARS index from Da, Engelberg, and Gao (2014). The FEARS (Financial and Economic Attitudes Revealed by Search) index is based on the volume of internet search queries for various economics-related terms, such as "gold prices", "recession" and "crisis". The authors aggregate the search volume for different terms into an index where the terms are chosen according to how well they explain negative returns. The authors show that a high level of FEARS is related to lower contemporaneous returns but predict higher future returns (over the next few days), and that the index explains volatility and the flow of mutual funds from equity to bonds. The authors argue that, contrary to sentiment measures based on market proxies, FEARS is not the equilibrium outcome of the economy. The authors also argue that FEARS is based on agents' attitudes and is possibly more truthful than indices based on surveys. FEARS is available at the daily frequency from 2007 to 2016.

The next section discusses the empirical methodology to analyze disagreement in the SPX options market.

6 Empirical Methodology

The impact of disagreement regarding new information is captured by the volume-volatility elasticity. According to Equation 2, we should observe an elasticity equal to unity in the case of no disagreement among investors. However, if there is disagreement, then the elasticity should be less than unity. Furthermore, the elasticity should decrease to the extent that disagreement increases.

To analyze these implications, we consider the following regression specification:

$$\ln(\text{Volume}) = (\alpha_0 + \alpha_1^T X_0) + (\beta_0 + \beta_1^T X_1) \cdot \ln(\text{Residual Volatility}) + \varepsilon \tag{3}$$

where α_0, β_0 are scalars, α_1, β_1 and X_0, X_1 are vectors, and the terms in X_0 and X_1 include different control and explanatory variables. Estimating the equation above allows us to directly recover the volume-volatility elasticity. Of particular importance to the analysis are the terms included in X_1. These terms include measures of disagreement, which allow us to evaluate the predictions regarding disagreement from the estimated β_1 coefficients.

[5] Data sourced from Wurgler's website.

[6] Originally, the authors also included NYSE turnover as a measure of liquidity in the index. However, they recently dropped it from the sentiment index because turnover is no longer a meaningful measure of liquidity due to high-frequency trading.

If the measures of disagreement included in X_1 capture investors' disagreement in the options market, then we expect the estimated value of β_0 to be close to unity, while those of β_1 to be negative.

The disagreement measures included in X_1 are the absolute value of moneyness, tenor and an interaction term between both. The absolute value of moneyness indicates how far from the money an option is. This measure captures disagreement regarding the distribution of the market index. For example, options far from the money will have values of moneyness far from zero, which indicates that the price of such options depend on the tails of the distribution of the market index. Since the probabilities of tail events are hard to predict, the impact of new information on the tails of this distribution should generate more disagreement. An underlying assumption is that the effect of decreases or increases in the moneyness have the same impact on the elasticity. This is plausible because the distribution of high-frequency returns is symmetric, and ITM puts are equivalent to OTM calls.[7]

We also expect the effect of tenor on elasticity to be negative, since higher tenor is related to uncertainty about the future distribution of the underlying asset. The interaction between the absolute value of moneyness and tenor allows us to capture the effect of moving away from the money on options that are close to expiration and on options that are far from expiration.

The next section discusses the empirical results.

7 Empirical Results

The main interest of the empirical analysis is to investigate the elasticity between trading volume and residual volatility under the predictions of Kandel and Pearson (1995) and the developments of Bollerslev, Li, and Xue (2018). The estimates are presented in Table 3 below.

The baseline elasticity is estimated to be 1.0252, which is statistically indistinguishable from unity. The baseline elasticity estimate indicates that there is no disagreement for options about to expire and that are at-the-money. The estimated coefficient for the absolute value of moneyness is negative, indicating that disagreement among investors increases for options far from the money (either OTM or ITM puts). This is consistent with the idea that there is more uncertainty about the tails of the distribution of the market returns.

The estimated coefficient for the tenor is negative, which indicates that disagreement increases for options that expire in more time. The coefficient for tenor is consistent with the idea that there is more uncertainty about the distribution of market returns over longer time-horizons than over shorter time-horizons.

Lastly, the coefficient of the interaction term between moneyness and tenor is positive. A positive coefficient implies that the effect of moneyness is attenuated for options that have a long time to expiration. In other words, while there is more disagreement on options that expire far in the future, the disagreement is less dependent on how far-from-the-money an option is. The attenuation of the impact of moneyness is consistent with the idea that over longer horizons the number of big moves in the market index gets averaged out, so that there is less disagreement for options far-from-the-money.

[7] An alternative specification where the negative and positive moneyness are allowed to have different impacts on the elasticity is explored in Section 9.4 of the Appendix. Similar results are obtained.

	Parameter Estimate	Standard Error
Baseline Estimates		
Intercept (α_0)	7.4682	0.057
Elasticity (β_0)	1.0252	0.013
Explanatory Variables for Elasticity		
|Moneyness|	-0.2893	0.007
ln (Tenor)	-0.1509	0.004
|Moneyness| · ln (Tenor)	0.0247	0.002
Adjusted R^2	0.24	
Number of Observations	446053	

Table 1: Volume-Volatility Elasticity Estimates. The table reports the estimated values of α_0, β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. Estimation is based on the full sample of cleaned data, spanning 2007 to 2016. We compute volume and volatility daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are heteroscedasticity robust as in White et al. (1980).

To get a better sense of the elasticity values at different moneyness and tenor levels, we use the parameter estimates to create Figure 2.

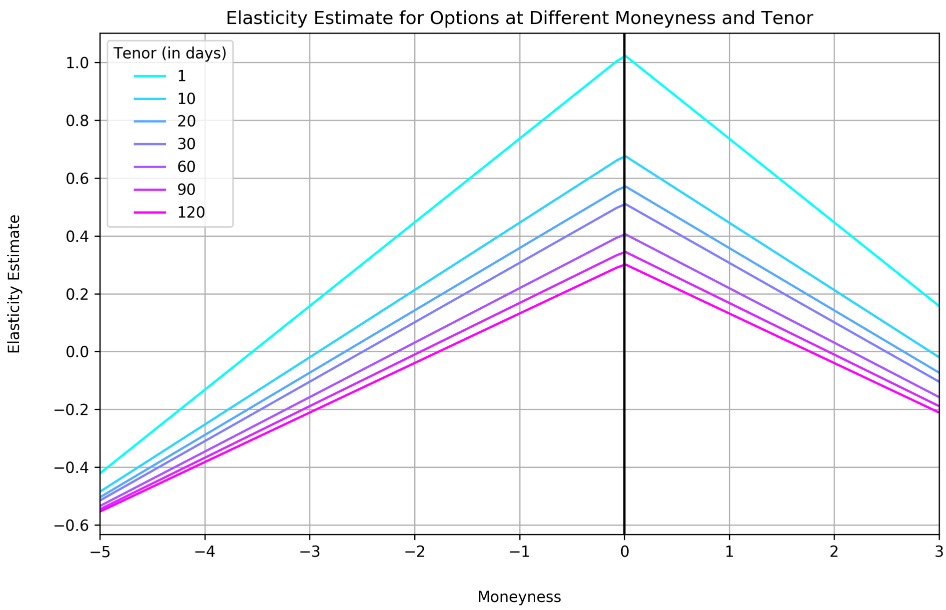

Figure 2: Elasticity Estimates for Options at Different Moneyness and Tenor. The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.

The case of no-disagreement occurs for at-the-money options that are about to expire (in a day). For any fixed tenor, there is a symmetric decrease in elasticity as we move away from-the-money. Since the tenor is fixed, this decrease in elasticity is interpreted as an increase in disagreement regarding the tails of the distribution of market returns. Fixing any moneyness level, there is a large drop in elasticity as the tenor increases, indicating a heightened disagreement regarding the future distribution of market returns. Lastly, notice that the decrease in elasticity when the tenor increases is smaller for options far-from-the-money than it is for options closer-to-the-money.

7.1 Impact of Sentiment on Elasticity

The volume-volatility elasticity was estimated taking into account the effect of options moneyness and tenor. While moneyness and tenor capture more specific aspects of the disagreement between investors, we also want to consider the impact of measures that capture a more general level of disagreement. To do so, we include two measures of sentiment as control variables when estimating the volume-volatility elasticity.

The first measure is the sentiment index from Baker and Wurgler (2006) and Baker and Wurgler (2007), henceforth referred to as the BW sentiment index. The BW sentiment index captures investors' sentiment by extracting a principal component from various market proxies for sentiment. The second measure is the FEARS index from Da, Engelberg, and Gao (2014). The FEARS index measures the Internet search volume for terms with negative economic connotations. A high value of FEARS means a higher than average search volume for these negative terms. While the BW sentiment index offers a sentiment measure based on market quantities, the FEARS index recovers sentiment from an alternative source only indirectly related to the financial markets.

We expect that when investors are optimistic, there should be less disagreement about new information, but when investors are pessimistic that new information leads to more disagreement. Therefore, we presume that the impact of the BW sentiment on the elasticity will be positive. However, for the FEARS index, we expect a negative impact on the elasticity, since higher FEARS means lower sentiment.

The elasticity estimate including the effect of the two sentiment measures are displayed in Table 2 below.

The elasticity estimate is in line with the estimates previously obtained in Table 2. We observe an elasticity that is decreasing with the absolute value of moneyness and with increases in tenor, indicating that new information leads to higher disagreement about tails of the market returns and about the future market returns farther in the future.

The impact of the sentiment measures in the elasticity estimate is as expected. At times of optimism, there is an increase in the elasticity, indicating that new information leads to less disagreement when investors are optimistic. Notice that optimism is captured by positive values in the BW index or negative values in the FEARS index. At times of pessimism, however, there is a decrease in the elasticity, which indicates that new information leads to more disagreement when investors are pessimistic. Observe that the effect of both sentiment measures are significant, which is a sign that sentiment measured from textual sources (such as the FEARS index) can complement the sentiment obtained from market proxies (as in the BW index).

A visual representation of the elasticity at different levels of moneyness and tenor is displayed in Figure 3 below.

The figure displays elasticity values similar to that shown in Figure 2. The elasticity is the highest for at-the-money options that are about to expire (in a day). There is a decrease in elasticity as we move away from-the-money and as tenor increases. However,

	Parameter Estimate	Standard Error
Baseline Estimates		
Intercept (α_0)	7.4260	0.072
Elasticity (β_0)	1.1744	0.017
Explanatory Variables for Elasticity		
$\lvert$Moneyness$\rvert$	-0.4048	0.010
ln (Tenor)	-0.1418	0.005
$\lvert$Moneyness$\rvert \cdot$ ln (Tenor)	0.0491	0.003
BW	0.0301	0.003
FEARS	-0.0238	0.004
Adjusted R^2	0.268	
Number of Observations	256453	

Table 2: Volume-Volatility Elasticity Estimates with the Inclusion of Sentiment Measures. The table reports the estimated values of α_0, β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. In addition to those control variables, we include the sentiment measure from Baker and Wurgler (2006) and Baker and Wurgler (2007) and the FEARS index from Da, Engelberg, and Gao (2014). The estimation is based on a partial sample of cleaned data, spanning 2007 to 2015. Values for volume and volatility are computed daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The two sentiment measures are standardized. The standard errors are heteroscedasticity robust as in White et al. (1980).

due to the effect of the interaction between moneyness and tenor, the effect of increases in tenor changes sign for options that are deep OTM, leading to increases in elasticity for such options.

7.2 Non-parametric Results

The results presented before are based on the estimation of Equation 3. We now apply a nonparametric approach to recover the volume-volatility elasticities. The idea is to allow for a more flexible impact of moneyness and tenor on the elasticity estimate. To do so, we categorize each option by moneyness and tenor. Moneyness and tenor are split in intervals of different sizes to accommodate a varying number of options available. The intervals for moneyness range from -5 to 3, which is the range of moneyness levels available in the data. The intervals for tenor range from 1 day to 120 days, which is also the range of tenors considered in this study.

After categorizing options by moneyness and tenor, we estimate the volume-volatility elasticity for each category. The elasticity estimates are presented as a surface in Figure 4 below.

The surface reveals similar results to the parametric estimates. The elasticity estimate achieves its maximum value for options close to the money (moneyness category $(-0.5, 0.5]$) and with short tenor (tenor category $(1, 20]$). The elasticity decreases as we move to moneyness categories that are far from the money. This decrease in elasticity occurs irrespective of the tenor category. Also consistent with previous estimates is the

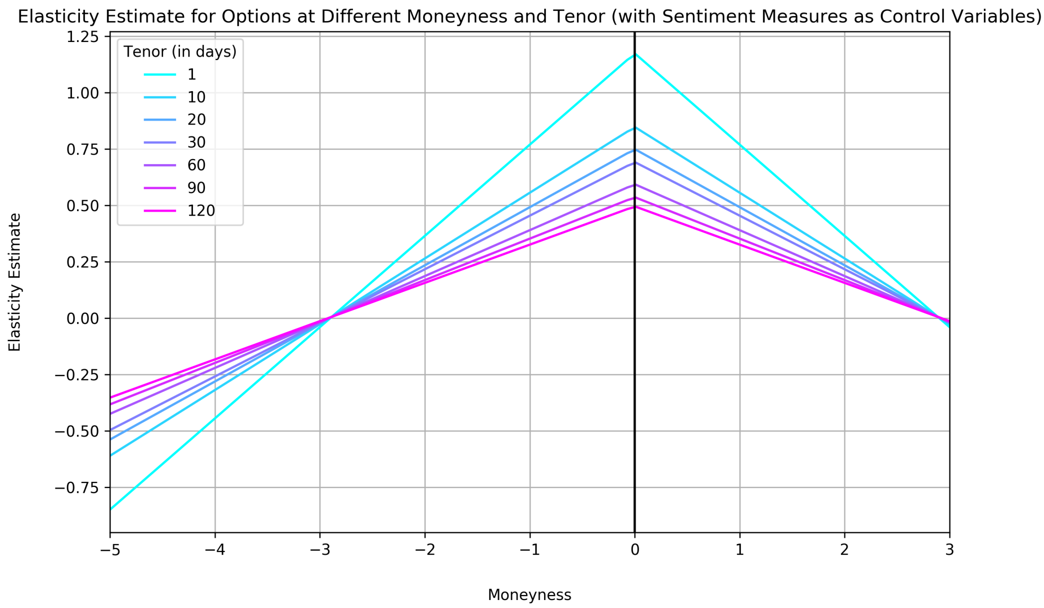

Figure 3: Elasticity Estimates for Options at Different Moneyness and Tenor with Sentiment Measures are Included as Control Variables. The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.

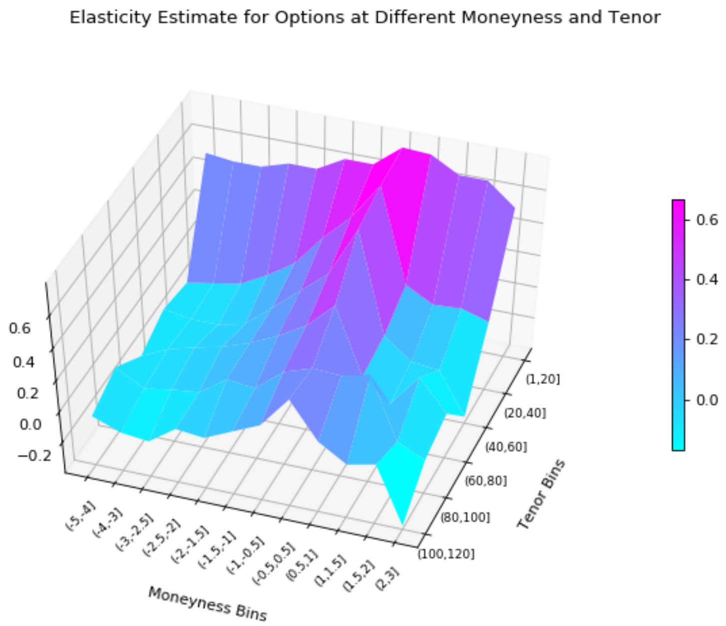

Figure 4: Elasticity Estimates for Options Categorized by Moneyness and Tenor. The figure shows a surface of volume-volatility elasticity estimates for different moneyness and tenor categories. Values at the left of the figure represent estimates for OTM options. Values deeper in the figure represent estimates for options with shorter tenors. The highest estimate is achieved for moneyness close to zero and short tenor. The elasticity decreases for options far from the money and for options with higher tenor.

decrease in elasticity as we move from options with short tenor to options with long tenor. This decrease also occurs irrespective of the options moneyness. Lastly, we observe that for options with high tenor (above 60 days) the decrease in the elasticity as the options get far from the money is attenuated in comparison to the effect for options with short tenor. This is also consistent with the previous estimates.

The nonparametric estimates reveal dynamics that were not captured by the parametric estimates. First, for options with short tenor (up to 20 days to expiration) we observe an elasticity estimate that changes less with the moneyness of the option. This is consistent with the idea that the probability of big moves in a short span of time is small, so that new information should not lead to higher disagreement about the tails.

Second, there is a sharper drop in the elasticity when the moneyness moves from zero to positive values (from ATM to ITM) than when it moves to negative values (from ATM to OTM). This implies that the release of new information generates more disagreement about the right-tail of the market returns than about the left-tail.

Third, while there is an overall increase in the disagreement as tenor increases, this decrease is attenuated for options ATM. Indeed, notice that there is a slight decrease in elasticity when tenor goes from the range $(1, 20]$ to $(20, 40]$, and almost zero decrease when tenor further increases to the range $(40, 60]$. The lack of a decrease in elasticity indicates that new information does not generate more disagreement regarding the valuation of ATM options that expire in less than 60 days.

Fourth, the highest elasticity estimate is not close to unity. In the parametric estimates, the elasticity is close to unity when moneyness is zero and tenor is one. However, the parametric estimates reflect the projection of the data on the model and take into account the downward trend when tenor increases. Therefore, the elasticity close to unity is possibly just an extrapolation of the negative effect of the tenor.

To better understand the nonparametric elasticity estimates, we plot them separately for each tenor category while allowing moneyness to change. See Figure 5 below.

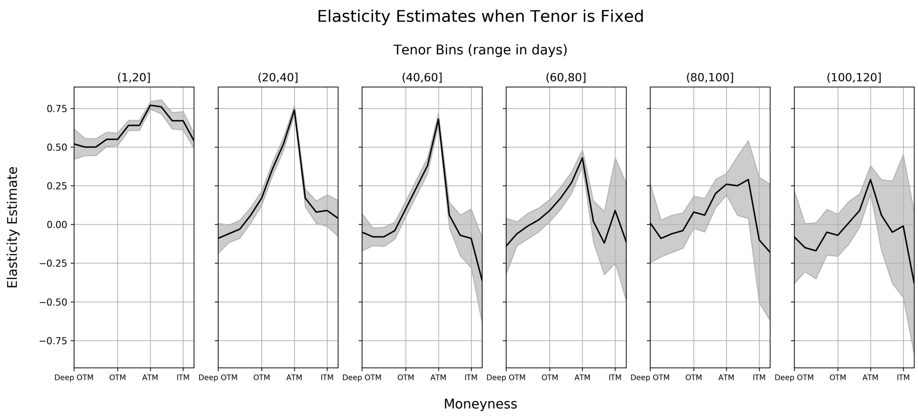

Figure 5: Elasticity Estimates for Options on Different Tenor Categories. The figure shows six plots. Each plot displays the elasticity estimates for varying degrees of moneyness. The first plot on the left estimates the elasticity using options of short tenor, while the last plot on the right uses options that expire in about 4 months. In each plot, the moneyness varies between Deep OTM (-5), OTM (-2), ATM (0) and ITM (2). The shaded region represents a 99% confidence interval.

The first plot on the left shows the volume-volatility elasticity for options that expire in less than 20 days. The elasticity is highest for ATM options and slowly decreases when moneyness moves away from zero. The elasticity estimates for these short tenor options are higher at every moneyness level when compared to the estimates for options with higher tenors. This implies that new information generates the least disagreement for options that are expiring soon.

The second plot from left-to-right shows the elasticity for options that expire in around 1 month. The least disagreement occurs for ATM options, and the elasticity estimate is close to that for ATM options with short tenor. However, contrary to the options with short tenor, the disagreement sharply increases as moneyness moves away from zero. Notice that the elasticity decreases almost linearly with moneyness, which is in line with the parametric estimates analyzed before. There is a sharper decrease in elasticity when moneyness increases from zero than when moneyness decreases from zero, indicating that there is more disagreement about the right tail of market returns.

The third plot from left-to-right displays the elasticity for options that expire in almost two months. This plot displays the same characteristics as options that expire close to 1 month.

The remaining three plots display the elasticity for options that expire in over two months. The behavior of the elasticity is similar to options with short tenor: elasticity is the highest when moneyness is zero and the elasticity almost linearly decreases when moneyness moves away from zero. However, there is a decrease in the highest elasticity, which is in line with tenor increases, leading to more overall disagreement.

Next, we analyze what happens to the elasticity as tenor increases, while fixing the moneyness of options. Figure 6 displays the results.

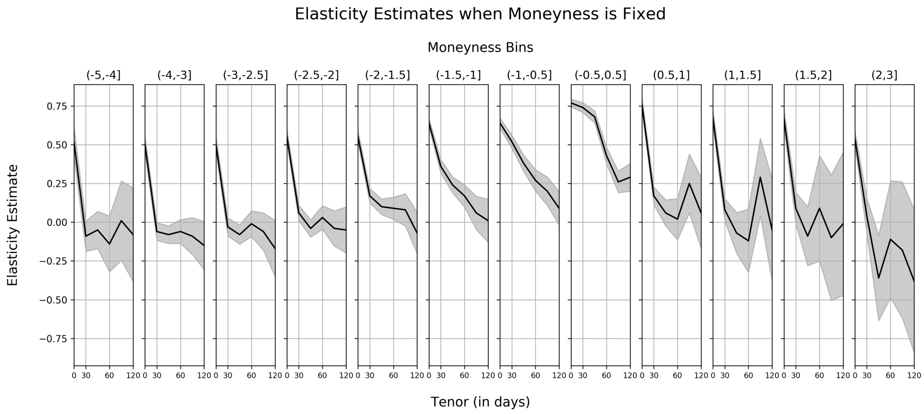

Figure 6: Elasticity Estimates for Options on Different Moneyness Categories. The figure shows twelve plots. Each plot displays the elasticity estimates for varying degrees of tenor. The first plot on the left estimates the elasticity using options that are deep OTM, while the last plot on the right uses options that are deep ITM. In each plot the tenor varies from 1 day to 120 days. The shaded region represents a 99% confidence interval.

All plots display a similar behavior. The elasticity sharply decreases when the tenor increases. This is consistent with the parametric estimates and indicates higher disagreement about future market returns. Options that are ITM or have moneyness smaller than -1.5 display a decrease in elasticity when tenor moves from 1 day to 30 days, but as the tenor further increases, there is only a small decrease in elasticity. This indicates a sharp increase in disagreement about the distribution of market returns in the short term, but there is not an increase in disagreement about market returns in the longer term (2 or 3 months). This is not the case with options that are ATM or OTM, where there is a linear decrease in elasticity when tenor increases. Last, notice that for options that are ATM, there is only a small decrease in elasticity when tenor increases from 1 day to 60 days, consistent with the previous findings. Overall, these findings indicate there is an almost linear (or piece-wise linear) relationship between disagreement and tenor.

The next section concludes the main findings of the present work.

8 Conclusion

In the Kandel-Pearson model, there are two motives for investors to trade a risky asset: changes in the asset's prices and disagreement between investors regarding new information. Bollerslev, Li, and Xue (2018) explicitly connects the disagreement between investors to the elasticity between trade volume and price volatility, resulting in two main implications. First, the volume-volatility elasticity is equal to unity only when there is no disagreement between traders. Second, the volume-volatility elasticity decreases if the disagreement between investors increases. The present work analyzes these two implications in the context of the S&P 500 options market.

We present empirical evidence consistent with the implications from the Kandel-Pearson model. We argue that there are two natural measures of disagreement for the options market: moneyness and tenor. These two measures speak to the distribution of the market index at different quantiles and at different times. We estimate a volume-volatility elasticity close to unit for options that are at-the-money and about to expire, which is consistent with the case of no disagreement among investors. We also find that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, implying that new information leads to more disagreement regarding tail events for the market index. Lastly, we find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index. These findings are also supported by nonparametric estimates.

References

Baker, Malcolm and Jeffrey Wurgler (2006). "Investor sentiment and the cross-section of stock returns". In: *The journal of finance* 61.4, pp. 1645–1680. URL: https://doi.org/10.1111/j.1540-6261.2006.00885.x.

— (2007). "Investor sentiment in the stock market". In: *Journal of economic perspectives* 21.2, pp. 129–152. URL: http://www.aeaweb.org/articles?id=10.1257/jep.21.2.129.

Bollen, Nicolas PB and Robert E Whaley (2004). "Does net buying pressure affect the shape of implied volatility functions?" In: *The Journal of Finance* 59.2, pp. 711–753. URL: https://doi.org/10.1111/j.1540-6261.2004.00647.x.

Bollerslev, Tim, Jia Li, and Yuan Xue (2018). "Volume, volatility, and public news announcements". In: *The Review of Economic Studies* 85.4, pp. 2005–2041. URL: https://doi.org/10.1093/restud/rdy003.

CBOE, Chicago Board Options Exchange (2014). *The CBOE Volatility Index - VIX.* Tech. rep. URL: https://web.archive.org/web/20180427204307/https://www.cboe.com/micro/vix/vixwhite.pdf.

Da, Zhi, Joseph Engelberg, and Pengjie Gao (2014). "The sum of all FEARS investor sentiment and asset prices". In: *The Review of Financial Studies* 28.1, pp. 1–32. URL: https://doi.org/10.1093/rfs/hhu072.

Duffie, Darrell (2001). *Dynamic asset pricing theory.* Princeton University Press.

Hamilton, James D (1995). "Time series analysis". In: *Economic Theory. II, Princeton University Press, USA*, pp. 625–630. URL: https://doi.org/10.1017/S0266466600009440.

Hong, Harrison and Jeremy C Stein (2007). "Disagreement and the stock market". In: *Journal of Economic perspectives* 21.2, pp. 109–128. URL: http://www.aeaweb.org/articles?id=10.1257/jep.21.2.109.

Hull, John C (2003). *Options futures and other derivatives.* 5th. Pearson Education India. URL: https://www.pearson.com/us/higher-education/product/Hull-Options-Futures-and-Other-Derivatives-5th-Edition/9780130090560.html.

Kandel, Eugene and Neil D Pearson (1995). "Differential interpretation of public signals and trade in speculative markets". In: *Journal of Political Economy* 103.4, pp. 831–872. URL: https://www.jstor.org/stable/2138584.

Karpoff, Jonathan M (1987). "The relation between price changes and trading volume: A survey". In: *Journal of Financial and quantitative Analysis* 22.1, pp. 109–126. URL: https://doi.org/10.2307/2330874.

Lemmon, Michael L and Sophie Xiaoyan Ni (2008). "The effects of investor sentiment on speculative trading and prices of stock and index options". In: URL: https://ssrn.com/abstract=1306237.

Ni, Sophie, Jun Pan, and Allen Poteshman (2008). "Volatility information trading in the option market". In: *The Journal of Finance* 63.3, pp. 1059–1091. URL: https://doi.org/10.1111/j.1540-6261.2008.01352.x.

Tauchen, George E and Mark Pitts (1983). "The price variability-volume relationship on speculative markets". In: *Econometrica: Journal of the Econometric Society*, pp. 485–505. URL: https://www.jstor.org/stable/1912002.

White, Halbert et al. (1980). "A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity". In: *Econometrica* 48.4, pp. 817–838. URL: https://www.jstor.org/stable/1912934.

9 Appendix

9.1 Cleaning Procedures

The computation of the residual volatility of options uses high-frequency data on the S&P 500 index options (SPX), and on the SPDR S&P 500 exchange-traded fund (SPY).

The data set on the SPX options was obtained from the Chicago Board Options Exchange (CBOE). The set contains quotes at the 1-minute interval level from 2007 to 2016. The market hours are from 9:30 am to 4:15 pm EST, resulting in 405 quotes per day. Partial trading days are removed from the sample (23 in total), and we base the analysis on the remaining 2458 days. On 28 trading days, there are missing values for the first minute or two at the market open. These missing values are filled with the first non-zero prices of the same day. As noted in Section 5, the options data are sparsely sampled to 5-minutes to mitigate the impact of microstructure noise on volatility estimates.

The data set on SPY was obtained from TickData[4]. The set contains the last closing price of every 5-minute interval, from 2007 to 2016.

To obtain the moneyness of the SPX options, we need their time to expiration. We follow the methodology of CBOE (2014) to compute the time to maturity in minutes, while dealing with the differences between AM and PM settled options and options that expire on holidays. We compute the moneyness of all options daily at the end of the day. We select options with moneyness ranging from -5 to 3, which captures a wide variety of options while eliminating options that have pricing issues due to extremely low liquidity.

Lastly, the composition of expiration dates of SPX options varies throughout the years and could lead to issues in prices and estimation of the residual volatility. The history of SPX options is relevant in understanding how so. SPX options were launched in 1987, and originally expired every 3rd Friday of the month. Their increasing success warranted the expansion of SPX options to include new expiration dates. In 2005, CBOE introduced SPX Weeklys, which expire on all other Fridays. However, at launch, these options had much lower liquidity than the original SPX options, but their liquidity increased substantially over the years[8]. Now, because the residual volatility is estimated from the prices of options, it is imperative to have prices that directly reflect the real value of options. For this reason, options with low liquidity, or higher spreads, are disregarded in favor of options with high liquidity. In the case of SPX options, the liquidity is related to the expiration dates, since the original SPX options had higher liquidity than the Weeklys at launch, and for some years after that. To take into account this difference, we only include options that do not have stale prices. An option is said to have stale prices if its intra-day prices do not change on more than 50% of the time intervals within a day.

9.2 Volatility Signature for SPX Options

Figure 7 displays a more complete volatility signature plot for the market options. The options are categorized by moneyness, and their realized variances are computed for different sampling frequencies.

We observe no increase in the realized variance when the sampling frequency increases for options that are ITM, ATM and slightly OTM. However, for options with moneyness smaller than -2.5 there is a clear indication of microstructure noise, since sampling frequencies smaller than 5-minutes lead to an ever increasing estimates of variance. Overall,

[8]See SPX Weeklys volume chart on CBOE's website.

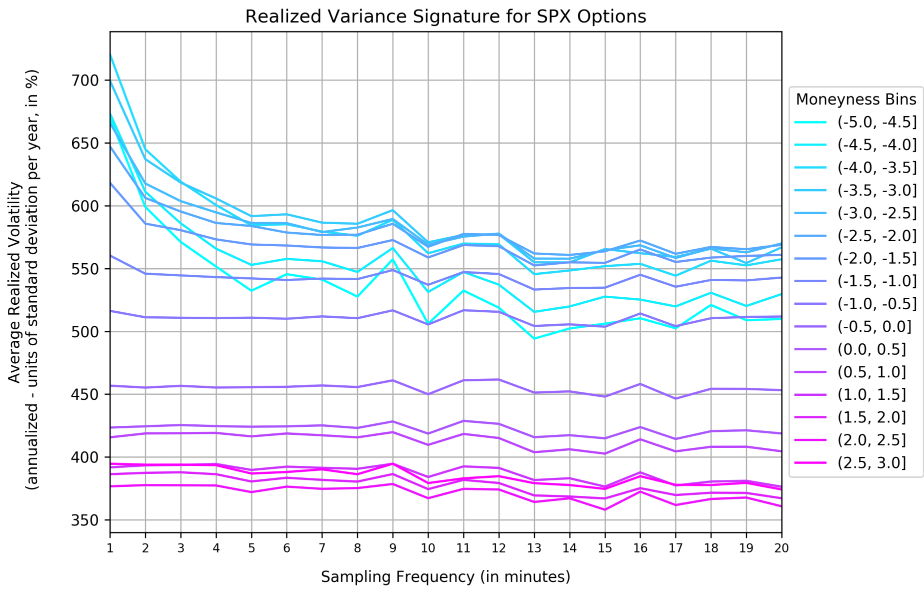

Figure 7: Volatility Signature for SPX Options. The figure shows the average realized volatility (annualized in percentage) for options of different moneyness based on different sampling frequencies. The realized volatility is estimated as the sum of squared returns over each day for each option, where the frequency of returns varies from 1-minute up to 20-minutes. The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

a sampling frequency of 5-minutes provides a good trade-off between avoiding microstructure noise and estimating the realized variance with enough precision.

9.3 Elasticity Surface from Parametric Estimates

Figure 8 displays a surface of the elasticity estimates for varying degrees of moneyness and tenors.

9.4 Parametric Estimates Plot and Surface Allowing Moneyness Effect to Change for OTM and ITM

The parametric specification analyzed in Section 7 assumed the effects of moneyness on elasticity were symmetrical, in the sense that increases in moneyness away from zero had the same impact as decreases in moneyness. However, the nonparametric results from 7.2 indicate that the decrease in elasticity is higher when moneyness increases away from zero than from when it decreases. We re-estimate the parametric specification allowing the impact of moneyness to be asymmetrical.

Figure 9 below displays the estimated elasticities for different levels of moneyness and tenor. The characteristics are similar to Figure 2, with the exception of a sharper drop in elasticity (higher increase in disagreement) as moneyness moves from zero to positive values.

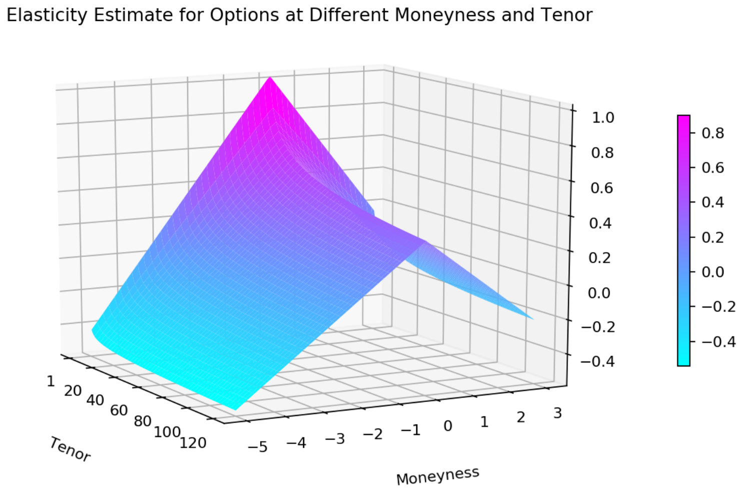

Figure 8: Surface of Elasticity Estimates for Options at Different Moneyness and Tenor. The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.

	Parameter Estimate	Standard Error
Baseline Estimates		
Intercept (α_0)	7.5726	0.058
Elasticity (β_0)	1.0186	0.013
Explanatory Variables for Elasticity		
$\mathbb{1}\{Moneyness > 0\} \cdot \lvert\text{Moneyness}\rvert$	-0.2258	0.007
$\mathbb{1}\{Moneyness \leq 0\} \cdot \lvert\text{Moneyness}\rvert$	-0.4873	0.009
$\ln(\text{Tenor})$	-0.1474	0.004
$\lvert\text{Moneyness}\rvert \cdot \ln(\text{Tenor})$	0.0138	0.002
Adjusted R^2	0.243	
Number of Observations	446053	

Table 3: Volume-Volatility Elasticity Estimates. The table reports the estimated values of α_0, β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. The absolute value of moneyness is interacted with an indicator for negative moneyness and for positive moneyness. Estimation is based on full sample of cleaned data, spanning 2007 to 2016. Values for volume and volatility are computed daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are heteroscedasticity robust as in White et al. (1980).

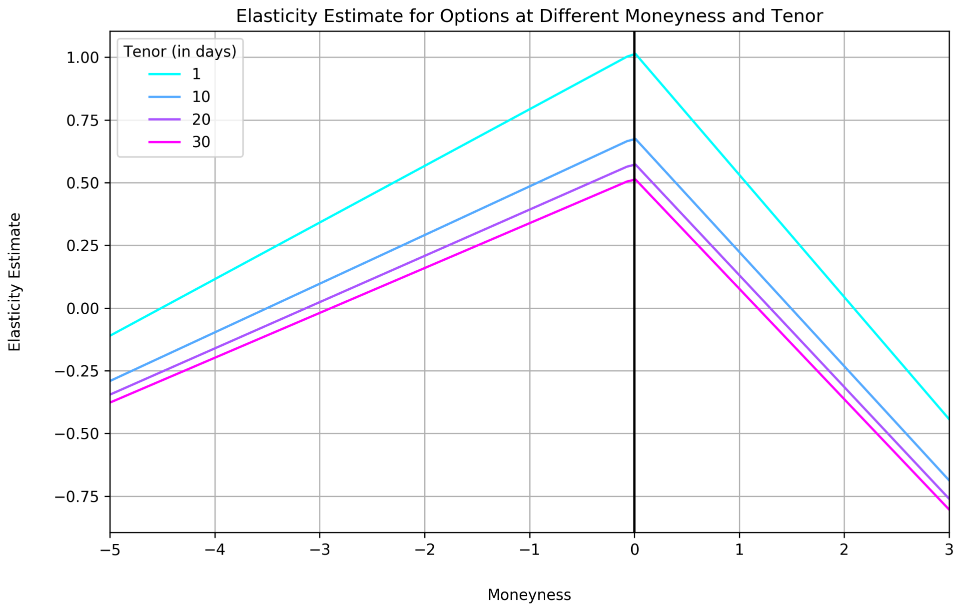

Figure 9: Elasticity Estimates for Options at Different Moneyness and Tenor. The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.