

Volume, Volatility, and Disagreement in Market Index Options

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Abstract

We provide new empirical evidence of disagreement between investors in the S&P 500 options market. Our findings rely on high-frequency intraday price and volume data for options on the S&P 500 index. We base our findings on a disagreement model where investors observe public information but agree to disagree on its interpretation. In this framework, the disagreement between investors is captured by the volume-volatility elasticity. The options market provides two natural variables that are sharply related to disagreement: moneyness and tenor. We argue that these variables speak to disagreement about the distribution of the market index at different quantiles and at different times. We estimate a volume-volatility elasticity equal to unity for options that are at-the-money and near expiration, which is consistent with the case of no disagreement among investors. The elasticity estimate for other options, however, are all below unity, consistent with the case of disagreement between investors. Our elasticity estimates decrease with increases in the absolute value of moneyness, indicating investors have a higher disagreement about rare events. We also find that the elasticity decreases with increases in tenor, implying in higher investors' disagreement about events farther into the future.

Keywords: SPX options, market index, high-frequency data, disagreement, volume-volatility elasticity, public information

1 Introduction

This paper studies disagreement between investors in the S&P 500 options market. The strength of the relationship between a financial asset's trade volume and its volatility reveals investors' disagreement about new information. This study uses high-frequency data on market index options to estimate the volume-volatility elasticity. Our results are consistent with the implications of a theoretical model where investors are subject to new information and may disagree about its interpretation, leading to trade based

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on such disagreement. The model implies that the volume-volatility elasticity is equal to unity in the case of no disagreement among investors, and that the elasticity should decrease as disagreement increases. We find volume-volatility elasticity estimates below unit, consistent with the case of disagreement among investors. We also find that the volume-volatility elasticity decreases with the measures of disagreement considered in the study. These disagreement measures also speak to the distribution of the market index, and the elasticity estimates reveal greater disagreement regarding the tails of the distribution, but a lack of disagreement regarding the center of the distribution.

According to Cochrane (2016):

Volume is the great unsolved problem of financial economics.

The problem stems from the no-trade theorem¹ of Milgrom and Stokey (1982), which implies that stock markets should see no trade. However, the reality of financial markets is that of a huge volume of trades occurring every year. Indeed, the United States' stock market turnover has been above 100% per year since 1998², implying that a value equivalent to the entire stock market changes hands at least once a year.

An important reason for the existence of financial markets is so that new information is incorporated into prices. Consider a market with passive investors (investors that buy and hold the market index), and informed and uninformed active investors. In this market, most of the trade volume should be due to trades by the informed and uninformed active investors. The trading between these investors reveals information, which is necessary to move prices to the efficient level.

In this market with informed and uninformed investors, the informed agents have a good reason to trade (information), but uninformed agents do not. In this context, understanding trading volume amounts to understanding why uninformed investors trade. French (2008) studies this problem and argues that, after accounting for active trading costs, informed and uninformed investors are playing a negative-sum game. However, this negative-sum game provides an upside for society, which is an improvement in the efficiency of financial prices. French (2008) also argues that price discovery is an externality, since investors have an effort cost. Due to the effort costs, if the rewards for finding information are not sufficient, then there might be too little trading volume in the market and prices might be far from efficient. Therefore, to understand how information is incorporated into prices, we need to understand the motives that generate trade volume and how trade volume relates to prices.

The relationship between trade volume and prices (or volatility) has been previously analyzed by several works. Karpoff (1987) surveys the earlier empirical evidence and argues that understanding the relationship between trade volume and prices is important due to four reasons. First, it allows us to distinguish between different hypotheses about the structure of financial markets. Second, trade volume can be key to quantifying the relevancy of a news event. Third, volume is related to stochastic variance and can be used to understand the distribution of speculative prices. Fourth, volume in the equities market can have an impact on the futures market. The surveyed empirical works find that there is a positive contemporaneous correlation between trade volume and returns (see tables 1 and 2 of Karpoff (1987)), and Karpoff concludes that:

¹See Mancini (2001) (pages 30 to 37) for a discussion of many no-trade theorems.

²See <https://fred.stlouisfed.org/series/DDEM01USA156NWDB> for the stock market turnover ratio. The turnover ratio measures the total value of stocks traded in a year divided by the total market capitalization. The turnover indicates how much of the stock market exchanges hands in a year.

It is likely that observations of simultaneous large volumes and large price changes [...] can be traced to their common ties to information flows [...].

An early explanation that ties volumes and returns to information flows is that of Tauchen and Pitts (1983). Tauchen and Pitts (1983) provide a statistical explanation for the positive correlation between volume and returns. Tauchen and Pitts argue that changes in price and in volume are both driven by a process that models the rate of information arrival. In their model, squared returns and volume are both positively related to the information arrival process, which leads to the positive correlation between volatility and volume found by many empirical works. Their model also implies that both the variance of price changes and trading volume depends on the extent to which investors disagree about new information (see equations 4, 5 and 6 in Tauchen and Pitts (1983)). However, their work is focused on understanding the relationship between returns and volume as more investors enter the market. The explanation based on the common information arrival process is known as the mixture of distribution hypothesis (MDH). The MDH provides a statistical explanation for the empirical findings, but it is deficient in providing the economic channel between new information and the relation of volume and volatility.

Other explanations that provide an economic channel are discussed in Cochrane (2016) and in Hong and Stein (2007). Cochrane (2016) argues that many of the explanations provided by the literature cannot account for the large trade volume observed in financial markets:

The sheer volume of trading is the puzzle. All these non-information mechanisms — life-cycle, preference shocks, rebalancing among heterogeneous agents [...], preference shifts, generate trading volume. But they do not generate the astronomical magnitude and concentration of volume that we see.

The arguments by Hong and Stein (2007) also revolve around the same idea: the volume in rational models is due to liquidity needs and portfolio rebalancing, which cannot account for the total volume observed in financial markets.

Hong and Stein (2007) surveys another strand of the literature on volume and prices that could account for the huge volume observed in the markets. This strand of the literature considers heterogeneous-agent models where agents disagree about new information (disagreement models). Different disagreement models have incorporated mechanisms that allow for gradual information flow (news arrive at different investors at different times), limited attention (investors process only a subset of all public information) and heterogeneous priors (different investors interpret same news differently). But, at the core of this class of models is the idea that different investors have different beliefs. When investors receive new information, their differences in beliefs leads to disagreement about the interpretation of the news. This disagreement between the investors generates an additional motive for trading, other than changes in the equilibrium price. And the trading due to the disagreement results in changes to prices and trading volume. Hong and Stein (2007) argue that disagreement models are compelling since they model the joint behavior of prices and volume, while providing the underlying mechanism (difference in beliefs) that connects new information to volatility and volume.

Kandel and Pearson (1995) develop a disagreement model. In the Kandel-Pearson model, investors trade a risky asset and observe public information. Each investor uses a different economic model to interpret the news, which leads to disagreement about the

value of the risky asset and generates trading. The trading generated by disagreement is not directly dependent on changes in the asset price. Therefore, in the Kandel-Pearson model, it is possible to have high trade volume even when there is no price change.

In Bollerslev, Li, and Xue (2018) the authors extend the implications of Kandel and Pearson (1995) and find an expression that directly connects disagreement between investors to the elasticity between volume and volatility. The expression that connects disagreement to the volume-volatility elasticity has two main implications. The first is that the volume-volatility elasticity is equal to unity only when there is no disagreement between traders. The second is that the elasticity decreases when the disagreement between traders increases. Bollerslev, Li, and Xue (2018) validate the two implications from the volume-volatility elasticity for the market index. The authors take into account proxies of investor’s disagreement and estimate a volume-volatility elasticity equal to unity and that decreases when the disagreement proxies increase.

The present work analyzes the connection between disagreement, trading volume and volatility in the context of the S&P 500 options market (SPX options). A benefit of using options data instead of stock data is the availability of two direct measures of disagreement: moneyness and tenor. We argue that moneyness and tenor speak to the distribution of the market index at different quantiles and at different times. This allows us to uncover disagreement between investors about different parts of the distribution of market returns.

We present empirical evidence consistent with the implications from the Kandel-Pearson model and Bollerslev, Li, and Xue (2018). We estimate a volume-volatility elasticity equal to unity for options that are at-the-money and near expiration, which is consistent with the case of no disagreement among investors. We also find that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, implying that new information leads to more disagreement regarding tail events for the market index. We find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index. Last, we estimate the volume-volatility elasticity in a nonparametric setting and obtain results that support our initial findings.

The article is organized as follows. Section 2 briefly discusses the literature on the trading volume of options and how it relates to disagreement. Section 3 discusses the Kandel-Pearson model and the extension proposed by Bollerslev, Li, and Xue (2018), including the volume-volatility elasticity and its interpretation regarding disagreement about new information. Section 4.1 argues why moneyness and tenor are measures of disagreement and how to interpret these measures with respect to the volume-volatility elasticity. Section 4.2 discusses how to measure the volatility of options and argues for the use of a measure of residual volatility. Section 4.3 describes the high-frequency options data used in the empirical section and discusses how to avoid microstructure noise effects. Section 4 discusses the parametric specification used to estimate the volume-volatility elasticity. Section 5 presents the main empirical findings using SPX options data. This section also discusses findings from a nonparametric estimation procedure that confirms the main findings from the parametric setting. Section 6 concludes the paper. Last, the Appendix offers details on cleaning procedures and additional figures.

2 Literature Review

The relationship between volume and volatility in the options market has been studied in different contexts. Several works in the literature analyze the trading volume of options and its information content. A strand of the literature studies the relationship between options volume and returns in the stock market and tries to establish whether there is a lead-lag relationship between both markets. Easley, O'hara, and Srinivas (1998) develop a model where informed investors trade in the stock market and in the options market in equilibrium. The model predicts that stock market returns lead³ trading volume in the options market due to hedging, which is supported by the authors' empirical findings. However, the authors also find evidence that the volume of some options lead the returns of the stock market.

The finding that trades in the options market influences the stock market is supported by several other works. Pan and Poteshman (2006) use a data set that identifies the types of investors in the options market and whether their investments open new positions or close existing ones. Pan and Poteshman find evidence that the trade volume of options predicts stock returns. The predictability is stronger when using the volume of out-of-the-money (OTM) options compared to in-the-money (ITM) options. The predictability is also stronger when there is a higher concentration of informed investors. However, the predictability is limited to firm-specific options and is not present for market index options. The authors argue that the return predictability is due to firm-specific nonpublic information.

C. Cao, Chen, and Griffin (2005) study the information content in options volume prior to takeover announcements. Cao, Chen and Griffin find that volume imbalances in the options market prior to takeover announcements predicts next-day stock returns. They also find that most trades take place with slightly OTM call options, since they provide the highest leverage. However, the authors find that options volume provide no predictability during times with no takeovers.

Hu (2014) analyze whether order imbalances in the options market leads to imbalances in the stock market. The author argues that due to the lower liquidity of the options market, when investors take new positions in options, the market maker often needs to hold the options position until expiration. Market makers then hedge their positions by trading in the stock market. Thus, an order imbalance in the options market generates and imbalance in the stock market. Hu finds that imbalances in the options market can predict next-day stock returns, and that the predictability is stronger for options that are close-to-the-money or in-the-money and for stocks of smaller companies.

Ge, Lin, and Pearson (2016) study whether trade volume that open new positions in options can predict stock returns. The authors find that volume that opens positions in calls predict positive stock returns, and volume that opens positions in puts predict

³Most of the literature on the lead-lag relationship between options and stocks finds that trading in options leads trading in stocks. In Easley, O'hara, and Srinivas (1998), the authors find that stock returns lead options' trade volume. Their finding is reasonable, since options are often used for hedging. Now, the work by Stephan and Whaley (1990) also finds that stocks lead options, but in the case of options' transaction prices. Specifically, Stephan and Whaley find that stock returns lead price changes in the options market by 15 minutes on average. However, the findings by Stephan and Whaley (1990) have been contested by Chan, Chung, and Johnson (1993). Chan, Chung and Johnson argue that the finding is due to the use of transaction prices (instead of bid-ask quotes) combined with infrequent trades, which generate the spurious relationship found in Stephan and Whaley (1990). Thus, the majority of the literature explores the impact of the options market on the stock market.

negative stock returns. The authors argue that the embedded leverage of options is what attracts informed investors to the options market and is what drives the predictability of the stock returns.

Cai and Du (2018) investigate the connection between options trading volume and stock factors. The authors argue that stocks with higher beta (higher correlation to the market index) also have higher volatility, which, due to short-sales constraints, leads to the stock being overvalued. On the other hand, high-volatility stocks have a higher option demand due to hedging. The authors argue that these two aspects link options trading volume to overvalued stocks that have lower expected returns. Cai and Du find a negative relationship between the trading volume of options and the stock returns. The authors also find that a strategy that sells stocks with high options trading volume and buys stocks with low options trading volume generates positive abnormal returns.

Ryu and Yang (2018) analyze the information content of options trading volume in the context of international markets. Ryu and Yang use a data set that specifies the type of investor (foreign or domestic) and whether their investments open or close positions. The authors find that trades by foreign investment firms that open a new position predict next-day stock returns, but the trades of domestic firms do not have predictive power. The predictability is greater when OTM and short-term options are traded by the foreign investment firms. The predictability also increases with the volume of orders and when the underlying asset has short-sale restrictions.

Although most papers argue in favor of price discovery in the options market, Muravyev, Pearson, and Broussard (2013) present evidence contrary to that. The authors analyze the price movements of stocks and options when there are deviations from the put-call parity. They find that stock prices do not adjust when deviations from the parity occur. Instead, the price of the associated option moves to eliminate the deviation from the parity. The authors argue that since prices implied by options do not affect the equity markets, options prices do not contain information that is not already incorporated in the stock prices.

Another strand analyzes the relationship between options volume and stock volatility. S. Ni, Pan, and Poteshman (2008) investigate whether options volume has information about future stock realized volatility. Ni, Pan and Poteshman argue that traders with information about stock volatility would trade in the options market, implying that excess demand for exposure to volatility should predict volatility in the stock market. To analyze this implication, the authors construct a measure of volatility demand based on the number of open positions on calls and puts. They find that volatility demand predicts stock volatility for up to five days. The authors also find that volatility demand has a price impact on options at moments of greater information asymmetry, which is consistent with the idea that market makers update prices to protect against investors with superior volatility information.

Wang (2013) finds evidence similar to that of S. Ni, Pan, and Poteshman (2008) but for the market index (S&P 500). Wang uses data on VIX and SPX options to analyze the trade volume impact. The author argues that trading in VIX options is purely motivated by trading information on market volatility. Wang finds that trade volume of VIX calls have information about the future realized volatility of the market index. The author also finds a stronger predictability when there is higher uncertainty and information asymmetry in the market. The author also argues that the predictability is weaker when using the volume of SPX options, since trading in SPX options is impacted by factors other than volatility.

Other works study how the trading volume of options relates to the release of new information. H. H. Cao and Ou-Yang (2009) develop a model of trading based on differences of opinion about public information. The work is purely theoretical, but their model implies that new information leads to an increase in trading volume both in the stock and options markets. In the stock market, the volume increases at the time the new information is made available and decreases slowly, but in the options market the increase in trade volume occurs even before the arrival of the information and decreases quickly.

Choy and Wei (2012) argue that trading volume in options is not due to pure information trading, but rather due to difference of opinions. The authors use EADs to control for informed trading, and the stock returns after the announcements serve as the "ultimate revelation" of private information. Choy and Wei argue that if options trading is purely informational, then there should be no change in trading volume before and after EADs with no new information (announcements where returns were close to zero). However, if there is a change in trading volume even after controlling for information, then it should be due to other factors, possibly disagreement between investors. The authors find that information alone is not enough to justify all the trading, and argue that trading around the EADs is speculative and dominated by retail investors. They also find that any predictability of post-announcement stock returns by pre-announcement options volume is explained by pre-announcement stock returns, which also indicates a lack of informed trading in options. The authors then use proxies for differences of opinion and find that they can explain the trading volume of options both in the cross-section and in time-series.

Lemmon and S. X. Ni (2011) study the trading volume of stock and market index options in relationship to stock returns and sentiment. The authors argue that stock options are traded by unsophisticated investors who use options to speculate on future stock returns, while index options are traded for hedging demands of sophisticated investors. The authors find that sentiment and past market returns are both positively related to the trading volume of stock options, but not to the volume of market index option. The authors argue that the differences in the relationship between volume is due to the differences in the composition of traders in stock and market index options. They argue this is an indication that assets traded due to speculation are influenced by sentiment, while assets traded for other motives are less impacted by changes in sentiment.

Andreou et al. (2018) build a measure of disagreement based on options trading volume and use it to explain stock returns. The authors argue that investors that speculate on directional moves trade at a moneyness level that corresponds to their views. Optimist investors buy calls with high strikes, while pessimist investors buy puts with low strikes. In this context, if investors differ in opinion there will be a range of pessimist and optimist investors, leading to trade volume across a wide range of strike prices. However, if there is no disagreement, investors' views are going to be concentrated and trade will occur over a smaller range of strike prices. Thus, the authors use the dispersion of trading volume across the different strikes as a measure for differences of opinion among investors. Now, under short-sales constraint, a higher disagreement in the options market implies the underlying asset is overvalued and its future expected returns should be small. The authors compute the disagreement measure for different stocks and find that the measure is persistent from month to month. They also find that stocks with high disagreement underperform stocks with low disagreement, and the relation is stronger for options with higher short-sale costs.

In summary, several works find evidence of price discovery in the options market. The evidence comes from the options of firms rather than options on the market index. The works argue that informed investors choose to trade on the options market due to the leverage embedded on options. There is also evidence that options contain information about future stock volatility and also about future market volatility, and the same information-based explanation applies. The evidence on understanding the trading volume of options indicates that options trading is concentrated around the release of new information and is mostly due to disagreement among investors.

The current paper relates to the existing literature that argues in favor of disagreement among investors as an explanation for the trading volume of options. We use the trading volume of options as a way to measure disagreement among investors, and, in this sense, the current work is related to Andreou et al. (2018). However, to measure the disagreement among investors we use the disagreement model of Kandel and Pearson (1995) and the extension by Bollerslev, Li, and Xue (2018). Our approach requires the use of volatility in addition to the options volume to measure disagreement, but allows us to investigate how the disagreement changes with the time-frame and the distribution of market returns.

3 Interpreting the Relationship Between Trade Volume and Volatility

We study the relationship between trade volume and volatility in the options market. The economic interpretation of this volume-volatility relationship originates in the model of Kandel and Pearson (1995) where investors disagree about the interpretation of new information. Bollerslev, Li, and Xue (2018) extend the findings from Kandel and Pearson (1995) and provide an interpretation for the volume-volatility elasticity. This section discusses the Kandel-Pearson model and the interpretation of the volume-volatility elasticity proposed by Bollerslev, Li, and Xue (2018). If the reader is familiar with both works, this section can be skipped.

3.1 Trade Motivated by Disagreement About New Information

In Kandel and Pearson (1995) the authors explore the relationship between the trading volume of an asset and its returns. More specifically, the authors seek to understand why the trading volume in financial markets is huge even in periods where prices does not change. The authors argue that this happens due to differences in the interpretation of new information by traders.

In the setting of Kandel and Pearson (1995), there is a competitive market with a risk-free and a risky asset. The risk-free asset has a zero return rate, but the risky asset has a random future payoff of X . There are two types of traders trading in this market: type 1 and type 2. The proportion of type 1 traders is given by $\alpha \in [0, 1]$, and that of type 2 traders is given by $1 - \alpha$.

Each trader has prior beliefs about the random payoff X of the risky asset. Traders of type 1 believe $X \stackrel{d}{\sim} \mathcal{N}(X_1, Z_1^{-1})$, while traders of type 2 believe $X \stackrel{d}{\sim} \mathcal{N}(X_2, Z_2^{-1})$. And $X_1 > X_2$ is assumed without loss of generality. The different types of traders expect different payoffs for the risky asset ($X_1 \neq X_2$). They disagree about the average payoff, and this disagreement captures the optimism or pessimism of each trader. The variables

Z_1 and Z_2 represent the precision of the traders with regards to their beliefs about the expected asset payoff. The difference in precision captures how heterogeneous is the confidence of the different investors with respect to their beliefs.

The model consists of three time periods. On period one the traders maximize their expected utility given their beliefs about the payoff of the risky asset, which is realized at the third period. The investors have a negative exponential utility function $U(w) = -e^{-\lambda w}$, where λ is the common risk aversion of the traders. A competitive market equilibrium is achieved, and the risky asset price P_1 is such that it is available in zero net supply. The demand for the risky asset for each type of investor at period 1 is given by:

$$\begin{aligned}\text{Type 1 : } m_{1,1} &= (X_1 - X_2) \cdot \alpha \frac{Z_1 Z_2}{\lambda \hat{Z}} \\ \text{Type 2 : } m_{2,1} &= (X_2 - X_1) \cdot (1 - \alpha) \frac{Z_1 Z_2}{\lambda \hat{Z}}\end{aligned}$$

where $\hat{Z} \equiv \alpha Z_1 + (1 - \alpha) Z_2$.

Investors of type 1 have a higher expectation for the payoff of the risky asset than investors of type 2. In equilibrium, investors of type 1 hold the asset, while those of type 2 sell it. The amount demanded (or supplied) by each investor depends on their confidence about their beliefs. For example, as the confidence of type 2 investors decrease ($Z_2 \approx 0$), the amount supplied by these investors converges to zero.

On the second period, the traders observe a public signal $L = X + \varepsilon$ about the risky asset payoff. The signal is a noisy version of the actual payoff. However, each type of investor interprets the signal differently. The difference in interpretation stems from each investor's assumption about the noise term ε . Traders of type 1 believe that $\varepsilon \stackrel{d}{\sim} \mathcal{N}(\mu_1, b_1^{-1})$, while traders of type 2 believe that $\varepsilon \stackrel{d}{\sim} \mathcal{N}(\mu_2, b_2^{-1})$. The parameters μ_1 and μ_2 represent the traders expectations about the impact of the noise on the public signal. The confidence of the different investors regarding that impact is given by the parameters b_1 and b_2 .

Different investors know that they have different interpretations of the signal, but they agree to disagree about the signal's interpretation. The investors take the new information into account and update their beliefs about the risky asset's future payoff. The updated expectations of the risky asset's payoff are given by:

$$\begin{aligned}\text{Type 1 : } \mathbb{E}_1(X|L) &= \rho_1 X_1 + (L - \mu_1)(1 - \rho_1), \rho_1 \equiv \frac{Z_1}{Z_1 + b_1} \\ \text{Type 2 : } \mathbb{E}_2(X|L) &= \rho_2 X_2 + (L - \mu_2)(1 - \rho_2), \rho_2 \equiv \frac{Z_2}{Z_2 + b_2}\end{aligned}$$

After observing the new information, each investor believes the expected asset payoff will be a combination of their prior beliefs and the public signal. Notice that each investor subtracts from the public signal what they believe is the expected value of the noise term. The combination of the priors with the new information depends on the investors' confidence regarding their priors and regarding the noise term. For example, if the precision of investors of type 1 regarding their prior beliefs is high (high Z_1), then when these investors observe the new information, they will revise their expectations about the payoff, but will attribute a higher weight to their prior beliefs instead of the new information. However, if these investors have a very high precision regarding the noise term (high b_1), then their revised expectations will be mostly derived from the new information.

The investors also revise their precision regarding the expected payoff, which is now given by:

$$\begin{aligned}\text{Type 1 : Var}_1(X|L) &= \frac{1}{Z_1 + b_1} \\ \text{Type 2 : Var}_2(X|L) &= \frac{1}{Z_2 + b_2}\end{aligned}$$

Investors' with a high precision on their priors and on the impact of noise in the new information will have more confidence on their updated expectations about the risky asset's payoff.

Given the investors updated beliefs, the traders again maximize their expected utility and a new competitive market equilibrium is achieved. The new price for the risky asset is denoted by P_2 and is such that the risky asset is available in zero net supply. The investors now have different demands for the risky asset, which can be used to compute the trading volume due to the arrival of the public signal. The trading volume is simply the change in the investors' demand for the risky asset from the first to the second period. If we denote the 2nd-period demand for the investor of type 1 by $m_{1,2}$ (and $m_{2,2}$ for the type-2 investor), then the trading volume is:

$$\text{Volume} = |m_{1,2} - m_{1,1}| = |m_{2,2} - m_{2,1}|$$

Given the demands for the second period, it is possible to compute the trading volume as a function of the change in the risky asset's price:

$$\begin{aligned}\text{Volume} &= |\beta_0 + \beta_1 \underbrace{(P_2 - P_1)}_{\Delta P}| \tag{1} \\ \beta_0 &= \frac{\alpha(1 - \alpha)}{\lambda(\alpha b_1 + (1 - \alpha)b_2)} b_1 b_2 (\mu_2 - \mu_1) \\ \beta_1 &= \frac{\alpha(1 - \alpha)}{\lambda(\alpha b_1 + (1 - \alpha)b_2)} (Z_2 b_1 - Z_1 b_2)\end{aligned}$$

Equation 1 above relates the risky asset's trade volume to two terms: a term (β_0) that captures disagreement among the investors and a term that depends on the return of the risky asset.

The equation above allows us to understand the relationship between trading volume, returns and new information. The return (ΔP) of the risky asset depends on the public signal made available on the second period, and on how the different investors interpret the signal. The effect of the signal implies that there are two cases to analyze when it comes to understanding Equation 1. The first case is when the public signal leads to a change in the price of the risky asset: $\Delta P \neq 0$. In this case, the trading volume depends on how big the asset return is. Given $\beta_1 \neq 0$ and fixing some β_0 , the model implies that high returns should be associated with high trading volumes.

The second case is when the public signal does not lead to a change in the price of the risky asset: $\Delta P = 0$. In this case, the trade volume is not necessarily zero and depends only on the term β_0 . The value of β_0 is a function of the investors disagreement about the interpretation of the public signal. As long as the traders disagree about the mean of ε , that is $\mu_1 \neq \mu_2$, the value of β_0 will be non-zero. In addition, as the disagreement between investors increases (higher $|\mu_1 - \mu_2|$), so does the trading volume.

The Kandel-Pearson model generates an additional motive for trading other than returns. The model implies that even when returns are zero, the disagreement among investors regarding the interpretation of public information generates trading volume. Furthermore, as the disagreement between investors increases, the relationship between trading volume and returns weakens. And, for a sufficiently high disagreement, most of the trading volume is driven by disagreement rather than returns. Thus, at times when investors' disagree about the interpretation of new public information, we expect a weak relationship between volume and returns. But, at times when investors' agree about the interpretation of new public information, we expect a strong relationship between volume and returns.

3.2 Measuring Disagreement by Volume-Volatility Elasticity

Bollerslev, Li, and Xue (2018) propose a weaker version of the volume equation from Kandel and Pearson (1995) (Equation 1). Bollerslev, Li and Xue argue that while the volume equation summarizes the relationship between trade volume and price changes in response to the release of new information, it also establishes an exact relationship between random quantities, which is a condition too strong to impose. A more realistic condition, according to the authors, is that Equation 1 should hold on average, resulting in a moment condition.

The setting in Bollerslev, Li, and Xue (2018) simplifies the model from Kandel and Pearson (1995) by assuming both types of investors have the same precision when interpreting the new information: $h \equiv b_1 = b_2$. This simplification implies that β_0 and β_1 can be written as:

$$\begin{aligned}\beta_0 &= r\alpha(1 - \alpha)h(\mu_2 - \mu_1) \\ \beta_1 &= r\alpha(1 - \alpha)(Z_2 - Z_1)\end{aligned}$$

where $r \equiv 1/\lambda$ is the degree of risk tolerance of traders. Now the disagreement regarding the public signal is captured by β_0 via $\mu_2 - \mu_1$.

Given Equation 1 and the additional assumption that $\Delta P \stackrel{d}{\sim} \mathcal{N}(0, \sigma)$, the authors derive an expression for the expected trading volume in function of the volatility σ :

$$\mathbb{E}[\text{Volume}] = \frac{2}{\pi} |\beta_1| \sigma e^{\frac{-\beta_0^2}{2\beta_1^2\sigma^2}} + |\beta_0| \left(2\Phi\left(\frac{|\beta_0|}{|\beta_1|\sigma}\right) - 1 \right)$$

where Φ is the cumulative density function of a standard normal distribution. The equation above relates the expected trading volume to the volatility of returns, but also the terms β_0 and β_1 . While it is possible to interpret the equation for the expected volume in terms of the disagreement between investors (β_0), a much clearer expression can be obtained by computing the volume-volatility elasticity:

$$\text{Volume-Volatility Elasticity} = \frac{1}{1 + \psi\left(\frac{|\beta_0|}{|\beta_1|\sigma}\right)} = \frac{1}{1 + \psi\left(\frac{h}{\sigma} \cdot \frac{|\mu_2 - \mu_1|}{|Z_2 - Z_1|}\right)} \quad (2)$$

where ψ is an increasing function with $\psi(0) = 0$ and $\lim_{x \rightarrow \infty} \psi(x) = \infty$. The denominator in the elasticity equation is a function of the investors' disagreement regarding new information: $|\mu_2 - \mu_1|$.

The expression for the volume-volatility elasticity has two important features. First, the elasticity is no greater than one and is equal to one only when $\mu_2 = \mu_1$. That is,

the volume-volatility elasticity is maximized only when there is no disagreement among traders regarding the public signal. Second, the elasticity is a decreasing function of the level of disagreement. Thus, a higher level of disagreement (larger $|\mu_2 - \mu_1|$) implies in a smaller volume-volatility elasticity.

Bollerslev, Li, and Xue (2018) analyze the implications of the volume-volatility elasticity regarding disagreement between investors. The authors study the behavior of the volume-volatility elasticity for the market index around macroeconomic news announcements. Initially, the authors estimate a volume-volatility elasticity below unity, which is in line with the predictions of Equation 2. The authors then control for disagreement by using proxies based on the survey of professional forecasters and economic uncertainty. When disagreement is accounted for, the volume-volatility elasticity estimates are close to unity, and the disagreement proxies have a negative impact on the elasticity.

The current work also focuses on understanding disagreement between investors via the volume-volatility elasticity, but centers the analysis on the options market. We discuss the advantages of analyzing disagreement in the options market in the next section.

4 Empirical Methodology

4.1 Measures of Disagreement for Options

The present work studies disagreement among investors in the options market for the S&P 500 index (market index). The study is motivated by the existence of two natural measures of disagreement for the options market: moneyness and tenor. These two measures speak to the distribution of the market index returns at different quantiles and at different times, and allow us to understand their impacts on investors' disagreement. We begin by defining moneyness and tenor for options. Then, we argue about the interpretation of these two measures with respect to investors' disagreement.

4.1.1 Moneyness and Tenor

Consider an options contract with strike price K and time to expiration τ . Let S denote the current price of the underlying (in this case the S&P 500 index) and σ the underlying's volatility. Then, we can define the moneyness of this option by:

$$\text{Moneyness} = \frac{\ln\left(\frac{K}{S}\right)}{\sigma\sqrt{\tau}}$$

This measure is commonly referred to as the normalized moneyness of an option, but henceforth we refer to it simply as the moneyness of an option.

The numerator in the expression above is the logarithm of the strike-to-underlying ratio, which we refer to as simply the strike-to-underlying ratio. The strike-to-underlying ratio is negative for out-of-the-money (OTM) puts, where $S > K$, but the ratio is positive for in-the-money (ITM) puts, where $S < K$. Notice that for call options the sign of the strike-to-underlying ratio inverts: OTM calls ($S < K$) have positive ratios, while ITM calls ($S > K$) have negative ratios. Since the current work uses only data on put options, we focus on the interpretation of moneyness for put options. Thus, references to options with negative moneyness should be understood as OTM put options, while options with positive moneyness refer to ITM put options.

The strike-to-underlying ratio represents by how much the price of the underlying asset would have to move so that the option is exactly at-the-money (ATM). Specifically, the ratio is the geometric return required from the underlying asset so that it is equal to the strike price. The denominator of the moneyness standardizes the strike-to-underlying ratio. Consider σ to be a measure of the yearly volatility of the underlying asset's prices, and let τ measure the time to expiration of the option also in years. Then, the term $\sigma\sqrt{\tau}$ represents the volatility of the underlying asset scaled to the total time duration of the option. When we divide the strike-to-underlying ratio by $\sigma\sqrt{\tau}$, we still obtain by how much the underlying asset has to move for the option to be ATM, but in units of standard deviations. Therefore, we interpret an option's moneyness as the number of standard deviations the underlying price needs to move for the option to be exactly ATM. For example, a put option with moneyness equal to -3 would need the underlying price to decrease by three times its standard deviation for the option to be ATM.

As stated by Bollen and Whaley (2004):

(...) an option's "moneyness" is intended to reflect its likelihood of being in the money at expiration.

Following this understanding, a put option that has a very negative moneyness (deep OTM) is unlikely to expire in the money. On the other hand, a put option with highly positive moneyness (deep ITM) is likely to expire in the money.

The tenor of an option is simply the amount of time until its expiration date. To obtain the tenor of options on the S&P 500 index (SPX options) special care needs to be taken due to differences in the types of expiration. There are two types of SPX options: standard and weekly. The standard SPX options have expiration dates set for the 3rd Friday of every month, and the actual expiration time is at the market open (9:30 a.m. Eastern Time). The weekly SPX options, however, have expiration dates at multiple days every week, but their actual expiration time is at the close of the trading day (4:00 p.m. ET)⁴. To obtain precise tenors for the SPX options, we compute the tenor at the minute level taking into account the differences in expiration times, and then convert the units to days.

4.1.2 Interpretation of Moneyness and Tenor as Measures of Disagreement

The moneyness and tenor of an SPX option speak to the distribution of market returns. To discuss the interpretation of these two measures, we need to understand the pricing of options.

The price of an option can be computed as the risk-neutral expectation of its payoff. The existence of a risk-neutral measure is assured under no-arbitrage conditions (see Section 6K of Duffie (2001)). Let \mathbb{Q} denote the risk-neutral distribution, then the price of an SPX put option is given by:

$$P_t(K, T) = e^{-(r-q)T} \mathbb{E}_t^{\mathbb{Q}}[\max\{K - S_T, 0\}]$$

where we denote the price of the market index at time t by S_t , K as the strike price of the put option and T its time to expiration, and r and q denote the corresponding interest rate and dividend yield.

The equation above shows that the price of a put option is given by the risk-neutral expectation of its payoff. The payoff of the options depends on the difference between

⁴For details see CBOE (2014).

the strike price and the market index at a later time T . And the expected value of the payoff depends on the distribution of the market index at expiration time.

Consider a put option where the strike price K is very low in comparison to the current market index value S_t . For this option to expire in-the-money, the value of the market index would have to decrease substantially before time T . Therefore, the price of this put option greatly depends on the left-tail of the probability distribution of the market index at time T . As the strike price increases, so does the option's moneyness. Indicating a higher likelihood of the option expiring at-the-money. The dependence of the option's price on the left-tail of the distribution of S_T decreases. At the same time, the price of the option depends increasingly more on the center of the distribution of S_T . On the other extreme, if the strike price K is very high in comparison to the current market index value S_t , then the option's moneyness will be positive and high. If the strike price is very high, then the price of the option is more dependent on the right tail of the distribution of S_T .

The distance between the strike price and the market index is captured by the option's moneyness. Therefore, moneyness measures how much the price of an option depends on the different parts of the distribution of the market index. From this perspective, depending on the moneyness of options we analyze, new information regarding the market index would lead to disagreement regarding different quantiles of the distribution of S_T . For this reason, we interpret moneyness as a measure of disagreement about different parts of the distribution of the market index. In addition, due to the difficulty of analyzing tail events, we expect new information to generate more disagreement regarding the tails of the distribution of the market index. Therefore, there should be a negative relationship between the absolute value of moneyness and the volume-volatility elasticity.

A similar explanation applies to the tenor of an option. The option pricing equation shows that the expectation depends on the distribution of S_T , which depends on the time of expiration T . Therefore, the pricing of the option at different tenors depends on how the distribution of S_T changes for different values of T . From this perspective, the tenor captures disagreement regarding the distribution of the market index at different times. Since events far in the future are subject to more uncertainty than events about to happen⁵, the value of options with longer times to expiration are subject to more uncertainty than options with short tenors. This is especially true for the case of the market index, where the uncertainty regarding the future price increases with time. Thus, we expect a negative relationship between tenor and the volume-volatility elasticity.

The moneyness and tenor are readily available for any option. These two measures are directly related to disagreement and are used in the analysis of the volume-volatility elasticity. While options have these direct measures of disagreement, the volatility of an option is not as straightforward to measure. This is discussed in the next section.

4.2 Measure of Volatility for Options

An option is an asset with a payoff that depends on the value of another asset, the underlying. Even though there is this dependence, we can think of an option as an asset

⁵Forecasting the expectation of a stochastic process may or may not be more difficult as the forecast horizon increases. Indeed, for a moving average process the mean-squared error of the forecast converges to the unconditional variance of the series as the forecasting horizon increases. However, for a unit-root process the mean-squared error of the forecast diverges as the forecasting horizon increases (see page 440 of Hamilton (1995)). Since the market index is a unit-root process, its forecasting error increases with the forecasting horizon.

on its own. In this case, the volatility of an option is just the volatility of this option's price.

We assume we observe the price of options at the high-frequency. Specifically, we assume that prices are observed at equi-spaced intervals (i.e., every 5 minutes) within each day and for several days. For each day, we can compute the returns for each option using the high-frequency price observations. If we let $r_{i,t}$ denote the geometric return of the price of an option at a time interval i of day t , then the volatility of this option can be estimated via the realized variance estimator (see Barndorff-Nielsen and Shephard (2002)):

$$RV_t^{\text{option}} = \sum_i \left(r_{i,t}^{\text{option}} \right)^2$$

where the summation is across all time intervals within a day. RV_t^{option} can be computed each day for each option and relies on the use of high-frequency returns. The realized variance is an estimate of the daily variance for a given option at day t .

We know, however, that an option is fundamentally linked to its underlying. Indeed, the value of an option is partially derived from the current price and volatility of the underlying. In the Black-Scholes world, this connection is made explicit by the Black-Scholes Delta:

$$\Delta_t^{\text{Black-Scholes}} \equiv \frac{\partial P_t}{\partial S_t} = \Phi \left(\frac{\ln \left(\frac{S_t}{K} \right) + (r + \sigma^2/2)(T - t)}{\sigma \sqrt{T - t}} \right)$$

where P_t is the price of a put option at time t with some strike price K and expiration time T , and S_t is the underlying price at time t . In this case, a price change in the underlying translates into a price change in the option. This connection implies that the volatility of an option can be partitioned into two terms. The first term is the price volatility of an option due to changes in the underlying price. In our case, the underlying is the market index, so we denote this first term as the impact of the market volatility. The second term is the price volatility of an option due to other changes, such as changes in the underlying volatility. This second term is denoted as the residual volatility. The partition of an option's volatility yields the following equation:

$$RV_t^{\text{option}} = RV_t^{\text{impact of market volatility}} + RV_t^{\text{residual volatility}}$$

Partitioning the volatility is relevant because different economic agents are interested in different parts of the total volatility. According to Hull (2003), traders in financial institutions maintain delta-neutral portfolios, by at least daily re-balancing their portfolios, but cannot maintain neutral portfolios with relation to the other Greeks due to the difficulty of finding financial instruments to hedge with at competitive prices. This suggests that portfolios of financial institutions are hedged against changes in the market index (via delta-hedging), but are still exposed to the residual volatility. This point of view is supported by S. Ni, Pan, and Poteshman (2008), who argue that market makers can delta-hedge directional moves but cannot avoid volatility risk. Since the position of market makers is delta-neutral, it is possible that the option volatility relevant to these agents is not the total volatility (RV_t^{option}), but actually the residual volatility ($RV_t^{\text{residual volatility}}$). On the other hand, the total option volatility would be relevant to speculators that trade options for directional moves, since the position of these agents would not be delta-neutral.

This work assumes that the sophisticated agents trading in the SPX options market are either maintaining a delta-neutral position or are speculators that trade on volatility information (as in S. Ni, Pan, and Poteshman (2008)). This is partially supported by the fact that financial institutions (sophisticated agents) maintain delta-neutral portfolios (Hull (2003)), that there is a high demand for SPX puts due to the hedging demand of institutional investors (Bollen and Whaley (2004)), and that unsophisticated traders account only for 3% of the total volume of trades in the SPX options market (Lemmon and S. X. Ni (2011)).

The discussion above motivates mapping the moves in an option's price into moves due to changes in the option's underlying and moves due to other (residual) changes:

$$r_{i,t}^{\text{option}} = \beta r_{i,t}^{\text{market}} + \varepsilon_i$$

To recover the impact of the market volatility on the option volatility, we estimate the equation above separately for each option and for each day in the sample. We then use the residuals to recover the impact of the market volatility and compute the residual volatility:

$$RV_t^{\text{impact of market volatility}} \equiv \sum_{i=1}^n \left(\hat{\beta} r_{i,t}^{\text{market}} \right)^2$$

$$RV_t^{\text{residual volatility}} \equiv \sum_{i=1}^n \hat{\varepsilon}_i^2$$

We use the residual volatility as the measure of volatility for an option. It represents how much variation in an option's price is due to factors not directly hedged by sophisticated agents.

The next section discusses the data used in the empirical analysis.

4.3 Data

This work uses data on S&P 500 options (SPX options) made available by the Chicago Board Options Exchange (CBOE). SPX options are European style and are cash-settled with a multiplier of \$100 on the market index. The multiplier implies that if the S&P 500 is at the 2,867.24 level, then the notional value of one option is \$286,724.00. The data set contains bid and ask quotes and trade volume at the 1-minute frequency for each option (varying strike and time to expiration), from 2007 to 2016, totaling 2458 days in the sample after cleaning procedures⁶. To provide some sense of the scale of the analysis, observe that there were 3,479,352,243 records in the raw data, and a number of techniques had to be employed to clean and condense the set.

The focus of the analysis is on put options (call options are excluded), since these options are heavily traded and provide a natural hedge against market downturns (Bollen and Whaley (2004)).

In the Kandel-Pearson model investors disagree about a public signal. In Bollerslev, Li, and Xue (2018), the public signals considered are macroeconomic announcements, and the authors compute the elasticity from data around the announcement times. In this work, however, we consider public signals as being the information disseminated throughout a trading day. Therefore, we analyze the volume and volatility at the daily frequency.

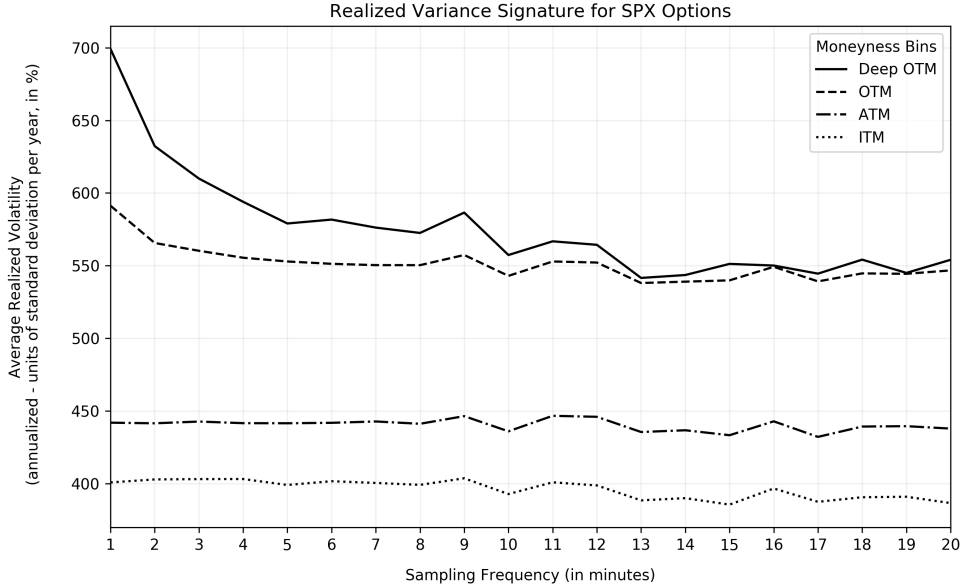
⁶See Section 7.1 in the Appendix for details.

The bid and ask quotes of each option are used to compute mid-prices. We estimate an option's residual variance from its high-frequency intra-day mid-prices. However, it is known that high-frequency prices can be affected by microstructure noise, which lead to high biases in variance estimates. To mitigate the effect of microstructure noise, we sparsely sample the mid-prices to the 5-minute frequency. Sampling sparsely is a standard practice in the literature and, in this case, is supported by a volatility signature plot for SPX options.

The volatility signature plot is displayed in Figure 1. The figure shows the average realized volatility for different sampling frequencies, with the options being categorized by their normalized moneyness. The volatility signature can indicate whether microstructure noise affects the return observations (see Zhou (1996), Andersen et al. (2000) and Hansen and Lunde (2006)). If there is no microstructure noise, then the volatility estimate should converge to a value as the sampling frequency increases. However, if microstructure noise is present, then the volatility estimate would diverge as the sampling frequency increases.

The figure indicates that the realized volatility estimator is stable for sampling frequencies as high as 5-minutes. For sampling frequencies above 5-minutes, we notice a sharp increase in the volatility estimates for out-of-the-money options, indicating the presence of microstructure noise at higher sampling frequencies. To avoid the effect of microstructure noise, we sparsely sample the options prices to the 5-minute frequency.

Figure 1: Volatility Signature for SPX Options.



Notes: The figure shows the average realized volatility (annualized in percentage) for options of different moneyness based on different sampling frequencies. The realized volatility is estimated as the sum of squared returns over each day for each option, where the frequency of returns varies from 1-minute up to 20-minutes. The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

To compute the residual volatility of each option, we use data at the 5-minutes fre-

quency. The residual volatility is estimated separately for each option at each day in the sample. The moneyness of each option is computed daily at the end of the day.

The trade volume is computed daily for each option and is given by its total volume of trades at that day. However, as discussed in Tauchen and Pitts (1983), the presence of a trend in the volume data could mislead the results of the volume-volatility analysis. We verify that indeed a trend exists in the data and mitigate its effects by detrending the volume with a moving average based on the volume on the previous 252 days.

The data set on SPY was obtained from TickData⁷, and it contains the last closing price of every 5-minute interval for all days from 2007 to 2016.

This study also considers two measures of sentiment. We want to explore the impact of sentiment on the disagreement between investors. The idea is that at times of optimism investors disagree less about new information, while at times of pessimism new information leads to greater disagreement.

The first sentiment measure is from Baker and Wurgler (2006) and Baker and Wurgler (2007)⁸. Baker and Wurgler construct a sentiment measure based on the principal component of various market proxies for sentiment. Specifically, they consider the dividend premium of stocks, the closed-end fund discount, the first-day return and trade volume on IPOs and the equity issues over total new issues⁹. The authors find evidence that their sentiment measure explains stock returns on the cross-section (increase in expected return on speculative and difficult-to-arbitrage stocks when sentiment increases, but the opposite for safe and easy-to-arbitrage stocks), is correlated with an equal-weighted market index and predict future average returns (lower average return for speculative stocks when past sentiment was high). The sentiment index is available at the monthly frequency from 2007 to September of 2015.

The second sentiment measure is the FEARS index from Da, Engelberg, and Gao (2014). The FEARS (Financial and Economic Attitudes Revealed by Search) index is based on the volume of internet search queries for various economics-related terms, such as "gold prices", "recession" and "crisis". The authors aggregate the search volume for different terms into an index where the terms are chosen according to how well they explain negative returns. The authors show that a high level of FEARS is related to lower contemporaneous returns but predict higher future returns (over the next few days), and that the index explains volatility and the flow of mutual funds from equity to bonds. The authors argue that, contrary to sentiment measures based on market proxies, FEARS is not the equilibrium outcome of the economy. The authors also argue that FEARS is based on agents' attitudes and is possibly more truthful than indices based on surveys. FEARS is available at the daily frequency from 2007 to 2016.

The next section discusses the empirical methodology to analyze disagreement in the SPX options market.

4.4 Regression Specification

The impact of disagreement regarding new information is captured by the volume-volatility elasticity. According to Equation 2, we should observe an elasticity equal to unity in the

⁷See <https://www.tickdata.com>

⁸Data sourced from Wurgler's website.

⁹Originally, the authors also included NYSE turnover as a measure of liquidity in the index. However, they recently dropped it from the sentiment index because turnover is no longer a meaningful measure of liquidity due to high-frequency trading.

case of no disagreement among investors. However, if there is disagreement, then the elasticity should be less than unity. Furthermore, the elasticity should decrease to the extent that disagreement increases.

To analyze these implications, we consider the following regression specification:

$$\ln(\text{Volume}) = (\alpha_0 + \alpha_1^T X_0) + (\beta_0 + \beta_1^T X_1) \cdot \ln(\text{Residual Volatility}) + \varepsilon \quad (3)$$

where α_0, β_0 are scalars, α_1, β_1 and X_0, X_1 are vectors, and the terms in X_0 and X_1 include different control and explanatory variables. Estimating the equation above allows us to directly recover the volume-volatility elasticity. Of particular importance to the analysis are the terms included in X_1 . These terms include measures of disagreement, which allow us to evaluate the predictions regarding disagreement from the estimated β_1 coefficients. If the measures of disagreement included in X_1 capture investors' disagreement in the options market, then we expect the estimated value of β_0 to be close to unity, while those of β_1 to be negative.

The disagreement measures included in X_1 are the absolute value of moneyness, tenor and an interaction term between both. The absolute value of moneyness indicates how far from the money an option is. This measure captures disagreement regarding the distribution of the market index. For example, options far from the money will have values of moneyness far from zero, which indicates that the price of such options depend on the tails of the distribution of the market index. Since the probabilities of tail events are hard to predict, the impact of new information on the tails of this distribution should generate more disagreement. An underlying assumption is that the effect of decreases or increases in the moneyness have the same impact on the elasticity. This is plausible because the distribution of high-frequency returns is symmetric, and ITM puts are equivalent to OTM calls.¹⁰

We also expect the effect of tenor on elasticity to be negative, since higher tenor is related to uncertainty about the future distribution of the underlying asset. The interaction between the absolute value of moneyness and tenor allows us to capture the effect of moving away from the money on options that are close to expiration and on options that are far from expiration. Since options far from the money and have higher tenor are also subject to more uncertainty, we expect the interaction term between the absolute value of moneyness and tenor to be negative.

The next section discusses the empirical results.

5 Empirical Results

The main interest of the empirical analysis is to investigate the elasticity between trading volume and residual volatility under the predictions of Kandel and Pearson (1995) and the developments of Bollerslev, Li, and Xue (2018). The volume-volatility elasticity measures the investors' disagreement with respect to new public information. Equation 2 implies that we should observe an elasticity equal to unity in the case of no disagreement among investors. However, in the case where investors do disagree we should observe an elasticity smaller than unity. The equation also implies that the elasticity should decrease as investors' disagreement strengthens.

¹⁰An alternative specification where the negative and positive moneyness are allowed to have different impacts on the elasticity is explored in Section 7.3 of the Appendix. Similar results are obtained.

The estimates for the volume-volatility elasticity are presented in Table 1 below. In column I, the elasticity estimate is denoted by β_0 . However, in columns II, III and IV we add measures of disagreement for options (moneyness and tenor), and allow these measures to impact the elasticity. In those columns, the elasticity is given by the combination of β_0 and the parameters β_1 . The standard errors are cluster-robust, where we assume options with similar moneyness and tenor are correlated.

Table 1: Volume-Volatility Elasticity Estimates.

	I	II	III	IV
Baseline Estimates				
Intercept (α_0)	4.2690 (0.3366)	4.9748 (0.3105)	6.9463 (0.3765)	7.4682 (0.6330)
Elasticity (β_0)	0.2305 (0.0247)	0.5035 (0.0279)	0.9110 (0.0834)	1.0252 (0.1437)
Explanatory Variables in Elasticity (β_1)				
Moneyness		-0.2031 (0.0156)	-0.2035 (0.0148)	-0.2893 (0.0603)
ln (Tenor)			-0.1182 (0.0214)	-0.1509 (0.0389)
Moneyness · ln (Tenor)				0.0247 (0.0165)
Adjusted R^2 (in %)	22.52	23.55	23.99	24.01
Number of Observations		446053		

Notes: The table reports the estimated values of α_0 , β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. Estimation is based on the full sample of cleaned data, spanning 2007 to 2016. We compute volume and volatility daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009). The total number of observations for all regressions is 446,053.

The elasticity estimate in the first column (I) is significant, but also smaller than unity. This finding is in line with the predictions of the disagreement model, and indicates there is disagreement among investors in the options market. The positive intercept is evidence that there is trading even when there are no changes in prices.

In columns II, III and IV, measures of disagreement for the options market are introduced as explanatory variables in the elasticity. In column II, the absolute value of moneyness is included in the regression. The elasticity estimate is now given by $0.5 - 0.2|\text{Moneyness}|$. By including a disagreement measure in the regression, the baseline elasticity (when $|\text{Moneyness}| \approx 0$) estimate has increased from 0.23 to 0.5. This increase suggests that moneyness captures a portion of the disagreement between investors. The baseline estimate also indicates that investors disagree on the interpretation of news and their impact on the distribution of the market index.

The coefficient for the absolute value of moneyness is significant and negative. As moneyness moves away from zero, the elasticity estimate decreases, which is in line with

our interpretation of the relationship between moneyness and disagreement. The negative coefficient for the absolute value of moneyness indicates that disagreement among investors increases for options far-from-the-money (either OTM or ITM puts). That is, for values of moneyness away from zero, the elasticity estimate is evidence of a higher disagreement with respect to the tails of the market index distribution. This is consistent with the idea that there is more uncertainty about the tails of the distribution of the market returns.

In column III, the tenor is included as an additional explanatory variable in the elasticity. The elasticity estimate is now given by $0.91 - 0.2|\text{Moneyness}| - 0.11\ln(\text{Tenor})$. Including another disagreement measure in the regression pushes the baseline elasticity estimate closer to unity. In fact, the baseline estimate (0.91) is now statistically indistinguishable from unity.

The baseline elasticity estimate in column III indicates that there is no disagreement for options that are at-the-money and about to expire. In terms of the distribution of the market index, the baseline elasticity indicates a lack of disagreement about new information when it comes to the distribution of the market index in the near future. The estimated coefficient for the tenor is significant and negative. The negative coefficient is also in line with our expectations and indicates that investors' disagreement increases for options with longer times to expiration. The coefficient for the tenor is consistent with the idea that there is more uncertainty about the distribution of market returns over longer time-horizons than over shorter time-horizons.

In column IV we also consider an interaction term between the absolute value of moneyness and the tenor. In this case, the baseline elasticity is estimated to be 1.0252, which is statistically indistinguishable from unity. Again, the baseline elasticity estimate indicates that there is no disagreement for options about to expire and that are at-the-money. The coefficient of the interaction term between moneyness and tenor is positive. A positive coefficient implies that the effect of moneyness is attenuated for options that have a long time to expiration. In other words, while there is more disagreement on options that expire far in the future, the disagreement is less dependent on how far-from-the-money an option is. The attenuation of the impact of moneyness is consistent with the idea that over longer horizons the number of big moves in the market index gets averaged out, so that there is less disagreement for options far-from-the-money. The coefficient for the interaction term, however, is statistically insignificant. Therefore, we focus on the analysis on the specification in column III.

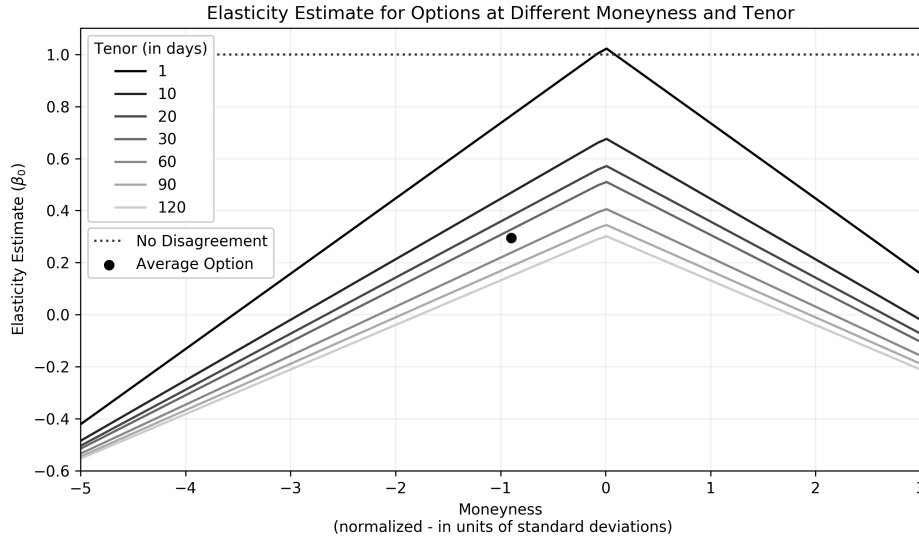
Table 2 displays the average values of moneyness and tenor for the options in the sample. We refer to options with moneyness and tenor similar to the ones displayed in the table as typical options. The typical option is slightly far-from-the-money, requiring the market index to move by slightly more than one standard deviation for the option to expire exactly ATM. Additionally, the time to expiration of the typical option is about 28 days. Under the estimates of column III, the volume-volatility elasticity estimate for the typical option is 0.24, contrasted to the baseline of 0.91. The disagreement indicated by the elasticity estimate is derived in part from the typical option being either OTM or ITM and in part from its time to expiration. The moneyness indicates disagreement about the higher quantiles of the distribution of the market index, while the tenor indicates the uncertainty regarding the future value of the market index.

Table 2: Representative Option.

Average Value	
Moneyiness	-0.898143
Tenor	38.548391
Moneyiness	1.336033
ln (Tenor)	3.341330

Notes: The table reports average values for the moneyiness and tenor of all options in the sample.

We use the parameter estimates to compute the elasticity values at different moneyiness and tenor levels. The values are displayed in Figure 2.

Figure 2: Elasticity Estimates for Options at Different Moneyiness and Tenor.

Notes: The figure shows the volume-volatility elasticity estimates for different moneyiness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyiness, the elasticity decreases as the time to expiration increases.

The case of no-disagreement occurs for at-the-money options that are about to expire (in a day). For any fixed tenor, there is a symmetric decrease in elasticity as we move away from-the-money. Since the tenor is fixed, this decrease in elasticity is interpreted as an increase in disagreement regarding the tails of the distribution of market returns. Fixing any moneyiness level, there is a large drop in elasticity as the tenor increases, indicating a heightened disagreement regarding the future distribution of market returns. Lastly, notice that the decrease in elasticity when the tenor increases is smaller for options far-from-the-money than it is for options closer-to-the-money.

5.1 Impact of Sentiment on Elasticity

The volume-volatility elasticity was estimated taking into account the effect of options moneyness and tenor. While moneyness and tenor capture more specific aspects of the disagreement between investors, we also want to consider the impact of measures that capture a more general level of disagreement. To do so, we include two measures of sentiment as control variables when estimating the volume-volatility elasticity.

The first measure is the sentiment index from Baker and Wurgler (2006) and Baker and Wurgler (2007), henceforth referred to as the BW sentiment index. The BW sentiment index captures investors' sentiment by extracting a principal component from various market proxies for sentiment. The second measure is the FEARS index from Da, Engelberg, and Gao (2014). The FEARS index measures the Internet search volume for terms with negative economic connotations. A high value of FEARS means a higher than average search volume for these negative terms. While the BW sentiment index offers a sentiment measure based on market quantities, the FEARS index recovers sentiment from an alternative source only indirectly related to the financial markets.

We expect that when investors are optimistic, there should be less disagreement about new information, but when investors are pessimistic that new information leads to more disagreement. Therefore, we presume that the impact of the BW sentiment on the elasticity will be positive. However, for the FEARS index, we expect a negative impact on the elasticity, since higher FEARS means lower sentiment.

The elasticity estimate including the effect of the two sentiment measures are displayed in Table 3 below.

Table 3: Volume-Volatility Elasticity Estimates with the Inclusion of Sentiment Measures.

	I	II	III
Baseline Estimates			
Intercept (α_0)	7.4682 (0.0568)	7.4301 (0.0724)	7.4260 (0.0723)
Elasticity (β_0)	1.0252 (0.0126)	1.1740 (0.0168)	1.1744 (0.0168)
Explanatory Variables for Elasticity			
Moneyness	-0.2893 (0.0072)	-0.4048 (0.0100)	-0.4048 (0.0100)
$\ln(\text{Tenor})$	-0.1509 (0.0036)	-0.1425 (0.0048)	-0.1418 (0.0048)
Moneyness $\cdot \ln(\text{Tenor})$	0.0247 (0.0021)	0.0494 (0.0030)	0.0491 (0.0030)
BW		0.0305 (0.0033)	0.0301 (0.0033)
FEARS			-0.0238 (0.0039)
Adjusted R-squared	0.2401	0.2679	0.2680
Number of Observations	446053	256453	

Notes: The table reports the estimated values of α_0 , β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. In addition to those control variables, we include the sentiment measure from Baker and Wurgler (2006) and Baker and Wurgler (2007) and the FEARS index from Da, Engelberg, and Gao (2014). The estimation is based on a partial sample of cleaned data, spanning 2007 to 2015. Values for volume and volatility are computed daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The two sentiment measures are standardized. The standard errors are heteroscedasticity robust as in White (1980).

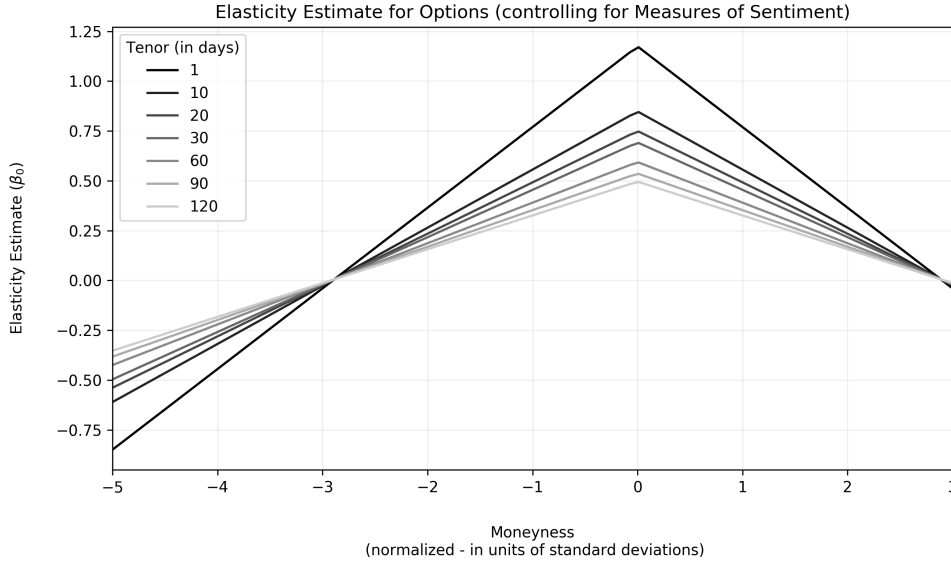
The elasticity estimate is in line with the estimates previously obtained in Table 3. We observe an elasticity that is decreasing with the absolute value of moneyness and with increases in tenor, indicating that new information leads to higher disagreement about tails of the market returns and about the future market returns farther in the future.

The impact of the sentiment measures in the elasticity estimate is as expected. At times of optimism, there is an increase in the elasticity, indicating that new information leads to less disagreement when investors are optimistic. Notice that optimism is captured by positive values in the BW index or negative values in the FEARS index. At times of pessimism, however, there is a decrease in the elasticity, which indicates that new information leads to more disagreement when investors are pessimistic. Observe that the effect of both sentiment measures are significant, which is a sign that sentiment measured from textual sources (such as the FEARS index) can complement the sentiment obtained

from market proxies (as in the BW index).

A visual representation of the elasticity at different levels of moneyness and tenor is displayed in Figure 3 below.

Figure 3: Elasticity Estimates for Options at Different Moneyness and Tenor with Sentiment Measures are Included as Control Variables.



Notes: The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.

The figure displays elasticity values similar to that shown in Figure 2. The elasticity is the highest for at-the-money options that are about to expire (in a day). There is a decrease in elasticity as we move away from-the-money and as tenor increases. However, due to the effect of the interaction between moneyness and tenor, the effect of increases in tenor changes sign for options that are deep OTM, leading to increases in elasticity for such options.

5.2 Time Variation in Disagreement

We now analyze whether the relationship between volume and volatility changes throughout the years. We estimate the volume-volatility relationship as in equation 2 using biyearly data. The estimates are presented in Table 4.

Table 4: Volume-Volatility Elasticity Estimates Biyearly.

	2007-2008	2009-2010	2011-2012	2013-2014	2015-2016
Baseline Estimates					
Intercept (α_0)	6.5674 (0.5843)	7.0969 (1.0657)	6.2563 (0.3803)	8.0892 (0.9272)	8.6087 (0.6940)
Elasticity (β_0)	1.3082 (0.1328)	1.1773 (0.2053)	1.3258 (0.1154)	1.0694 (0.2364)	0.9894 (0.1467)
Explanatory Variables in Elasticity (β_1)					
Moneyiness	-0.2873 (0.0763)	-0.4516 (0.0980)	-0.3761 (0.0534)	-0.3437 (0.1111)	-0.2440 (0.0573)
ln (Tenor)	-0.1146 (0.0356)	-0.1206 (0.0586)	-0.1330 (0.0323)	-0.0873 (0.0668)	-0.1770 (0.0392)
Moneyiness · ln (Tenor)	-0.0322 (0.0228)	0.0393 (0.0283)	0.0116 (0.0183)	0.0156 (0.0328)	0.0265 (0.0155)
Adjusted R-squared	0.2654	0.2542	0.2358	0.3578	0.2461
Observations	40850	30391	35109	72692	267011

Notes: The table reports the estimated values of α_0 , β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyiness, absolute value of moneyiness, logarithm of tenor and interaction terms. Estimation is done on a bi-yearly basis, spanning 2007 to 2016. We compute volume and volatility daily for each option from high-frequency data. Moneyiness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009).

The baseline estimates (estimate of β_0) represent the disagreement for options close-to-the-money and about to expire. The estimates vary from one estimation period to another, but move closer to unity on the estimates from the more recent periods. Except on the period 2009-2010, on all other periods the elasticity estimate is statistically indistinguishable from unity. When controlling for the measures of disagreement, we find the baseline estimates are consistently close to unity, which is in line with the case of no disagreement among investors.

The estimated coefficient for the absolute value of moneyiness is significant and negative for all time periods. The coefficient varies from period to period, being the highest in the 2009-2010 period, and the lowest in the period 2015-2016. The estimates indicate that the disagreement about the tails of the market index distribution could be time varying. However, the standard errors do not allow us to conclude that the estimated coefficients for the absolute value of moneyiness are statistically different from one another.

The estimated coefficients for the tenor are negative for all periods, but not significant on 2009-2010 and 2013-2014. The findings indicate that disagreement increases with tenor, suggesting greater uncertainty about the market index farther into the future. However, the relationship is weakened at some of the periods, indicating a lack of disagreement as tenor increases.

The findings in Table 2 are in accordance with the previous findings. The elasticity estimate is indifferent from unity for options close-to-the-money and about to expire. The measures of disagreement for options (moneyiness and tenor) have the expected sign and are for the most part significant.

5.3 Total Variance

The previous discussions are based on the volume-volatility elasticity. The elasticity was estimated using the residual volatility of options. In this section we show we obtain similar conclusions when we compute the elasticity using the total option variance instead. The elasticity estimates are presented in Table 5.

Table 5: Volume-Volatility Elasticity Estimates Using the Total Variance.

	I	II	III	IV
Baseline				
Intercept (α_0)	3.6663 (0.3258)	3.7457 (0.3004)	5.3555 (0.3294)	5.8422 (0.5431)
Elasticity (β_0)	0.0528 (0.0279)	0.3589 (0.0519)	1.1170 (0.1047)	1.3379 (0.1956)
Explanatory Variables for Elasticity (β_1)				
Moneyiness		-0.2176 (0.0251)	-0.2196 (0.0205)	-0.3834 (0.0827)
ln (Tenor)			-0.2079 (0.0274)	-0.2695 (0.0535)
Moneyiness · ln (Tenor)				0.0459 (0.0225)
Adjusted R-squared	0.2171	0.2227	0.2325	0.2330
Number of Observations		446053		

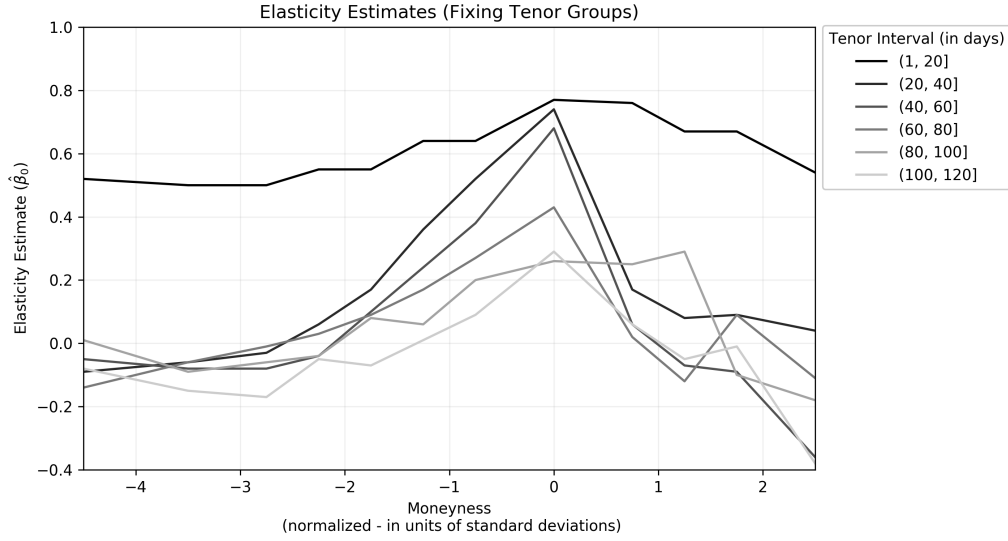
Notes: The table reports the estimated values of α_0 , β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyiness, absolute value of moneyiness, logarithm of tenor and interaction terms. Estimation is based on the full sample of cleaned data, spanning 2007 to 2016. We compute volume and volatility daily for each option from high-frequency data. Moneyiness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009).

The baseline estimates follow the same pattern as the estimates in Table 1. When no disagreement measure is included (column I) in the elasticity, we obtain an estimate that indistinguishable from zero. This low coefficient indicates disagreement between investors when new information is released.

In column II, we add the absolute value of moneyiness as a disagreement measure in the elasticity. Its addition pushes the baseline estimate closer to one. The coefficient of the absolute value of moneyiness is significant and negative. The negative coefficient indicates more disagreement on the interpretation of news and how they affect options far-from-the-money.

In column III, both the absolute value of moneyiness and tenor are explanatory variables in the elasticity. In this case, the baseline estimate is indistinguishable from unity. This is the case of no disagreement among investors, and suggests that the options' moneyiness and tenor capture the disagreement among investors. The coefficient for the tenor is also significant and negative, which indicates there is more disagreement about options with longer times to expiration.

Figure 4: Elasticity Estimates for Options Categorized by Moneyness and Tenor.



Notes: The figure shows a surface of volume-volatility elasticity estimates for different moneyness and tenor categories. Values at the left of the figure represent estimates for OTM options. Values deeper in the figure represent estimates for options with shorter tenors. The highest estimate is achieved for moneyness close to zero and short tenor. The elasticity decreases for options far from the money and for options with higher tenor.

In column IV, we also include an interaction term between the disagreement measures for options. However, the parameter estimate is indistinguishable from zero.

The estimates using the total variance of options presents the same evidence as the estimates using the residual variance. There is disagreement among investors, and there is greater disagreement about the tails of the market index distribution. The disagreement also increases with increases in the time frame.

5.4 Non-parametric Results

The results presented before are based on the estimation of Equation 3. We now apply a nonparametric approach to recover the volume-volatility elasticities. The idea is to allow for a more flexible impact of moneyness and tenor on the elasticity estimate. To do so, we categorize each option by moneyness and tenor. Moneyness and tenor are split in intervals of different sizes to accommodate a varying number of options available. The intervals for moneyness range from -5 to 3, which is the range of moneyness levels available in the data. The intervals for tenor range from 1 day to 120 days, which is also the range of tenors considered in this study.

After categorizing options by moneyness and tenor, we estimate the volume-volatility elasticity for each category. The elasticity estimates are presented as a surface in Figure 4 below.

The surface reveals similar results to the parametric estimates. The elasticity esti-

mate achieves its maximum value for options close to the money (moneyness category $(-0.5, 0.5]$) and with short tenor (tenor category $(1, 20]$). The elasticity decreases as we move to moneyness categories that are far from the money. This decrease in elasticity occurs irrespective of the tenor category. Also consistent with previous estimates is the decrease in elasticity as we move from options with short tenor to options with long tenor. This decrease also occurs irrespective of the options moneyness. Lastly, we observe that for options with high tenor (above 60 days) the decrease in the elasticity as the options get far from the money is attenuated in comparison to the effect for options with short tenor. This is also consistent with the previous estimates.

The nonparametric estimates reveal dynamics that were not captured by the parametric estimates. First, for options with short tenor (up to 20 days to expiration) we observe an elasticity estimate that changes less with the moneyness of the option. This is consistent with the idea that the probability of big moves in a short span of time is small, so that new information should not lead to higher disagreement about the tails.

Second, there is a sharper drop in the elasticity when the moneyness moves from zero to positive values (from ATM to ITM) than when it moves to negative values (from ATM to OTM). This implies that the release of new information generates more disagreement about the right-tail of the market returns than about the left-tail.

Third, while there is an overall increase in the disagreement as tenor increases, this decrease is attenuated for options ATM. Indeed, notice that there is a slight decrease in elasticity when tenor goes from the range $(1, 20]$ to $(20, 40]$, and almost zero decrease when tenor further increases to the range $(40, 60]$. The lack of a decrease in elasticity indicates that new information does not generate more disagreement regarding the valuation of ATM options that expire in less than 60 days.

Fourth, the highest elasticity estimate is not close to unity. In the parametric estimates, the elasticity is close to unity when moneyness is zero and tenor is one. However, the parametric estimates reflect the projection of the data on the model and take into account the downward trend when tenor increases. Therefore, the elasticity close to unity is possibly just an extrapolation of the negative effect of the tenor.

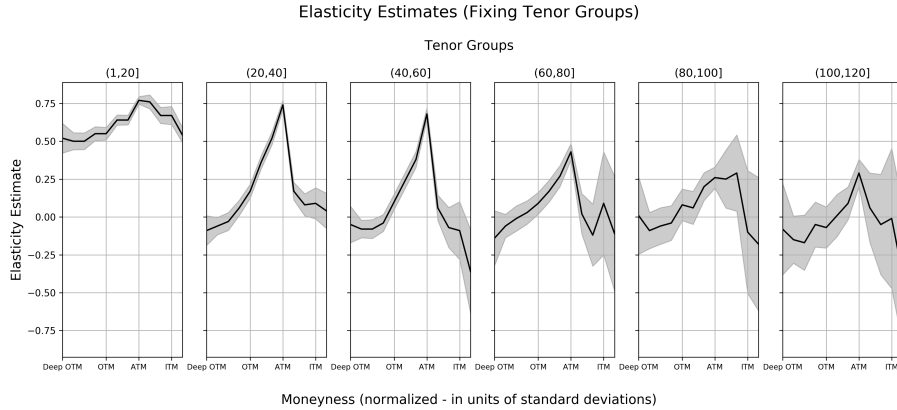
To better understand the nonparametric elasticity estimates, we plot them separately for each tenor category while allowing moneyness to change. See Figure 5 below.

The first plot on the left shows the volume-volatility elasticity for options that expire in less than 20 days. The elasticity is highest for ATM options and slowly decreases when moneyness moves away from zero. The elasticity estimates for these short tenor options are higher at every moneyness level when compared to the estimates for options with higher tenors. This implies that new information generates the least disagreement for options that are expiring soon.

The second plot from left-to-right shows the elasticity for options that expire in around 1 month. The least disagreement occurs for ATM options, and the elasticity estimate is close to that for ATM options with short tenor. However, contrary to the options with short tenor, the disagreement sharply increases as moneyness moves away from zero. Notice that the elasticity decreases almost linearly with moneyness, which is in line with the parametric estimates analyzed before. There is a sharper decrease in elasticity when moneyness increases from zero than when moneyness decreases from zero, indicating that there is more disagreement about the right tail of market returns.

The third plot from left-to-right displays the elasticity for options that expire in almost two months. This plot displays the same characteristics as options that expire close to 1 month.

Figure 5: Elasticity Estimates for Options on Different Tenor Categories.



Notes: The figure shows six plots. Each plot displays the elasticity estimates for varying degrees of moneyness. The first plot on the left estimates the elasticity using options of short tenor, while the last plot on the right uses options that expire in about 4 months. In each plot, the moneyness varies between Deep OTM (-5), OTM (-2), ATM (0) and ITM (2). The shaded region represents a 99% confidence interval.

The remaining three plots display the elasticity for options that expire in over two months. The behavior of the elasticity is similar to options with short tenor: elasticity is the highest when moneyness is zero and the elasticity almost linearly decreases when moneyness moves away from zero. However, there is a decrease in the highest elasticity, which is in line with tenor increases, leading to more overall disagreement.

Next, we analyze what happens to the elasticity as tenor increases, while fixing the moneyness of options. Figure 6 displays the results.

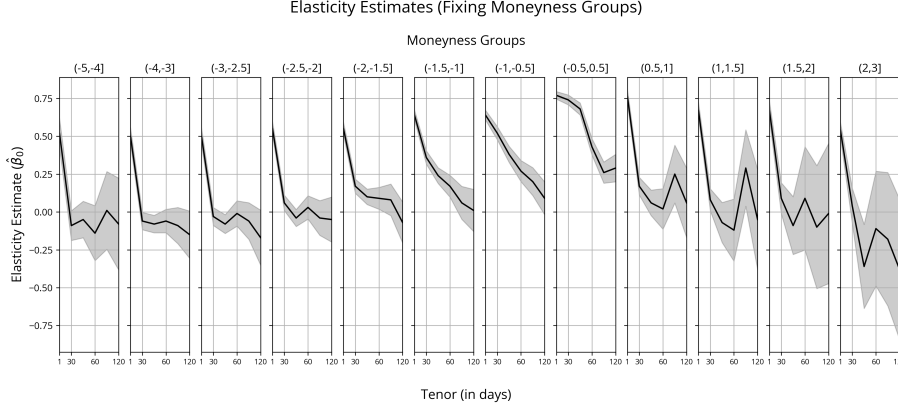
All plots display a similar behavior. The elasticity sharply decreases when the tenor increases. This is consistent with the parametric estimates and indicates higher disagreement about future market returns. Options that are ITM or have moneyness smaller than -1.5 display a decrease in elasticity when tenor moves from 1 day to 30 days, but as the tenor further increases, there is only a small decrease in elasticity. This indicates a sharp increase in disagreement about the distribution of market returns in the short term, but there is not an increase in disagreement about market returns in the longer term (2 or 3 months). This is not the case with options that are ATM or OTM, where there is a linear decrease in elasticity when tenor increases. Last, notice that for options that are ATM, there is only a small decrease in elasticity when tenor increases from 1 day to 60 days, consistent with the previous findings. Overall, these findings indicate there is an almost linear (or piece-wise linear) relationship between disagreement and tenor.

The next section concludes the main findings of the present work.

6 Conclusion

In the Kandel-Pearson model, there are two motives for investors to trade a risky asset: changes in the asset's prices and disagreement between investors regarding new information. Bollerslev, Li, and Xue (2018) explicitly connects the disagreement between investors to the elasticity between trade volume and price volatility, resulting in two

Figure 6: Elasticity Estimates for Options on Different Moneyness Categories.



Notes: The figure shows twelve plots. Each plot displays the elasticity estimates for varying degrees of tenor. The first plot on the left estimates the elasticity using options that are deep OTM, while the last plot on the right uses options that are deep ITM. In each plot the tenor varies from 1 day to 120 days. The shaded region represents a 99% confidence interval.

main implications. First, the volume-volatility elasticity is equal to unity only when there is no disagreement between traders. Second, the volume-volatility elasticity decreases if the disagreement between investors increases. The present work analyzes these two implications in the context of the S&P 500 options market.

We present empirical evidence consistent with the implications from the Kandel-Pearson model. We argue that there are two natural measures of disagreement for the options market: moneyness and tenor. These two measures speak to the distribution of the market index at different quantiles and at different times. We estimate a volume-volatility elasticity close to unit for options that are at-the-money and about to expire, which is consistent with the case of no disagreement among investors. We also find that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, implying that new information leads to more disagreement regarding tail events for the market index. Lastly, we find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index. These findings are also supported by nonparametric estimates.

References

- Andersen, Torbin et al. (2000). “Great realisations”. In: *Risk* 13, pp. 105–108. URL: http://public.econ.duke.edu/~boller/Published_Papers/risk_00.pdf.
- Andreou, Panayiotis C et al. (2018). “Differences in options investors’ expectations and the cross-section of stock returns”. In: *Journal of Banking & Finance* 94, pp. 315–336. URL: <https://doi.org/10.1016/j.jbankfin.2018.07.016>.
- Baker, Malcolm and Jeffrey Wurgler (2006). “Investor sentiment and the cross-section of stock returns”. In: *The journal of finance* 61.4, pp. 1645–1680. URL: <https://doi.org/10.1111/j.1540-6261.2006.00885.x>.
- (2007). “Investor sentiment in the stock market”. In: *Journal of economic perspectives* 21.2, pp. 129–152. URL: <http://www.aeaweb.org/articles?id=10.1257/jep.21.2.129>.
- Barndorff-Nielsen, Ole E. and Neil Shephard (2002). “Estimating Quadratic Variation Using Realized Variance.” In: *Journal of Applied Econometrics* 17.5, pp. 457–477. ISSN: 08837252. URL: <http://search.ebscohost.com.proxy.lib.duke.edu/login.aspx?direct=true&db=eoh&AN=0628412&site=ehost-live&scope=site>.
- Bollen, Nicolas PB and Robert E Whaley (2004). “Does net buying pressure affect the shape of implied volatility functions?” In: *The Journal of Finance* 59.2, pp. 711–753. URL: <https://doi.org/10.1111/j.1540-6261.2004.00647.x>.
- Bollerslev, Tim, Jia Li, and Yuan Xue (2018). “Volume, volatility, and public news announcements”. In: *The Review of Economic Studies* 85.4, pp. 2005–2041. URL: <https://doi.org/10.1093/restud/rdy003>.
- Cai, Li and Jian Du (2018). “Excess returns to buying low options-volume stocks and selling high options-volume stocks: Information or characteristics?” In: *Journal of Futures Markets* 38.12, pp. 1487–1513. URL: <https://doi.org/10.1002/fut.21957>.
- Cao, Charles, Zhiwu Chen, and John M Griffin (2005). “Informational content of option volume prior to takeovers”. In: *The Journal of Business* 78.3, pp. 1073–1109. URL: <https://www.jstor.org/stable/10.1086/429654>.
- Cao, H. Henry and Hui Ou-Yang (2009). “Differences of Opinion of Public Information and Speculative Trading in Stocks and Options”. In: *The Review of Financial Studies* 22.1, pp. 299–335. URL: <https://www.jstor.org/stable/40056912>.
- CBOE, Chicago Board Options Exchange (2014). *The CBOE Volatility Index - VIX*. Tech. rep. URL: <https://web.archive.org/web/20180427204307/https://www.cboe.com/micro/vix/vixwhite.pdf>.

- Chan, Kalok, Y Peter Chung, and Herb Johnson (1993). “Why option prices lag stock prices: A trading-based explanation”. In: *The Journal of Finance* 48.5, pp. 1957–1967. URL: <https://doi.org/10.1111/j.1540-6261.1993.tb05136.x>.
- Choy, Siu Kai and Jason Wei (2012). “Option trading: Information or differences of opinion?” In: *Journal of Banking & Finance* 36.8, pp. 2299–2322. URL: <https://doi.org/10.1016/j.jbankfin.2012.04.010>.
- Cochrane, John (2016). “Volume and Information. Comments on “Random Risk Aversion and Liquidity: a Model of Asset Pricing and Trade Volumes” by Fernando Alvarez and Andy Atkeson”. In: URL: https://faculty.chicagobooth.edu/john.cochrane/research/papers/Atkeson_Alvarez_comments.pdf.
- Da, Zhi, Joseph Engelberg, and Pengjie Gao (2014). “The sum of all FEARS investor sentiment and asset prices”. In: *The Review of Financial Studies* 28.1, pp. 1–32. URL: <https://doi.org/10.1093/rfs/hhu072>.
- Duffie, Darrell (2001). *Dynamic asset pricing theory*. Princeton University Press.
- Easley, David, Maureen O’hara, and Pulle Subrahmanya Srinivas (1998). “Option volume and stock prices: Evidence on where informed traders trade”. In: *The Journal of Finance* 53.2, pp. 431–465. URL: <https://doi.org/10.1111/0022-1082.194060>.
- French, Kenneth R (2008). “Presidential address: The cost of active investing”. In: *The Journal of Finance* 63.4, pp. 1537–1573. URL: <https://doi.org/10.1111/j.1540-6261.2008.01368.x>.
- Ge, Li, Tse-Chun Lin, and Neil D Pearson (2016). “Why does the option to stock volume ratio predict stock returns?” In: *Journal of Financial Economics* 120.3, pp. 601–622. URL: <https://doi.org/10.1016/j.jfineco.2015.08.019>.
- Hamilton, James D (1995). “Time series analysis”. In: *Economic Theory. II, Princeton University Press, USA*, pp. 625–630. URL: <https://doi.org/10.1017/S0266466600009440>.
- Hansen, Peter R and Asger Lunde (2006). “Realized variance and market microstructure noise”. In: *Journal of Business & Economic Statistics* 24.2, pp. 127–161. URL: <https://doi.org/10.1198/073500106000000071>.
- Hong, Harrison and Jeremy C Stein (2007). “Disagreement and the stock market”. In: *Journal of Economic perspectives* 21.2, pp. 109–128. URL: <http://www.aeaweb.org/articles?id=10.1257/jep.21.2.109>.
- Hu, Jianfeng (2014). “Does option trading convey stock price information?” In: *Journal of Financial Economics* 111.3, pp. 625–645. URL: <https://doi.org/10.1016/j.jfineco.2013.12.004>.

- Hull, John C (2003). *Options futures and other derivatives*. 5th. Pearson Education India.
URL: <https://www.pearson.com/us/higher-education/product/Hull-Options-Futures-and-Other-Derivatives-5th-Edition/9780130090560.html>.
- Kandel, Eugene and Neil D Pearson (1995). “Differential interpretation of public signals and trade in speculative markets”. In: *Journal of Political Economy* 103.4, pp. 831–872. URL: <https://www.jstor.org/stable/2138584>.
- Karpoff, Jonathan M (1987). “The relation between price changes and trading volume: A survey”. In: *Journal of Financial and quantitative Analysis* 22.1, pp. 109–126. URL: <https://doi.org/10.2307/2330874>.
- Lemmon, Michael L and Sophie Xiaoyan Ni (2011). “The effects of investor sentiment on speculative trading and prices of stock and index options”. In: *SSRN 1340762*. URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1340762.
- Mancini, Cecilia (2001). “Disentangling the jumps of the diffusion in a geometric jumping Brownian motion”. In: *Giornale della Istituto Italiano degli Attuari* 64.19-47, p. 44.
- Milgrom, Paul and Nancy Stokey (1982). “Information, trade and common knowledge”. In: *Journal of economic theory* 26.1, pp. 17–27. URL: [https://doi.org/10.1016/0022-0531\(82\)90046-1](https://doi.org/10.1016/0022-0531(82)90046-1).
- Muravyev, Dmitriy, Neil D Pearson, and John Paul Broussard (2013). “Is there price discovery in equity options?” In: *Journal of Financial Economics* 107.2, pp. 259–283. URL: <https://doi.org/10.1016/j.jfineco.2012.09.003>.
- Ni, Sophie, Jun Pan, and Allen Poteshman (2008). “Volatility information trading in the option market”. In: *The Journal of Finance* 63.3, pp. 1059–1091. URL: <https://doi.org/10.1111/j.1540-6261.2008.01352.x>.
- Pan, Jun and Allen Poteshman (2006). “The information in option volume for future stock prices”. In: *The Review of Financial Studies* 19.3, pp. 871–908. URL: <https://doi.org/10.1093/rfs/hhj024>.
- Petersen, Mitchell A (2009). “Estimating standard errors in finance panel data sets: Comparing approaches”. In: *The Review of Financial Studies* 22.1, pp. 435–480. URL: <https://doi.org/10.1093/rfs/hhn053>.
- Ryu, Doojin and Heejin Yang (2018). “The directional information content of options volumes”. In: *Journal of Futures Markets* 38.12, pp. 1533–1548. URL: <https://doi.org/10.1002/fut.21960>.
- Stephan, Jens A and Robert E Whaley (1990). “Intraday price change and trading volume relations in the stock and stock option markets”. In: *The Journal of Finance* 45.1, pp. 191–220. URL: <https://doi.org/10.1111/j.1540-6261.1990.tb05087.x>.

- Tauchen, George E and Mark Pitts (1983). “The price variability-volume relationship on speculative markets”. In: *Econometrica: Journal of the Econometric Society*, pp. 485–505. URL: <https://www.jstor.org/stable/1912002>.
- Wang, Yaw-Huei (2013). “Volatility information in the trading activity of stocks, options, and volatility options”. In: *Journal of Futures Markets* 33.8, pp. 752–773. URL: <https://doi.org/10.1002/fut.21601>.
- White, Halbert (1980). “A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity”. In: *Econometrica* 48.4, pp. 817–838. URL: <https://www.jstor.org/stable/1912934>.
- Zhou, Bin (1996). “High-frequency data and volatility in foreign-exchange rates”. In: *Journal of Business & Economic Statistics* 14.1, pp. 45–52. URL: <https://www.jstor.org/stable/1392098>.

7 Appendix

7.1 Cleaning Procedures

The computation of the residual volatility of options uses high-frequency data on the S&P 500 index options (SPX), and on the SPDR S&P 500 exchange-traded fund (SPY).

The data set on the SPX options was obtained from the Chicago Board Options Exchange (CBOE). The set contains quotes at the 1-minute interval level from 2007 to 2016. The market hours are from 9:30 am to 4:15 pm EST, resulting in 405 quotes per day. Partial trading days are removed from the sample (23 in total), and we base the analysis on the remaining 2458 days. On 28 trading days, there are missing values for the first minute or two at the market open. These missing values are filled with the first non-zero prices of the same day. As noted in Section 4.3, the options data are sparsely sampled to 5-minutes to mitigate the impact of microstructure noise on volatility estimates.

The data set on SPY was obtained from TickData⁷. The set contains the last closing price of every 5-minute interval, from 2007 to 2016.

To obtain the moneyness of the SPX options, we need their time to expiration. We follow the methodology of CBOE (2014) to compute the time to maturity in minutes, while dealing with the differences between AM and PM settled options and options that expire on holidays. We compute the moneyness of all options daily at the end of the day. We select options with moneyness ranging from -5 to 3, which captures a wide variety of options while eliminating options that have pricing issues due to extremely low liquidity.

Lastly, the composition of expiration dates of SPX options varies throughout the years and could lead to issues in prices and estimation of the residual volatility. The history of SPX options is relevant in understanding how so. SPX options were launched in 1987, and originally expired every 3rd Friday of the month. Their increasing success warranted the expansion of SPX options to include new expiration dates. In 2005, CBOE introduced SPX Weeklys, which expire on all other Fridays. However, at launch, these options had much lower liquidity than the original SPX options, but their liquidity increased substantially over the years¹¹. Now, because the residual volatility is estimated from the prices of options, it is imperative to have prices that directly reflect the real value of options. For this reason, options with low liquidity, or higher spreads, are disregarded in favor of options with high liquidity. In the case of SPX options, the liquidity is related to the expiration dates, since the original SPX options had higher liquidity than the Weeklys at launch, and for some years after that. To take into account this difference, we only include options that do not have stale prices. An option is said to have stale prices if its intra-day prices do not change on more than 50% of the time intervals within a day.

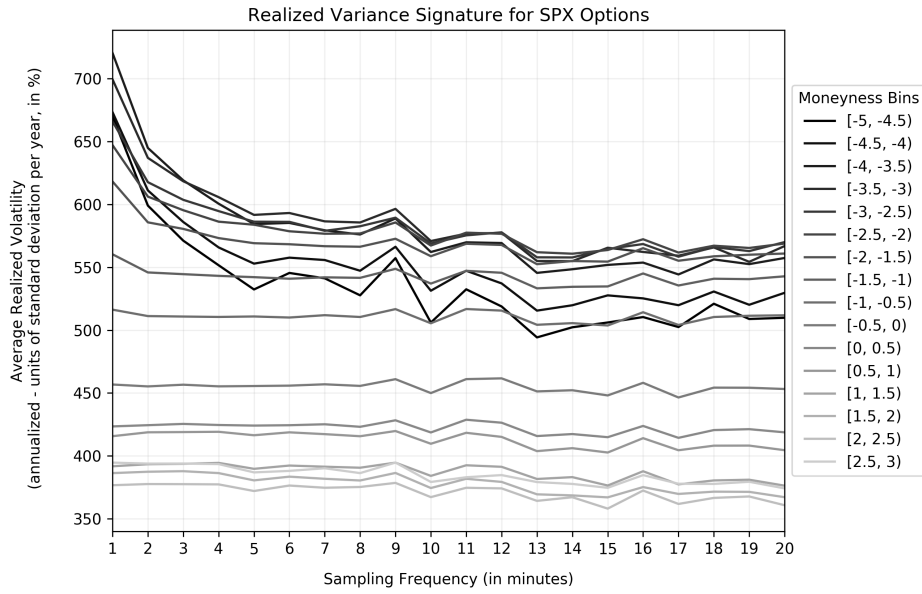
7.2 Volatility Signature for SPX Options

Figure 7 displays the volatility signature plot for the market options using a finer division for moneyness. The options are categorized by moneyness, and their realized variances are computed for different sampling frequencies.

We observe no increase in the realized variance when the sampling frequency increases for options that are ITM, ATM and slightly OTM. However, for options with moneyness smaller than -2.5 there is a clear indication of microstructure noise, since sampling frequencies smaller than 5-minutes lead to an ever increasing estimates of variance. Overall,

¹¹See SPX Weeklys volume chart on CBOE's website.

Figure 7: Volatility Signature for SPX Options.



Notes: The figure shows the average realized volatility (annualized in percentage) for options of different moneyness based on different sampling frequencies. The realized volatility is estimated as the sum of squared returns over each day for each option, where the frequency of returns varies from 1-minute up to 20-minutes. The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

a sampling frequency of 5-minutes provides a good trade-off between avoiding microstructure noise and estimating the realized variance with enough precision.

7.3 Parametric Estimates Plot and Surface Allowing Moneyness Effect to Change for OTM and ITM

The parametric specification analyzed in Section 5 assumed the effects of moneyness on elasticity were symmetrical, in the sense that increases in moneyness away from zero had the same impact as decreases in moneyness. However, the nonparametric results from 5.4 indicate that the decrease in elasticity is higher when moneyness increases away from zero than from when it decreases. We re-estimate the parametric specification allowing the impact of moneyness to be asymmetrical. Table 6 below displays the estimates.

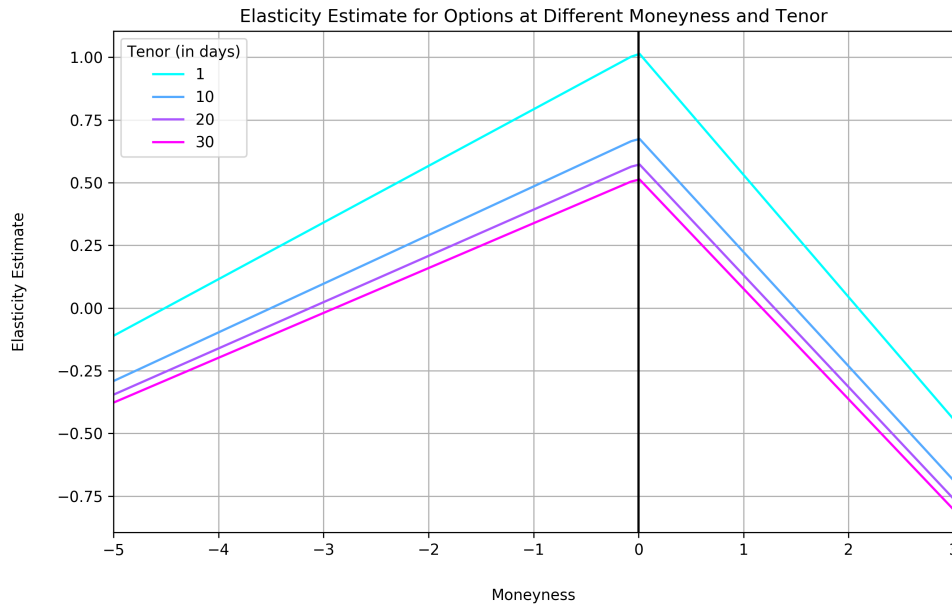
Table 6: Volume-Volatility Elasticity Estimates.

	Parameter Estimate	Standard Error
Baseline Estimates		
Intercept (α_0)	7.5726	0.058
Elasticity (β_0)	1.0186	0.013
Explanatory Variables for Elasticity		
$\mathbb{1}\{Moneyness > 0\} \cdot Moneyness $	-0.2258	0.007
$\mathbb{1}\{Moneyness \leq 0\} \cdot Moneyness $	-0.4873	0.009
$\ln(\text{Tenor})$	-0.1474	0.004
$ Moneyness \cdot \ln(\text{Tenor})$	0.0138	0.002
Adjusted R^2	0.243	
Number of Observations	446053	

Notes: The table reports the estimated values of α_0 , β_0 and β_1 in equation (3). The estimated values of α_1 are not reported for brevity. The control variables in X_0 include moneyness, absolute value of moneyness, logarithm of tenor and interaction terms. The absolute value of moneyness is interacted with an indicator for negative moneyness and for positive moneyness. Estimation is based on full sample of cleaned data, spanning 2007 to 2016. Values for volume and volatility are computed daily for each option from high-frequency data. Moneyness is measured in standard deviations to be at-the-money. Tenor is measured in days. The standard errors are heteroscedasticity robust as in White (1980).

Figure 8 below displays the estimated elasticities for different levels of moneyness and tenor. The characteristics are similar to Figure 2, with the exception of a sharper drop in elasticity (higher increase in disagreement) as moneyness moves from zero to positive values.

Figure 8: Elasticity Estimates for Options at Different Moneyness and Tenor.



Notes: The figure shows the volume-volatility elasticity estimates for different moneyness levels and tenors. Fixing any time to expiration, the elasticity decreases as options move away from the money. Fixing any moneyness, the elasticity decreases as the time to expiration increases.