

## Kidney Exchange Optimization

The Kidney Paired Donation (KPD) program is a voluntary kidney exchange program operated by Canadian Blood Services (CBS) (you can read more about it [here](#)). Periodically, when the number of donor/patient pairs in the KPD pool reaches a certain size (e.g., 25 pairs), CBS runs an algorithm to maximize the number of transplants achievable through compatible exchange cycles. This question involves a slightly simplified version of the real process, but captures the essence of the underlying optimization problem.

Before receiving a kidney transplant, both patient and donor must undergo a thorough medical evaluation to make sure that they are both blood type compatible and tissue type compatible. For instance, a patient with “A blood type” and “tissue type 1” kidney can only receive a kidney from a donor with A or O blood type and tissue type 1 kidney. The following table gives the compatibility of blood types (BT) between donors and patients, with a check mark meaning a donor with the blood type shown in that row can donate to a patient with the blood type shown in that column. Assume that in addition to the blood type compatibility rules, the donor’s tissue type must match exactly with the patient’s tissue type. Also, assume that a donor in one pair won’t give to another patient in another pair unless their loved one also receives a transplant from some donor in the pool. In other words, we are not considering altruistic donors in this setting.

Donor’s BT	Patient’s BT			
	A	B	AB	O
A	✓	?	✓	?
B	?	✓	✓	?
AB	?	?	✓	?
O	✓	✓	✓	✓

Currently, there are 25 pairs in the KPD pool with 5 different tissue types; see ‘KPD\_pool.csv’ for details.

- Formulate (algebraically) an optimization problem to help the KPD program maximize the number of achievable transplants.
- Ask ChatGPT to do the same for part a) (the actual data from the .csv file is not needed for this part). That’s right...go ahead and copy and paste, starting from “The Kidney Paired Donation...” to “Currently, there are...details.” Then add the sentence “Formulate (algebraically) an optimization problem to maximize the number of transplants achievable,” and see what ChatGPT returns. Share screen shots of its response and indicate up to two things that it either: 1) gets wrong, and/or 2) is logically correct about, but it could have done in a cleaner/simpler way. If you do not find two things, then put one thing. If you don’t find one thing, then say you think it got everything correct.

- c) Solve your formulation from part a) using Python/Gurobi to determine the maximum number of transplants achievable from this pool, along with an indication of which donors should give to which patients to achieve this total. Provide some managerial discussion of your solution; e.g., any interesting observations regarding the solution, any potential practical challenges in implementing it, any concerns about fairness, etc.
- d) A new biotechnology is being developed that can turn donor kidneys of any blood type into kidneys of blood type O<sup>1</sup>. Update your optimization model to create a plot of “total transplants vs. # of donor blood type conversions,” and discuss your findings for an audience of transplant program managers.