Surprise Quiz: Probability and Statistics (30 Marks)

Each question: 5 marks

1. Suppose $\mathbf{X} \sim \mathcal{N}(\mu, \mathbf{\Sigma})$. Let $\mathbf{Y} = W^{-1}(\mathbf{X} - \mu)$ where W is a matrix that satisfies $W^2 = \Sigma$. Obtain the distribution for the random vector $\mathbf{Y}$.

2. (Memory test:)
   - State the Central limit theorem. (2.5 marks)
   - Write the three equivalent definitions for a multivariate Gaussian. (2.5 mks)

3. Find the stationary distribution π for Markov Chain with the following transition probability matrix.(3 marks) State if π is unique. (1 mark) Is the chain irreducible? Give reasons (1mark)

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0 & 0.9 & 0.1 \\ 0.1 & 0.9 & 0 \end{bmatrix}$$

4. Suppose $\mathbf{X}$ is a standard normal vector of size n. Let $\mathbf{Y} = A\mathbf{X}+\mathbf{b}$ where A is a square symmetric invertible matrix. Derive the expression for $f_{\mathbf{Y}}(\mathbf{y})$.

Each question: 10 marks

1. Let (X, Y) be a pair of continuous random variables with joint PDF $f_{X,Y}(x, y)$. Define a transformation from (X, Y) to (U, V) given by

$$U = X + Y, \quad V = X - Y.$$

   (a) Find the joint PDF $f_{U,V}(u, v)$ of the transformed random vector (U, V). (5mks)

   (b) Assume $f_{X,Y}(x, y) = ce^{-(x^2+y^2)}$ for all $x, y \in \mathbb{R}$. Find $f_{U,V}(u, v)$ for this specific case and elaborate on what kind of random variables are U and V. (5mks)