INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Monsoon Semester (Aug-Nov), 2023
Discrete Structures (DS, Section B)

Quiz I
30.08.2023

1. **(Sets)** [4]

   1. For all sets A and B, prove that
   $$(A \cap B) \cup (A \cap B') = A.$$
   Here A' denotes complement of set A.

   $(1, 1)$ $\Rightarrow$

   2. Suppose $A = \{1, 2\}$ and $B = \{2, 3\}$. Find power set $P(A \times B)$. Here $A \times B$ denotes the Cartesian product of A and B.
   3. Let $\mathbb{R}$ be the set of real numbers. Is $\{\mathbb{R}^+, \mathbb{R}^-, \{0\}\}$ a partition of $\mathbb{R}$? Here $\mathbb{R}^+$ denote set of positive reals, $\mathbb{R}^-$ denote set of negative reals. Explain your answer.
   4. Let $S_i = \{x \in \mathbb{R} \,|\, 1 < x < 1 + \frac{1}{i}\} = (1, 1 + \frac{1}{i})$ for all **positive** integers i. Then find the following:
      (a) $\cup_{i=1}^{\infty} S_i = ?$
      (b) $\cap_{i=1}^{\infty} S_i = ?$

   $x \in (1, 1 + \frac{1}{i})$ $\qquad \bigcup_{i=1}^{\infty} S_i$

   $\sum \frac{i+1}{i} \Rightarrow \frac{\sum(i+1)}{\sum i}$

2. **(Induction Proofs)** [4]

   Prove the following using induction.

   1. Suppose $e_0, e_1, \ldots,$ is a sequence defined as follows
   $$e_0 = 12,\ e_1 = 29,\ e_k = 5e_{k-1} - 6e_{k-2},\ \forall k \geq 2$$
   Prove that $e_n = 5 \cdot 3^n + 7 \cdot 2^n$ for all integers $n \geq 0$.
   2. Show that
   $$\frac{m!}{0!} + \frac{(m+1)!}{1!} + \cdots + \frac{(m+n)!}{n!} = \frac{(m+n+1)!}{n!(m+1)},$$
   where $m, n = 0, 1, 2, \ldots$

3. **(Pigeon hole principle)** [3]

   The **pigeon-hole principle** states that:

   If we put $N + 1$ pigeons in N pigeon-holes, then there will be atleast one pigeon hole with at least two pigeons. Prove this statement using contrapositive proof.

   A **general pigeon-hole principle** is stated as follows:

   If we must put $Nk + 1$ or more pigeons into N pigeon holes, then some pigeon-hole must contain at least $k + 1$ pigeons. Prove this using contrapositive proof.

   Prove the following using pigeon-hole principle.

   1. Show that among 5 people at a dinner table, there are two that have an identical number of friends among those at the table.