Quiz1: Probability and Statistics (20 Marks)

Instruction:

- Please state reasons wherever applicable.
- Use precise mathematical arguments, no speeches.

Each question: 5 marks

1. For a non-negative discrete random variable X, prove that $E[X] = \sum_{i=0}^{\infty} P(X > i)$.
2. Consider a Geometric random variable X with parameter p. Derive the expression for its mean, second moment and variance.
3. Suppose that we roll a die twice. Consider the following three events
   - A = Second roll is 4
   - B = Difference between the two rolls is 4
   - C = Difference between the two rolls is 3

   Are the three events pairwise independent ? Are they also mutually independent?
4. Let X denote a Gaussian random variables with parameters c and d. Let Y denote a Binomial random variable with parameters n and p. Derive the expression for their respective variance. (2.5 mks each)