Midsem: Probability and Statistics (50 Marks)

Each question: 6 marks

1. Let X be a continuous random variable with distribution $F_X(\cdot)$ and density $f_X(x)$. Find the probability density and cumulative distribution for $Z = X^2 + 4$.

2. Suppose X and Y are independent and exponential random variables with parameter a and b respectively. Then find $P(X < Y)$.

3. Assume that random variables $X_1, X_2, \ldots X_n$ each have a finite variance and are not independent of each other. Let $S_n = \sum_{i=1}^{n} X_i$. Derive an expression for the Variance of S_n. How does the expression change when X_i's are independent.

4. Let X be a random variable having Binomial distribution with parameters N and p where N is itself a random variable having Poisson distribution with mean λ. Find the probability mass function of the random variable X. Also find $E[X]$.

5. Let X be a standard normal variable (Gaussian with zero mean and unit variance.) Let $Z = \sigma X + \mu$. Obtain the pdf and cdf of Z.

Each question: 10 marks

1. Suppose U_1 and U_2 are independent uniform random variables on segments $[-1, 1]$ and $[0, 1]$ respectively. Let $Z = U_1 + U_2$. Derive an expression for the pdf and cdf of Z.

2. Let X and Y be jointly continuous random variables with joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 6e^{-(2x+3y)} & \text{if } x, y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $E[X]$ and $E[Y]$. (4 mks)

   (b) Are X and Y independent? Justify. (2mks)

   (c) Find $E[Y|X > 2]$.(2mks)

   (d) Find $P(X > Y)$.(2mks)