

Instructions:

- Calculator is not required and not allowed in this exam.
 - Question paper consists of 14 questions, points indicated for each question.
 - Answer precisely; over-verbosity will be penalized.
 - No questions/doubts will be entertained during exam; assume appropriately, state assumptions and answer the question.
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1. (3 points) State the three laws of mechanics by Newton.
2. (2 points) State the assumptions of Special Theory of Relativity.
3. (2 points) State the Zeroth Law of thermodynamics.
4. (2 points) State the First Law of thermodynamics; how is it different or specialized from classical mechanical law of conservation of energy.
5. (3 points) Classical mechanical systems have several conservation laws. State these conserved quantities, and for projectile motion, write the mathematical formula for them.
6. (4 points) State the differences between Newtonian and Lagrangian mechanics.
7. (4 points) For the projectile motion, write the Lagrangian and Hamiltonian. Also write the equation of motion in Lagrangian and Hamiltonian formulations.
8. (4 points) A rocket engine exhausts gases with a constant velocity u (relative to the rocket body and in backward direction) to provide forward thrust to rocket. Assuming vertical motion, constant burning rate of fuel and constant gravity, find the equation of motion of the rocket. From equation of motion, find the velocity of rocket as a function of current mass of rocket. Comment on when lift-off is achieved.
9. (4 points) Consider a rope of mass per unit length ρ and length a suspended just above the table (i.e held hanging from the higher end). When the rope is released, as the rope falls, it forms a pile on the table. Find the force on the table when a length x of the rope has dropped to the table.
10. (3 points) A hook attached to ceiling of the room has a small loop, through which a rope can pass easily. Now, the rope is hung with equal lengths on either side so it balances. A small tug is given to one side, so that the rope will start sliding. Find the velocity of the rope when the whole rope slides off the loop.
11. (5 points) Consider light source (frequency ν_0 in its rest frame) located distance l away from the observer. The source is moving perpendicular to the line of sight of the observer with velocity v , find the apparent frequency of light source.

12. (5 points) A double pendulum is constructed by using two straight rods of length l_1 and l_2 , connected by a hinge. One end of the first rod is attached to fixed point of support by an hinge; when set in motion, the first rod is a pendulum with a fixed point of support and the second rod is pendulum with "moving point" of support ("free" end of the first rod). Find the equation of motion of this double pendulum, assuming no friction at hinges.
13. (3 points) Show how the classical mechanical kinetic energy formula can be arrived at from relativistic kinetic energy formula. Find the range of speeds for which the kinetic energy in classical and relativistic mechanics differ by less than 1%.
14. (6 points) (a) For a perfect gas, using the definition of perfect gas, find the change in internal energy in a constant volume process when temperature changes from T_1 to T_2 . And hence show that internal energy is function of temperature alone. (b) For a perfect gas, assume that ideal gas holds, calculate the work done in reversible isothermal expansion from V_1 to V_2 . (c) In thermodynamics, in an adiabatic process there is no heat flow allowed between system and surroundings (for example, by use of insulating boundary). Such an adiabatic process brings the system from (T_1, V_1) to (T_2, V_2) . You should see that processes part-a and part-b above do the same change but in a different path, using which show that $TV^c = \text{constant}$, and find c .
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