Quiz2: Probability and Statistics (30 Marks)

Each question: 5 marks

1. Let A and B be two independent Poisson random variables with parameter a and b respectively. Let $C = A + B$. Using MGF, show that C is also a Poisson random variable.

2. Derive an expression for the k^{th} moment of an exponential random variable with parameter λ using MGF.

3. $X_1, X_2, \ldots, X_n \ldots$ are i.i.d Uniform [-1,1]. Define $Y_n = \frac{X_n}{n}$. Show that Y_n converges in probability to some limit and identify the limit.

4. Let $X_1, X_2 \ldots$ be sequence of random variables such that

$$F_{X_n}(x) = \frac{e^{n(x-1)}}{1 + e^{n(x-1)}} \text{for } x > 0.$$

Show that X_n converges in distribution. Identify the limiting random variable.

Each question: 10 marks

1. Given samples $x_1, \ldots x_n$ from an exponential random variable X with parameter λ, convert it into samples from another exponential random variable Y with parameter μ. Explain the procedure in detail with justifications.