Quiz II

Consider the following system of congruent-recurrences:

$$a_n \equiv a_{n-1} + 2n \quad \text{with} \quad a_0 \equiv 0 \quad (\text{mod } 3)$$
$$\overline{a_n} \equiv 4a_{n-1} - 4a_{n-2} \quad \text{with} \quad a_0 \equiv 1, a_1 \equiv 3 \quad (\text{mod } 5)$$
$$a_n \equiv 3a_{n-1} + 5(3^n) \quad \text{with} \quad a_0 \equiv 2 \quad (\text{mod } 7)$$

Answer the following:

1. What is $a_0 \bmod 105$? **5 marks**

2. What is $a_2 \bmod 105$? **5 marks**

3. What is $(a_{100} \bmod 3)$? **15 marks**

4. What is $(a_{150} \bmod 15)$? **20 marks**

5. What is $(a_{200} \bmod 105)$? **25 marks**

6. With the same initial/boundary conditions, how many values between 0 and 104 can $(a_3 \bmod 105)$ take, if:

   - all the three congruences are satisfied? **3 marks**
   - *none of the three* congrunences are satsfied? **12 marks**
   - *exactly one of the three* congrunences are satsfied? **15 marks**