INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

MID SEMESTER EXAMINATION
IIIT, Hyderabad, Monsoon 2023
Subject: Discrete Structures
(Code: MA05.101)

20.09.2023
8:30 - 10:00

Duration: 90 minutes

Maximum Marks: 30

Instructions and Notations

There are 3 pages and 14 questions. All questions are compulsory. Calculator is not allowed. You are not allowed to ask questions in exams to anyone including invigilators. If you find that there are doubts or errors in questions, then please write it in your answer script with proper justification. If the justification is correct, you will be awarded full marks.

1. Let $A = \mathbb{Z}$ and let $\mathcal{R} = \{(x, y) \in A \times A \mid xy + x^2 = x^2 + 1\}$. Which of the following are true? [3]

   a). $0\mathcal{R}0$.

   b). $1\mathcal{R}1$.

   c). $1\mathcal{R}0$.

   d). $1\mathcal{R}(-2)$.

   e). $3\mathcal{R}2$.

   Note: Here $x\mathcal{R}y$ means that $(x, y) \in \mathcal{R}$.

   A relation is called **symmetric** if whenever $x\mathcal{R}y$, then $y\mathcal{R}x$. A relation is called **transitive** if whenever $x\mathcal{R}y$ and $y\mathcal{R}z$, then $x\mathcal{R}z$. Is the given relation $\mathcal{R}$ symmetric and transitive? Justify.

2. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$. Define a function $f : A \to B$ as follows: [2]

$$f(1) = b, \quad f(2) = a, \quad f(3) = c, \quad f(4) = a.$$

   Let $g : B \to A$ be the function defined by the rule

$$g(a) = 2, \quad g(b) = 3, \quad g(c) = 1, \quad g(d) = 3.$$

   a). Find $(g \circ f)(x)$ for each element x of A.

   b). What is the domain, co-domain, and range of $g \circ f$?

   c). Does inverse of f, g, and $g \circ f$ exists? Justify.

   d). Is it possible to define functions p and q with suitable domain and co-domain such that $p \circ g \circ f \circ q$ is an identity map from A to A? Justify.

3. Let $\mathbb{N}_+ = \mathbb{N} \cup \{0\}$, and define the function $f : \mathbb{N}_+ \times \mathbb{N}_+ \to \mathbb{N}$ by [3]

$$f(m, n) = 2^m(2n + 1).$$

   Prove or disprove the following:

   a). f is injective.

   b). f is surjective.

Student's name:

4. For three sets $A, B, C \subseteq U$ we have the following: [2]

$$B \subseteq A, \quad C^c \cup A = U, \quad |C^c \cap A| = 7, \quad |B \Delta C| = 5,$$
$$|A \setminus (B \setminus C)| = 6, \quad |A \setminus (C \setminus B)| = 9, \quad |(B \cup C)^c| = 5.$$

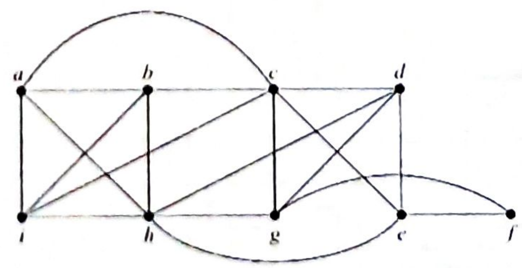
Find $|A|, |B|$, and $|C|$. If the given conditions above are inconsistent, then justify.

Here X^c denotes complement of the set X, and $\setminus$ denotes set difference, i.e., $X \setminus Y = \{x \in X \mid x \notin Y\}$, and Δ denotes symmetric set difference, i.e., $X \Delta Y = (X \setminus Y) \cup (Y \setminus X)$. Also, $|X|$ denotes number of elements in set X. You may use Venn diagram.

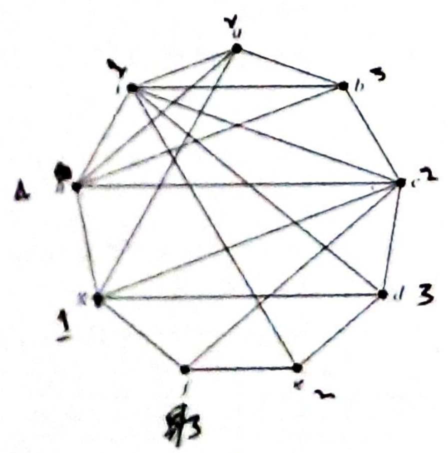
5. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 3. [2]

6. Using division algorithm, show that there exists integers x and y such that $19x + 43y = 1$. [2]

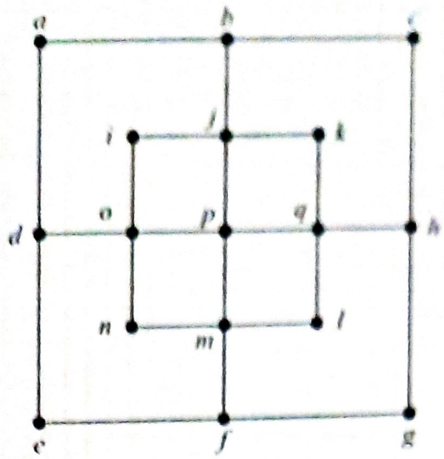
7. Determine whether the Euler tour (also called circuit) exists for the following graph. If Euler tour does not exist, then is there any Euler path? Justify your answer. [1]



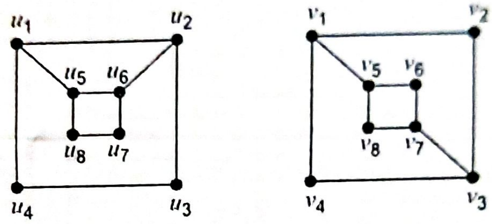
8. Recall that the chromatic number of a graph is the minimum number of colors needed to color the vertices of a graph, so that no two adjacent vertices have same color. Find the chromatic number of the following graph. 4 [1]



9. Determine whether the following graph has Hamiltonian circuit. State reasons why? [2]



10. Show whether the following graphs are isomorphic or not? [2]



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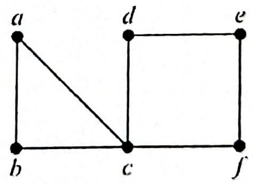
11. Find all the cut vertices and cut edges of the following graph. A cut vertex is a vertex which when removed increases the number of components of graph. Similarly, cut edge is an edge which when removed increases the components of the graph. [Note when vertex gets removed, all edges incident on it is removed as well, whereas, when an edge is removed, the endpoints (vertices joining that edge) of the edges are not removed.] [2]



12. How many non-isomorphic connected simple graphs are possible with n vertices when n is 4? [2]

13. Show that a simple graph with at least two vertices has at least two vertices that are not cut vertices. See cut vertex defined above. [3]

14. Show that if G is a connected graph, then it is possible to remove vertices (along with edges incident on it) to disconnect G if and only if G is not a complete graph. [3]