

1. (Sets)

[4]

1. For all sets A and B , prove that

$$(A \cap B) \cup (A \cap B') = A.$$

Here A' denotes complement of set A .

2. Suppose $A = \{1, 2\}$ and $B = \{2, 3\}$. Find power set $P(A \times B)$. Here $A \times B$ denotes the Cartesian product of A and B .
3. Let \mathbb{R} be the set of real numbers. Is $\{\mathbb{R}^+, \mathbb{R}^-, \{0\}\}$ a partition of \mathbb{R} ? Here \mathbb{R}^+ denote set of positive reals, \mathbb{R}^- denote set of negative reals. Explain your answer.
4. Let $S_i = \{x \in \mathbb{R} \mid 1 < x < 1 + \frac{1}{i}\} = (1, 1 + \frac{1}{i})$ for all positive integers i . Then find the following:
- (a) $\cup_{i=1}^{\infty} S_i = ?$
- (b) $\cap_{i=1}^{\infty} S_i = ?$

2. (Induction Proofs)

[4]

Prove the following using induction.

1. Suppose e_0, e_1, \dots , is a sequence defined as follows

$$e_0 = 12, e_1 = 29, e_k = 5e_{k-1} - 6e_{k-2}, \forall k \geq 2$$

Prove that $e_n = 5 \cdot 3^n + 7 \cdot 2^n$ for all integers $n \geq 0$.

2. Show that

$$\frac{m!}{0!} + \frac{(m+1)!}{1!} + \dots + \frac{(m+n)!}{n!} = \frac{(m+n+1)!}{n!(m+1)},$$

where $m, n = 0, 1, 2, \dots$

3. (Pigeon hole principle)

[3]

The pigeon-hole principle states that:

If we put $N + 1$ pigeons in N pigeon-holes, then there will be atleast one pigeon hole with at least two pigeons. Prove this statement using contrapositive proof.

A general pigeon-hole principle is stated as follows:

If we must put $Nk + 1$ or more pigeons into N pigeon holes, then some pigeon-hole must contain at least $k + 1$ pigeons. Prove this using contrapositive proof.

Prove the following using pigeon-hole principle.

1. Show that among 5 people at a dinner table, there are two that have an identical number of friends among those at the table.