Tutorial Sheet 3

II B. Tech. (Common)
Mathematics-II
(MCI102)

1. Test whether the below equations are exact and hence solve it.

   (a) $\left(2x^2+4y\right)dx+(4x+y-1)dy=0$, (b) $\left(1+2xy\cos x^2-2xy\right)dx+\left(\sin x^2-x^2\right)dy=0$,
   (c) $xdx+ydy+\frac{xdy-ydx}{x^2+y^2}=0$, (d) $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$,
   (e) $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+[x+\log x-x\sin y]dy=0$.

   Answers: (a) $\left[\left(x^4+y^2\right)/2\right]+4xy-y=c$, (b) $x+y\sin x^2-yx^2=c$, (c) $x^2+y^2-2\tan^{-1}\left(\frac{x}{y}\right)=-c$,
   (d) $e^{xy^2}+x^4-y^3=c$, (e) $yx+y\log x+x\cos y=c$

2. Find the integrating factors and solve the following differential equations:

   (a) $\left(2x^2+y^2+x\right)dx+xydy=0$, (b) $x^2\frac{dy}{dx}+xy=\sqrt{1-x^2y^2}$, (c) $\left(3xy^2-y^3\right)dx-\left(2x^2y-xy^2\right)dy=0$,
   (d) $\left(y+xy^2\right)dx+\left(x-x^2y\right)dy=0$, (e) $\left(x^2+y^2+x\right)dx+xydy=0$.

   Answers: (a) $3x^4+2x^3+3x^2y^2=c$, (b) $\sin^{-1}(xy)=\log x+c$, (c) $3\log x-2\log y+\frac{y}{x}=c$, (d) $-\frac{1}{xy}+\log\frac{x}{y}=c$,
   (e) $\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}=c$

3. Solve the following Bernoulli's equations:

   (a) $\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$, (b) $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ (c) $x\left(\frac{dy}{dx}\right)+y\log y=xye^x$ (d) $\frac{dy}{dx}+2xy=xy^3$
   (e) $\frac{dy}{dx}\sin x-y\cos x+y^2=0$, (f) $\left(1+y^2\right)+\left(x-e^{\tan^{-1}x}\right)\frac{dy}{dx}=0$, (g) $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$, (h) $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$,
   (i) $\frac{dy}{dx}+\frac{1}{x}\sin 2y=x^3\cos^2 y$, (j) $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$.

   Answers: (a) $\sqrt{y}=-\frac{1}{3}\left(1-x^2\right)+c\left(1-x^2\right)^{\frac{1}{4}}$, (b) $e^{x^2}\tan y=\frac{1}{2}e^{x^2}\left(x^2-1\right)+c$, (c) $x\log y=e^x(x-1)+c$,
   (d) $\frac{1}{y^2}=\frac{1}{2}+ce^{2x^2}$, (e) $\sin x=y(x+c)$, (f) $x=\left(c+\tan^{-1}y\right)e^{-\tan^{-1}y}$, (g) $x=\frac{y}{2}+cx^2y$,
   (h) $2x=e^y\left(1+cx^2\right)$, (i) $6x^2\tan y=x^6+c$, (j) $x=\log y\left(cx^2+\frac{1}{2}\right)$.

4. Find the orthogonal trajectories of the following family of curves, where a is the parameter

   (a) $xy=a^2$, (b) $3xy=x^3-a^3$, (c) $x^2+y^2+2ax+b=0$, (d) $r=a(1-\cos\theta)$, (e) $r^n=a^n\cos(n\theta)$,
   (f) $x^2-y^2=a^2$, (g) $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda^2}=1$ (where λ is a parameter), (h) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, (i) $r^2=a^2\cos 2\theta$,
   (j) $r^n\sin n\theta=a^n$, (k) $r=a(1+\cos\theta)$, (l) $r=\frac{2a}{(1+\cos\theta)}$

   Answers: (a) $x^2-y^2=c^2$, (b) $x^2=y-\frac{1}{2}+ce^{-2y}$, (c) $x^2+y^2-cy-b=0$, (d) $r=c(1+\cos\theta)$,
   (e) $r^n=a^n\sin(n\theta)$, (f) $xy=c$, (g) $x^2+y^2=2a^2\log x+c$, (h) $x^{\frac{4}{3}}-y^{\frac{4}{3}}=c^{\frac{4}{3}}$, (i) $r^2=c^2\sin 2\theta$,
   (j) $r^n\cos n\theta=c$, (k) $r=c(1-\cos\theta)$, (l) $r=\frac{2c}{1-\cos\theta}$

5. (a) Show that the family of parabolas $x^2=4a(y+a)$ is self-orthogonal.
   (b) Prove that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, λ being the parameter is self-orthogonal.

6. If S is defined by the rectangle $|x|\le a, |y|\le b$, show that function $f(x,y)=x\sin y+y\cos x$, satisfy Lipschitz condition. Find the Lipschitz constant. Answer: $a+1$

7. Can we drop the Lipschitz condition in the equation $f(x,y)=y^{2/3}$ on $R:|x|\le 1, |y|\le 1$. Answer: No

8. Prove that the differential equation $Mdx+Ndy=0$ possess on infinite number of integrating factors.

9. Find the largest interval in which Picard's theorem generates for unique solution $\frac{dy}{dx}=16+y^2, y(0)=0$.
   Answer: $|x|<\frac{1}{8}$.

10. Find the second approximation of the solution of the equation $\frac{dy}{dx}=2-\frac{y}{x}$, $y(1)=2$ by Picard's method.
    Answer: $2+(\log x)^2=y_2$.

11. By using Picard iteration method find the solution of following initial value problems:

    (a) $y'=2xy,\ y(0)=1$, (b) $y'=y-x,\ y(0)=2$

    Also find the solution analytically and compare it with the said iteration scheme.