

Tutorial Sheet 4
II B. Tech. (Common)
Mathematics-II
(MCI102)

1. Consider the vector space P_2 consisting of all polynomials of degree at most two together with the zero polynomial. Let $S = \{p_1(t), p_2(t)\}$ be a set of polynomials in P_2 where :
 $p_1(t) = -t^2 + 3$, $p_2(t) = -2t^2 - 2t + 3$.
Show that the set $S = \{p_1(t), p_2(t)\}$ is linearly independent in P_2 .
2. Compute the wronskian of the following sets of functions and state whether or not the two functions are linearly independent:
(a) $\{e^x, e^x \sin x\}$ **Ans:** Linearly independent
(b) $\{x \ln |x|, x^2 \ln |x|\}$ **Ans:** Linearly independent
(c) $\{1, e^x\}$ **Ans:** Linearly independent
3. Find the general solution of the following second order homogenous differential equations:
(a) $y'' + 3y' - 10y = 0$ **Ans:** $y(x) = Ae^{-5x} + Be^{2x}$
(b) $2y'' - 3y' = 0$ **Ans:** $y(x) = C_1 + C_2 e^{\frac{3}{2}x}$
4. Use variation of parameters to find the general solution to the differential equation $y'' - 9y = \frac{9x}{e^x}$.
Ans:
$$y(x) = C_1 e^{3x} + C_2 e^{-3x} + \frac{9}{8} e^x \left(-x + \frac{1}{4}\right)$$
5. Solve the given initial value problem $X'(t) = \begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix} X(t)$, $X(0) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$.
Ans:
$$x(t) = \begin{bmatrix} 2e^{-2t} + 5e^{8t} \\ -2e^{-2t} + 5e^{8t} \end{bmatrix}$$
6. Find the general solution of the second order non-homogenous differential equation $x^2 y'' - 5xy' + 8y = 8x^6$.
Ans:
$$y(x) = C_1 x^2 + C_2 x^4 + x^6$$
7. Use variation of parameters to find the general solution of the differential equation $y'' + 4y = \tan 2x$.
Ans:
$$y(x) = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} \log(\sec 2x + \tan 2x)$$
8. Compute the wronskian of the sets of functions and state whether or not the two functions are linearly dependent: $\{e^x, e^{5+x}\}$ **Ans:** Linearly dependent.
9. Find a particular solution for the differential equation $y'' - 4y = e^x + \sin 2x$ by using undetermined coefficient.
Ans: $y(x) = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{3} e^x - \frac{1}{8} \sin 2x$
10. Solve the system of equations $X' = \begin{bmatrix} 6 & 1 \\ 4 & 3 \end{bmatrix} X + \begin{bmatrix} 6 \\ -10 \end{bmatrix} t + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$.
Ans:
$$X(t) = \frac{2}{5} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{7t} + \frac{3}{5} \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{2t} + \begin{bmatrix} -2 \\ 6 \end{bmatrix} t + \begin{bmatrix} \frac{-4}{7} \\ \frac{10}{7} \end{bmatrix}$$
11. Find the general solution of the equation $y'' - 2y' - 3y = 6e^{-x} - 8e^x$.
Ans:
$$y(x) = Ae^{-x} + Be^{3x} - \frac{3}{2} x e^{-x} + 2e^x$$
12. Find the general solution of the equation $y'' + 9y = \cos 3x$.
Ans:
$$y(x) = A \cos 3x + B \sin 3x + \frac{1}{6} x \sin 3x$$

13. Find the general solution of the equation $y'' + 4y' + 4y = 12e^{-2x}$.

Ans:

$$y(x) = (Ax + B)e^{-2x} + 6x^2e^{-2x}$$

14. Find the general solution of the equation $y'' - 4y' + 13y = 12e^{2x} \sin 3x$.

Ans:

$$y(x) = e^{2x}[A \cos 3x + B \sin 3x - 2x \cos 3x]$$

15. Solve the initial value problem $y''' + 3y'' - 4y = 0$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = \frac{1}{2}$.

Ans:

$$y(x) = \frac{[e^x + (x+1)e^{-2x}]}{2}$$

16. It is given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are two linearly independent solutions of the associated homogenous equation of $x^2y'' + xy' - y = x$, $x \neq 0$. Find a particular integral and the general solution of the equation.

Ans:

$$y(x) = C_1^*x + \frac{1}{x}C_2 + \frac{x}{2} \ln |x|, \text{ where } C_1^* = C_1 - \frac{1}{4}$$

17. Find the general solution of the equation $x^2y'' + 5xy' + 3y = \log x$, $x > 0$.

Ans:

$$y(x) = \frac{A}{x} + \frac{B}{x^3} + \frac{1}{3} \log x - \frac{4}{3}$$

18. Using the method of undetermined coefficients find the general solution of differential equation $y'' + y = 32x^3$.

Ans: $y(x) = A \cos 3x + B \sin 3x + 32x(x^2 - 6)$.

19. Solve the differential equation $4y^{iv} - 12y''' - y'' + 27y' - 18y = 0$.

Ans:

$$y(x) = Ae^x + Be^{2x} + Ce^{\frac{-3}{2}x} + De^{\frac{3}{2}x}$$

20. Using variation of parameters find the general solution of the equation $y'' + 3y' + 2y = 2e^x$.

Ans:

$$y(x) = C_1e^{-x} + C_2e^{-2x} + \frac{1}{3}e^x$$

21. Solve the initial value problem

$$\frac{dx}{dt} = 3x$$

$$\frac{dy}{dt} = x - 2y$$

with initial condition $(x(0), y(0)) = (1, 0)$.

Ans:

$$(x(t), y(t)) = (e^{3t}, \frac{1}{5}(e^{3t} - e^{-2t}))$$

22. Find fundamental set of solutions $\{y_1(x), y_2(x)\}$ of the equation $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 5y = 0$ in the interval $x > 0$.

Ans:

$$y_1(x) = \frac{1}{x} \cos(2 \ln x), y_2(x) = \frac{1}{x} \sin(2 \ln x)$$

23. Solve the differential equation $y'' + a^2y = \sec ax$.

Ans:

$$y(x) = c_1 \cos ax + c_2 \sin ax + \left(\frac{x}{a} \sin ax + \frac{\ln(\cos ax)}{a^2} \cos ax \right)$$

24. Solve $y'' - 4y = \cosh(2x - 1) + 3^x$.

Ans:

$$y = c_1e^{2x} + c_2e^{-2x} + \frac{x}{4} \sinh(2x - 1) + \frac{3^x}{(\log 3)^2 - 4}$$

25. Solve $y''' - y'' - 6y' = 1 + x^2$.

Ans:

$$y = c_1 + c_2 e^{3x} + c_3 e^{-2x} - \frac{1}{108}(6x^3 - 3x^2 + 25x)$$

26. Solve the differential equation $y^{iv} + 16y'' = x^2 + 5$.

Ans:

$$y = c_1 + c_2 x + c_3 \cos 4x + c_4 \sin 4x + \frac{x^2}{64} \left(\frac{x^2}{3} + \frac{39}{4} \right)$$

27. Solve $y^{iv} - y = e^x \cos x$.

Ans:

$$y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x - \frac{1}{5} e^x \cos x$$

28. Solve the differential equation $y''' + 2y'' + y' = \sin^2 x$.

Ans:

$$y = c_1 + (c_2 x + c_3) e^{-x} + \frac{x}{2} + \frac{3 \sin 2x + 4 \cos 2x}{100}$$

29. Solve the differential equation $y''' - 3y'' + 4y' - 2y = e^x$.

Ans:

$$y = c_1 e^x + e^x (c_2 \cos x + c_3 \sin x)$$

30. Solve $y'' + 3y' + 2y = x e^x \sin x$.

Ans:

$$y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x} \left[\frac{x}{10} (\sin x + \cos x) + \frac{1}{50} (2 \sin x - 5 \cos x) \right]$$