Tutorial Sheet 2

II B. Tech. (Common)
Mathematics-II
(MCI102)

1. Find the eigenvalues and eigenvectors of the following matrices:

   (a) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

2. Verify the Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.
   Also (a) obtain A^{-1} and A^3, (b) find the eigenvalues of A and A^2 and verify that the eigenvalues of A^2 are squares of those of A, (c) find the spectral radius of A.

3. By using the Cayley-Hamilton theroem, find A^8 for $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and find A^4 for $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

4. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ and then find A^{-1} by using the Cayley-Hamilton theorem. Also, find the value of $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.

5. Examine whether A is similar to B, where
   (a) $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.
   (b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

6. Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Then find a matrix P such that $P^{-1}AP$ is a diagonal matrix. Then obtain the matrix $B = A^2 + 5A + 3I$.

7. Examine whether the matrix A is diagonalizable, where A is given by
   (a) $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
   If A is diagonalizable, then find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

8. The eigenvectors of a 3×3 matrix A corresponding to the eigenvalues $1, 1, 3$ are $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}^T$, $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$ respectively. Find the matrix A.

9. Obtain the symmetric matrix for the quadratic form:
   (a) $2x_1^2 + 3x_1x_2 + x_2^2$ (b) $x_1^2 + 2x_1x_2 - 4x_1x_3 + 6x_2x_3 - 5x_2^2 + 4x_3^2$

10. Reduce the following quadratic form to the canonical form by an orthogonal transformation. Also, specify the matrix of transformation in each case.
    (a) $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ (b) $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx$

11. Determine the nature, index, and signature of the following:
    (a) $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx$ (b) $2x_1x_2 + 2x_1x_3 + 2x_2x_3$

12. Solve the following differential equations (initial value problems) by matrix method:
    (a) $x'' - 2x' - 3x = 0,\ x(0) = 0, x'(0) = 1$ (b) $y'' + \mu^2 y = 0,\ y(0) = 1, y'(0) = \mu$
    (c) $\frac{dx_1}{dt} = x_1 + x_2,\ \frac{dx_2}{dt} = lx_1 + x_2,\ x_1(0) = 10, x_2(0) = 70$

13. Show that:

(a) The eigenvalues of a Hermitian matrix are real.
(b) The eigenvalues of a skew-Hermitian matrix are zero or pure imaginary.
(c) The eigenvalues of a unitary matrix are of magnitude 1.
(d) $\begin{bmatrix} 2 & -2 & -4 \\ -3 & 3 & 4 \\ 1 & -2 & -2 \end{bmatrix}$ is an idempotent matrix. Also, show that $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is involutory.
(e) Every skew-Hermitian matrix A can be expressed as $B + iC$, where B is real skew-symmetric matrix and C is real symmetric matrix.
(f) $\lambda = 0$ is an eigenvalue of a matrix A if and only if A is a singular matrix.

14. Determine the values of a, b, and c if the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.