A review on emerging variants of the multi-period vehicle routing problem

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Abstract

The extension of the classical Vehicle Routing Problem (VRP) to cover multiple periods is among the most common features within the VRP literature, due to the nature of many problem settings the VRP is applied to. Within the field of multi-period VRPs, especially applications with periodically recurring customer visits have been extensively studied. However, the strictly periodic VRP (PVRP) constitutes a special case of the more generic multi-period VRP (MPVRP). So far, the MPVRP in its non-periodic version has received less attention, although it arises in numerous applications, like dynamic problem settings and rich VRPs. Moreover, current literature misses a clear differentiation between MPVRP and PVRP as both terms are often used interchangeably. This paper provides a formulation for a general MPVRP and a comprehensive taxonomy for the recent literature devoted to periodic and non-periodic MPVRPs. As a field of special interest, it focuses on settings where the decisions on allowed visit periods and frequency are explicitly considered in the objective function.

Keywords: Periodic Vehicle Routing, Multi-period Vehicle Routing, Typology, Review

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1. Introduction

The well-known *vehicle routing problem* (VRP) seeks to group a given number of customers to tours and decide on the visiting sequence of each tour in a way that overall driven distance is minimized. It usually considers a restricted fleet of vehicles with a certain capacity needed to fulfill customers' demands and a single period only. Since its introduction, numerous publications investigated solution methods and various extensions.[1] This paper deals with one of the most studied extensions, which is its generalization to multiple periods. The feature of multiple periods needs to be addressed in many typical real-world applications of the VRP. For instance, online retailers often announce a latest delivery date for orders or their customers can choose a specific delivery day which needs to be respected for all customers in delivery tour planning (e.g., Archetti et al., 2015). Grocery stores require a supply of fresh goods a fixed number of times in a week, but deliveries should not be on consecutive days to ensure product availability throughout the whole week (e.g., Frank et al., 2021). In home healthcare services, patients may need help three times a week with the limitation that visits should have a minimum time lag of two days (e.g., Liu et al., 2014). Another example is given by Schrotenboer et al. (2018) where maintenance activities at offshore wind farms arise and need to be conducted before a deadline and technicians need to be assigned and routed accordingly.

It becomes apparent that problem settings can be differentiated by the characteristic of either periodically recurring visits or irregular visits, both possibly restricted by constraints on the allowable visit periods. The former variant is commonly known as the *periodic vehicle routing problem* (PVRP) which focuses on the strategic or tactical level, as for example delivery patterns of grocery stores are operated unchanged for several months, whereas the latter problem setting is mostly referred to as *multi-period vehicle routing problem* (MPVRP). It is typically used for tour planning in a more operational short-term planning horizon, but does not necessarily restrict to single visits only (e.g., Pasha et al., 2016). However, current literature lacks a clear differentiation and the terms PVRP and MPVRP are often used differently and interchangeably. The MPVRP clearly has its origins in the PVRP, but so far, no general problem formulation and model for the non-periodic MPVRP have been proposed. In this paper we want to take a look at the various definitions of a "multi-period VRP",

[1] For a recent survey, see Vidal et al. (2020).

propose a clear distinction to the PVRP and review publications that include a VRP setting with multiple periods, based on an appropriate typology and focusing on works in which the feature multi-period is of particular importance. The remainder of this paper is structured as follows. Section 2 describes the different understandings of the MPVRP in current literature, introduces the general problem setting and provides a corresponding general model. Section 3 presents a typology of MPVRP attributes to be used to structure the subsequent literature review. The recent literature of periodic and non-periodic VRPs with multiple periods are analyzed to obtain a clear view of the developments in the field. We conclude our findings and indicate future research directions in Section 4.

2. A definition of the multi-period vehicle routing problem

When reviewing the current literature it becomes apparent that the terms MPVRP and PVRP are used inconsistently for VRPs with multiple periods. We therefore give a short overview of the different understandings to be found and propose a unified point of view to be able to differentiate between the two in Section 2.1. We then present a definitional model formulation for a general MPVRP in Section 2.2.

2.1. "PVRP" versus "MPVRP"

In order to find criteria when to use which term, the relevant differences between MPVRP and PVRP need to be identified. For this reason, we first recall the formal description of the PVRP as the older, more precisely defined variant and more consistently used term and then take a look at the occurring usage of the terms PVRP and MPVRP.

As stated in the survey by Campbell and Wilson (2014), the PVRP is defined as follows. The objective is to minimize total transportation costs by assigning each customer to one of its allowable visit patterns and create corresponding daily routes (clustering & routing decision). A visit pattern contains a subset of periods[2] whose combination is allowed for visiting the customer. Also, it is standard to define a visit frequency that gives the number of needed visits within the planning horizon. In most publications, the visit frequency is given additionally to the allowed visit patterns, but the visit frequency can also be the only constraint (e.g., Russell and Igo, 1979). For a given visit frequency, the corresponding demands are typically

[2]Note that we use the terms day and period interchangeably.

fixed and equal for all visits or dependent on the spacing between two visits. For instance, if a customer is visited every three days, the demand for three days must be satisfied at each visit. Constraints include a given number of vehicles available on each day, with a limited capacity and a limitation on the total travel time, which start and end all of their tours at a single depot. Also, no customer demand should be greater than a vehicle's capacity. Concerning the allowable visit patterns, Campbell and Wilson (2014) find three different variants for defining the set of feasible combinations.

- A predetermined set of allowable visit patterns is used.
- A fixed spacing between visits is given, e.g., a visit needs to be conducted every two days.
- A minimum and maximum spacing is given, which means the time between two visits is flexible to a certain extent.

Note that the latter two alternatives clearly state the periodicity of visits. Periodicity means that it is usually assumed that the last day of the given planning horizon is followed by the first one again and the created tour plan cyclically repeats. Even though the literature on the PVRP is not completely consistent regarding the problem definition, the term PVRP is used continuously since its introduction by Christofides and Beasley (1984) and Gaudioso and Paletta (1992). This is in contrast to the usage of the term MPVRP which does not refer to a specifically defined problem that has been introduced formally. In the next section, we provide a general MPVRP to close this gap. The term MPVRP is mostly used in the field of dynamic VRPs, where a number of publications considers the *dynamic multi-period vehicle routing problem* (DMPVRP) (e.g., Wen et al., 2010; Albareda-Sambola et al., 2014; Archetti et al., 2015) and in the field of rich VRPs, which are characterized by a great number of side constraints, very often including the multi-period feature (Lahyani et al., 2015). The mentioned features that differentiate a MPVRP from a PVRP are a visit frequency of one for each customer and that there is no periodicity in the service (e.g., Rothenbächer, 2019; Archetti et al., 2015). This would define the MPVRP as a special case of the PVRP. However, both terms have been used differently.

- The term PVRP is used for all multi-period VRP settings, including those where visit frequency is equal to one and no periodicity is assumed.
- The term MPVRP is used for all multi-period VRP settings, including those with periodicity in visits (e.g., Estrada-Moreno et al., 2019).

- The term MPVRP is used for multi-period VRP settings in which each customer is visited exactly once and the term PVRP for settings with higher visit frequencies (e.g., Rothenbächer, 2019).
- The term *tactical planning vehicle routing problem* (TPVRP) is used for multi-period VRP settings in which each customer is visited exactly once and the term PVRP for settings with higher visit frequencies (Baldacci et al., 2011).
- The term MPVRP is used as long as there is no periodicity in the service, regardless of visit frequency (e.g., Archetti et al., 2015).

Most works on the PVRP structure their input data quite strictly and assume certain periodical structures. The visit frequency is set to the same for all customers, all allowable visit patterns for each customer are non-overlapping, show equal intervals between two visits and every day is contained in at least one pattern (Rothenbächer, 2019). Therefore the PVRP can not only be seen as a "VRP with [multiple] periods", but has periodic demands as a core aspect. Visit frequencies of one are rare, but possible in the PVRP, especially in PVRP extensions which also decide on the visit frequency (e.g., Francis et al., 2006; Rothenbächer, 2019; Frank et al., 2021). But weekly visits are also periodically repeated visits. Beyond that, there also exist non-periodic multi-period VRPs variants that consider multiple visits (i.e., a visit frequency higher than one) within the planning horizon (e.g., Pasha et al., 2016). That is why we propose to follow the point of view of Archetti et al. (2015) and define the MPVRP to be a special case of the PVRP where no periodicity is considered. Thereby, the MPVRP states the short-term planning problem (as opposed to the long-term PVRP), where the planning period is not meant to be continuously repeated, but finite. This means the MPVRP's planning period ends at a certain point when used for short term planning only or it would be repeatedly solved in a rolling planning horizon, depending on the planning environment. As before, we use the term MPVRP for the non-periodic VRP with multiple periods, the term PVRP for the periodic VRP with multiple periods, and we speak of "VRPs with multiple periods" to address both and other problem classes with the feature multiple periods.

2.2. A general multi-period vehicle routing problem and its formulation

As a foundation for discussing different problem variants in the following sections, we formulate a general MPVRP based on our findings above in which customers may be visited on every day and transportation costs are to be minimized. We then show two of the most common extensions, i.e., visit period intervals and day-dependent service costs.

A general version of the MPVRP can be defined on a directed graph $G(N, A)$ with node set N and arc set A. Node set N consists of the depot 0 and the customer set C, $N = C \cup \{0\}$. The arc set is defined as $A = \{(i, j) : i \neq j,\ i, j \in N\}$. c_{ij}^{trans} denotes the associated transportation costs with each arc. Let $T = \{1, ..., |T|\}$ be the planning horizon consisting of periods t. In each period, a given fleet of K delivery trucks with the same restricted vehicle capacity Q is available to perform routes starting and ending at the depot. $|K|$ states the maximum number of tours in each period. We define a customer to have a single demand d_i that can be fulfilled on any day (split-delivery is not allowed). If multiple visits at the same customer are needed, this can be modelled by co-locating customers with individual demands. Three decision have to be taken: (i) which days the customers are assigned to, (ii) the clustering of customers to tours and (iii) the sequence of customers on each tour. The following binary decision variables are applied:

- x_{ijkt} indicating whether arc (i, j) is traversed by vehicle k in period t; $i, j \in N, k \in K, t \in T$.
- z_{ikt} indicates whether customer i is served by vehicle k in period t; $i \in C, k \in K, t \in T$.

The objective function and the constraints of the general MPVRP can then be formulated as follows:

$$Minimize \quad \text{TC} = \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{t=1}^{T} c_{ij}^{\text{tran}} \cdot x_{ijkt} \tag{1}$$

s.t.

$$\sum_{k=1}^{K}\sum_{j=0}^{N}\sum_{t=1}^{T} x_{ijkt} = 1 \qquad \forall i \in C \tag{2}$$

$$\sum_{i=1}^{C} d_i \sum_{j=0}^{N} x_{ijkt} \leq Q \qquad \forall k \in K, t \in T \tag{3}$$

$$\sum_{j=0}^{N} x_{ijkt} = \sum_{j=0}^{N} x_{jikt} \qquad \forall i \in N, k \in K, t \in T \tag{4}$$

$$z_{ikt} = \sum_{j=0}^{N} x_{ijkt} \qquad \forall i \in N, k \in K, t \in T \tag{5}$$

$$\sum_{i,j \in S} x_{ijkt} \leq \sum_{i \in S} z_{ikt} - z_{jkt} \qquad S \subset C, \forall j \in S, k \in K, t \in T \tag{6}$$

$$\sum_{j=0}^{N} x_{1jkt} \leq 1 \qquad \forall k \in K, t \in T \qquad (7)$$

$$\sum_{j=0}^{N} x_{j1kt} \leq 1 \qquad \forall k \in K, t \in T \qquad (8)$$

$$x_{ijkt} \in \{0,1\} \qquad \forall i,j \in N, k \in K, t \in T \qquad (9)$$

$$z_{ikt} \in \{0,1\} \qquad \forall i \in N, k \in K, t \in T \qquad (10)$$

The objective function (1) minimizes overall transportation costs. Constraints (2) ensure that every customer is visited exactly once during the planning horizon. Constraints (3) prohibit the vehicle capacities to be exceeded on any day. Equations (4) conserve flow. Equations (5) link the assignment variables z_{ikt} with the flow variables x_{ijkt}. Constraints (6) are the subtour elimination constraints, formulated in terms of z_{ikt} variables. Equations (7) and (8) ensure that only one tour is performed per vehicle and day. Equations (9) and (10) define the variable domains. This model is a generalization of the VRP that is known to be an NP-hard optimization problem (see, e.g., Toth and Vigo, 2014).

Extension by delivery intervals. One of the most frequent extensions of the MPVRP is the introduction of delivery intervals that restrict the visit of each customer to specified, consecutive days (see, e.g., Bostel et al., 2008). This can be done by setting an earliest delivery day e_i and a latest delivery day l_i for each customer $i \in C$. Constraints (2) are then modified to allow exactly one visit within the delivery interval instead of the whole planning horizon.

$$\sum_{k=1}^{K} \sum_{j=0}^{N} \sum_{t=e_i}^{l_i} x_{ijkt} = 1 \qquad \forall i \in C \qquad (11)$$

Extension by service costs. In its basic form formulated above, the MPVRP only takes transportation costs into account. It is therefore beneficial to visit customers close to each other on the some day and tour. However, service costs c_{it}^{serv} can be added (see, e.g., Baldacci et al., 2011) to reflect the common situation that visiting a customer i occurs costs depending on the chosen day t (or profits in the case of a negative c_i^{serv}). The objective function (1) is then complemented as follows.

$$\textit{Minimize} \quad \mathrm{TC} = \sum_{k=1}^{K} \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{t=1}^{T} c_{ij}^{\mathrm{tran}} \cdot x_{ijkt} + \sum_{k=1}^{K} \sum_{i=1}^{C} \sum_{t=1}^{T} c_{it}^{\mathrm{serv}} \cdot z_{ikt} \qquad (12)$$

The presented model can be easily adapted further to account for other common constraints and extensions, e.g., waiting or inventory costs, heterogeneous vehicles and multiple customer visits.

3. Literature review

In this section, we provide a comprehensive literature review that focuses on VRPs with multiple periods. Based on our findings in Section 2.1, we distinguish between MPVRP and PVRP settings according to their assumptions on the periodicity of visits. We review MPVRPs in Section 3.1 first and PVRPs in Section 3.2 second and finally give an overview of used solution approaches in Section 3.3. In order to find reasonable structures for the reviews, we introduce a typology for the MPVRP problem class which is later adapted to PVRPs. Our paper retrieval procedure for the whole survey is based on an initial Web of Science search via keywords ”vehicle routing problem with multiple periods”, “multi-period vehicle routing problem” and “period[ic] vehicle routing problem”, followed by a snowball search of literature cited by relevant publications.

3.1. Literature on the MPVRP

As mentioned throughout the paper, the term MPVRP was never formally introduced but merely separated from the PVRP over time to be used for non-periodic short-term planning. Many short-term problem settings feature some kind of dynamism which may be the reason for the wide-spread usage of the term MPVRP within the field of dynamic VRPs. Overall, there is a high number of publications considering MPVRPs, we therefore first present a typology of the MPVRP (Section 3.1.1) and then divide the characterization of its literature body into the two subsections of static (Section 3.1.2) and dynamic MPVRPs (Section 3.1.3).

3.1.1. A typology for multi-period vehicle routing problems

The problem settings discussed in literature vary strongly, therefore it is no straightforward task to develop a comprehensive typology for the MPVRP. Some typologies including VRPs with multiple periods have already been suggested. For instance, Mourgaya and Vanderbeck (2006) present a classification for the PVRP focusing on different objective functions. Vidal et al. (2020) develops a typology for various VRP extensions. Mor and Speranza (2022) give an overview of “VRPs over time”, which also includes the basic PVRP. However, none of these publications addresses the attributes specific to the feature multiple periods and provides a comprehensive literature review on the MPVRP. After scanning the literature, we therefore present a typology based on multi-period-related attributes. In the following, we give a short explanation of each attribute.

Visiting frequency. A first criterion for distinguishing MPVRP is the required number of visits at the customer. The multiple visits attribute is a necessary criterion for PVRPs (which we cover in Section 3.2). However, for MPVRPs, customers can be visited only once. Visits of customers in a PVRP with frequency one can be seen as multiple visits, because they are cyclically repeated, in contrast to a MPVRP with a finite planning horizon.

Allowable visit periods. A second criterion arises with the feature of allowable visit periods, i.e., whether the set of periods in which a customer may be visited is restricted (see Section 2.1). If that is the case, the type of restriction can be further specified. The two possible types are hard constraints, where a visit on any other day than the allowed ones is not possible, and soft constraints which allow the violation of given visit restrictions at a cost. This includes setting the visit day to a day outside of the planning horizon, i.e., not delivering at all.

Objective function and visit period-related components. Similar to Mourgaya and Vanderbeck (2006), we classify works according to their objectives considered. The three main streams can be divided in minimizing routing costs, maximizing the number of customers served and multi-objective formulations with application-specific objectives that do not coincide. As we focus our review to settings in which considering multiple periods plays a major role, it is reasonable to additionally indicate works in which the objective function includes a visit-period-related component, such as day-dependent service costs, penalty costs for late or no visit, waiting times or service frequency, all directly determined by the decisions on when and how often to visit customers and independent from subsequent routing decisions. We distinguish between such models with an explicit visit period-related component and models where the decisions on frequency and specific visit days only have an implicit impact on the objective function, e.g., by influencing the routing costs or the number of vehicles to be used in a period.

All considered multi-period-related attributes for the MPVRP are summarized in Figure 1. The used attributes are explained in Table 1. In addition, we also indicate the most encountered VRP-related attributes in the literature characterizations below. They are defined in the respective sections.

Table 1: Legend for Tables 2-3

Attribute	Abbreviation	Explanation
Multiple Visits	MV	Checked, if customer requires multiple visits. If not checked, single visits are considered.
Visit periods	VP	**H** for hard constraints, **S** for soft constraints.
Objective function	Obj	**R** for routing costs (distance, time, transportation costs, etc.) **FS** for fleet size/fleet mix objectives **NC** for number of served customers **O** for other single-objective function **MO** for multi-objective function
Visit-period-related component	VC	Checked, if objective function contains an explicit visit-period related component such as day-dependent service costs

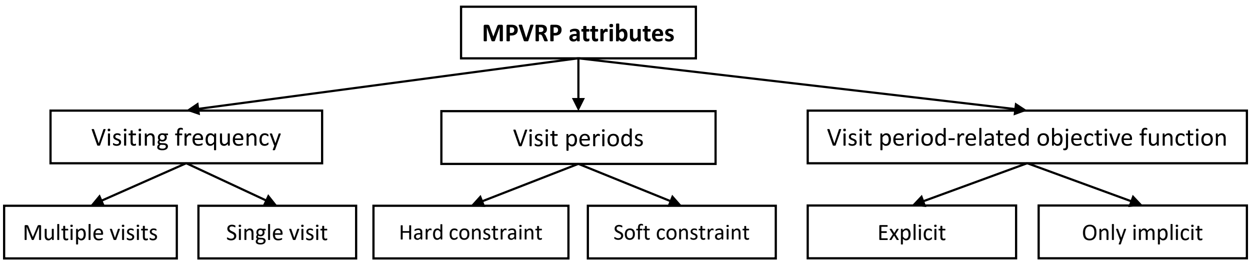

Figure 1: Attributes of MPVRPs.

3.1.2. Static MPVRP

First, we review the literature on static MPVRPs, which means all input data is known in advance, however, not necessarily with certainty. Overall, 14 publications can be attributed to this MPVRP type. A majority of papers (8) addresses specific applications. Table 2 shows the attributes of the reviewed works. In the follwing, we shortly describe the main contributions and indicate Some interrelated streams of literature.

Baldacci et al. (2011) can be seen as first work on the MPVRP as the authors explicitly consider a variant of the PVRP for short-term planning which is referred to as TPVRP (see Section 2.1). However, their formulation already includes allowable visit intervals consisting of consecutive days and service costs. By using a branch-and-bound algorithm, they are able to optimally solve instances up to 100 customers.

A static-stochastic VRP with multiple periods, but without delivery intervals is discussed by Dayarian et al. (2015). The problem arises in the collection of milk at local producers. The

Table 2: Characterization of Static MPVRPs

			MP-specific			VRP-related [a]			
Paper	Application	Obj	VC	MV	VP	HF	TW	DC	WB
Baldacci et al. (2011)	Generic	R+O	x		H				
Pacheco et al. (2012)	Distribution	R			H				
Athanasopoulos and Minis (2013)	Generic	R			H	x	x	x	
Dayarian et al. (2015)	Collection	R		x					
Archetti et al. (2015)	Parcel	R+O	x		S				
Schönberger (2016)	Collection	R						x	x
Pasha et al. (2016)	Generic	FS+R		x	H				
Dayarian et al. (2016)	Collection	R		x					
Cantu-Funes et al. (2018)	Distribution	R		x	H	x	x	x	x
Kim (2019)	Green VRP	R+O		x	H	x			x
Estrada-Moreno et al. (2019)	Generic	R+O	x		S				
Larrain et al. (2019)	Generic	R+O	x		S				
Darvish et al. (2020)	Generic	R+O	x		H				
Muñoz-Villamizar et al. (2021)	Green VRP	MO	x		S				
Share in %	-	-	50	36	79	21	14	21	21

[a] HF: heterogeneous fleet, TW: time windows, DC: distance/duration constrained, MD: multi-depot

quantities to be collected are subject to weekly and seasonal fluctuations. Due to contractual agreements, a single vehicle routing plan must be designed for the whole planning horizon and not be changed afterwards. They solve the proposed model by a branch-and-price approach. Dayarian et al. (2016) develops an adaptive large neighborhood search in order to generate solutions for larger instances.

Another stream is on the MPVRP with due dates (MVRPD). Motivated by a problem setting in city logistics, it is introduced by Archetti et al. (2015). Each customer demand is associated with an earliest and latest delivery date (i.e., release and due date). Orders may be postponed at a penalty cost. Additionally, inventory costs for each day an order stays at the depot after its release date incur. They also generate a first test instance set and solve it by using a branch-an-bound procedure. Building on their findings, Larrain et al. (2019) present two new solution approaches for the MVRPD, an optimized new branch-and-bound algorithm and a variable MIP neighborhood descent algorithm, which embeds a local search within a branch-and-bound procedure and clearly outperforms the previous approach. Muñoz-Villamizar et al. (2021) integrate ecological factors into the MVRPD. In order to investigate shipment consolidation strategies for green home delivery, three different objective functions are used.

Various other MPVRP variants are introduced. Pacheco et al. (2012) seek to design routes for delivering bakery products to satisfy the orders placed for the week, which are associated with deadlines. Athanasopoulos and Minis (2013) introduce the MPVRP with time windows and also consider a heterogeneous fleet, its size varying from period to period. They develop an exact solution method based on column generation. Schönberger (2016) consider a MPVRP with restricted capacity per period. Due to that restriction, quantities cannot be collected immediately but are visited some periods later, however, within a specified deadline. Pasha et al. (2016) introduce the MPVRP including the fleet size and mix problem, where the demands of the customers vary between periods, and all customers may have to be visited in each period. Cantu-Funes et al. (2018) formulate an extensive MPVRP motivated by the case of a brewing company supplying DCs from production plants. In order to not run out of inventory, the DCs impose due dates to be served. Additionally, the model is characterized by a higher costs for using additional external vehicles, which can be seen as a variant of penalty costs. Kim (2019) extend the MPVRP by introducing a limit of carbon emissions per period. Trading costs for additional carbon emission certificates incur when this limit is exceeded, like a penalized soft constraint on the overall tour length per period. Estrada-Moreno et al. (2019) consider the MPVRP with price discounts for delivery flexibility. Customers indicate a single preferred delivery day which is not guaranteed by the service provider. Instead, the customer gets a fixed price discount if the delivery takes place on any other day than the preferred one.

3.1.3. Dynamic MPVRP

In the dynamic MPVRP, input data is not (fully) available when planning takes place, but gets (partially) revealed over time. As additional information arrives on every further day, replanning actions with regard to the first tour plan are necessary, conducted either instantly as new information becomes available or periodically (e.g. daily), in a rolling horizon. Thereby, dynamic MPVRPs are usually applied on an operational planning level which may be the reason for widespread usage of the term MPVRP in this field (as opposed to the mid-term-oriented planning approach of the PVRP). Additionally, several works in this field also assume that some problem information is only available in a stochastic manner, as shown in Table 3. For complete reviews on the dynamic VRP, we refer to Psaraftis et al. (2016) and for dynamic and stochastic VRPs to Ritzinger et al. (2016) and Soeffker et al. (2021). In total, we can retrieve 17 publications on dynamic MPVRPs. However, we match some works together in the table due to identical model formulations.

Table 3: Characterization of Dynamic MPVRPs

			MP-specific			VRP-related[a]			
Paper	Application	Obj	VC	MV	VP	ST	TW	DC	WB
Angelelli et al. (2007b,a)	Same-day	R							
Tricoire (2007)	Home services	NC+R			H		x	x	
Bostel et al. (2008)	Home services	R			H		x	x	
Angelelli et al. (2009a)	Same-day	NC+R							
Angelelli et al. (2009b, 2010)	Same-day	R+O	x		S			x	
Wen et al. (2010)	Agriculture	R+O	x		H			x	x
Albareda-Sambola et al. (2014)	Generic	R+O	x		H	x			
Ninikas et al. (2014)	Parcel	R			H		x		x
Cordeau et al. (2015)	Auto-carrier	FS+R+O	x	x	H				
Billing et al. (2018)	Auto-carrier	FS+R+O		x	H	x			
Ulmer et al. (2018)	Same-day	NC			H	x		x	
Bonassa et al. (2019)	Auto-carrier	R	x	x	H				
Ulmer (2020)	Same-day	NC			H	x		x	
Laganà et al. (2021)	Parcel	R			H				
Subramanyam et al. (2021)	Gases	R			H	x			
Share in %	-	-	29	18	94	29	18	41	12

[a] ST: stochastic demand, TW: time windows, DC: distance/duration constrained, WB: workload balancing

Numerous publications focus the dynamic MPVRP including same-day delivery: Angelelli et al. (2007a) are the first to introduce the dynamic MPVRP (DMPVRP), where it is decided on each day which orders to fulfill on the current day and which orders to postpone without knowing the set of new requests that will be issued the day after. They consider a special case of delivery intervals where orders can be postponed only for one day, so every order has to be delivered either on the day the order arrived or the next one. Three simple algorithms are compared, one of them being optimal for the case of a two-day planning horizon and outperforming the other two for longer horizons. These algorithms are further analyzed in Angelelli et al. (2007b) for a special case. Furthermore, Angelelli et al. (2009b) builds on these model definition and findings and extend the problem setting by introducing a set of vehicles and a maximum route length constraint. Additionally, some orders may arrive during the day and tour plans can therefore be modified while vehicles are already traveling between customers. Angelelli et al. (2009a) proposes a rolling horizon solution framework and different short-term strategies, formulated as optimization models and solved by a variable large neighborhood search. Angelelli et al. (2010) tests additional numerical experiments based on the existing model. Ulmer et al. (2018) incorporates stochastic information, as customer orders arrive during the day according to a known stochastic distribution. The problem is formulated by a markov decision process and solved by a time budgeting approach which anticipates the number and location of future orders intra- and inter-periodically by means of a value function approximation approach that simulates future realizations based on historic data. Finally, Ulmer (2020) define a new solution approach with the objective to combine the benefits of an online algorithm that is able to respond quickly in the dynamic same-day setting and those of an offline algorithm that has more computation time available to pre-determine possible outcomes. Both algorithms use value function approximation by simulation.

Concerning other applications, Wen et al. (2010) consider a case study involving the deliver of fodder to farmers, the authors formulate another extended variant of the DMPVRP. The objective function takes customer waiting costs and workload balancing into account. Albareda-Sambola et al. (2014) build on the deterministic model of Wen et al. (2010) and add the assumption that for each customer and day there is a probability given that the day is an allowed day, i.e. it is included in the customer's delivery interval. An adaptive policy is proposed to determine which customer to serve on each day and which to postpone.

Several papers are motivated by the delivery of produced vehicles to dealerships which are requested dynamically and with deadlines. In these problems, the set of customers is known and multiple visits to each one are assumed. Additionally, the loading of the vehicles on carriers is considered. Cordeau et al. (2015) propose an ILS, applied in a rolling horizon framework to capture the dynamic nature of the problem. For increased efficiency, Billing et al. (2018) use the historic orders of a customer to forecast future ones. Bonassa et al. (2019) focuses on finding the best combination of vehicles to be loaded into the carriers, permitting deadline violations at a cost.

Tricoire (2007) address a problem arising in the scheduling and routing of technicians. They encounter two kinds of demands, which must either be served at a specific day at any cost or can be scheduled flexibly within a given number of periods. Bostel et al. (2008) study a similar application, however embedded in a rolling horizon planning framework. In a related manner, Ninikas et al. (2014) analyze a setting in hybrid courier operations, where dynamic pick-up requests arise throughout each period, while standard deliveries with a certain deadline are made. Another problem in courier operations is studied by Laganà et al. (2021). The dynamic multi-period general routing problem, where vertices and (un)directed edges of the considered mixed graph can both be delivery destinations, is defined in this work. The problem setting arises in combined postal service and parcel delivery systems when no estimate on the future demand is known. Finally, Subramanyam et al. (2021) study a problem setting in the delivery of industry gases, where customers may place their orders on any given day in the planning horizon and thereby reveal a demand quantity and the allowed delivery days. Only customers who have already placed their orders are known and future orders are modeled as binary random variables with known distribution. The decision problem is modelled as a multistage robust optimization problem with binary recourse decisions.

3.2. Literature review on the PVRP

The PVRP is a generalization of the MPVRP, assuming a stationary customer demand and therefore considering a repeating planning horizon. Decisions on the delivery days are made for the long term and the defined visit schedules are assumed to repeat themselves cyclically (see Section 2.1).

Taxonomy for the PVRP. We adapt the taxonomy developed for the MPVRP in Section 3.1.1 in several ways for the PVRP. First, we exclude the attribute "multiple visits" as all PVRPs consider the possibility to visit a customer multiple times (at least once in each cycle). Moreover, we introduce another classification of allowable visit periods in column "VP". In the PVRP, there are two possible types of restrictions, either there is a required spacing between two visits or a set of visit patterns is predefined, indicated by an **S** or **P**, respectively. Again, spacing restrictions can take the flexible form, i.e., a lower and an upper bound on the spacing between two visits (e.g. Fauske et al., 2020) or the fixed form, where the required spacing is set to a fixed value (e.g., Cordeau et al., 1997). Also, the use of a predefined set of visit patterns can be further divided in separate days (e.g. Estrada-Moreno et al., 2019) and consecutive days only, which refers to an allowable visit interval (e.g., Baldacci et al., 2011). Figure 2 shows an example with seven periods, where in the flexible case, the spacing between visits is at least one day and at most two days and in the fixed case a visit is conducted every second day. For the predefined cases, either the separate days $\{1,2,4,6\}$ can be given, or a visit interval of consecutive days $\{1,2,3,4,5\}$. The last alternative can also be used to model deadlines. Note that allowable visit periods only specify when a visit may take place and not necessarily, if a visit has to be conducted in every allowable period. This is defined together with the visiting frequency. We additionally indicate in column "VF" if the visiting frequency is a decision variable or not (e.g., Francis et al., 2006).

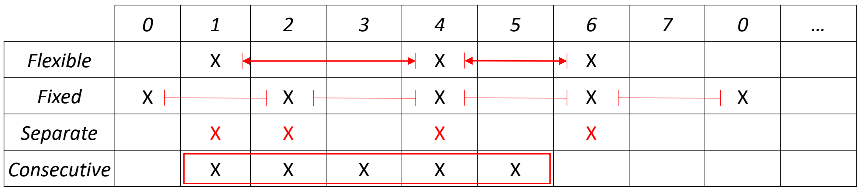

Figure 2: Classification of allowable visit periods.

Reviewed literature. The PVRP was introduced by Beltrami and Bodin (1974). Since then, there have been a variety of publications and different variants. Naturally, a number of surveys on the PVRP have already been suggested. The evolution of the PVRP and some variants, namely the PVRP with time windows (PVRPTW), the PVRP with service choice and the multi-depot PVRP (MDPVRP) is studied by Francis et al. (2008). Irnich et al. (2014) propose another survey on the PVRP, with an emphasis on solution methods. The most recent comprehensive review is Campbell and Wilson (2014), who summarize research on the PVRP of forty years and reviewed the wide range of applications of the PVRP. Therefore we concentrate on works published later than 2014. Some more recent publications on the basic PVRP can also be found in Mor and Speranza (2022). However, we are not only interested in the basic version, but want to give an overview of different PVRP settings and their attributes. In total, 23 relevant publications can be found, of which 15 are motivated by substantially different real-world applications. Multiple related streams in literature can be identified. Table 4 shows all reviewed publications on the PVRP.

One of the most studied variants is the PVRPTW. Liu et al. (2014) study an extension of the PVRPTW that arises in the transportation of drugs and medical devices to patients' homes. Michallet et al. (2014) address an unusual PVRPTW that occurs in the field of high-value shipments, where arrival times need to be spread out within customers' time windows for a higher unpredictability to make robberies more difficult. Norouzi et al. (2015) study a PVRPTW with competition, where sales can be maximized by arriving earlier than a competitor at the customer. A variant of the PVRPTW which includes five different objectives is addressed by Wang et al. (2020), which is the only paper found to consider a type of service costs and penalty costs as well as costs dependent on the chosen visit frequency simultaneously. A container transportation problem where the planning horizon can be divided into

Table 4: Characterization of PVRPs

Paper	Application	Obj	MP-specific			VRP-related[a]				
			VC	VP	VF	HF	TW	DC	WB	MD
Nguyen et al. (2014)	Generic	R		P		x	x	x		
Cacchiani et al. (2014)	Generic	R		P						
Liu et al. (2014)	Healthcare	MO		P			x		x	
Michallet et al. (2014)	Security	R		P			x			
Ramos et al. (2014)	Waste	MO		S				x	x	x
Norouzi et al. (2015)	Distribution	R+O	x	P			x			
Lahrichi et al. (2015)	Generic	R		P						x
Gómez et al. (2015)	Waste	MO	x	S	x	x			x	
Nair et al. (2016)	Distribution	R		P				x		
Archetti et al. (2017)	Generic	R		P						
Lei et al. (2017)	Surveillance	MO	x	P						
Archetti et al. (2018)	Generic	R		P						
Kisialiou et al. (2018)	Maintenance	FS+R		S		x	x	x	x	
Borthen et al. (2019)	Maintenance	FS+R		P		x		x	x	
Rothenbächer (2019)	Generic	R		P	x		x			
Borthen et al. (2019)	Maintenance	MO	x	P		x		x	x	
Chen et al. (2020)	Container	R		P			x			
Wang et al. (2020)	Generic	MO	x	P			x	x		
Fauske et al. (2020)	Surveillance	O		S	x	x		x		
Frank et al. (2021)	Distribution	R	x	P	x			x		
Vieira et al. (2021)	Maintenance	FS+R		P		x				
López-Sánchez et al. (2021)	Green VRP	MO	x	P	x					
Huerta-Muñoz et al. (2022)	Generic	R		P	x	x				
Share in %	-	-	26	93(P)	26	35	35	39	26	9

[a] HF: heterogeneous fleet, TW: time windows, DC: distance/duration constrained, SD: split delivery, WB: workload balancing, MD: multi-depot

multiple shifts is studied by Chen et al. (2020). The problem structure makes it possible to model it as a PVRPTW with open routes in which vehicles do not have to return to the depot at the end of their tours.

Another common application for the PVRP is the scheduling and routing of supply vessels for offshore installations. However, these problem settings are usually more complicated since the supply vessels make multiple-day voyages. Kisialiou et al. (2018) address a variant with flexible departure times, Borthen et al. (2018) develop a new genetic algorithm for the basic setting and outperform older solution approaches. Borthen et al. (2019) extend the problem to include a persistence objective. Recently, Vieira et al. (2021) develop a branch-and-cut algorithm as well as an adaptive large neighborhood search for these applications and achieve new best-known solutions on corresponding benchmarks.

A specific literature stream on the flexible PVRP is created by Archetti et al. (2017). They introduce this PVRP variant in which the number and timing of visits and the delivered quantities can be chosen freely (only subject to a maximum receiving capacity per visit) and solve small instance with branch-and-bound procedures. Archetti et al. (2018) develops a matheuristic for the problem in order to solve larger instances and Huerta-Muñoz et al. (2022) extends the problem to consider a heterogeneous vehicle fleet.

Works with generic problem settings are either devoted to introducing a new variant of the PVRP or focus on achieving better solution methods. Nguyen et al. (2014) find new best known solutions for the PVRPTW by means of a genetic algorithm with local search education operators. Cacchiani et al. (2014) are able to reach new best known solutions for the PVRP using a column generation procedure in which the sub-problem is solved by an iterated local search (ILS) heuristic. Lahrichi et al. (2015) develop a new type of solution approach which consists of multiple, parallel-running, exact or heuristic solution approaches which cooperate trough an adaptive guidance mechanism, use it to solve the MDPVRP and yield new best known solutions. Rothenbächer (2019) present an exact branch-and-price-and-cut algorithm for the PVRPTW and also for a variant where they explicitly allow all kinds of schedule structures and the choice of the visit frequency.

Finally, numerous other applications show the versatility of the PVRP. A multi-objective MD-PVRP arising in the collection of recyclables from drop-off containers is addressed by Ramos et al. (2014), including economic, ecological and social objectives. Nair et al. (2016) introduce a combination of PVRP and the pickup and delivery problem with unassigned quantities of a single product that can be delivered to any customer. Lei et al. (2017) use an adapted version of the PVRP to optimize the schedule of parking enforcement patrols, where the visit at each parking lot and the routing plan of patrol vehicles are determined simultaneously. An adapted version of the PVRP is also used by Fauske et al. (2020) in order to schedule maritime surveillance activities carried out by the Norwegian military. Instead of visiting customers, maritime regions are to be scanned in certain intervals to keep information sufficiently updated. Frank et al. (2021) use a combination of the PVRP with multi-compartment vehicles to supply stores with grocery products of different segments, directly accounting for costs associated with the chosen frequency and day-combination in the objective function. Finally, López-Sánchez et al. (2021) consider a generalization of the PVRP considering the two conflicting objectives minimization of total emissions produced by all vehicles and maximization of the service frequency.

3.3. *Solution Approaches for VRPs with multiple periods*

We further analyze the reviewed literature streams according to the applied solution methods. Table 5 gives an overview of the used procedures. We introduce a separate column for a specific approach, if it is used in at least three publications. All approaches used less frequently are summarized by the columns "Others Exact" and "Others Heuristic", respectively. Note that some papers develop multiple solution approaches, e.g., an exact procedure for smaller instances and a heuristic for larger ones.

Table 5: Overview of solution approaches

	Exact approaches[a]				Heuristic approaches[b]							
Class	B&C	B&P /CG	Others Exact	Exact Total	TS	LNS	VNS	GA	ILS	Math	Others Heuristic	Heuristic Total
PVRP	3	1	1	5	2	2	1	4	2	5	4	20
MPVRP	5	2	0	7	2	2	1	0	1	0	2	8
DMPVRP	2	3	0	5	0	1	3	2	1	0	6	13
Sum	10	6	1	17	4	5	5	6	4	5	12	41

[a] B&C: branch-and-cut, B&P/CG: branch-and-price/column generation

[b] TS: tabu search, LNS: large neighborhood search, VNS: variable neighborhood search, GA: genetic algorithm, ILS: iterated local search, Math: matheuristic

One can observe that many papers use one of two extremes: branch-and-cut procedures on the one side which mostly refer to the use of common solvers, and specialized, self-developed heuristic approaches on the other side. In summary however, a variety of solution methods have already been applied to all three problem classes, without a single approach dominating the others.

4. Conclusion

In this paper we provide a comprehensive view on VRPs with multiple periods, namely the PVRP and the MPVRP. We examine the inconsistent use of the terms PVRP and MPVRP and propose the criterion of periodicity to distinguish between the two problem variants, as many publications already do. We formulate a general MPVRP as a basis for following discussions that is easy to complement with typical constraints arising in the field of multi-period tour planning. We structure our comprehensive literature review according to a typology based on attributes characteristic to the feature of multiple periods.

The literature on the MPVRP can be subdivided in static and dynamic problem settings.

Specifically the DMPVRP has recently been studied extensively. It is to be expected that especially the combination of dynamic MPVRPs with stochastic elements will continue to see much research, as the large amounts of data often available today enable decision makers to predict information that was previously only revealed over time. For the PVRP, numerous recent publications can be found, arising in a broad range of applications, showing the versatility of this problem. Since the last survey by Campbell and Wilson (2014), particularly the extension of with time windows and the incorporation of multiple objectives have gained importance. As a future research avenue, the areas of other related problem classes where multi-period settings arise, but are not a core feature nor of special interest like the consistent VRP (e.g. Kovacs et al., 2014), the rich VRP (e.g. Lahyani et al., 2015) or the multi-compartment VRP (e.g. Ostermeier et al., 2021) are to be examined for the influence of the multi-period feature.

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