Chapter 8

Neuro-Fuzzy Systems

8.1. Similitude between the RBFNN and the TSK fuzzy systems

8.2. The ANFIS architecture

8.3. Neuro-Fuzzy systems type Mamdani

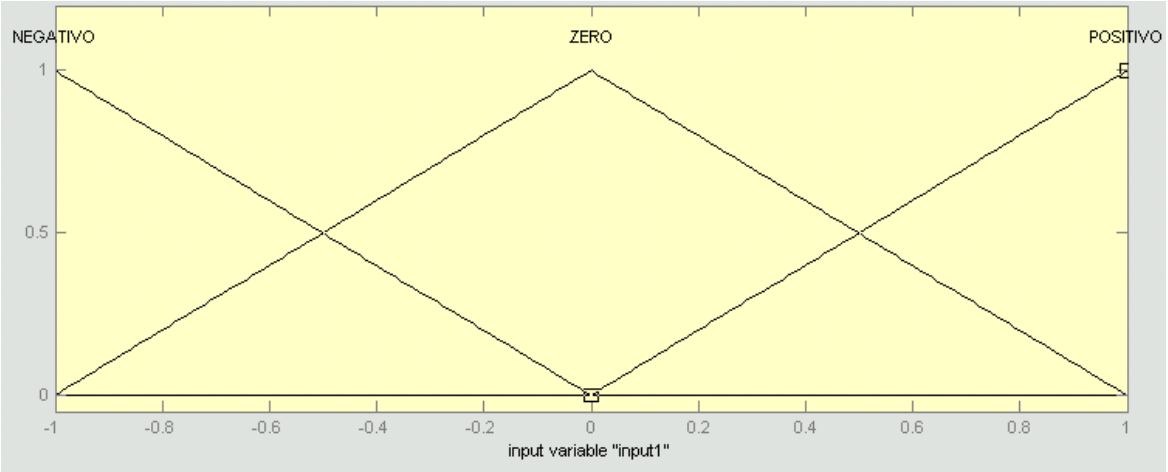
8.4. ANFIS in Matlab

8.5. Derivation of the rules from the fuzzy partitions

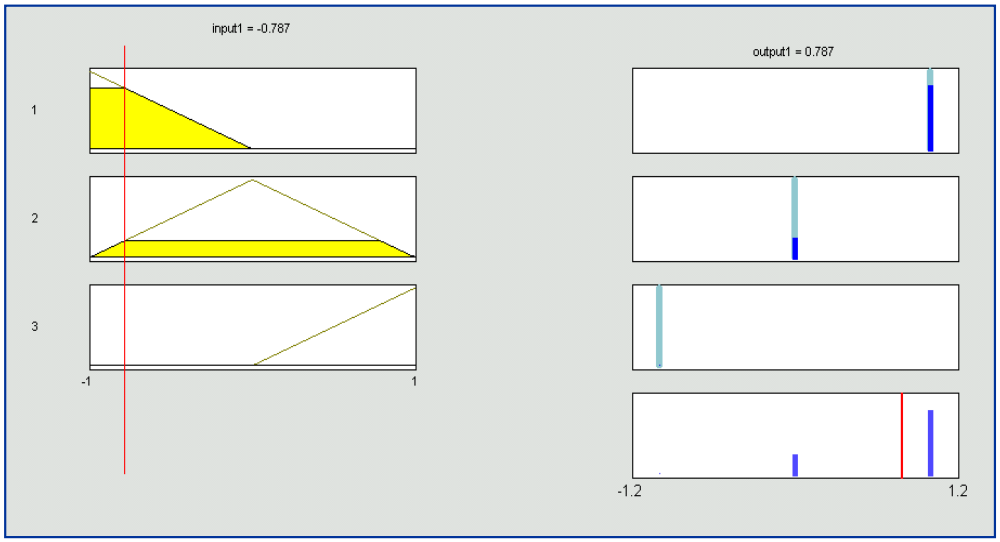
8.6. Derivation of the rules from subtractive clustering

8.7. Conclusions

8.1. Similitude between the RBFNN and the zero order TSK systems



1. IF p is NEGATIVE THEN y is 1
2. IF p is ZERO THEN y is 0
3. IF p is POSITIVE THEN y is -1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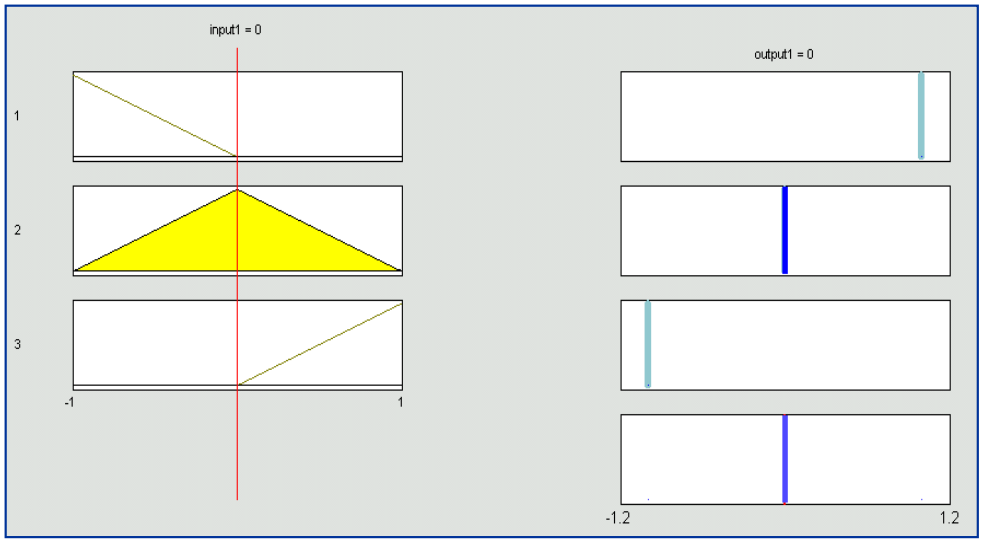
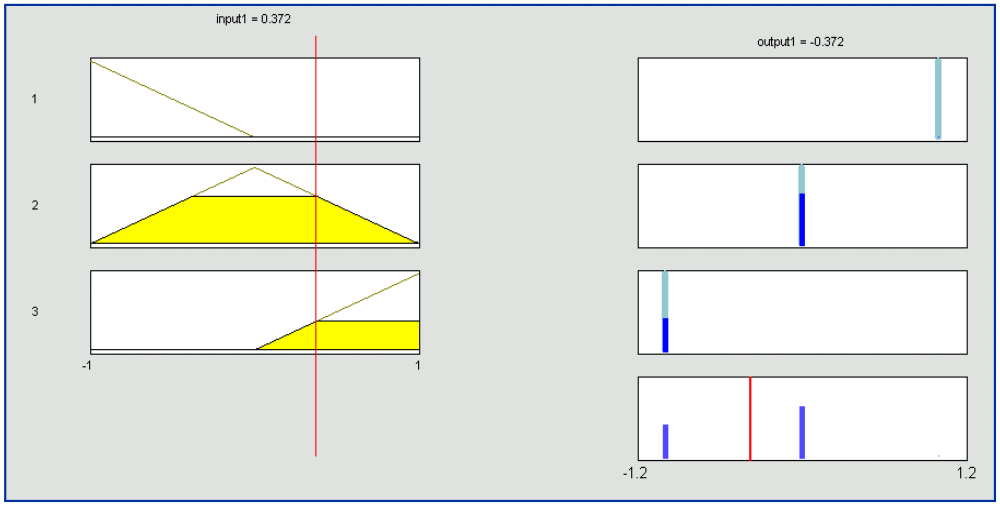
1
2
3
-1
1
-1.2
1.2


input1 = 0
output1 = 0
1
2
3
-1
1
-1.2
1.2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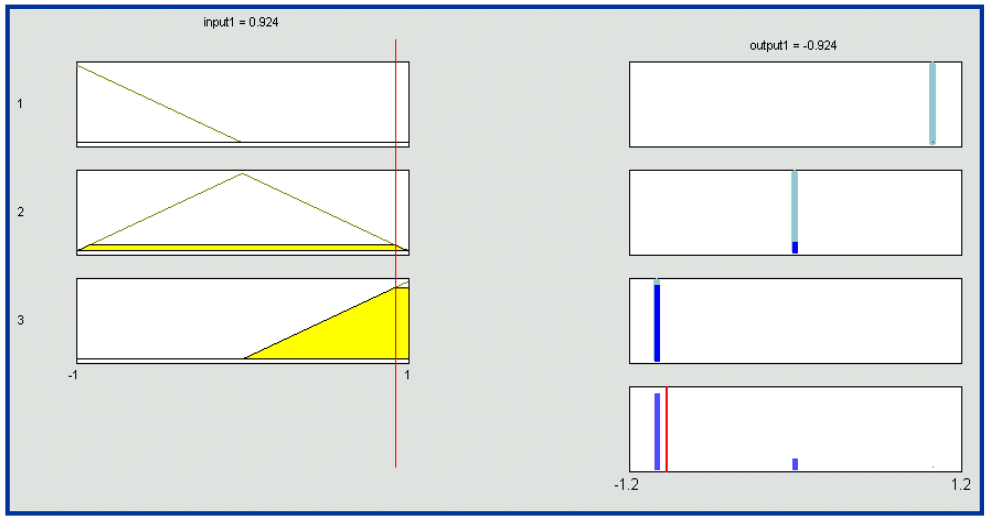
2
3
-1
1
-1.2
1.2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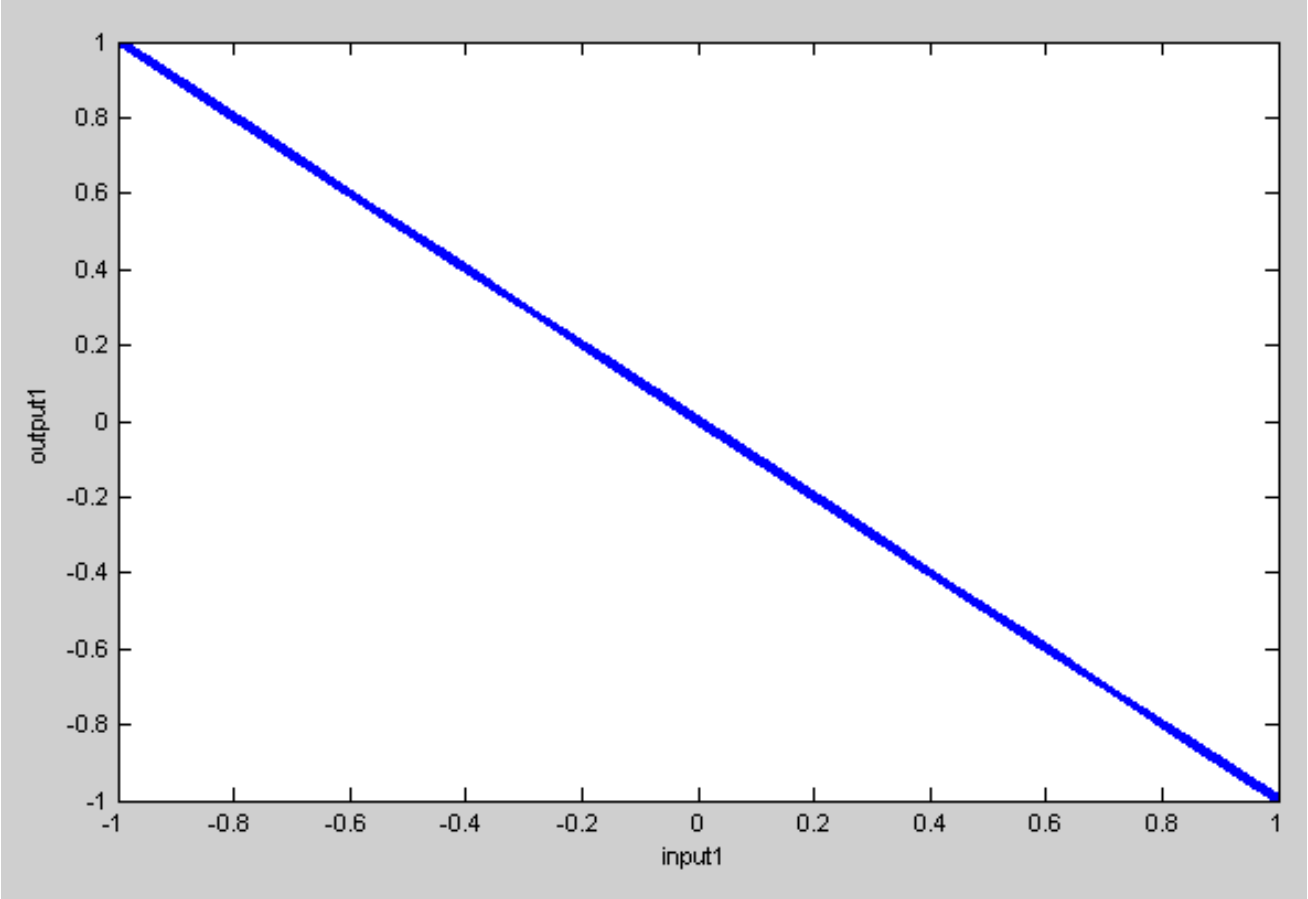
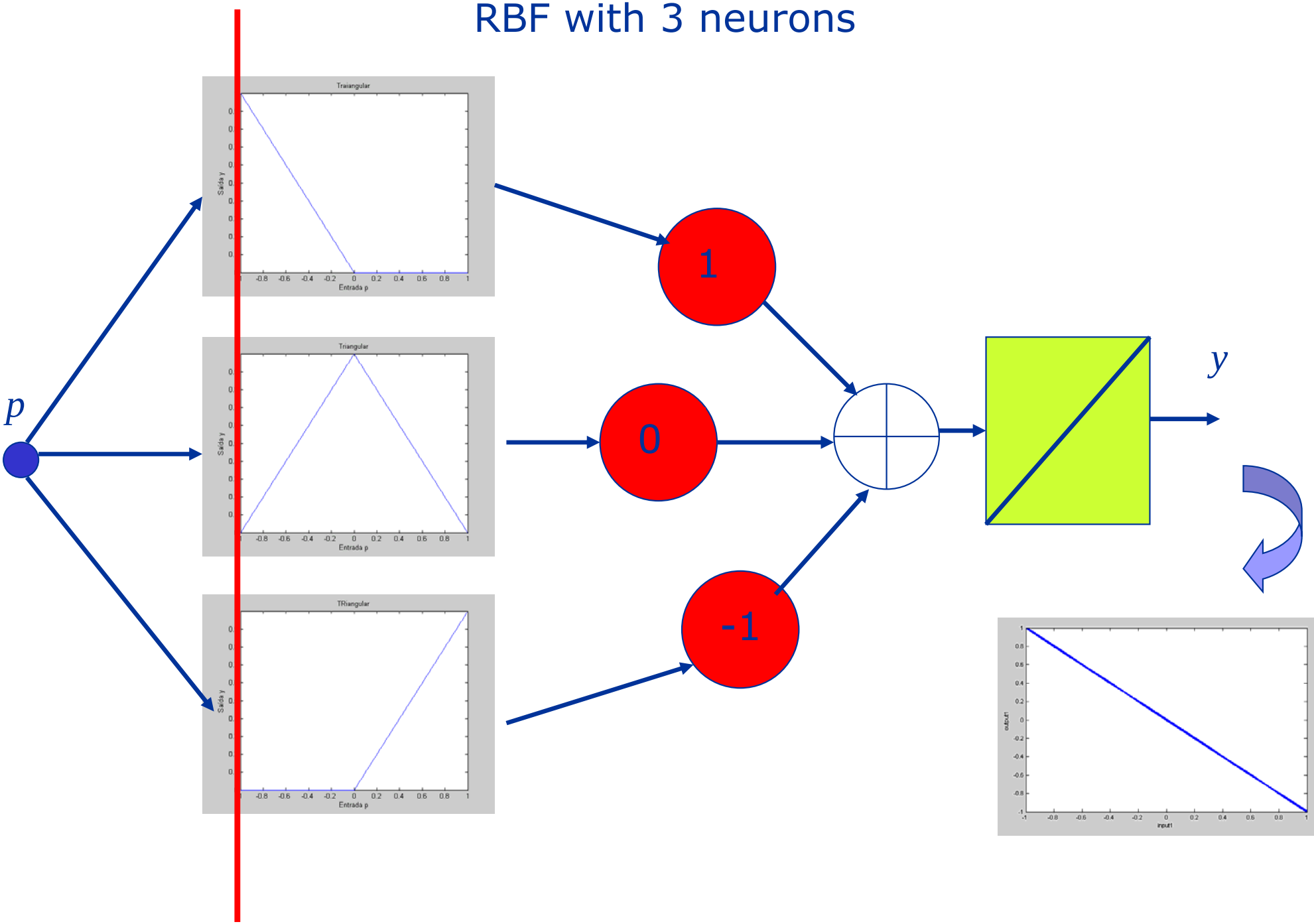
output1 = -0.924
1
2
3
-1
1
-1.2
1.2

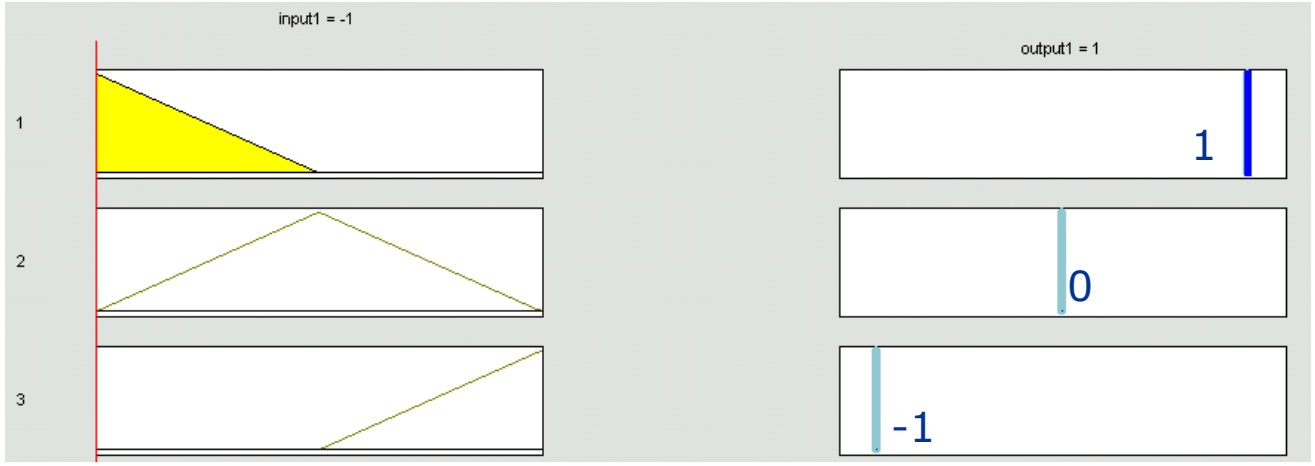
Input-output relation (*view surface*)

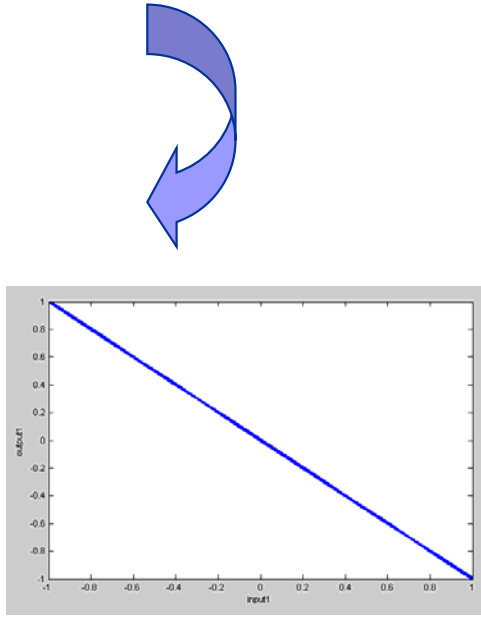


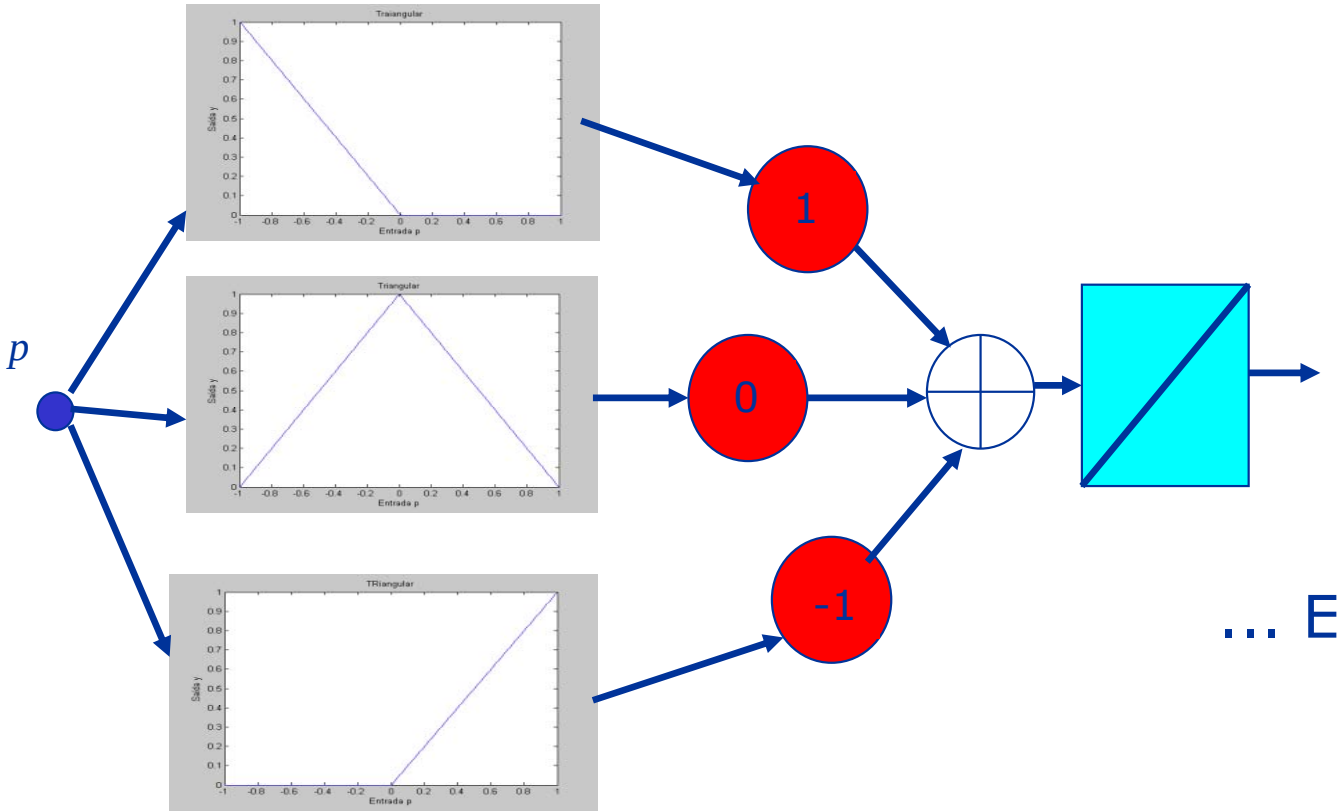
Note the (almost) perfect approximation of the function $y = -p$

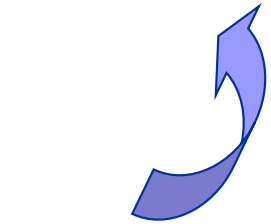
RBF with 3 neurons



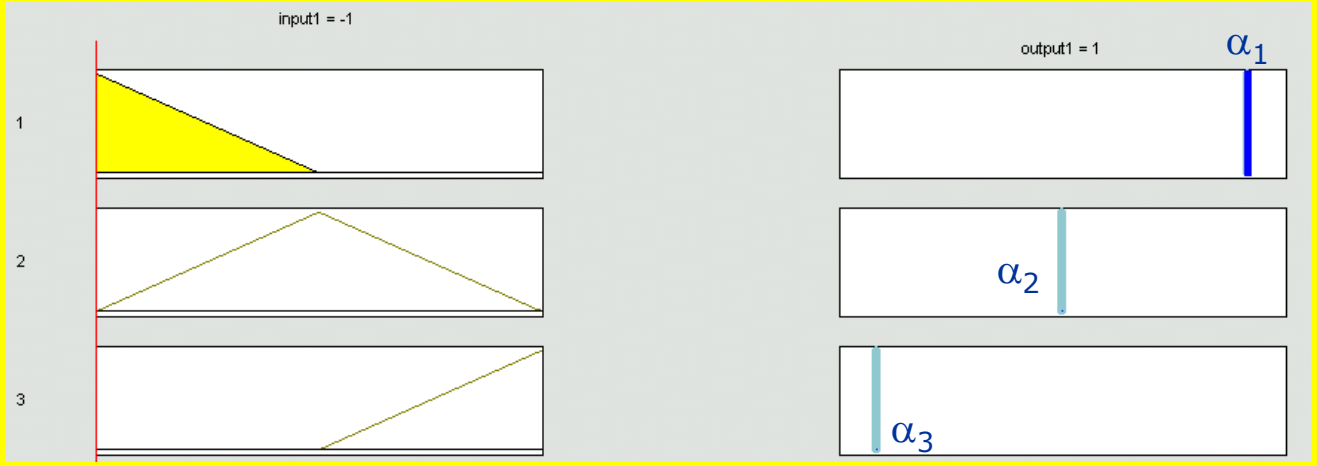




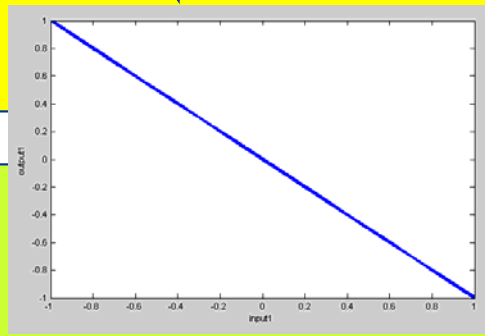


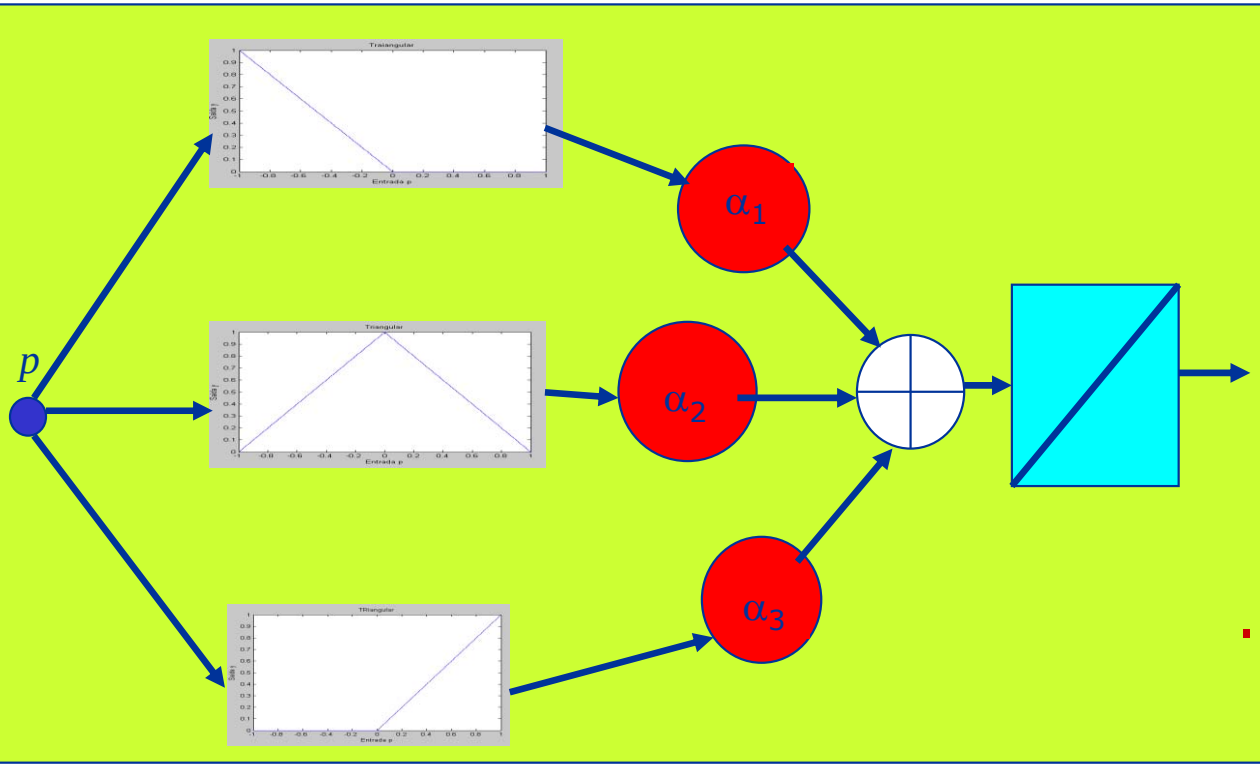


... Equivalent !



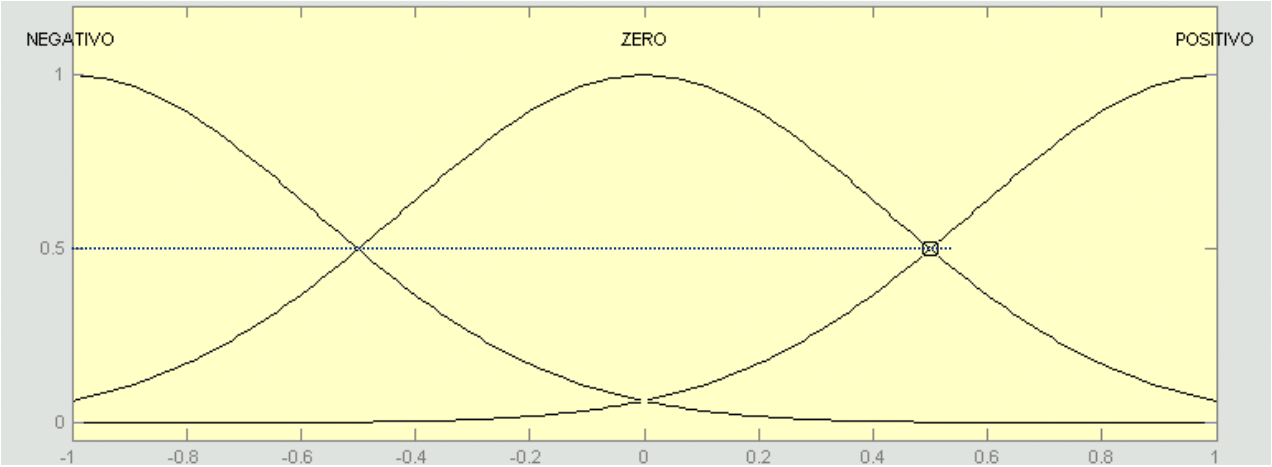
1. IF p is NEGATIVE THEN y is α_1
2. IF p is ZERO THEN y is α_2
3. IF p is POSITIVE THEN y is α_3





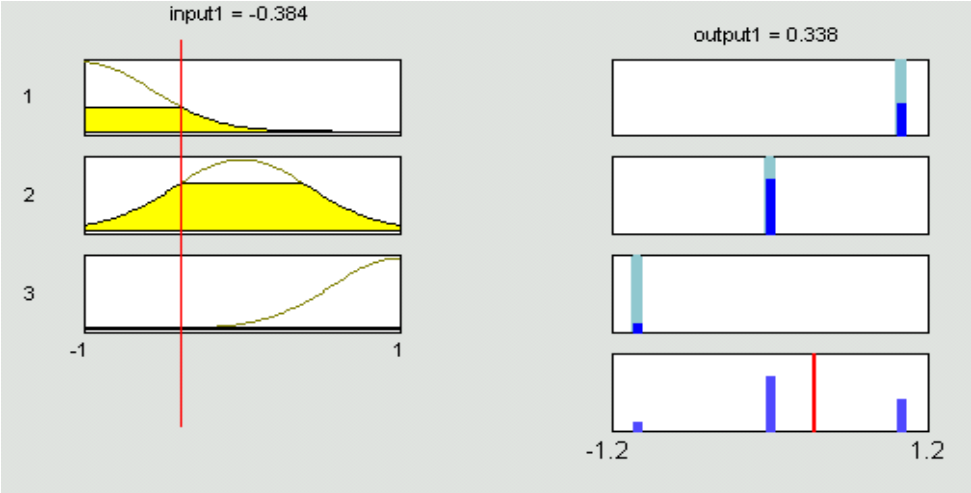
... Equivalent !

Case of Gaussian functions



[SIGMA, c]=[0.4246 1]

1. IF p is NEGATIVE THEN y is *1*
2. IF p is ZERO THEN y is 0
3. IF p is POSITIVE THEN y is -1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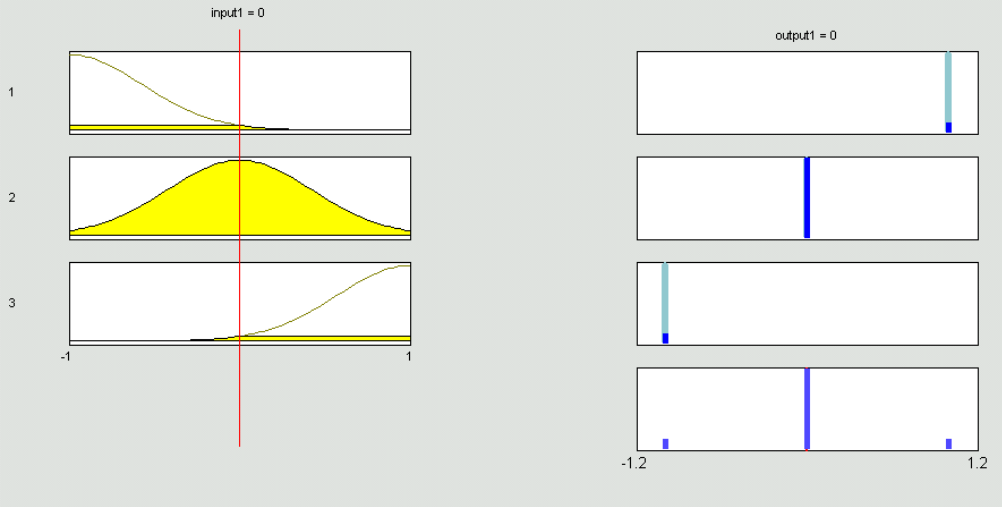
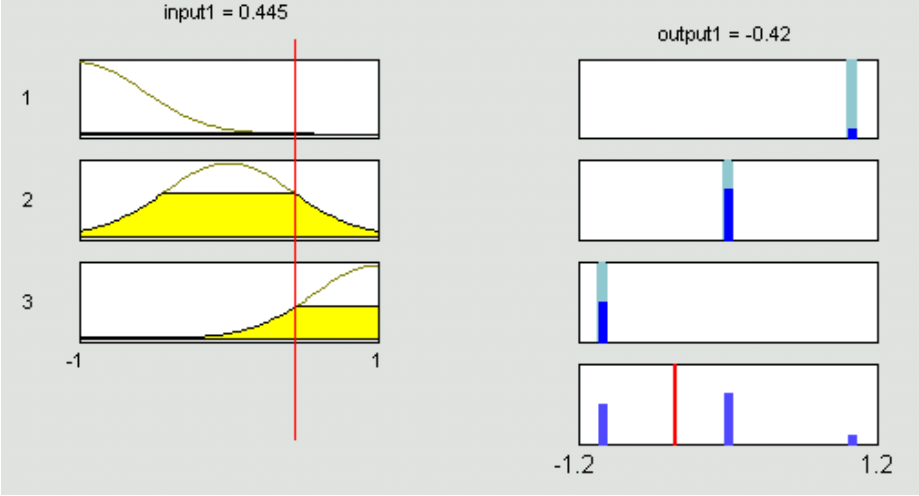
2
3
-1
1
-1.2
1.2


input1 = 0
output1 = 0
1
2
3
-1
1
-1.2
1.2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output1 = -0.42
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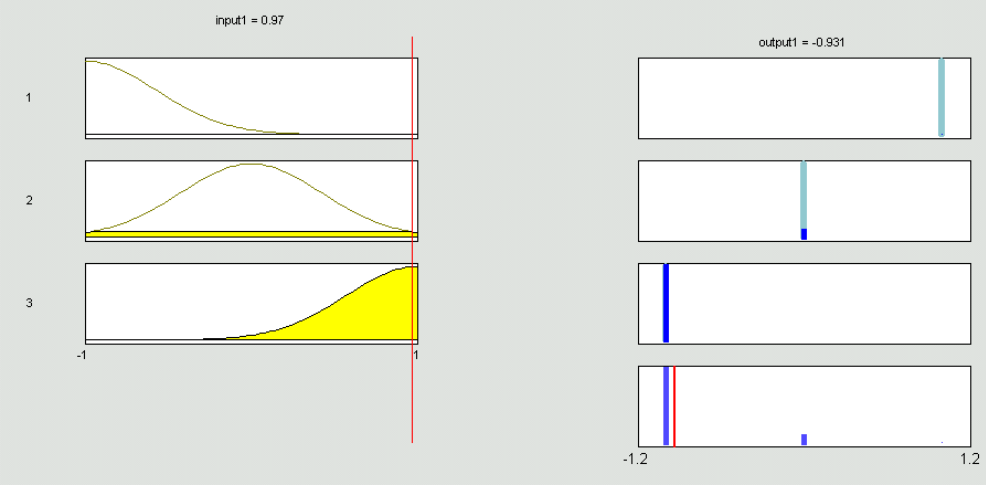
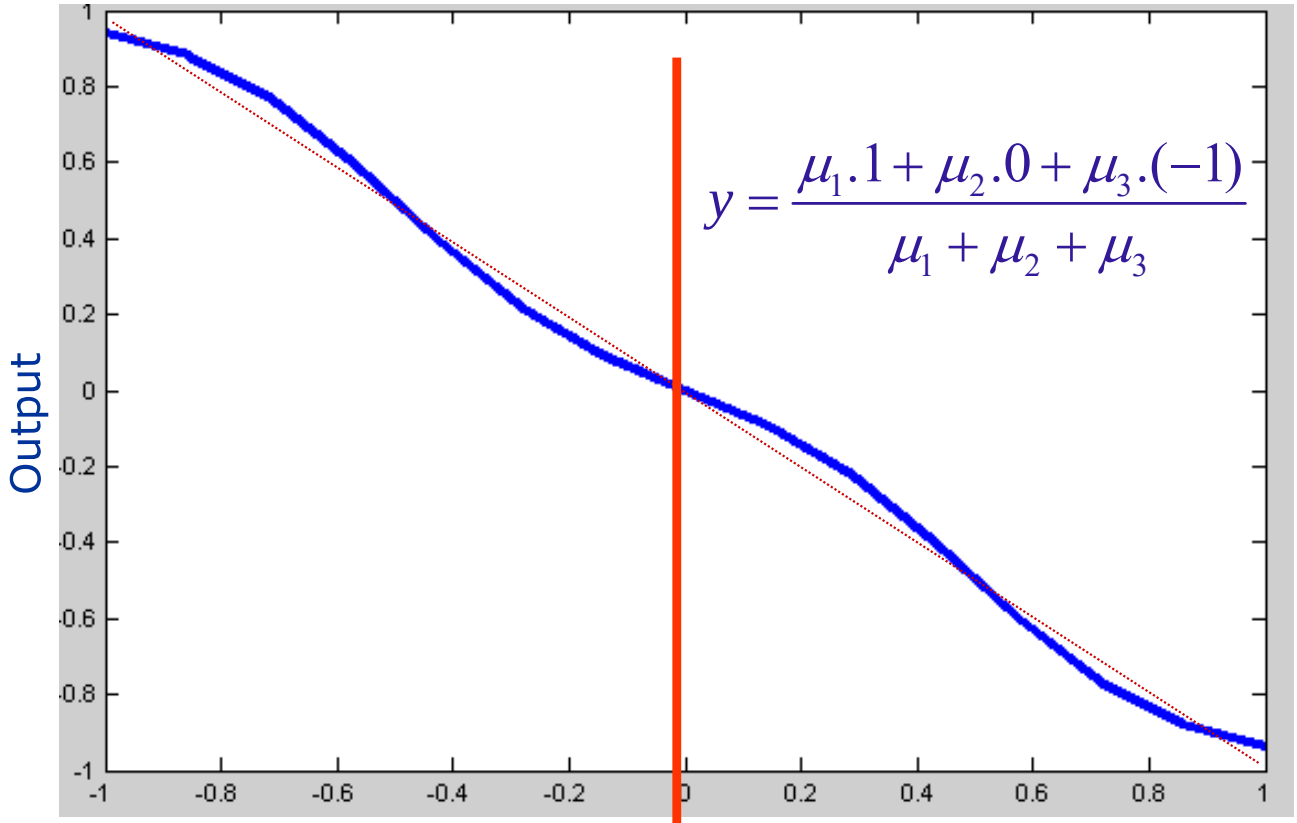
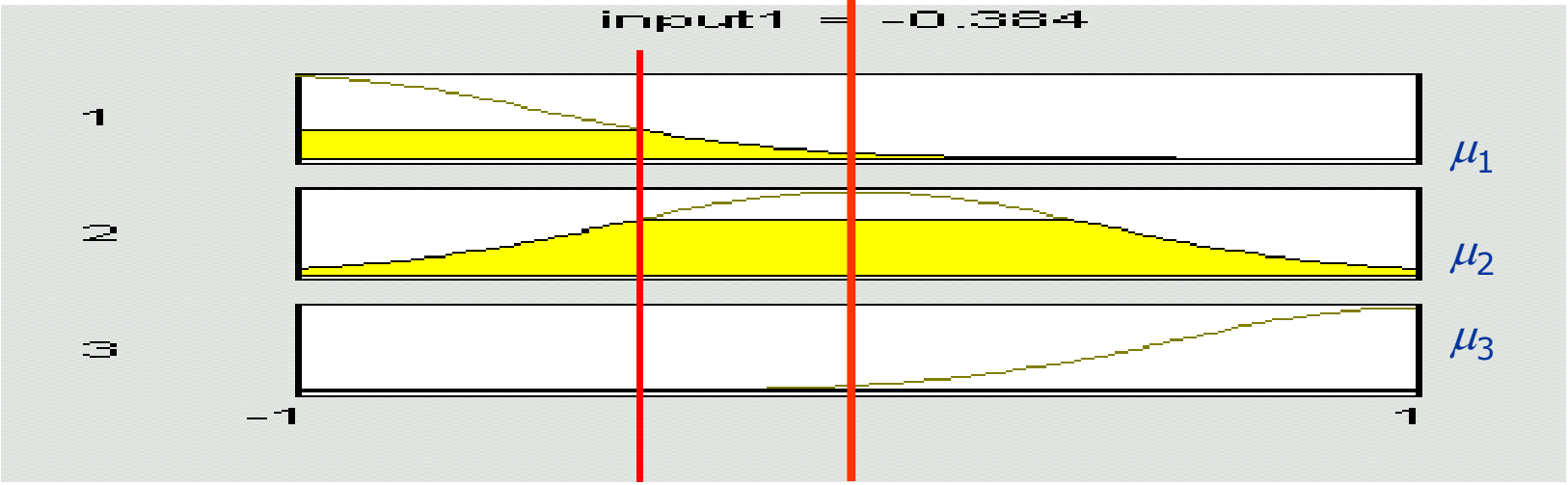
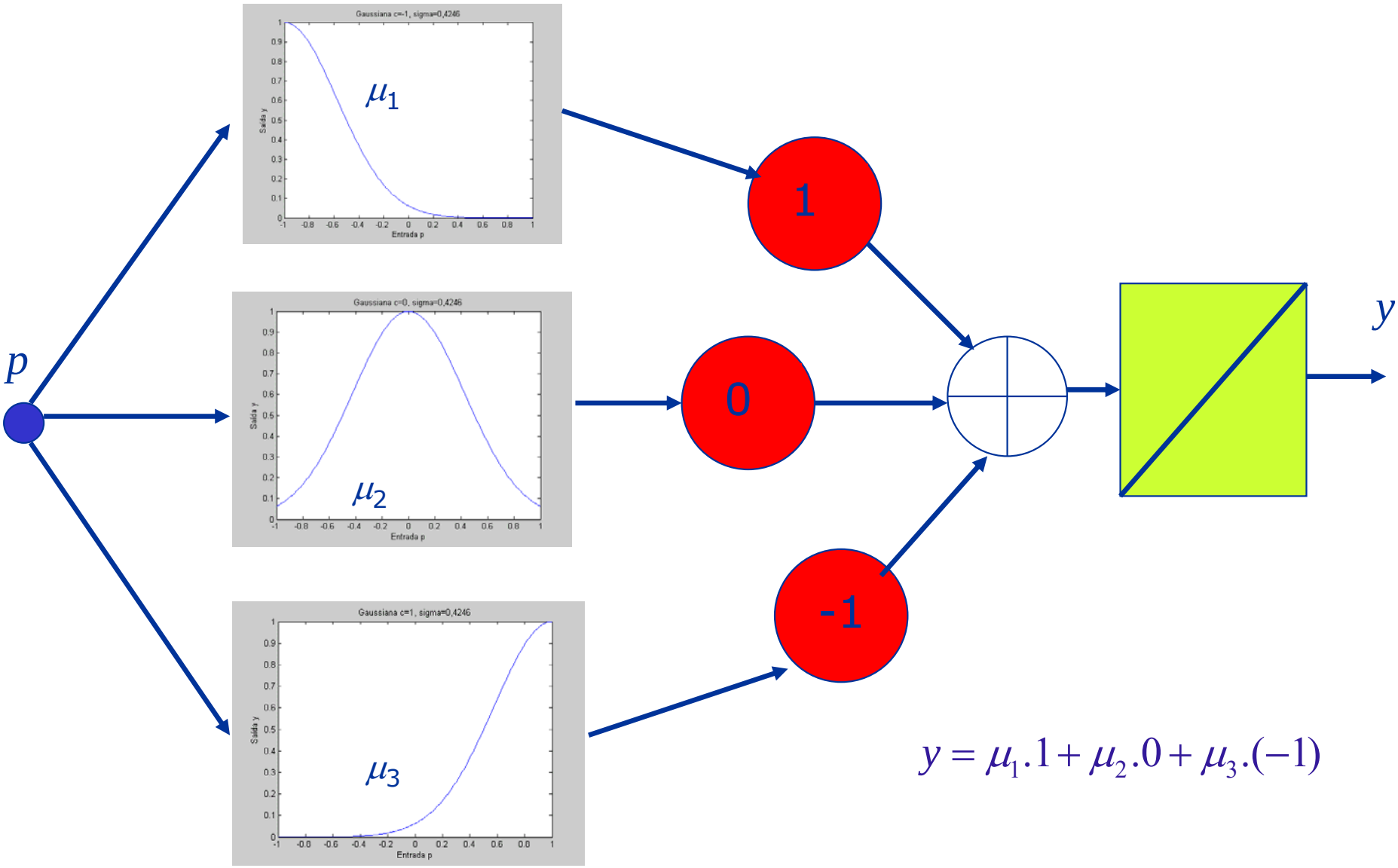
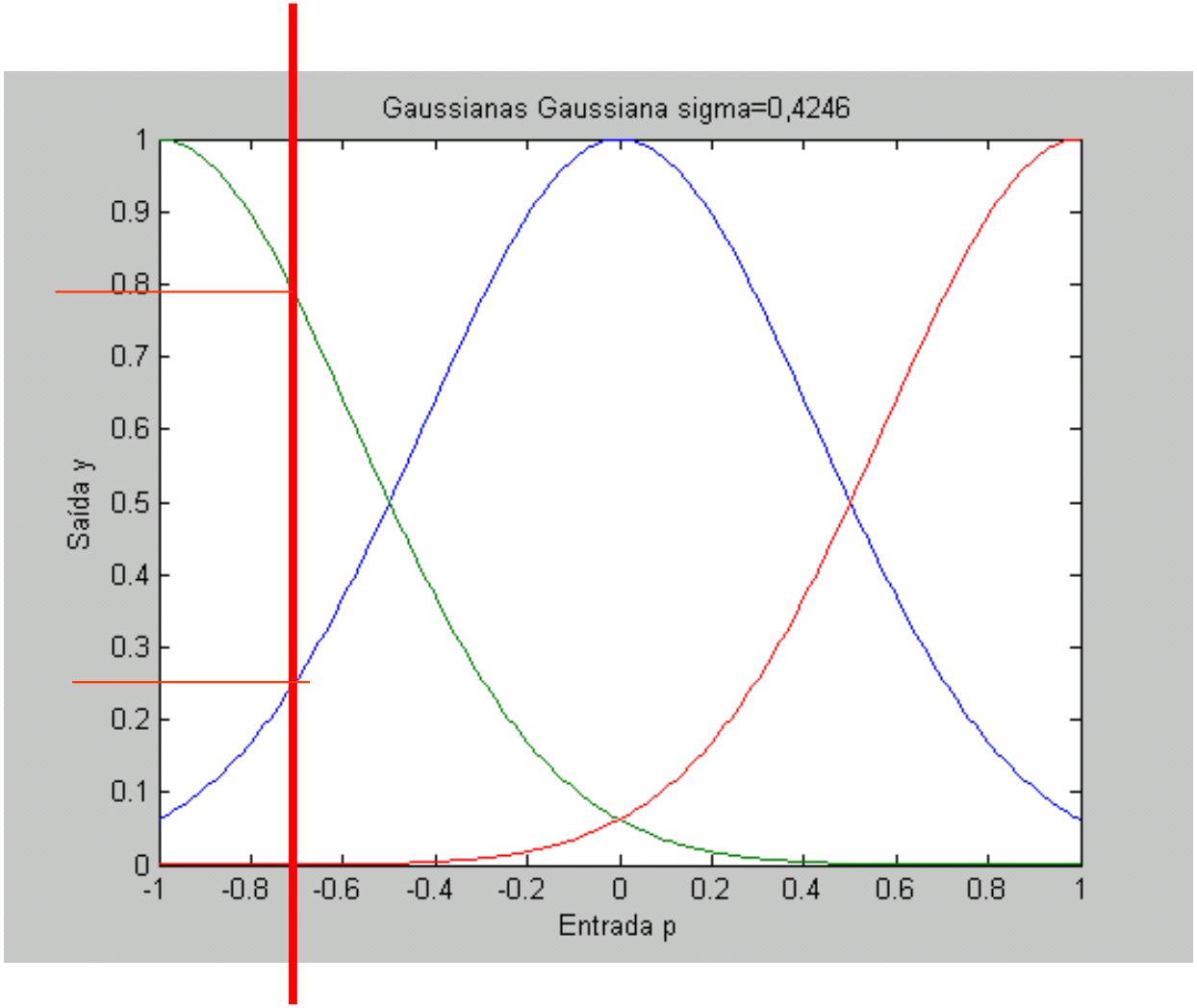
2
3
-1
1
-1.2
1.2

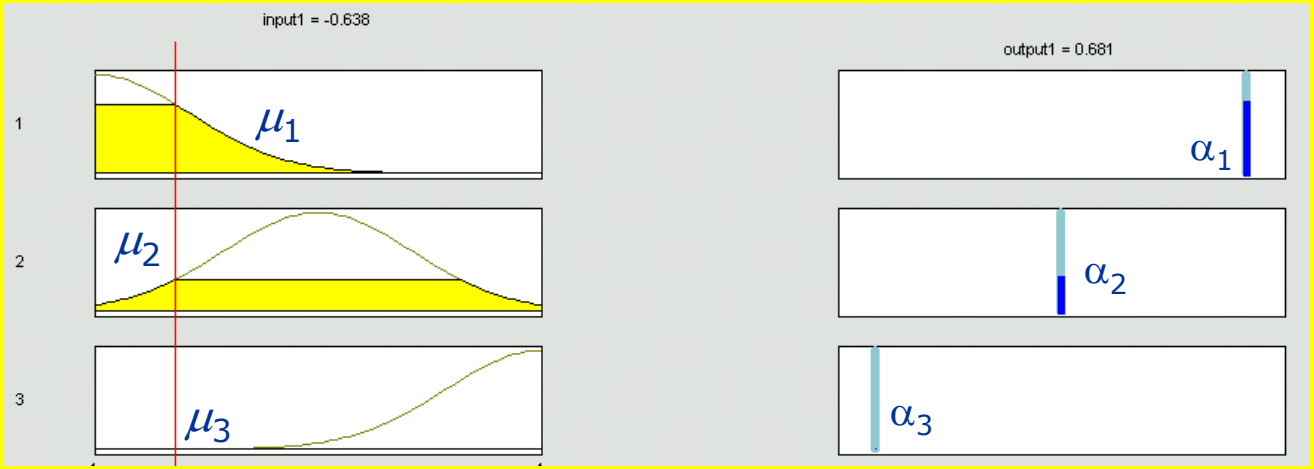

input1 = 0.97
output1 = -0.931
1
2
3
-1
1
-1.2
1.2


Output
Input
$y = \frac{\mu_1.1 + \mu_2.0 + \mu_3.(-1)}{\mu_1 + \mu_2 + \mu_3}$


input1 = -0.384
1
2
3
μ_1
μ_2
μ_3
-1
1


Gaussiana c=-1, sigma=0,4246
Gaussiana c=0, sigma=0,4246
Gaussiana c=1, sigma=0,4246
Saída y
Entrada p
μ_1
μ_2
μ_3
p
1
0
-1
y
$y = \mu_1.1 + \mu_2.0 + \mu_3.(-1)$


Gaussianas Gaussiana sigma=0,4246
Saída y
Entrada p
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

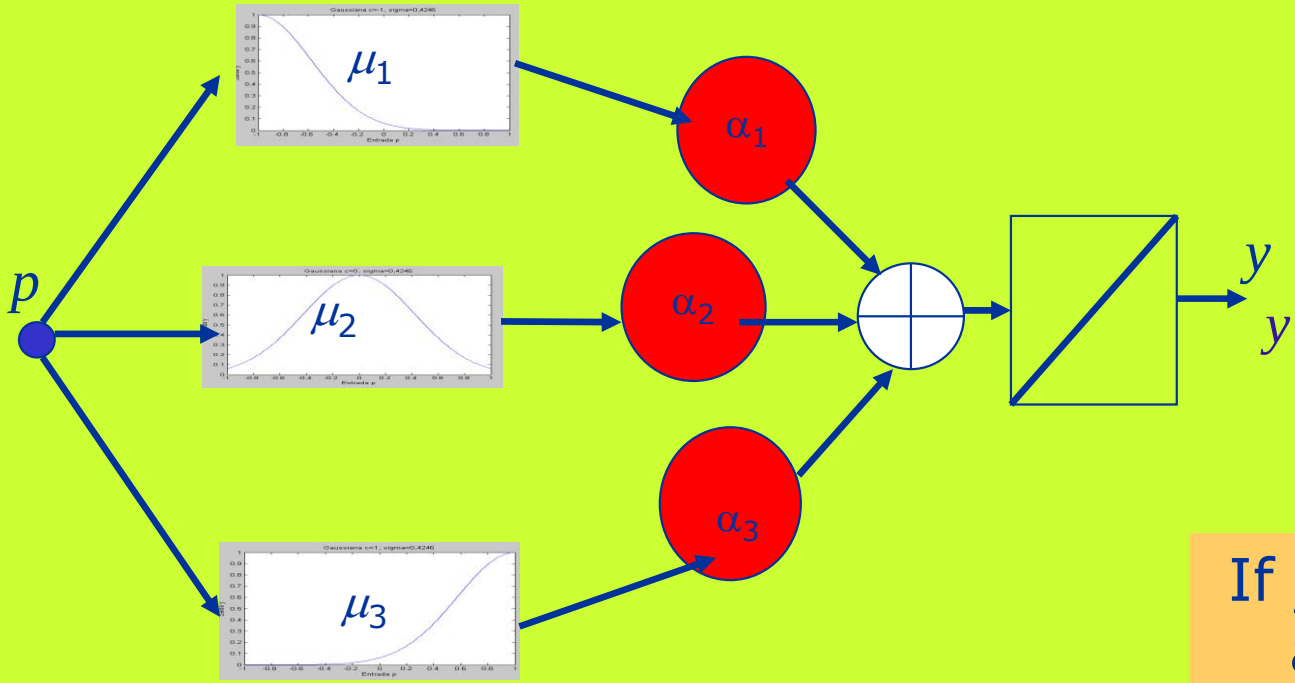


1. IF p is NEGATIVE THEN y is α_1

2. IF p is ZERO THEN y is α_2

3. IF p is POSITIVE THEN y is α_3

$$y = \frac{\mu_1.\alpha_1 + \mu_2.\alpha_2 + \mu_3.\alpha_3}{\mu_1 + \mu_2 + \mu_3}$$

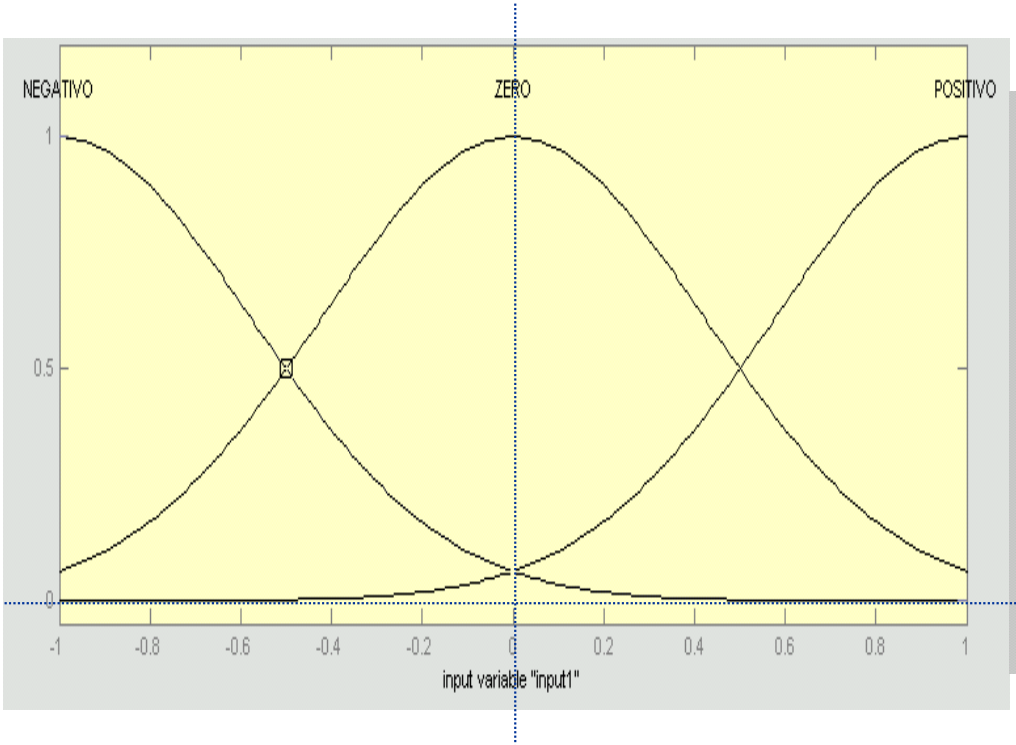


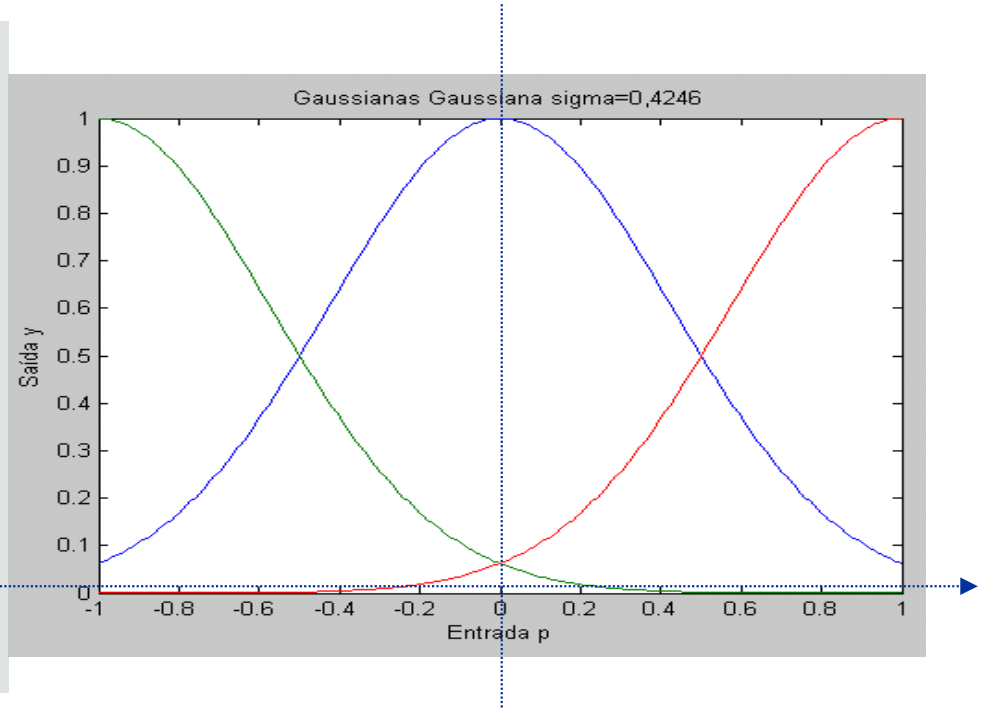
$$y = \mu_1.\alpha_1 + \mu_2.\alpha_2 + \mu_3.\alpha_3$$

If $\mu_1 + \mu_2 + \mu_3 = 1$, they are equivalent

Membership functions

RBFs





In some intervals, the sum of membership is greater than 1.

Conditions of equivalence:

1- the membership functions are equal to the radial basis functions.

2- The sum of memberships for each value of the input is 1.

And if each rule has more than one antecedent ?

And if the consequents are not of zero order ?

Does it exist any NN equivalent ?

8.2. The architecture ANFIS (*Adaptive Network Fuzzy Inference System*)

System TSK of 1st order:

Rule 1: IF p_1 is $\underset{\sim 1}{A}$ AND p_2 is $\underset{\sim 1}{B}$ THEN $y^1 = \alpha_1 + \beta_1 p_1 + \gamma_1 p_2$

Rule 2: IF p_1 is $\underset{\sim 2}{A}$ AND p_2 is $\underset{\sim 2}{B}$ THEN $y^2 = \alpha_2 + \beta_2 p_1 + \gamma_2 p_2$

Two inputs p_1^* and p_2^* are presented and the rules are fired with intensities r_1 and r_2, (conjunction by the product), producing the outputs y^1 and y^2 :

$$r_1 = \underset{\sim 1}{A}(p_1^*) \times \underset{\sim 1}{B}(p_2^*)$$

$$r_2 = \underset{\sim 2}{A}(p_1^*) \times \underset{\sim 2}{B}(p_2^*)$$

$$y^1 = \alpha_1 + \beta_1 p_1^* + \gamma_1 p_2^*$$

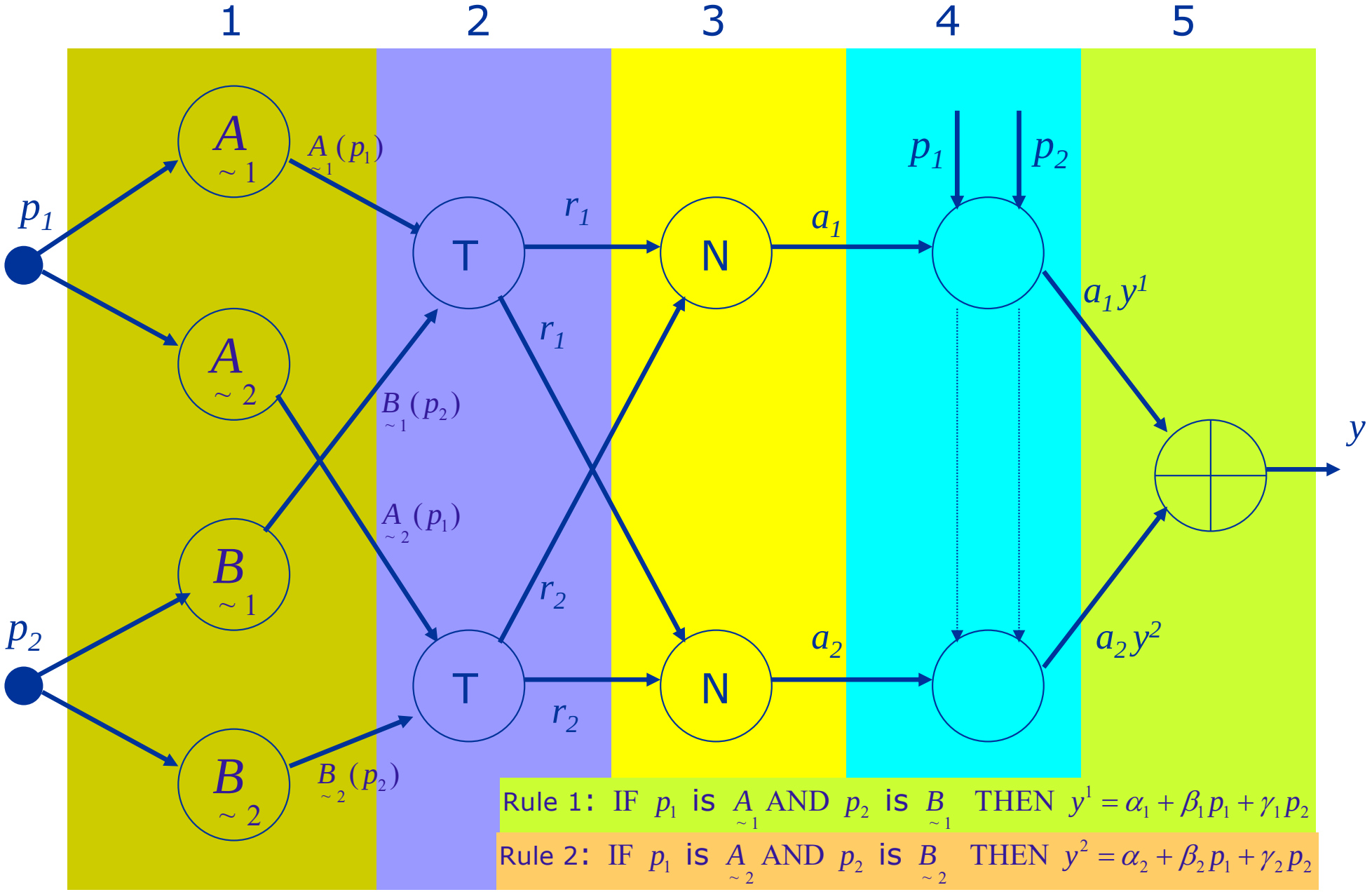
$$y^2 = \alpha_2 + \beta_2 p_1^* + \gamma_2 p_2^*$$

The total output will be

$$y^* = \frac{r_1 y^1 + r_2 y^2}{r_1 + r_2} = a_1 y^1 + a_2 y^2, \qquad a_1 = \frac{r_1}{r_1 + r_2} \qquad a_2 = \frac{r_2}{r_1 + r_2}$$

To realize these operations a neural network with five layers is implemented.

ANFIS example of two rules



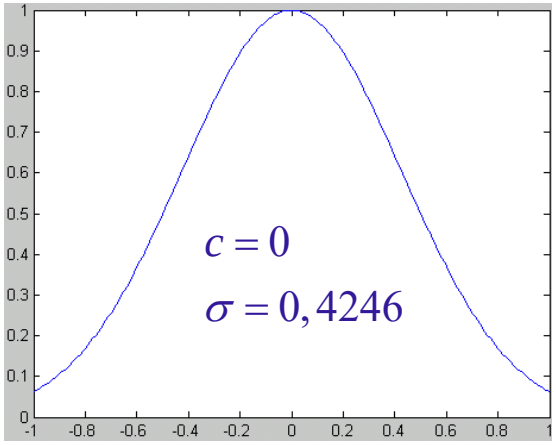
Layer 1: fuzzification

The output of the neuron is the membership value of the crisp inputs to the respective fuzzy sets

$(p_1$ to $\underset{\sim 1}{A}$ and $\underset{\sim 2}{A}$, p_2 to $\underset{\sim 1}{B}$ and $\underset{\sim 2}{B})$

Frequently, because of differentiability, the membership functions of $\underset{\sim 1}{A}, \underset{\sim 2}{A}, \underset{\sim 1}{B}, \underset{\sim 2}{B}$ are Gaussians.

$$\underset{\sim i}{A}(p) = e^{-\frac{(p-c_{i1})^2}{2\sigma_{i1}^2}} \qquad \underset{\sim i}{B}(p) = e^{-\frac{(p-c_{i2})^2}{2\sigma_{i2}^2}}$$



$\{c_{i1} \quad c_{i2} \quad \sigma_{i1} \quad \sigma_{i1}\} \triangleq$ parameters of layer 1

parameters of the antecedents of the rules.

Layer 2 : firing

Each neuron computes the fire strength of the rule it is associated with. The outputs of the first neuron (upper) and of the second neuron (lower) are respectively r_1 and r_2:

$$r_1 = \underset{\sim 1}{A}(p_1^*) \times \underset{\sim 1}{B}(p_2^*)$$

$$r_2 = \underset{\sim 2}{A}(p_1^*) \times \underset{\sim 2}{B}(p_2^*)$$

Usually, the algebraic product is used as the conjunction operator, because it is differentiable (advantageous for training using backpropagation).

Layer 3:normalization

Each neuron normalizes (N) the firing strengths of the rules

$$a_1 = \frac{r_1}{r_1 + r_2} \qquad a_2 = \frac{r_2}{r_1 + r_2}$$

Layer 4: consequent

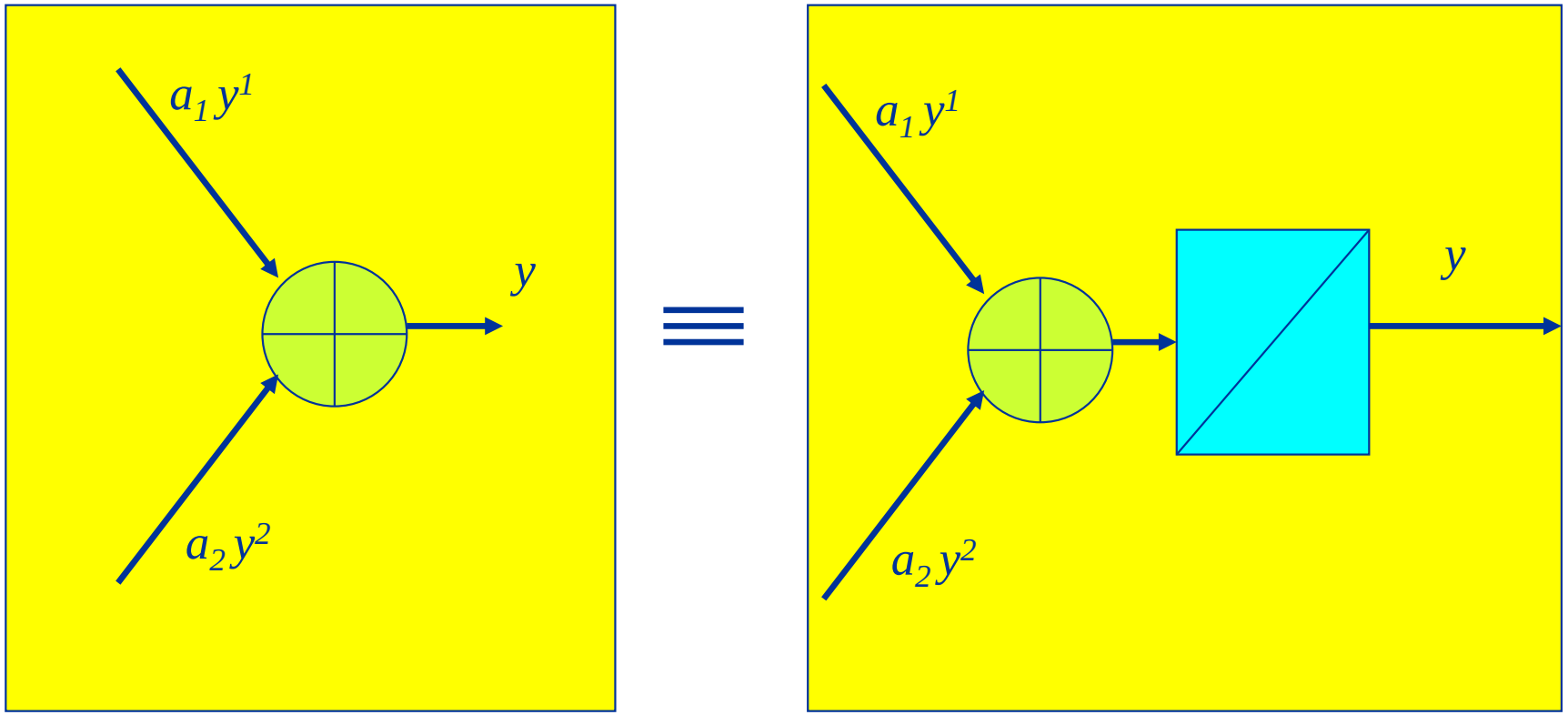
The output of each neuron is the product of a_i by the individual output of each rule

$$a_1 y^1 = a_1(\alpha_1 + \beta_1 p_1^* + \gamma_1 p_2^*)$$
$$a_2 y^2 = a_2(\alpha_2 + \beta_2 p_1^* + \gamma_2 p_2^*)$$

Layer 5: aggregation and defuzzification

The single neuron (in the case there is only one output) of this layer computes the overall output of the system:

$$y^* = a_1 y^1 + a_2 y^2$$



How does it work:

Give a training set $\{(p_1^k, p_2^k), k=1,...Q\}$.

Compute the output of the network y^k.

Compute the error of the output $e_k = y_d^k - y_k$, y_d^k is the desired (target) output.

A learning procedure is implemented, using backpropagation (from this comes the name of the architecture ANFIS- *Adaptive Network Fuzzy Inference System*).

For this aim the different activation functions in each layer must be differentiable.

Initialization

The parameters of the antecedents are chosen such that in each dimension the membership functions fulfil the requirements:

Completeness: they cover all the possible values of the inputs

Normality: they are normal fuzzy sets

Convexity: the fuzzy sets are convex.

The final result depends on the initialization.

A global minimum cannot be guaranteed (usually one attains a local minimum).

Minimization algorithms implemented in ANFIS (*Fuzzy Logic Toolbox MatLab*)

(i) backpropagation

All parameters are optimized:

- of the antecedents (membership functions)
- of the consequents (the polynomial coefficients)

order zero: α_i

order 1: α_i, β_i, γ_i , i= 1,..., number of rules

Requires the derivatives of all activation functions.

(ii) Least squares

If the antecedents are fixed, (i.e., the layers 1, 2 and 3) then

$$y^* = a_1 y^1 + a_2 y^2 = a_1(\alpha_1 + \beta_1 p_1^* + \gamma_1 p_2^*) + a_2(\alpha_2 + \beta_2 p_1^* + \gamma_2 p_2^*)$$

$$= a_1\alpha_1 + (a_1 p_1^*)\beta_1 + (a_1 p_2^*)\gamma_1 + a_2\alpha_2 + (a_2 p_1^*)\beta_2 + (a_2 p_2^*)\gamma_2 =$$

$$= \underbrace{\begin{bmatrix} a_1 & (a_1 p_1^*) & (a_1 p_2^*) & a_2 & (a_2 p_1^*) & (a_2 p_2^*) \end{bmatrix}}_{\mathbf{p}^T} \underbrace{\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \\ \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix}}_{\boldsymbol{\theta}}$$

$$= \mathbf{p}^T\boldsymbol{\theta} = \boldsymbol{\theta}^T\mathbf{p}$$

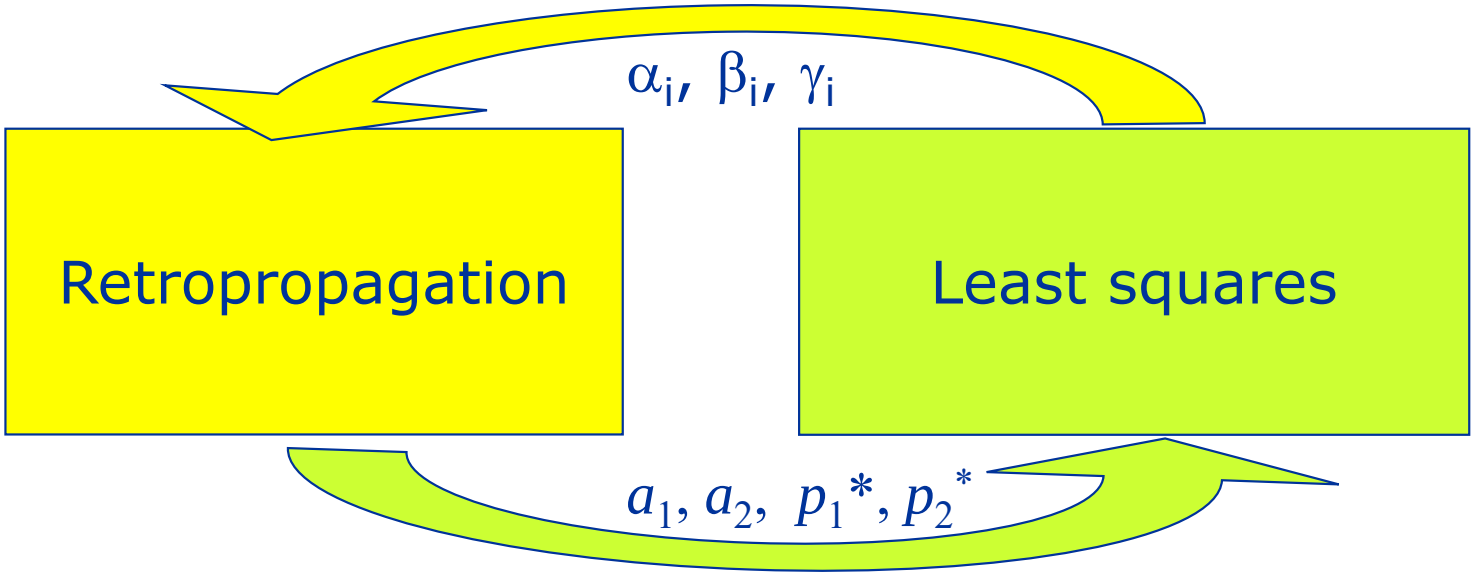
Given a_1, a_2, p_1^*, p_2^* the Widrow-Hoff or the RLS algorithm can be applied.

(ii) Hybrid method

-consequents (α_i, β_i, γ_i , i= 1,..., number of rules):

least squares (recursive or not) (fixing the antecedents)

- antecedents (coefficients of the membership functions):
backpropagation (fixing the consequents)



8.3. Mamdani type neuro-fuzzy systems

Rule 1: IF p_1 is $\underset{\sim 1}{A}$ AND p_2 is $\underset{\sim 1}{B}$ THEN y^1 is $\underset{\sim 1}{C}$

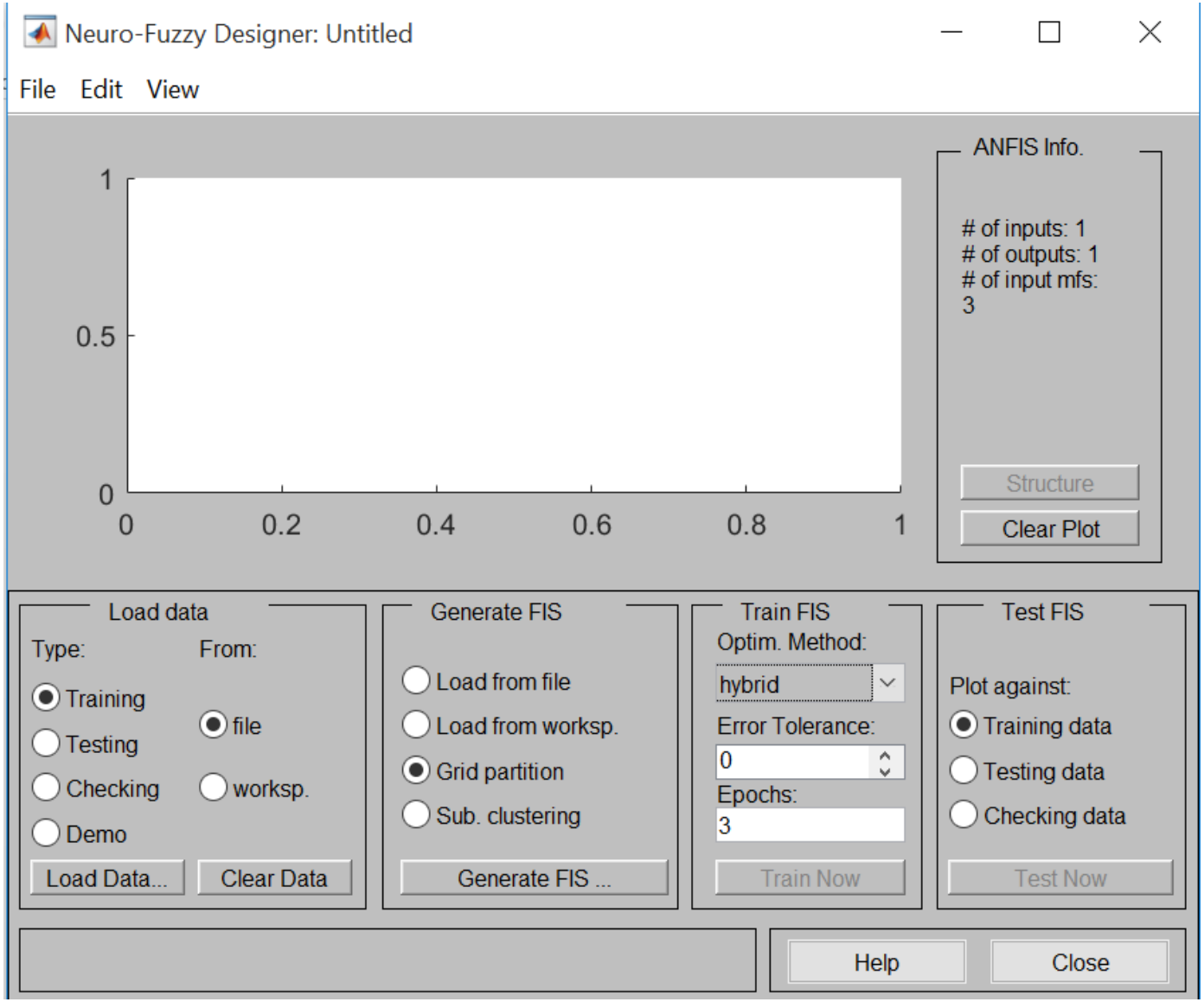
Rule 2: IF p_1 is $\underset{\sim 2}{A}$ AND p_2 is $\underset{\sim 2}{B}$ THEN y^1 is $\underset{\sim 2}{C}$

Several architectures of NN proposed in literature.

Learning difficulties not yet overcome.

Less used that TSK.

8.4. ANFIS in MatLab (*anfisedit*)

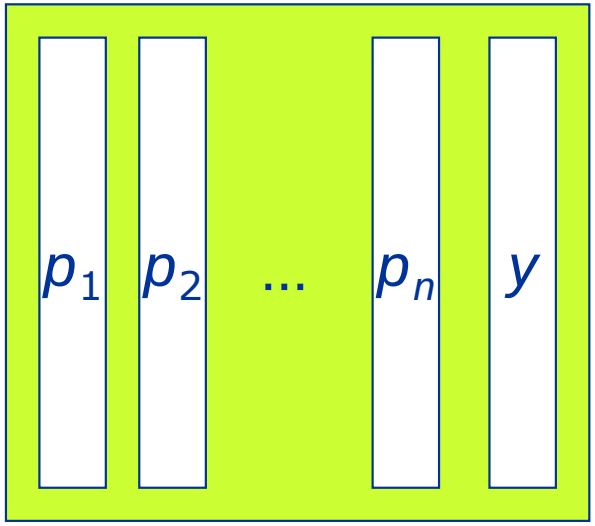


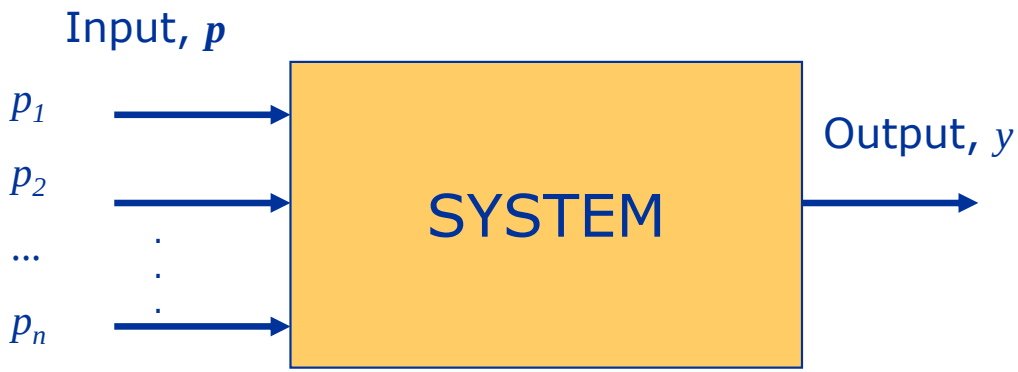
Data format

-Matrices n+1 columns:

-n columns: dimension of the input data p_1, p_2, ..p_n

- last column is the output y data





Initialization

Create a *fis* initial file using

- Grid partition
- Subtractive clustering (see Chapt.3 Clustering)

see also function genfis (can use grid partition, subtractive of fcm clustering)
FIS = genfis(XIN,XOUT,OPTIONS)

Training

- Backpropagation
- Hybrid (backpropagation + least squares)

8.5. Derivation of the rules from fuzzy partitions

The fuzzy c-means clustering may be applied to obtain the initial membership functions in ANFIS.

one input,
one output

i) Collect the training data made up by pairs (p,y)

ii) Represent these data in the plane (p,y)

iii) Apply the clustering method

Each center $v_i = (p_i, y_i)$ obtained will define a rule

Definition of the rule from the center

Center $v_i=(p_i, y_i)$

Rule TSK zero order

IF input is $\underset{\sim i}{p}$ THEN output is y_i

$\underset{\sim i}{p}$ Fuzzy set obtained by defining, in the dimension of p , a membership function centered in p_i

y_i value of the coordinate of the center v_i in the dimension of y , *i.e.*, y_i

Case of two inputs

i) Collect the training data made up by 3-tuples (p_1,p_2,y)

ii) Represent these data in the tridimensional space (p_1,p_2,y)

iii) Apply the clustering method

Each obtained center (p_{i1},p_{i2},y_i) will define one rule

Definition of a rule from the center

Center $v_i=(p_{i1}, p_{i2}, y_i)$

Rule Mamdani type:

IF p_1 is $\underset{\sim i1}{p}$ AND p_2 is $\underset{\sim i2}{p}$ THEN output is $\underset{\sim i}{y}$

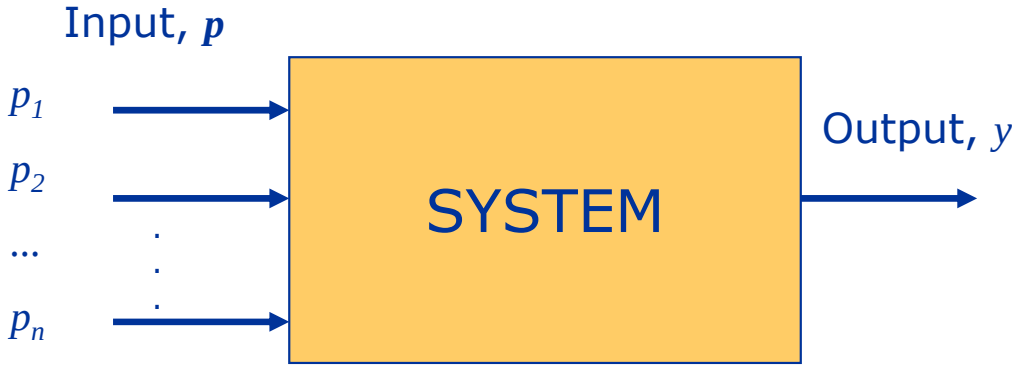
$\underset{\sim i1}{p}, \underset{\sim i2}{p}, \underset{\sim i}{y}$ Fuzzy sets obtained projecting the center v_i in the dimensions p_1, p_2, y, respectively; these projections are the centers of the membership functions

Rule TSK of order zero:

IF p_1 is $\underset{\sim i1}{p}$ AND p_2 is $\underset{\sim i2}{p}$ THEN output is y_i

y_i value of the coordinate of the center in the dimension y.

Case de n inputs



Training data : tuples $(p_1, p_2, \ldots, p_n, y)$

Centers: $v_i = (p_{i1}, p_{i2}, \ldots, p_{in}, y_i)$

Rule TSK of order zero:

IF Input1 is $\underset{\sim i1}{p}$ AND Input2 is $\underset{\sim i2}{p}$ AND ... AND Input$_n$ is $\underset{\sim in}{p}$ THEN output is y_i

Rules type Mamdani

- For the antecedents the same procedure as TSK is applied.
- For the consequents the cluster is projected in the y dimension.
- Algorithms improving o FCM
  - Gustafson-Kessel
  - Gath-Geva

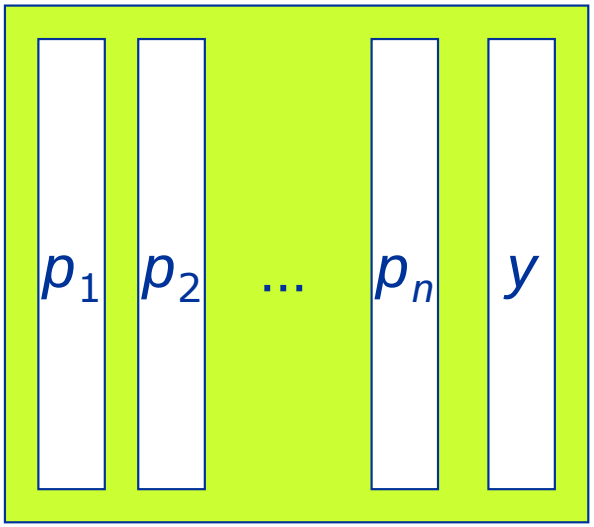
See:

Graves D., Pedrycz W. (2007) Fuzzy C-Means, Gustafson-Kessel FCM, and Kernel-Based FCM: A Comparative Study. In: Melin P., Castillo O., Ramírez E.G., Kacprzyk J., Pedrycz W. (eds) Analysis and Design of Intelligent Systems using Soft Computing Techniques. Advances in Soft Computing, vol 41. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-72432-2_15

I. Gath, and A.B. Geva (1989), Unsupervised optimal fuzzy clustering,, IEEE Transactions on Pattern Analysis and Machine Intelligence (Volume: 11, Issue: 7, Jul 1989), . https://doi.org/10.1109/34.192473

8.6. Derivation of the rules from subtractive clustering

- The subtractive clustering produces a set of centers.



- Each center has n+1 coordinates
  - n for the inputs
  - 1 for the output

- center one membership function in each center

- the openness is computed as in the case of RBF.

- project the membership function in each dimension

- The resulting fuzzy sets define the rules (one for each center).

see also function genfis (can use grid partition, subtractive of fcm clustering)
FIS = genfis(XIN,XOUT,OPTIONS)

Heuristics to compute σ (Hassoun, 290)

1- if equal to all membership functions

compute the distance between each center c_i e and its nearest neighbor c_j

$\sigma =$ average of the distances between the neighbors centers $=< \left\| c_i - c_j \right\| >$

2- if proper to each membership function

$\sigma_i = \alpha \left\| v_j - v_i \right\|$, where v_j is the center closest to v_i

$$1,0 < \alpha < 1,5$$

8.7. Conclusions

Neuro-fuzzy systems are useful whenever there exist available experimental data end there isn’t available sufficient knowledge (theoretical or empirical) about the system to write directly fuzzy rules.

They are used to optimize rule-based fuzzy systems.

The rules issued form optimization should preferably have a semantic meaning for the human user, i.e., they should be transparent and interpretable. This leads to special learning techniques, an actual research subject.

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