INSTITUTO SUPERIOR TÉCNICO | UNIVERSIDADE DE LISBOA
NETWORK SCIENCE 2019/20

1st Exam | 15 January 2020 (2h)

This exam can be answered in Portuguese or in English in a standard IST exam sheet.
All exercises are equally valued and cannot be graded with a negative value.

1. The idea of "date" and "party" hubs has been influential in the study of social and biological networks, being used to identify nodes that work as higher-level connectors between groups and communities. How would you measure the relevance of a vertex with respect to this feature? Justify.

R: Betweeness centrality (BC). The BC of a vertex is the number of shortest paths that pass through the vertex. It represents the degree of which nodes stand between each other. Other option would be to apply a community finding algorithm that allows for overlapping communities, and check those nodes belonging to a larger number of communities.

2. Which of the following problems in graph analysis is known to be solved in polynomial time (select one): a) Compute the partitioning with maximum modularity; b) Given a pattern, enumerate all its occurrences in a given graph; c) Identify motifs of arbitrary size; d) Compute the betweenness centrality for all vertices.

R: d

3. Briefly explain the main contribution of the Watts-Strogatz model.

R: The Watts–Strogatz model is a random graph generation model that produces graphs with small-world properties, including short average path lengths and high clustering. It shows that it takes a lot of randomness to reduce clustering and locality, and just a little to drastically reduce the average distance among nodes. Just to recall, in this model we start by constructing a regular ring lattice, a graph with N nodes each connected to z neighbors, z/2 on each side. For every node i, take every edge connecting to its z/2 rightmost neighbors. Randomly rewire each these edges with a probability p, selecting a new partner for i from the entire population of nodes. Varying p makes it possible to interpolate between a regular ring (p=0) and a structure close to an Erdős–Rényi random graph (p=1). Interestingly, for small p the network maintains high clustering but the random long-range links can drastically decrease the distances between the nodes, leading to the main message above.

4. The Barabási-Albert (BA) model suggests that growth and preferential attachment are two fundamental principles underlying the emergence of power-law degree distributions in real systems. However, contrary to networks generated through the BA model, most empirically observed networks also portray a significant saturation for low degrees and prominent exponential cutoffs for high degrees. Suggest one hypothesis for the origin of each of these features, and briefly describe how the BA model can be modified to test your hypothesis.

R: In the BA model an isolated node cannot acquire links, as according to preferential attachment the likelihood that a new node attaches to a k=0 node is strictly zero. In real networks even isolated nodes acquire links. To allow unconnected nodes to acquire links we may add a constant to the preferential attachment function. Such initial attractiveness adds a random component to preferential attachment. Consequently, the degree distribution develops a small-degree saturation, as observed in real-world networks. Furthermore, often nodes have a limited lifetime or a threshold above which cannot receive more ties. To cope with that, node and link removal, present in many real systems, can be added to the BA, inducing exponential high-degree cutoffs in the degree distribution. A similar effect is obtained if, in the BA model, one precludes nodes above a given degree to receive additional ties.

5. Suggest a method to measure degree-degree correlations in a network.

R: Please see our class on degree-degree correlation. Any of the proposed methods (e.g., Pearson correlation coeff.) would be a good answer.

6. Propose a measure to assess the robustness of a network against *i) random failures* (R_F) and *ii) targeted attacks* (R_A).

R: i) One possibility would be to compute the size G(f) of the giant component after randomly removing a fraction f of nodes. ii) Equal to i), yet successively removing (e.g.) the highest degree nodes. For each case, we could get a single value of the robustness by summing the size of the giant component for all values of f — $R = \int_0^1 G(f)\,df$ *.We may also define R, for each case, as the critical fraction* f_c *above which G(f)≈0 (as in percolation thresholds).*

7. Suggest a vaccination strategy for scale-free networks that is likely to perform better than random vaccination and does not require complete information on the network topology.

R: Basic idea: Vaccinate the acquaintances of randomly selected individuals, indirectly targeting the hubs without having to know precisely which individuals are hubs. Example: 1) Choose randomly a fraction p of nodes (Group 0). Select randomly a link for each node in Group 0. Let us call Group 1 to this new set of nodes. Immunize the Group 1 individuals. Proceed like this, creating a group 2 if needed, from the neighbors of the vertices in group 1.

8. Consider the following 2-player game played in large populations of self-regarding agents. One player, the proposer, is endowed with a sum of money. The proposer is tasked with splitting it with another player, the responder. Once the proposer communicates their decision, the responder may accept it or reject it. If the responder accepts, the money is split as proposed; if the responder rejects, both players receive nothing. Consider two strategies available to the proposer: propose a fair split, or propose an unfair split. The responder may choose to accept or reject, following one of 4 strategies: always accept, always reject, accept only a fair split, or accept only an unfair split. If we consider a well-mixed population where agents play both as a receiver and as a proposer and revise their strategies through social learning, indicate one evolutionary stable strategy (if any) associated with this dynamics? Justify.

R: This game is called the Ultimatum game, a game-theoretical framework to address bargaining situations. Proponents can offer a high (fair) (H) or low (unfair) offer (L). As a responder, the four strategies above can be reduced to two — accept low and high offers (L), and accept only high offers (H) — as "always reject" is always disadvantageous and may be discarded, and "always accept" can be seen as equivalent to L. If agents may play both as a proposers and as a receivers with equal probability, we get four strategies defined as a tuple (donor-strategy, receiver-strategy): (L, L), (H, L), (H, H), and (L, H), which we enumerate, in this order, by S1 to S4. S1 is the rational or asocial strategy of offering little and rejecting nothing. S2 makes a high offer but is willing to accept a low offer. S3 is the "fair" strategy, offering and demanding a high share. S4 is a paradoxical strategy. To describe the change in the frequencies of these strategies, we can use the replicator equation. An evolutionarily stable strategy (ESS) is a strategy which, if adopted by a population, cannot be invaded by any alternative strategy that are initially rare (i.e., a mutant). Under this dynamics, any mutant adopting the S2, S3 or S4 strategies will have a lower fitness than resident players adopting the reasonable strategy S1, and, for this reason, cannot invade the population. Thus, S1 is an Evolutionary Stable Strategy (ESS). S3 is also an ESS. Indeed, a population consisting only of S1 and S3 players will converge to pure S1 or S3 populations depending on the initial frequencies of the two strategies. A mutant S1 cannot invade a population of S3, since their offers would never get accepted. Moreover, S3 is neutral with S2 and cannot be invaded by any S4 mutant. S4 is invaded by any of the other strategies and S2 is invaded by an asocial mutant. As a side note, starting with a mixture of S1, S2, and S3 players, evolution will lead to a population that consists entirely of S1 players. Fortunately, reason only dominates fairness in theory ☺.

> Note: We considered as correct (100%) any answer indicating <u>one</u> of the possible Evolutionary Stable Strategies (ESS) and showing a good understanding of what an ESS is.

9. Please indicate whether each of the following statements is TRUE or FALSE.
<u>Note: For each wrong answer we discount a correct one.</u>

   a) Average Path Length (APL) of a network is given by the average over all shortest paths between all pairs of nodes.

   b) A k-core of a graph G is a maximal connected sub-graph of G in which all vertices have degree at least k.

   c) Complex contagion is the phenomenon in social networks in which multiple sources of exposure to an innovation are required before an individual adopts the change of behavior.

   d) The famous small-world experiments conducted by Stanley Milgram and other researchers aimed at analyzing the clustering coefficient of social networks of people in the United States.

   e) Consider a growing network model in which at each time step we add a new node to the network; each new node selects a link e at random, and connects itself to the two ends of e. This model gives rise to a network with a power-law degree distribution and a high clustering coefficient.

   R: V, V, V, F, V