1. The basic reproductive number or ratio (R_0, r naught) of an infection is a key value in epidemiology. Briefly explain the concept.

2. The simplest way of modeling an epidemic outbreak may be to consider a population of individuals with one of two states — Susceptible and Infected — where Susceptible may become infected in the presence of Infected with a given rate. Moreover, pathogens may be eventually defeated, such that those that were infected may eventually become susceptible again with a given rate. These are the fundamental assumptions of the SIS model.
   a) The SIS model has two possible outcomes: *endemic* and *disease-free* states. Explain these outcomes and the conditions underlying their emergence.
   b) For many pathogens, like most strains of influenza, individuals develop immunity after they recover from the infection. Thus, recovered individuals cease to spread the disease and will no longer be susceptible reaching a third state "R" absent in the SIS model. Describe the outcome(s) of this "SIR" model.

3. Consider the SIS model and that pathogens spreads on a network.
   a) Resort to the idea of epidemic threshold to highlight the differences obtained from a random network (Erdős–Rényi model) and from a scale-free network. What is the origin of such difference?
   b) Discuss 2 vaccination strategies likely to perform better than random vaccination.

4. Recent access to accurate real time data on human travel and demographics, has allowed constructing more complex models, where one reconstruct the mobility network that is responsible for the global spread of a pathogen. Take some time to learn the essence of how these models are built and used (see slides and suggested pointers, such as `http://www.gleamviz.org`).

5. The Prisoner's dilemma is one of the most famous dilemmas of cooperation. In its simplest form, it defines pairwise interactions involving costs (c) and benefits (b) associated with a cooperative act. Propose a payoff matrix associated with the problem and justify why mutual Defection is the only Nash Equilibria.

6. The payoff matrix indicated below defines a canonical 2-player, 2-strategy symmetric cooperation game. Depending on the ranking of T, R, S and P, different dilemmas emerge. Indicate the ranking associated with the a) stag-hunt game, b) snowdrift-game and c) prisoner's dilemma.

	Cooperation	**Defection**
Cooperation	Reward for mutual cooperation (R)	Sucker's Payoff (S)
Defection	Temptation to defect (T)	Punishment for mutual defection (P)

7. Resorting to the replicator equation, describe the emerging evolutionary dynamics and compute the position of the fixed points associated with each dilemma (see slides).

8. Jean-Jacques Rousseau described a paradigmatic situation of conflict between safety and social cooperation. Imagine that two individuals go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. Each player must choose an action without knowing the choice of the other. If an individual hunts a stag, they must have the cooperation of their partner in order to succeed. An individual can get a hare by himself or herself, but a hare is worth less than a stag. Let us say that if successful, hunting a stag provides 10 times more meat than if both engage in hare hunting.
   a) Resort to the language of Game Theory to propose one possible payoff matrix for this 2-player, 2-strategy game.

b) Let us consider a well-mixed population of individuals that interact following this game. Resort to Evolutionary Game Theory (EGT) to describe the evolutionary dynamics associated with the adoption of each strategy. Please indicate the evolutionary stable strategy/strategies of this dilemma.

9. Direct and indirect reciprocity have been proposed as mechanisms for the evolution of cooperation.
    c) Briefly explain the main idea and the differences among the two.
    d) Suggest one possible individual strategy for direct reciprocity and for indirect reciprocity.
    e) Explain what is a social norm in the context of indirect reciprocity.

10. When there are more than two players, we enter the realm of N-person games. The public goods game (a.k.a. N-person Prisoner's dilemma) N individuals have to decide to contribute (Cooperate) a value c or not (Defect) to a common pot. The sum of all contributions is multiplied by a Factor F>1 and evenly divided among all players irrespectively if they contributed or not.
    f) Use the concept of Nash Equilibria to predict the outcome of this game when F<N and F>N.
    g) Consider a well-mixed population of size Z>>N; discuss the evolutionary stable strategy in the same conditions. Suggestion: please check our slides. The average fitness of Cs and Ds may be computed from a binomial sampling of groups.

Solutions/suggestions:

. 1. R_0[1] represents the average number of susceptible individuals infected by an infected individual during its infectious period in a fully susceptible population. In other words, R_0 is the number of new infections each infected individual causes under ideal circumstances.

. 2a. The SIS has to states: susceptible and infected. individuals are transmitting the pathogen, each at rate β. Infected individuals are eventually cured, becoming susceptible again at rate μ. For $\mu < \beta<k>$, i.e., for low recovery rate, the system reaches an *endemic state*, in which the fraction of infected individuals is constant. for high recovery rate $\mu > \beta<k>$ the number of infected individuals decreases exponentially and the disease dies out. We reach a disease free-state. If you look at the equations (see slides) describing the time evolution of the fraction of infected you will see the above inequalities emerging, such that $R_0 = \beta<k>/\mu$. If R_0 exceeds the unit the epidemic will be at the endemic state, and will disappear otherwise.

. 5. Check our slides and try to understand why no player has anything to gain by changing their strategy (i.e., the Nash Eq.) in the case of mutual defection.

	Cooperation	Defection
Cooperation	b-c	-c
Defection	b	0

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. Can you find any other configuration where the Nash is fulfilled?

. 6. Snowdrift game: T>R>S>P; Prisoner's dilemma: T>R>P>S; Stag-hunt game: R>T>P>S. Can you identify the Nash equilibria or the Evolutionary Stable strategies emerging from these 3 games?

	Cooperation	Defection
Cooperation	R	S
Defection	T	P

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. 8. Again, this is a stag-hunt game[2], with two Nash equilibria (Stag-Stag and Hare-Hare). From a Evolutionary Game Theory perspective both Stag and Hare hunting are evolutionary stable strategies. Here's one possible payoff matrix:

	Stag	Hare
Stag	10	0
Hare	1	1

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[1] Check our slides for more information.
[2] Check the slides of our first class on evolutionary game theory.