

# Foundations of Planning for $LTL_f$ and $LDL_f$ Goals

Giuseppe De Giacomo

Sapienza Università di Roma  
Rome, Italy

Rice University  
Houston, TX, USA, January 31, 2018

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/LDL$  on finite traces
- 3  $LTL_f/LDL_f$  and automata
- 4 Planning for  $LTL_f/LDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/LDL$  on finite traces
- 3  $LTL_f/LDL_f$  and automata
- 4 Planning for  $LTL_f/LDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion

# Introduction and background

Reasoning about Actions and Planning community well aware of temporal logics since a long time:

- Temporally extended goals [BacchusKabanza96] - infinite/finite
- Temporal constraints on trajectories [GereviniHaslumLongSaettiDimopoulos09 - PDDL3.0 2009] - finite
- Declarative control knowledge on trajectories [BaierMcIlraith06] - finite
- Procedural control knowledge on trajectories [BaierFrizMcIlraith07] - finite
- Temporal specification in planning domains [CalvaneseDeGiacomoVardi02] - infinite
- Planning via model checking - infinite
  - ▶ Branching time (CTL) [CimattiGiunchigliaGiunchigliaTraverso97]
  - ▶ Linear time (LTL) [DeGiacomoVardi99]

## Foundations for temporal extended goals and constraints

We borrow foundations from temporal logics studied in CS, in particular: Linear Temporal Logic (LTL) [Pnueli77].

### However

- Often, we interpret temporal logic on finite trajectories/traces.
- Often, we blur the distinction interpreting LTL on infinite or on finite traces.

# Introduction and background

Also BPM (Business Process Management) advocates the use of LTL on finite traces.

## Declarative Business Processes

Basic idea: Drop explicit representation of processes, and use **LTL on finite traces** to specify allowed (finite) traces. [VanPesciBovsnavkiDraganVanDerAalst10].

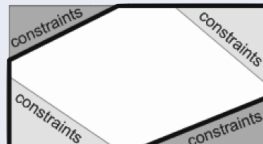


(a) forbidden, optional and allowed in business processes

 possible



(b) procedural workflow



(c) declarative workflow

# Introduction and background

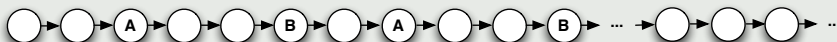
Assuming finite vs. infinite traces has **big impact**.

## Example

Consider the following formula:

$$\Box(A \supset \Diamond B)$$

- On **infinite** traces:



- On **finite** traces:



# Introduction and background

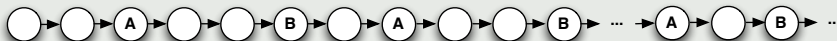
Assuming finite vs. infinite traces has **big impact**.

## Example

Consider the following formula:

$$\Box(A \supset \Diamond B) \wedge \Box(B \supset \Diamond A)$$

- On **infinite** traces:



- On **finite** traces:



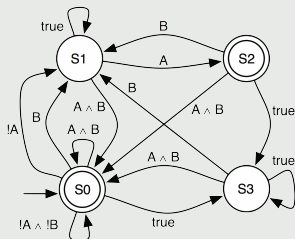
## Introduction and background

Assuming finite or infinite traces has **big impact**.

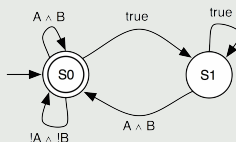
### Example

Consider again the formula:  $\Box(A \supset \Diamond B) \wedge \Box(B \supset \Diamond A)$

- Buchi automaton accepting its **infinite** traces:



- NFA accepting its **finite** traces:





# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/DDL$  on finite traces
- 3  $LTL_f/DDL_f$  and automata
- 4 Planning for  $LTL_f/DDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion

# LTL<sub>f</sub>: LTL over finite traces

## LTL<sub>f</sub>: the language

$$\varphi ::= A \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \circ\varphi \mid \varphi_1 \mathcal{U} \varphi_2$$

- $A$ : **atomic** propositions
- $\neg\varphi, \varphi_1 \wedge \varphi_2$ : **boolean** connectives
- $\circ\varphi$ : “**next step exists** and at **next step** (of the trace)  $\varphi$  holds”
- $\varphi_1 \mathcal{U} \varphi_2$ : “**eventually**  $\varphi_2$  holds, and  $\varphi_1$  holds **until**  $\varphi_2$  does”
- $\bullet\varphi \doteq \neg\circ\neg\varphi$ : “**if next step exists** then at **next step**  $\varphi$  holds” (*weak next*)
- $\Diamond\varphi \doteq \text{true} \mathcal{U} \varphi$ : “ $\varphi$  will **eventually** hold”
- $\Box\varphi \doteq \neg\Diamond\neg\varphi$ : “from current till last instant  $\varphi$  will **always** hold”
- $\text{Last} \doteq \neg\circ\text{true}$ : denotes **last** instant of trace.

## Main formal properties:

- **Expressibility**: FOL over finite sequences or Star-free RE
- **Reasoning**: satisfiability, validity, entailment PSPACE-complete
- **Model Checking**: linear on TS, PSPACE-complete on formula

# LT<sub>L<sub>f</sub></sub>: LTL over finite traces

Some interesting LT<sub>L<sub>f</sub></sub> formulas:

<i>name of template</i>	<i>LTL semantics</i>
<i>responded existence</i> (A, B)	$\Diamond A \Rightarrow \Diamond B$
<i>co-existence</i> (A, B)	$\Diamond A \Leftrightarrow \Diamond B$
<i>response</i> (A, B)	$\Box(A \Rightarrow \Diamond B)$
<i>precedence</i> (A, B)	$(\neg B \cup A) \vee \Box(\neg B)$
<i>succession</i> (A, B)	$\text{response}(A, B) \wedge \text{precedence}(A, B)$
<i>alternate response</i> (A, B)	$\Box(A \Rightarrow \bigcirc(\neg A \cup B))$
<i>alternate precedence</i> (A, B)	$\text{precedence}(A, B) \wedge \Box(B \Rightarrow \bigcirc(\text{precedence}(A, B)))$
<i>alternate succession</i> (A, B)	$\text{alternate response}(A, B) \wedge \text{alternate precedence}(A, B)$
<i>chain response</i> (A, B)	$\Box(A \Rightarrow \bigcirc B)$
<i>chain precedence</i> (A, B)	$\Box(\bigcirc B \Rightarrow A)$
<i>chain succession</i> (A, B)	$\Box(A \Leftrightarrow \bigcirc B)$

<i>name of template</i>	<i>LTL semantics</i>
<i>not co-existence</i> (A, B)	$\neg(\Diamond A \wedge \Diamond B)$
<i>not succession</i> (A, B)	$\Box(A \Rightarrow \neg(\Diamond B))$
<i>not chain succession</i> (A, B)	$\Box(A \Rightarrow \bigcirc(\neg B))$

<i>name of template</i>	<i>LTL semantics</i>
<i>existence</i> (1, A)	$\Diamond A$
<i>existence</i> (2, A)	$\Diamond(A \wedge \bigcirc(\text{existence}(1, A)))$
...	...
<i>existence</i> (n, A)	$\Diamond(A \wedge \bigcirc(\text{existence}(n-1, A)))$
<i>absence</i> (A)	$\neg \text{existence}(1, A)$
<i>absence</i> (2, A)	$\neg \text{existence}(2, A)$
<i>absence</i> (3, A)	$\neg \text{existence}(3, A)$
...	...
<i>absence</i> (n+1, A)	$\neg \text{existence}(n+1, A)$
<i>init</i> (A)	A



## LDL<sub>f</sub>: LDL over finite traces

### LDL<sub>f</sub>: the language

$$\varphi ::= \phi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle \rho \rangle \varphi \mid [\rho] \varphi \quad \rho ::= \phi \mid \varphi? \mid \rho_1 + \rho_2 \mid \rho_1; \rho_2 \mid \rho^*$$

- $\phi$ : **propositional formula** on current state/instant
- $\neg\varphi, \varphi_1 \wedge \varphi_2$ : **boolean connectives**
- $\rho$  is a **regular expression** on propositional formulas
- $\langle \rho \rangle \varphi$ : **exists** an “execution” of RE  $\rho$  that ends with  $\varphi$  holding
- $[\rho] \varphi$ : **all** “executions” of RE  $\rho$  (along the trace!) end with  $\varphi$  holding

*In the infinite trace setting, such enhancement strongly advocated by industrial model checking [ForSpec, PSL].*

### Main formal properties:

- **Expressibility**: MSO over finite sequences: adds the power of recursion (as RE)
- **Reasoning**: satisfiability, validity, entailment PSPACE-complete
- **Model Checking**: linear on TS, PSPACE-complete on formula



## Example

- All coffee requests from person  $p$  will eventually be served:

$$[\mathbf{true}^*](request_p \supset \langle \mathbf{true}^* \rangle coffee_p)$$

- Every time the robot opens door  $d$  it closes it immediately after:

$$[\mathbf{true}^*]([openDoor_d]closeDoor_d)$$

- Before entering restricted area  $a$  the robot must have permission for  $a$ :

$$\langle (\neg inArea_a^*; getPermission_a; \neg inArea_a^*; inArea_a)^*; \neg inArea_a^* \rangle end$$

Note that the first two properties (not the third one) can be expressed also in LTL<sub>f</sub>:

$$\Box(request_p \supset \Diamond coffee_p)$$

$$\Box(openDoor_d \supset \circ closeDoor_d)$$

## LDL<sub>f</sub>: Linear Dynamic Logic on finite traces

LDL<sub>f</sub>, not LTL<sub>f</sub>, is able to easily express procedural constraints [BaierFritzMcIlraith07].

Let's introduce a sort of propositional variant of GOLOG

$$\delta ::= A \mid \varphi? \mid \delta_1 + \delta_2 \mid \delta_1; \delta_2 \mid \delta^* \mid \text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \mid \text{while } \phi \text{ do } \delta$$

where **if** and **while** can be seen as abbreviations for LDL<sub>f</sub> path expression, namely:

$$\text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \doteq (\phi?; \delta_1) + (\neg\phi?; \delta_2) \quad \text{while } \phi \text{ do } \delta \doteq (\phi?; \delta)^*; \neg\phi?$$

### Example (LDL<sub>f</sub> procedural constraints)

- “At every point, if it is hot then, if the air-conditioning system is off, turn it on, else don't turn it off”:

$$[\text{true}^*](\text{if } (\text{hot}) \text{ then} \\ \quad \text{if } (\neg \text{airOn}) \text{ then } \text{turnOnAir} \\ \quad \text{else } \neg \text{turnOffAir}) \text{true}$$

- “alternate till the end the following two instructions: (1) while is hot if the air-conditioning system is off turn it on, else don't turn it off; (2) do something for one step”

$$\langle (\text{while } (\text{hot}) \text{ do} \\ \quad \text{if } (\neg \text{airOn}) \text{ then } \text{turnOnAir} \\ \quad \text{else } \neg \text{turnOffAir}; \\ \text{true})^* \rangle \text{end}$$

Example (LDL<sub>f</sub> captures finite domain variant of GOLOG in SitCalc)

## GOLOG – finite domain variant

$\delta ::= A \mid \varphi? \mid \delta_1 + \delta_2 \mid \delta_1; \delta_2 \mid \delta^* \mid \pi x. \delta(x) \mid \text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \mid \text{while } \phi \text{ do } \delta$

- $\pi x. \delta(x)$  stands for  $\sum_{o \in Obj} \delta(o)$
- $\text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2$  stands for  $(\phi?; \delta_1) + (\neg\phi?; \delta_2)$
- $\text{while } \phi \text{ do } \delta$  stands for  $(\phi?; \delta)^* \neg\phi?$

- $\langle \delta \rangle \phi$  in LDL<sub>f</sub> captures SitCalc formula  $\exists s'. Do(\delta, s, s') \wedge s \leq s' \leq last \wedge \phi(s')$ .
- $[\delta] \phi$  in LDL<sub>f</sub> captures SitCalc formula  $\forall s'. Do(\delta, s, s') \wedge s \leq s' \leq last \supset \phi(s')$ .

( $\phi(s)$  “uniform” in  $s$ .)

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/DDL$  on finite traces
- 3  $LTL_f/DDL_f$  and automata**
- 4 Planning for  $LTL_f/DDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion





### Key point

LTL<sub>f</sub>/LDL<sub>f</sub> formulas can be translated into **nondeterministic finite state automata (NFA)**.

$$t \models \varphi \text{ iff } t \in \mathcal{L}(\mathcal{A}_\varphi)$$

where  $\mathcal{A}_\varphi$  is the NFA  $\varphi$  is translated into.

*We can compile reasoning into automata based procedures!*

# LTL<sub>f</sub>/LDL<sub>f</sub> and automata

Both LTL<sub>f</sub> and LDL<sub>f</sub> formulas can be translated in **exponential time** to **nondeterministic automata on finite words (NFA)**.

NFA  $\mathcal{A}_\varphi$  associated with an LTL<sub>f</sub> formula  $\varphi$  (in NNF)

## Auxiliary rules

$\delta(a, \Pi)$	=	true if $a \in \Pi$
$\delta(a, \Pi)$	=	false if $a \notin \Pi$
$\delta(\neg a, \Pi)$	=	false if $a \in \Pi$
$\delta(\neg a, \Pi)$	=	true if $a \notin \Pi$
$\delta(\varphi_1 \wedge \varphi_2, \Pi)$	=	$\delta(\varphi_1, \Pi) \wedge \delta(\varphi_2, \Pi)$
$\delta(\varphi_1 \vee \varphi_2, \Pi)$	=	$\delta(\varphi_1, \Pi) \vee \delta(\varphi_2, \Pi)$
$\delta(\bigcirc \varphi, \Pi)$	=	$\begin{cases} \varphi & \text{if } Last \notin \Pi \\ \text{false} & \text{if } Last \in \Pi \end{cases}$
$\delta(\diamond \varphi, \Pi)$	=	$\delta(\varphi, \Pi) \vee \delta(\bigcirc \diamond \varphi, \Pi)$
$\delta(\varphi_1 \mathcal{U} \varphi_2, \Pi)$	=	$\delta(\varphi_2, \Pi) \vee$ $(\delta(\varphi_1, \Pi) \wedge \delta(\bigcirc(\varphi_1 \mathcal{U} \varphi_2), \Pi))$
$\delta(\bullet \varphi, \Pi)$	=	$\begin{cases} \varphi & \text{if } Last \notin \Pi \\ \text{true} & \text{if } Last \in \Pi \end{cases}$
$\delta(\Box \varphi, \Pi)$	=	$\delta(\varphi, \Pi) \wedge \delta(\bullet \Box \varphi, \Pi)$
$\delta(\varphi_1 \mathcal{R} \varphi_2, \Pi)$	=	$\delta(\varphi_2, \Pi) \wedge (\delta(\varphi_1, \Pi) \vee$ $\delta(\bullet(\varphi_1 \mathcal{R} \varphi_2), \Pi))$

( $\varepsilon(\varphi)$  replaces in  $\varphi$  all occurrences of  $\mathbf{tt}_{\psi}$  and  $\mathbf{ff}_{\psi}$  by  $\varepsilon(\psi)$ )

## Algorithm

**algorithm** LDL<sub>f</sub>2NFA

**input** LTL<sub>f</sub> formula  $\varphi$

**output** NFA  $\mathcal{A}_\varphi = (2^{\mathcal{P}}, S, \{s_0\}, q, \{s_f\})$

$s_0 \leftarrow \{\varphi\}$

▷ single initial state

$s_f \leftarrow \emptyset$

▷ single final state

$S \leftarrow \{s_0, s_f\}, q \leftarrow \emptyset$

**while** ( $S$  or  $q$  change) **do**

**if** ( $q \in S$  and  $q' \models \bigwedge_{(\psi \in q)} \delta(\psi, \Pi)$ )

$S \leftarrow S \cup \{q'\}$

▷ update set of states

$q \leftarrow q \cup \{(q, \Pi, q')\}$

▷ update transition relation



NFA  $\mathcal{A}_\varphi$  associated with an LDL<sub>f</sub> formula  $\varphi$  (in NNF)

## Auxiliary rules

$\delta(A, \Pi)$	=	true if $A \in \Pi$
$\delta(A, \Pi)$	=	false if $A \notin \Pi$
$\delta(\varphi_1 \wedge \varphi_2, \Pi)$	=	$\delta(\varphi_1, \Pi) \wedge \delta(\varphi_2, \Pi)$
$\delta(\varphi_1 \vee \varphi_2, \Pi)$	=	$\delta(\varphi_1, \Pi) \vee \delta(\varphi_2, \Pi)$
$\delta(\langle \phi \rangle \varphi, \Pi)$	=	$\begin{cases} \text{false} & \text{if } \Pi \not\models \phi \text{ or } Last \in \Pi \\ \varepsilon(\varphi) & \text{o/w } (\phi \text{ propositional}) \end{cases}$
$\delta(\langle \psi? \rangle \varphi, \Pi)$	=	$\delta(\psi, \Pi) \wedge \delta(\varphi, \Pi)$
$\delta(\langle \rho_1 + \rho_2 \rangle \varphi, \Pi)$	=	$\delta(\langle \rho_1 \rangle \varphi, \Pi) \vee \delta(\langle \rho_2 \rangle \varphi, \Pi)$
$\delta(\langle \rho_1; \rho_2 \rangle \varphi, \Pi)$	=	$\delta(\langle \rho_1 \rangle \langle \rho_2 \rangle \varphi, \Pi)$
$\delta(\langle \rho^* \rangle \varphi, \Pi)$	=	$\delta(\varphi, \Pi) \vee \delta(\langle \rho \rangle \text{ff} \langle \rho^* \rangle \varphi, \Pi)$
$\delta([\phi] \varphi, \Pi)$	=	$\begin{cases} \text{true} & \text{if } \Pi \models \phi \text{ or } Last \in \Pi \\ \varepsilon(\varphi) & \text{o/w } (\phi \text{ propositional}) \end{cases}$
$\delta([\psi?] \varphi, \Pi)$	=	$\delta(\text{nnf}(\neg \psi), \Pi) \vee \delta(\varphi, \Pi)$
$\delta([\rho_1 + \rho_2] \varphi, \Pi)$	=	$\delta([\rho_1] \varphi, \Pi) \wedge \delta([\rho_2] \varphi, \Pi)$
$\delta([\rho_1; \rho_2] \varphi, \Pi)$	=	$\delta([\rho_1][\rho_2] \varphi, \Pi)$
$\delta([\rho^*] \varphi, \Pi)$	=	$\delta(\varphi, \Pi) \wedge \delta([\rho] \text{tt} [\rho^*] \varphi, \Pi)$
$\delta(\text{ff} \psi, \Pi)$	=	false
$\delta(\text{tt} \psi, \Pi)$	=	true

( $\varepsilon(\varphi)$  replaces in  $\varphi$  all occurrences of  $\text{tt}_\psi$  and  $\text{ff}_\psi$  by  $\varepsilon(\psi)$ )

## Algorithm

**algorithm** LDL<sub>f</sub>2NFA

**input** LTL<sub>f</sub> formula  $\varphi$

**output** NFA  $\mathcal{A}_\varphi = (2^P, \mathcal{S}, \{s_0\}, \varrho, \{s_f\})$

$s_0 \leftarrow \{\varphi\}$

▷ single initial state

$s_f \leftarrow \emptyset$

▷ single final state

$\mathcal{S} \leftarrow \{s_0, s_f\}, \varrho \leftarrow \emptyset$

**while** ( $\mathcal{S}$  or  $\varrho$  change) **do**

**if** ( $q \in \mathcal{S}$  and  $q' \models \bigwedge_{(\psi \in q)} \delta(\psi, \Pi)$ )

$\mathcal{S} \leftarrow \mathcal{S} \cup \{q'\}$

▷ update set of states

$\varrho \leftarrow \varrho \cup \{(q, \Pi, q')\}$

▷ update transition relation



# LTL<sub>f</sub>/LDL<sub>f</sub> reasoning

## LTL<sub>f</sub>/LDL<sub>f</sub> satisfiability ( $\varphi$ SAT)

- 1: Given LTL<sub>f</sub>/LDL<sub>f</sub> formula  $\varphi$
- 2:     Compute NFA for  $\varphi$  (*exponential*)
- 3:     Check NFA for nonemptiness (*NLOGSPACE*)
- 4:     Return result of check

## LTL<sub>f</sub>/LDL<sub>f</sub> validity ( $\varphi$ VAL)

- 1: Given LTL<sub>f</sub>/LDL<sub>f</sub> formula  $\varphi$
- 2:     Compute NFA for  $\neg\varphi$  (*exponential*)
- 3:     Check NFA for nonemptiness (*NLOGSPACE*)
- 4:     Return complemented result of check

## LTL<sub>f</sub>/LDL<sub>f</sub> logical implication ( $\Gamma \models \varphi$ )

- 1: Given LTL<sub>f</sub>/LDL<sub>f</sub> formulas  $\Gamma$  and  $\varphi$
- 2:     Compute NFA for  $\Gamma \wedge \neg\varphi$  (*exponential*)
- 3:     Check NFA for nonemptiness (*NLOGSPACE*)
- 4:     Return complemented result of check

**Thm:**[IJCAI13] All above reasoning tasks are PSPACE-complete. (*As for infinite traces.*)  
(Construction of NFA can be done while checking nonemptiness.)

## Relationship to Classical Planning

Let  $\Psi_{domain}$  describe action domain (LTL<sub>f</sub> formula),  $\phi_{init}$  initial state (prop. formula), and  $G$  goal (prop. formula). **Classical planning** amounts to LTL<sub>f</sub> satisfiability of:

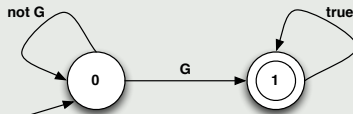
$$\phi_{init} \wedge \Psi_{domain} \wedge \Diamond G$$

*Complexity: PSPACE-complete.*

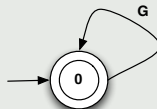
# Automata for some $LTL_f$ formulas

## Example (Automata for some $LTL_f$ formulas)

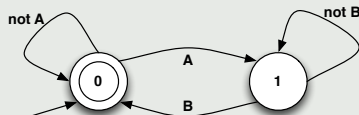
- $\Diamond G$ :



- $\Box G$



- $\Box(A \supset \circ \Diamond B)$



*Observe all of these automata are DFAs!*

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/DDL$  on finite traces
- 3  $LTL_f/DDL_f$  and automata
- 4 Planning for  $LTL_f/DDL_f$  goals: deterministic domains**
- 5  $FOND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion



# Planning in deterministic domain

## Deterministic domain (including initial state)

$\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  where:

- $\mathcal{F}$  **fluents** (atomic propositions)
- $\mathcal{A}$  **actions** (atomic symbols)
- $2^{\mathcal{F}}$  set of states
- $s_0$  initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$  represents **action preconditions**
- $\delta(s, a) = s'$  with  $a \in \alpha(s)$  represents **action effects (including frame)**.

## Traces

A **trace** for  $\mathcal{D}$  is a finite sequence:

$$s_0, a_1, s_1, \dots, a_n, s_n$$

where  $s_0$  is the initial state, and  $a_i \in \alpha(s_i)$  and  $s_{i+1} = \delta(s_i, a_{i+1})$  for each  $i$ .

## Goals, planning, and plans

**Goal** = propositional formula  $G$  on fluents

**Planning** = find a trace  $s_0, a_1, s_1, \dots, a_n, s_n$  such that  $s_n \models G$ . (PSPACE-complete)

**Plan** = project traces on actions, i.e., return  $a_1, \dots, a_n$ .

# Deterministic planning domains as automata

Let's transform the planning domain  $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  into a DFA recognizing all its traces.

## DFA $A_{\mathcal{D}}$ for $\mathcal{D}$

$A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \rho, F)$  where:

- $2^{\mathcal{F} \cup \mathcal{A}}$  alphabet (actions  $\mathcal{A}$  include dummy *start* action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$  set of states
- $s_{init}$  dummy initial state
- $F = 2^{\mathcal{F}}$  (all states of the domain are final)
- $\rho(s, [a, s']) = s'$  **with**  $a \in \alpha(s)$ , **and**  $\delta(s, a) = s'$   
 $\rho(s_{init}, [start, s_0]) = s_0$

(notation:  $[a, s']$  stands for  $\{a\} \cup s'$ )

## Traces

Each trace  $s_0, a_1, s_1, \dots, a_n, s_n$  of the domain  $\mathcal{D}$  becomes a finite sequence:

$$[start, s_0], [a_1, s_1], \dots, [a_n, s_n]$$

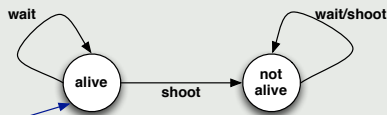
recognized by the DFA  $A_{\mathcal{D}}$ .



# Deterministic planning domains as automata

## Example (Simplified Yale shooting domain)

- Domain  $\mathcal{D}$ :



- DFA  $A_{\mathcal{D}}$ :



# Deterministic planning domains as automata

## Planning in deterministic domains

Planning = find a trace of DFA  $A_{\mathcal{D}}$  for deterministic domain  $\mathcal{D}$  such that is also a trace for the DFA for  $\diamond G$  where  $G$  is the goal. That is:

*CHECK for nonemptiness  $A_{\mathcal{D}} \cap A_{\diamond G}$ : extract plan from witness.*

*(Computable on-the-fly, PSPACE in  $\mathcal{D}$ , constant in  $G$ . i.e., optimal)*

## Example (Simplified Yale shooting domain)

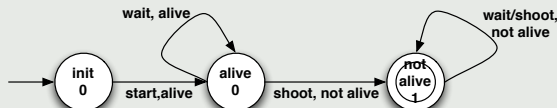
$A_{\mathcal{D}}$



$A_{\diamond \neg \text{alive}}$



$A_{\mathcal{D}} \cap A_{\diamond G}$ :



# Generalization: planning for $LTL_f/LDL_f$ goals in deterministic domains

## Planning in deterministic domains for $LTL_f/LDL_f$ goals

Planning = find a trace of DFA  $A_{\mathcal{D}}$  for deterministic domain  $\mathcal{D}$  such that is also accepted by NFA  $A_{\varphi}$  for the  $LTL_f/LDL_f$  formula  $\varphi$ . That is:

*CHECK for nonemptiness  $A_{\mathcal{D}} \cap A_{\varphi}$ : extract plan from witness.*

*(Computable on-the-fly, PSPACE in  $\mathcal{D}$ , PSPACE also in  $\varphi$  i.e., optimal)  
(We can use NFA directly since we are checking for **existence** of a trace satisfying  $\varphi$ )*

## Example (Simplified Yale shooting domain)

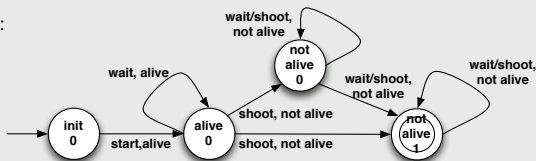
$A_{\mathcal{D}}$



$A_{\Diamond\Box\neg\text{alive}}$



$A_{\mathcal{D}} \cap A_{\Diamond\Box\neg\text{alive}}$



## Generalization: planning for $LTL_f/LDL_f$ goals in deterministic domains

### Planning for $LTL_f/LDL_f$ goals

#### Algorithm: Planning for $LDL_f/LTL_f$ goals

- 1: Given  $LTL_f/LDL_f$  domain  $\mathcal{D}$  and goal  $\varphi$
- 2:   Compute corresponding NFA (exponential)
- 3:   Compute intersection with DFA of  $\mathcal{D}$  (polynomial)
- 5:   Check nonemptiness of resulting NFA (NLOGSPACE)
- 6: Return plan

### Theorem

Planning for  $LTL_f/LDL_f$  goals is:

- *PSPACE-complete in the domain;*
- *PSPACE-complete in the goal.*

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/LDL$  on finite traces
- 3  $LTL_f/LDL_f$  and automata
- 4 Planning for  $LTL_f/LDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains**
- 6  $FOND_{sc}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion

FOND<sub>sp</sub>: strong planning in nondeterministic domains

## Nondeterministic domain (including initial state)

$\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  where:

- $\mathcal{F}$  **fluents** (atomic propositions)
- $\mathcal{A}$  **actions** (atomic symbols)
- $2^{\mathcal{F}}$  set of states
- $s_0$  initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$  represents **action preconditions**
- $\delta(s, a, s')$  with  $a \in \alpha(s)$  represents **action effects (including frame)**.

## Who controls what?

Fluents controlled by **environment**

Actions controlled by **agent**

*Observe:  $\delta(s, a, s')$*

## Goals, planning, and plans

**Goal** = propositional formula  $G$  on fluents

**Planning** = **game** between two players:

**agent** tries to force eventually reaching  $G$  no matter how other **environment** behave.

**Plan** = **strategy** to **win** the game.

*(FOND<sub>sp</sub> is EXPTIME-complete)*

# Nondeterministic domains as automata

Let's transform the nondeterministic domain  $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  into an automaton recognizing all its traces.

Automaton  $A_{\mathcal{D}}$  for  $\mathcal{D}$  is a **DFA!!!**

$A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$  where:

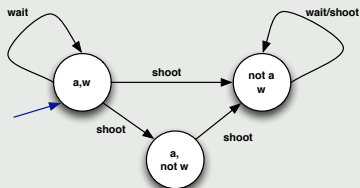
- $2^{\mathcal{F} \cup \mathcal{A}}$  alphabet (actions  $\mathcal{A}$  include dummy *start* action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$  set of states
- $s_{init}$  dummy initial state
- $F = 2^{\mathcal{F}}$  (all states of the domain are final)
- $\rho(s, [a, s']) = s'$  **with**  $a \in \alpha(s)$ , **and**  $\delta(s, a, s')$   
 $\rho(s_{init}, [start, s_0]) = s_0$

(notation:  $[a, s']$  stands for  $\{a\} \cup s'$ )

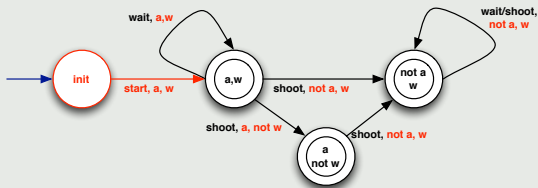
# Nondeterministic domains as automata

## Example (Simplified Yale shooting domain variant)

- Domain  $\mathcal{D}$ :



- DFA  $A_{\mathcal{D}}$ :



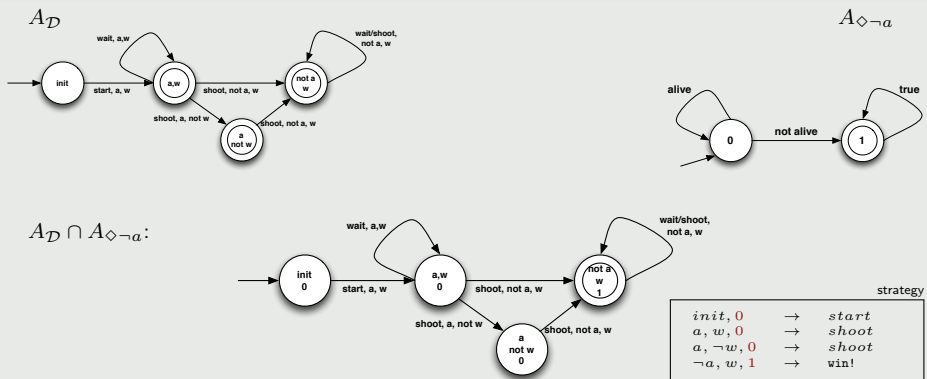


# Nondeterministic domains as automata

## FOND<sub>sp</sub>: strong planning in nondeterministic domains

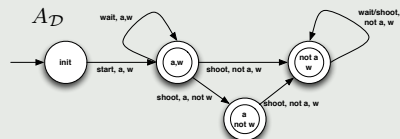
- Set the **arena** formed by all traces that satisfy both the DFA  $A_{\mathcal{D}}$  for  $\mathcal{D}$  and the DFA for  $\Diamond G$  where  $G$  is the goal.
- Compute a **winning strategy**. (EXPTIME-complete in  $\mathcal{D}$ , constant in  $G$ )

## Example (Simplified Yale shooting domain)

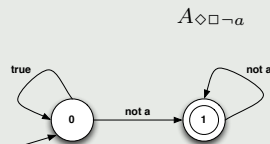
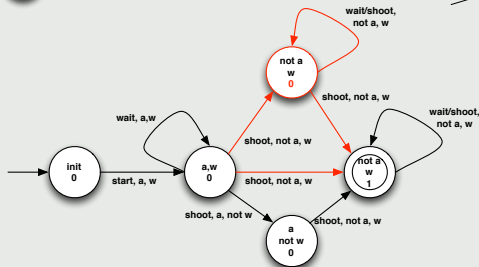


# Generalization: $\text{FOND}_{sp}$ for $\text{LTL}_f/\text{LDL}_f$ goals

## Example (Simplified Yale shooting domain)



$A_{\mathcal{D}} \cap A_{\Diamond\Box\neg a}$ :



Can we use directly NFA's?

No, because of a basic mismatch

- NFA have perfect **foresight**, or **clairvoyance** *(angelic nondeterminism)*
- Strategies must be runnable: **depend only on past**, not future *(devilish nondeterminism)*

# Generalization: $\text{FOND}_{sp}$ for $\text{LTL}_f/\text{LDL}_f$ goals

We need first to determinize the NFA for  $\text{LTL}_f/\text{LDL}_f$  formula

NFA for  $\Diamond\Box\neg a$



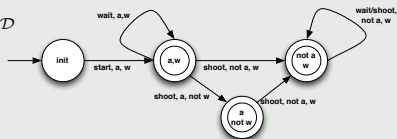
corresponding DFA



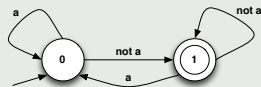
(DFA can be exponential in NFA in general)

Example (Simplified Yale shooting domain)

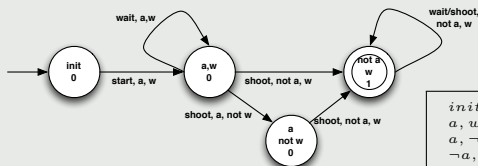
$A_D$



$A_{\Diamond\Box\neg a}$



$A_D \cap A_{\Diamond\Box\neg a}$ :



strategy

$init, 0$	$\rightarrow$	$start$
$a, w, 0$	$\rightarrow$	$shoot$
$a, \neg w, 0$	$\rightarrow$	$shoot$
$\neg a, w, 1$	$\rightarrow$	$win!$

# Generalization: DFA Games

## DFA games

A **DFA game**  $\mathcal{G} = (2^{\mathcal{F} \cup \mathcal{A}}, S, s_{init}, \varrho, F)$ , is such that:

- $\mathcal{F}$  controlled by **environment**;  $\mathcal{A}$  controlled by **agent**;
- $2^{\mathcal{F} \cup \mathcal{A}}$ , alphabet of game;
- $S$ , states of game;
- $s_{init}$ , initial state of game;
- $\varrho : S \times 2^{\mathcal{F} \cup \mathcal{A}} \rightarrow S$ , transition function of the game: given current state  $s$  and a choice of action  $a$  and resulting fluents values  $E$  the resulting state of game is  $\varrho(s, [a, E]) = s'$ ;
- $F$ , final states of game, where game can be considered terminated.

## Winning Strategy:

- A play is **winning** for the agent if such a play leads from the initial to a final state.
- A **strategy** for the agent is a function  $f : (2^{\mathcal{F}})^* \rightarrow \mathcal{A}$  that, given a **history of choices from the environment**, decides which action  $\mathcal{A}$  to do next.
- A **winning strategy** is a strategy  $f : (2^{\mathcal{F}})^* \rightarrow \mathcal{A}$  such that for all traces  $\pi$  with  $a_i = f(\pi_{\mathcal{F}}|_i)$  we have that  $\pi$  leads to a final state of  $\mathcal{G}$ .

# Generalization: DFA Games

## Winning condition for DFA games

Let

$$PreC(S) = \{s \in S \mid \exists a \in \mathcal{A}. \forall E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S\}$$

Compute the set  $Win$  of winning states of a DFA game  $\mathcal{G}$ , i.e., states from which the agent can win the game  $\mathcal{G}$ , by **least-fixpoint**:

- $Win_0 = F$  (the final states of  $\mathcal{G}$ )
- $Win_{i+1} = Win_i \cup PreC(Win_i)$
- $Win = \bigcup_i Win_i$

*(Computing  $Win$  is linear in the number of states in  $\mathcal{G}$ )*

## Computing the winning strategy

Let's define  $\omega : S \rightarrow 2^{\mathcal{A}}$  as:

$$\omega(s) = \{a \mid \text{if } s \in Win_{i+1} - Win_i \text{ then } \forall E. \varrho(s, [a, E]) \in Win_i\}$$

- **Every way** of restricting  $\omega(s)$  to return only one action (chosen arbitrarily) gives a **winning strategy** for  $\mathcal{G}$ .
- Note  **$s$  is a state of the game!** not of the domain only!  
To phrase  $\omega$  wrt the domain only, we need to return a **stateful transducer** with transitions from the game.

## Generalization: $\text{FOND}_{sp}$ for $\text{LTL}_f/\text{LDL}_f$ goals

### $\text{FOND}_{sp}$ for $\text{LTL}_f/\text{LDL}_f$ goals

#### Algorithm: $\text{FOND}_{sp}$ for $\text{LDL}_f/\text{LTL}_f$ goals

- 1: Given  $\text{LTL}_f/\text{LDL}_f$  domain  $\mathcal{D}$  and goal  $\varphi$
- 2:     Compute NFA for  $\varphi$  (*exponential*)
- 3:     Determinize NFA to DFA (*exponential*)
- 4:     Compute intersection with DFA of  $\mathcal{D}$  (*polynomial*)
- 5:     Synthesize winning strategy for DFA game (*linear*)
- 6:     Return strategy

### Theorem

$\text{FOND}_{sp}$  for  $\text{LTL}_f/\text{LDL}_f$  goals is:

- *EXPTIME-complete in the domain;*
- *2-EXPTIME-complete in the goal.*

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/DDL$  on finite traces
- 3  $LTL_f/DDL_f$  and automata
- 4 Planning for  $LTL_f/DDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains**
- 7  $POND_{sp}$  for  $LTL_f/DDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion

## FOND<sub>sc</sub>: strong cyclic planning in nondeterministic domains

### Nondeterministic domain (including initial state)

$\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  where:

- $\mathcal{F}$  **fluents** (atomic propositions)
- $\mathcal{A}$  **actions** (atomic symbols)
- $2^{\mathcal{F}}$  set of states
- $s_0$  initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$  represents **action preconditions**
- $\delta(s, a, s')$  with  $a \in \alpha(s)$  represents **action effects (including frame)**.

### Who controls what?

**Fluents** controlled by **environment**, though under **fairness assumption**:

*(i.e., all effects will eventually happen)*

**Actions** controlled by **agent**

*Observe:  $\delta(s, a, s')$*

### Goals, planning, and plans

**Goal** = propositional formula  $G$  on fluents

**Planning** = **agent**, in spite of the **environment**, stays in an area from where is possible to reach  $G$   
*(with the cooperation of **environment**! it is not a pure adversarial game!)*

**Plan** = **strategy** to stay within the good area.

*(FOND<sub>sc</sub> is EXPTIME-complete)*



## Nondeterministic domains as automata

Let's transform the nondeterministic domain  $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$  into an automaton recognizing all its traces as before.

Automaton  $A_{\mathcal{D}}$  for  $\mathcal{D}$  is a **DFA!!!**

$A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$  where:

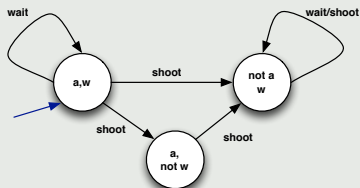
- $2^{\mathcal{F} \cup \mathcal{A}}$  alphabet (actions  $\mathcal{A}$  include dummy *start* action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$  set of states
- $s_{init}$  dummy initial state
- $F = 2^{\mathcal{F}}$  (all states of the domain are final)
- $\rho(s, [a, s']) = s'$  **with**  $a \in \alpha(s)$ , **and**  $\delta(s, a, s')$   
 $\rho(s_{init}, [start, s_0]) = s_0$

(notation:  $[a, s']$  stands for  $\{a\} \cup s'$ )

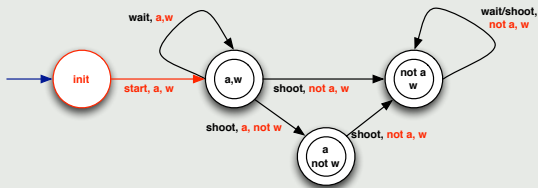
# Nondeterministic domains as automata

## Example (Simplified Yale shooting domain variant)

- Domain  $\mathcal{D}$ :



- DFA  $A_{\mathcal{D}}$ :

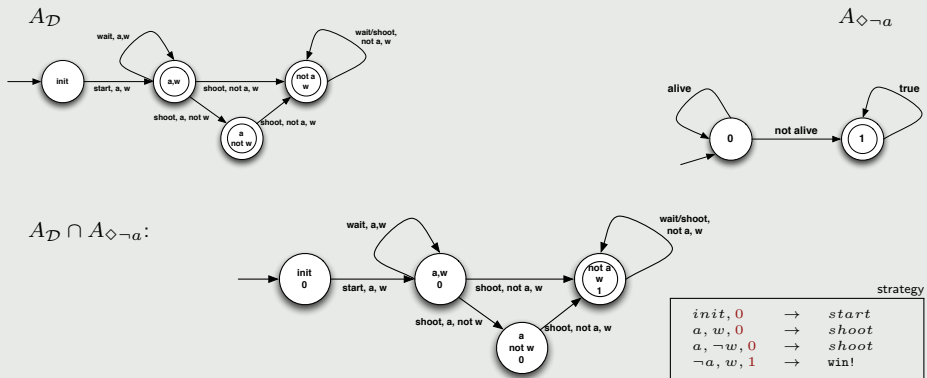


# Nondeterministic domains as automata

## FOND<sub>SC</sub>: strong cyclic planning in nondeterministic domains

- Set the **arena** formed by all traces that satisfy both the DFA  $A_{\mathcal{D}}$  for  $\mathcal{D}$  and the DFA for  $\Diamond G$  where  $G$  is the goal.
- Compute a **winning strategy**. (EXPTIME-complete in  $\mathcal{D}$ , constant in  $G$ )

## Example (Simplified Yale shooting domain)



# Generalization: $\text{FOND}_{sc}$ for $\text{LTL}_f/\text{LDL}_f$ goals

We need first to determinize the NFA for  $\text{LTL}_f/\text{LDL}_f$  formula

NFA for  $\Diamond\Box\neg a$



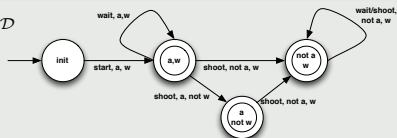
corresponding DFA



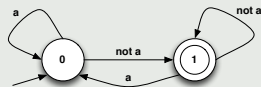
(DFA can be exponential in NFA in general)

Example (Simplified Yale shooting domain)

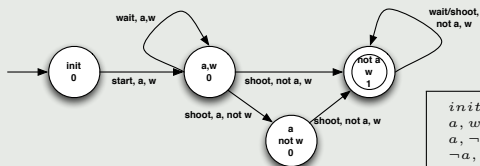
$A_D$



$A_{\Diamond\Box\neg a}$



$A_D \cap A_{\Diamond\Box\neg a}$ :



strategy

$init, 0$	$\rightarrow$	$start$
$a, w, 0$	$\rightarrow$	$shoot$
$a, \neg w, 0$	$\rightarrow$	$shoot$
$\neg a, w, 1$	$\rightarrow$	$win!$

# Generalization: Fair DFA Games

## Fair DFA games

A **fair DFA game**  $\mathcal{G} = (2^{\mathcal{F} \cup \mathcal{A}}, S, s_{init}, \varrho, F)$ , is such that:

- $\mathcal{F}$  controlled by **environment**;  $\mathcal{A}$  controlled by **agent**;
- $2^{\mathcal{F} \cup \mathcal{A}}$ , alphabet of game;
- $S$ , states of game;
- $s_{init}$ , initial state of game;
- $\varrho : S \times 2^{\mathcal{F} \cup \mathcal{A}} \rightarrow S$ , transition relation of the game: given current state  $s$  and a choice of action  $a$  and resulting fluents values  $E$  the resulting state of game is  $s' = \varrho(s, [a, E])$ ;
- $F$ , final states of game, where game can be considered terminated.

## Winning Strategy:

- A play is **winning** for the agent if from the initial the agent can force to remain in a safe area from which is possible to cooperatively reach the final state.
- A **strategy** for the agent is a function  $f : (2^{\mathcal{F}})^* \rightarrow \mathcal{A}$  that, given a **history of choices from the environment**, decides which action  $\mathcal{A}$  to do next.
- A **winning strategy** is a strategy  $f : (2^{\mathcal{F}})^* \rightarrow \mathcal{A}$  such that all traces  $\pi$  with  $a_i = f(\pi_{\mathcal{F}}|_i)$  are winning for the agent.

# Generalization: Fair DFA Games

## Existential and universal preimages wrt the environment

$$PreE(a, S) = \{s \in S \mid a \in \mathcal{A}. \exists E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S\}$$

$$PreA(a, S) = \{s \in S \mid a \in \mathcal{A}. \forall E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S\}$$

## Agent forces that always agent reach a final state if environment cooperates

The winning condition of the game is defined by two nested fixpoints, a **greatest-fixpoint** (for safety) and **least fixpoint** (for reachability):

$$Safe = \nu X. \mu Y. F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X) \cap PreE(a, Y))$$

This gives rise to the following nested fixpoint computation:

- $X_0 = S$  (all states of  $\mathcal{G}$ )
- $X_{i+1} = Y_{i+1} = \mu Y. F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X_i) \cap PreE(a, Y))$
- $Safe = \bigcap_i X_i$

where  $\mu Y. F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X_i) \cap PreE(a, Y))$  is computed as

- $Y_{i,0} = F$  (the final states of  $\mathcal{G}$ )
- $Y_{i,j+1} = F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X_i) \cap PreE(a, Y_{i,j}))$
- $Y_i = \bigcup_j Y_{i,j}$

*(Computing each  $Y_i$  is linear in the number of states in  $\mathcal{G}$ , hence computing  $Safe$  is quadratic.)*

## Computing the winning strategy

We can stratify *Safe* according to when a state enters the least fixpoint:

- $Reach_1 = F$ ,
- $Reach_{j+1} = Reach_j \cup PreA(a, Safe) \cap PreE(a, Reach_j)$ .

Note that  $Safe = \bigcup_{j \leq |S|} Reach_j$ .

Let's define  $\omega : S \rightarrow 2^A$  as:

$$\omega(s) = \{a \mid \text{if } s \in Reach_{j+1} - Reach_j \text{ then } \exists E. \varrho(s, [a, E]) \in Reach_j\}$$

- **Every way** of restricting  $\omega(s)$  to return only one action (chosen arbitrarily) gives a **winning strategy** for  $\mathcal{G}$ .
- Note  **$s$  is a state of the game!** not of the domain only!  
To phrase  $\omega$  wrt the domain only, we need to return a **stateful transducer** with transitions from the game.

## Generalization: $\text{FOND}_{sc}$ for $\text{LTL}_f/\text{LDL}_f$ goals

### $\text{FOND}_{sc}$ for $\text{LTL}_f/\text{LDL}_f$ goals

#### Algorithm: $\text{FOND}_{sc}$ for $\text{LDL}_f/\text{LTL}_f$ goals

- 1: Given  $\text{LTL}_f/\text{LDL}_f$  domain  $\mathcal{D}$  and goal  $\varphi$
- 2:     Compute NFA for  $\varphi$  (exponential)
- 3:     Determinize NFA to DFA (exponential)
- 4:     Compute intersection with DFA of  $\mathcal{D}$  (polynomial)
- 5:     Synthesize winning strategy for resulting fairDFA game (quadratic)
- 6:     Return strategy

### Theorem

$\text{FOND}_{sc}$  for  $\text{LTL}_f/\text{LDL}_f$  goals is:

- *EXPTIME-complete in the domain;*
- *2-EXPTIME-complete in the goal.*



# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/LDL$  on finite traces
- 3  $LTL_f/LDL_f$  and automata
- 4 Planning for  $LTL_f/LDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains with partial observability**
- 8 Conclusion

POND<sub>sp</sub>: strong planning in nondeter. domain with partial observability

Nondeterministic partially observable domain (including initial state)

$\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, Obs, s_0, \delta, \alpha, obs)$  where:

- $\mathcal{F}$  **fluents** (atomic propositions)
- $Obs \subseteq \mathcal{F}$  **observable fluents**
- $\mathcal{A}$  **actions** (atomic symbols)
- $2^{\mathcal{F}}$  set of states
- $s_0$  initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$  represents **action preconditions**
- $\delta(s, a, s')$  with  $a \in \alpha(s)$  represents **action effects (including frame)**
- $obs(s) \subseteq Obs$  project  $s$  on the **observable fluents only**.

Goals, planning, and plans

**Goal** = propositional formula  $G$  on fluents

**Planning** = **game** between two players:

**agent** tries to force eventually reaching  $G$  no matter how other **environment** behave and in spite of seeing only **observable fluents**.

**Plan** = **strategy** (depending on observable fluents only) to **win** the game.

Strategies  $f : (2^{Obs})^* \rightarrow \mathcal{A}$  such that for all generated traces  $\pi$  with  $E_i$  compatible with observations  $O_i$  and  $a_i = f(\pi_{Obs|_i})$ , we have that  $\pi$  satisfies eventually  $G$ .

(POND, strong plans, 2-EXPTIME-complete)

# Does Belief-State Construction for $POND_{sp}$ work for $LTL_f/ LDL_f$ ? Yes ...

## Belief-states DFA game

Given a DFA game  $\mathcal{A} = (2^{\mathcal{F} \cup \mathcal{A}}, S, s_{init}, \varrho, F)$  the associated **belief-states DFA game** is the following DFA game:  $\mathcal{G}_A^{Obs} = (2^{\mathcal{F} \cup \mathcal{A}}, \mathcal{B}, B_{init}, \partial, \mathcal{F})$ , where:

- $2^{\mathcal{F} \cup \mathcal{A}}$  is the alphabet which is the same of the original game;
- $\mathcal{B} = 2^S$  are the **belief states**, corresponding to sets of the states of the original game;
- $B_{init} = \{s_{init}\}$  is the initial belief-state, formed by initial state of the original game;
- $\partial : \mathcal{B} \times 2^{\mathcal{F} \cup \mathcal{A}} \rightarrow \mathcal{B}$  is the transition function: given the current belief-state  $B$  and a choice of action  $a$  and propositions  $E$ , respectively for the agent and the environment:

$$\partial(B, [a, E]) = \{s' \mid \exists s, E'. s \in B \wedge obs(E') = obs(E) \wedge \delta(s, [a, E]) = s'\}$$

- $\mathcal{F} = 2^F$  are the final belief-states, formed only by final states of the original game.

**NB:** **belief-states DFA game**  $\mathcal{G}_A^{Obs}$  is itself a DFA game (with full observability) over the  $\mathcal{F}$  and  $\mathcal{A}$ .

... Belief-State Construction is 3EXPTIME in the goal!

### Belief-State Algorithm for $\text{POND}_{sp}$ for $\text{LDL}_f/\text{LTL}_f$ goals

- 1: Given domain  $\mathcal{D}$  and  $\text{LTL}_f/\text{LDL}_f$  formula  $\varphi$
- 2:     Compute NFA for  $\varphi$  (exponential)
- 3:     Determinize NFA to DFA (exponential)
- 4:     Compute intersection with DFA of  $\mathcal{D}$  (polynomial)
- 5:     Compute the **belief-state DFA game** (exponential)
- 6:     Synthesize winning strategy for resulting DFA game (linear)
- 7:     Return strategy

*Cost is 2-EXPTIME in  $\mathcal{D}$  and 3-EXPTIME in  $\varphi$ !*

Can we do better? **YES!**

# Projection-based Construction

$\text{POND}_{sp}$  for  $\text{LTL}_f/\text{LDL}_f$  goals

Projection-based Algorithm for  $\text{POND}_{sp}$  for  $\text{LDL}_f/\text{LTL}_f$  goals

- 1: Given domain  $\mathcal{D}$  and  $\text{LTL}_f/\text{LDL}_f$  formula  $\varphi$
- 2: Compute NFA for  $\neg\varphi$  (exponential)
- 4: Compute NFA for union with complement of DFA for  $\mathcal{D}$  (polynomial)
- 3: Project out unobservable props, getting NFA  $\overline{A}$  (linear)  
*(behaves existentially on unobservables)*
- 4: Complement NFA  $\overline{A}$  getting DFA  $A$  (exponential)  
*(behaves universally on unobservables)*
- 5: Synthesize winning strategy for resulting DFA game  $A$  (linear)
- 6: Return strategy

*(Inspired by [DeGiacomoVardi-IJCAI16])*

## Theorem

$\text{POND}_{sp}$  for  $\text{LTL}_f/\text{LDL}_f$  goals is:

- 2-EXPTIME-complete in the domain;
- 2-EXPTIME-complete in the goal.

# Outline

- 1 Introduction and background
- 2  $LTL_f/LDL_f$ :  $LTL/LDL$  on finite traces
- 3  $LTL_f/LDL_f$  and automata
- 4 Planning for  $LTL_f/LDL_f$  goals: deterministic domains
- 5  $FOND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 6  $FOND_{sc}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains
- 7  $POND_{sp}$  for  $LTL_f/LDL_f$  goals: nondeterministic domains with partial observability
- 8 Conclusion



# What to bring home

- Interpreting temporal constraints/goals on finite traces is different than interpreting them on infinite traces (and much more well-behaved)
- When expressing temporal constraints and temporally extend goals we can add to usual  $LTL_f$  more powerful constructs a la  $LDL_f$  at no cost (possibly for future versions of PDDL).
- There are very general and effective techniques for reasoning, verification and synthesis in this setting – it's not just theory.
- In perspective, the Planning community may come up with a new generation of performing algorithms to deal with these basic tasks (after all, these are all compilable to reachability in large search spaces).

# Thanks

## Verification and Planning

- Moshe Vardi
- Sasha Rubin
- Benjamin Aminof
- Hector Geffner
- Blai Bonet
- Ronen Brafman
- Nello Murano
- Alessio Lomuscio
- ...

## UoT

This work [started](#) from discussions with people from UoT

- Jorge Baier
- Sheila McIlraith
- Yves Lesperance
- Hector Levesque
- Sebastian Sardina
- ...

## BPM

EU ACSI project and [DECLARE](#)

- Francesco Maria Maggi
- Marco Montali
- Diego Calvanese
- Marlon Dumas
- Lior Limonad
- Wil van der Aalst
- ...

## Sapienza

An intellectually lively environment formed by people working in [Reasoning about Action](#)

- Fabio Patrizi
- Stavros Vassos
- Riccardo De Masellis
- Paolo Felli
- Marco Grasso (Ms)
- Antonella Iacomino (Ms)
- ...

and in [Data and Services](#)

- Andrea Marrella
- Claudio Di Ciccio
- Alessandro Russo
- Massimo Mecella
- Domenico Lembo
- Maurizio Lenzerini
- Antonella Poggi
- Riccardo Rosati
- Francesco Leotta
- ...

