Planning for \mathtt{LTL}_f and \mathtt{LDL}_f Goals

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Outline

- ① LTL $_f$ /LDL $_f$: LTL/LDL on finite traces
- 2 LTL_f/LDL_f and automata
- 3 Planning for LTL_f/LDL_f goals: deterministic domains
- \P FOND $_{sp}$ for LTL $_f$ /LDL $_f$ goals: nondeterministic domains
- FOND $_{sc}$ for LTL $_f/$ LDL $_f$ goals: nondeteministic fair domains
- $oldsymbol{oldsymbol{eta}}$ POND $_{sp}$ for LTL $_f/$ LDL $_f$ goals: nondeterministic domains with partial observability
- Conclusion



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- 5 FOND_{sc} for LTL_f/LDL_f goals: nondeteministic fair domains
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LTL $_f$: LTL over finite traces

LTL_f : the language

$$\varphi ::= A \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \bigcirc \varphi \mid \varphi_1 \mathcal{U} \varphi_2$$

- A: atomic propositions
- $\neg \varphi$, $\varphi_1 \wedge \varphi_2$: boolean connectives
- $\bigcirc \varphi$: "next step exists and at next step (of the trace) φ holds"
- $\varphi_1 \mathcal{U} \varphi_2$: "eventually φ_2 holds, and φ_1 holds until φ_2 does"
- • $\varphi \doteq \neg \bigcirc \neg \varphi$: "if next step exists then at next step φ holds" (weak next)
- $\Diamond \varphi \doteq \mathsf{true} \, \mathcal{U} \, \varphi$: " φ will eventually hold"
- $\Box \varphi \doteq \neg \Diamond \neg \varphi$: "from current till last instant φ will always hold"
- $Last \doteq \neg \bigcirc true$: denotes last instant of trace.

Main formal properties:

- Expressibility: FOL over finite sequences or Star-free RE
- Reasoning: satisfiability, validity, entailment PSPACE-complete
- Model Checking: linear on TS, PSPACE-complete on formula



LTL_f : LTL over finite traces

Some interesting LTL_f formulas:

name of template	LTL semantics	
$responded\ existence(A,B)$	$\Diamond A \Rightarrow \Diamond B$	
co-existence(A,B)	$\Diamond A \Leftrightarrow \Diamond B$	
response(A,B)	$\Box(A\Rightarrow\Diamond B)$	
precedence(A, B)	$(\neg B\ UA) \lor \Box(\neg B)$	
succession(A, B)	$response(A, B) \land precedence(A, B)$	
$alternate\ response(A,B)$	$\Box(A\Rightarrow\bigcirc(\neg A\;UB))$	
$alternate\ precedence(A,B)$	$precedence(A, B) \land \\ \Box(B \Rightarrow \bigcirc(precedence(A, B)))$	
$alternate\ succession(A,B)$	$\begin{array}{c} alternate \ response(A,B) \ \land \\ alternate \ precedence(A,B) \end{array}$	
$chain\ response(A,B)$	$\Box(A\Rightarrow\bigcirc B)$	
$chain\ precedence(A,B)$	$\Box(\bigcirc B\Rightarrow A)$	
$chain\ succession(A,B)$	$\Box(A \Leftrightarrow \bigcirc B)$	

name of template	LTL semantics
$not\ co\text{-}existence(A,B)$	$\neg(\Diamond A \land \Diamond B)$
$not\ succession(A,B)$	$\Box(A\Rightarrow\neg(\Diamond B))$
$not\ chain\ succession(A,B)$	$\Box(A\Rightarrow\bigcirc(\neg B))$

name of template	LTL semantics
existence(1, A)	$\Diamond A$
existence(2, A)	$\Diamond(A \land \bigcirc(existence(1, A)))$
existence(n, A)	$\Diamond(A \land \bigcirc(existence(n-1,A)))$
absence(A)	$\neg existence(1,A)$
absence(2, A)	$\neg existence(2, A)$
absence(3, A)	$\neg existence(3, A)$
`	
absence(n+1,A)	$\neg existence(n+1,A)$
init(A)	Α
(11)	**
'	

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\mathtt{LDL}_f : \mathtt{LDL} over finite traces

LDL_f : the language

$$\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid \neg \varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle \rho \rangle \varphi \mid [\rho] \varphi \qquad \rho ::= \phi \mid \varphi? \mid \rho_1 + \rho_2 \mid \rho_1; \rho_2 \mid \rho^*$$

- tt and ff stand for true and false
- ϕ : propositional formula on current state/instant
- $\neg \varphi$, $\varphi_1 \wedge \varphi_2$: boolean connectives
- ullet ho is a regular expression on propositional formulas
- $\langle \rho \rangle \varphi$: exists an "execution" of RE ρ that ends with φ holding
- $[\rho]\varphi$: all "executions" of RE ρ (along the trace!) end with φ holding

In the infinite trace setting, such enhancement strongly advocated by industrial model checking (ForSpec, PSL).

Main formal properties:

- Expressibility: MSO over finite sequences: adds the power of recursion (as RE)
- Reasoning: satisfiability, validity, entailment PSPACE-complete
- Model Checking: linear on TS, PSPACE-complete on formula

LDL_f: Linear Dynamic Logic on finite traces

Example

• All coffee requests from person p will eventually be served:

$$[\mathsf{true}^*](request_p \supset \langle \mathsf{true}^* \rangle coffee_p)$$

ullet Every time the robot opens door d it closes it immediately after:

$$[true^*]([openDoor_d]closeDoor_d)$$

• Before entering restricted area a the robot must have permission for a:

$$\langle (\neg inArea_a{}^*; getPermission_a; \neg inArea_a{}^*; inArea_a)^*; \neg inArea_a{}^*\rangle end$$

Note that the first two properties (not the third one) can be expressed also in LTL $_f$:

$$\Box(request_p \supset \Diamond coffee_p) \qquad \qquad \Box(openDoor_d \supset \bigcirc closeDoor_d)$$



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LDL_f : Linear Dynamic Logic on finite traces

 LDL_f , not LTL_f , is able to easily express procedural constraints [BaierFritzMcllraith07].

Let's introduce a sort of propositional variant of Golog

 $\delta ::= A \mid \varphi? \mid \delta_1 + \delta_2 \mid \delta_1; \delta_2 \mid \delta^* \mid \text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \mid \text{while } \phi \text{ do } \delta$

where if and while can be seen as abbreviations for LDL_f path expression, namely:

if
$$\phi$$
 then δ_1 else $\delta_2 \doteq (\phi?; \delta_1) + (\neg \phi?; \delta_2)$ while ϕ do $\delta \doteq (\phi?; \delta)^*; \neg \phi?$

Example (LDL $_f$ procedural constraints)

• "At every point, if it is hot then, if the air-conditioning system is off, turn it on, else don't turn it off":

$$[true^*]\langle if (hot) then if (\neg airOn) then turnOnAir else $\neg turnOffAir \rangle true$$$

• "alternate till the end the following two instractions: (1) while is hot if the air-conditioning system is off turn it on, else don't turn it off; (2) do something for one step"

LDL_f : Linear Dynamic Logic on finite traces

Example (LDL_f captures finite domain variant of GOLOG in SitCalc)

GOLOG - finite domain variant

 $\delta \quad ::= \quad A \mid \varphi? \mid \delta_1 + \delta_2 \mid \delta_1; \delta_2 \mid \delta^* \mid \pi x. \delta(x) \mid \text{if } \phi \text{ then } \delta_1 \text{else } \delta_2 \mid \text{while } \phi \text{do } \delta$

- $\bullet \ \ \pi x. \delta(x) \ \text{stands for} \ \overline{\Sigma_{o \in Obj}} \ \overline{\delta(o)}$
- if ϕ then δ_1 else δ_2 stands for $(\phi?; \delta_1) + (\neg \phi?; \delta_2)$
- while ϕ do δ stands for $(\phi?; \delta)^* \neg \phi$?
- $\langle \delta \rangle \phi$ in LDL_f captures SitCalc formula $\exists s'. Do(\delta, s, s') \land s \leq s' \leq last \land \phi(s')$.
- $[\delta]\phi$ in LDL_f captures SitCalc formula $\forall s'.Do(\delta, s, s') \land s \leq s' \leq last \supset \phi(s')$.

($\phi(s)$ "uniform" in s.)



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LTL_f/LDL and automata

Key point

 LTL_f/LDL_f formulas can be translated into nondeterministic finite state automata (NFA).

$$t \models \varphi \text{ iff } t \in \mathcal{L}(\mathcal{A}_{\varphi})$$

where \mathcal{A}_{arphi} is the NFA arphi is translated into.

We can compile reasoning into automata based procedures!



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$\mathrm{LTL}_f/\mathrm{LDL}_f$ and automata

Both LTL_f and LDL_f formulas can be translated in exponential time to nondetermistic automata on finite words (NFA).

NFA \mathcal{A}_{φ} associated with an LTL $_f$ formula φ (in NNF)

Auxiliary rules $\begin{array}{l} = \text{true if } A \in \Pi \\ = \text{false if } A \not \in \Pi \\ = \text{false if } A \in \Pi \end{array}$ $\delta(A,\Pi)$ $\overset{\circ}{\delta(A,\Pi)} \\ \delta(\neg A,\Pi)$ $\delta(\neg A, \Pi)$ = true if $A \not\in \Pi$ $\begin{array}{l} \delta(\Pi,\Pi) = \delta(\omega_1,\Pi) + \delta(\omega_2,\Pi) \\ \delta(\varphi_1 \wedge \varphi_2,\Pi) = \delta(\varphi_1,\Pi) \wedge \delta(\varphi_2,\Pi) \\ \delta(\varphi_1 \vee \varphi_2,\Pi) = \delta(\varphi_1,\Pi) \vee \delta(\varphi_2,\Pi) \end{array}$ $= \begin{cases} \varphi & \text{if } Last \not\in \Pi \\ \text{false} & \text{if } Last \in \Pi \end{cases}$ $\delta(\bigcirc \varphi, \Pi)$ $\begin{array}{ll} \delta(\Diamond\varphi,\Pi) &= \delta(\varphi,\Pi) \vee \delta(\Diamond\Diamond\varphi,\Pi) \\ \delta(\varphi_1\,\mathcal{U}\,\varphi_2,\Pi) &= \delta(\varphi_2,\Pi) \vee (\delta(\varphi_1,\Pi) \wedge \delta(\bigcirc(\varphi_1\,\mathcal{U}\,\varphi_2),\Pi)) \end{array}$ $= \begin{cases} \varphi & \text{if } Last \not\in \Pi \\ \text{true} & \text{if } Last \in \Pi \end{cases}$ $\delta(\bullet\varphi,\Pi)$ $=\delta(\varphi,\Pi) \wedge \delta(\bullet \square \varphi,\Pi)$ $\delta(\Box\varphi,\Pi)$ $\delta(\varphi_1 \mathcal{R} \varphi_2, \Pi) = \delta(\varphi_2, \Pi) \wedge (\delta(\varphi_1, \Pi) \vee \delta(\bullet(\varphi_1 \mathcal{R} \varphi_2), \Pi))$ Observe these are the rules defining the transition function of the AFW!

Algorithm

algorithm LTL f 2NFA

 $\begin{array}{l} \text{input } \operatorname{LTL}_f \text{ formula } \varphi \\ \text{output } \operatorname{NFA} A_\varphi = (2^{\mathcal{P}}, \mathcal{S}, \{s_0\}, \varrho, \{s_f\}) \\ s_0 \leftarrow \{\varphi\} & \rhd \text{ single initial state } \\ s_f \leftarrow \emptyset & \rhd \text{ single final state } \\ \mathcal{S} \leftarrow \{s_0, s_f\}, \varrho \leftarrow \emptyset \\ \text{while } (\mathcal{S} \text{ or } \varrho \text{ change}) \text{ do} \\ \\ \text{if}(q \in \mathcal{S} \text{ and } q' \models \bigwedge_{(\psi \in q)} \delta(\psi, \Pi)) \\ & \mathcal{S} \leftarrow \mathcal{S} \cup \{q'\} & \rhd \text{ update set of states } \\ \varrho \leftarrow \varrho \cup \{(q, \Pi, q')\} & \rhd \text{ update transition } \\ \text{relation} \end{array}$



LTL_f/LDL_f and automata

NFA \mathcal{A}_{φ} associated with an LDL_f formula φ (in NNF)

```
Auxiliary rules
       \delta(\mathsf{tt},\Pi)
                                                                                                                              true
       \delta(ff,\Pi)
                                                                                                                            false
                                                                                                                            \begin{matrix} \overline{\delta(\varphi_1,\Pi)} \wedge \delta(\varphi_2,\Pi) \\ \delta(\varphi_1,\Pi) \vee \delta(\varphi_2,\Pi) \end{matrix}
      \begin{matrix} \delta(\varphi_1 \wedge \varphi_2, \Pi) \\ \delta(\varphi_1 \vee \varphi_2, \Pi) \end{matrix}
                                                                                                                                 \left\{ \begin{array}{ll} \text{false} & \text{if } \Pi \not\models \phi \text{ or } \Pi = \epsilon \text{ (trace ended)} \\ \mathbf{e}(\varphi) & \text{o/w} & (\phi \text{ propositional)} \end{array} \right. 
       \delta(\langle \phi \rangle \varphi, \Pi)
                                                                                                                           \begin{array}{l} \delta(\psi,\Pi) \wedge \delta(\varphi,\Pi) \\ \delta(\langle \rho_1 \rangle \varphi,\Pi) \vee \delta(\langle \rho_2 \rangle \varphi,\Pi) \\ \delta(\langle \rho_1 \rangle \langle \rho_2 \rangle \varphi,\Pi) \\ \delta(\varphi,\Pi) \vee \delta(\langle \rho \rangle \mathbf{f}_{\langle \rho^* \rangle \varphi},\Pi) \end{array}
       \delta(\langle \psi? \rangle \varphi, \Pi)
      \delta(\langle \rho_1 + \rho_2 \rangle \varphi, \Pi)
      \begin{array}{l} \delta(\langle \rho_1; \rho_2 \rangle \varphi, \Pi) \\ \delta(\langle \rho^* \rangle \varphi, \Pi) \end{array}
                                                                                                                                 \left\{ \begin{array}{ll} \mathsf{true} & \mathsf{if} \ \Pi \not\models \phi \ \mathsf{or} \ \Pi = \epsilon \ \mathsf{(trace \ ended)} \\ \mathbf{e}(\varphi) & \mathsf{o/w} \ \ (\phi \ \mathsf{propositional)} \end{array} \right. 
       \delta([\phi]\varphi,\Pi)
                                                                                                                              \delta(\mathit{nnf}(\neg\psi),\Pi) \vee \delta(\varphi,\Pi)
       \delta([\psi?]\varphi,\Pi)
      \begin{array}{l} \delta([\rho_1+\rho_2]\varphi,\Pi) \\ \delta([\rho_1;\rho_2]\varphi,\Pi) \end{array}
                                                                                                                             \begin{array}{l} \delta([\rho_1]\varphi,\Pi) \wedge \delta([\rho_2]\varphi,\Pi) \\ \delta([\rho_1][\rho_2]\varphi,\Pi) \end{array}
                                                                                                                              \delta(\varphi,\Pi) \wedge \delta([\rho] \mathbf{t}_{\left[\rho^*\right] \varphi}, \ \Pi)
       \delta([\rho^*]\varphi,\Pi)
       \delta(\mathbf{f}_{\psi}\,,\,\Pi)
      \delta(\mathbf{t}_{\psi}^{'},\Pi)
                                                                                                                              true
                                                                                          (\mathbf{e}(\varphi) \text{ replaces in } \varphi \text{ all occurrences of } \mathbf{t}_{\psi} \text{ and } \mathbf{f}_{\psi} \text{ by } \mathbf{e}(\psi))
```

Algorithm

```
algorithm LDL _f 2NFA
 input LDL _f formula arphi
output NFA A_{arphi}=(2^{\mathcal{P}},\mathcal{S},\{s_0\},\varrho,\{s_f\})
s_0 \leftarrow \{\varphi\}
s_f \leftarrow \emptyset
S \leftarrow \{s_0, s_f\}, \varrho \leftarrow \emptyset
                                                                               \triangleright single final state
while (S or \varrho change) do
        \mathsf{if}(q \in \mathcal{S} \; \mathsf{and} \; q' \; \models \bigwedge\nolimits_{(\psi \in q)} \delta(\psi, \Pi))
            \mathcal{S} \leftarrow \mathcal{S} \cup \{q'\}

    □ update set of states

            \varrho \leftarrow \varrho \cup \{(q,\Pi,q')\}
                                                                 > update transition relation
```



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LTL_f/LDL_f reasoning

LTL_f/LDL_f satisfiability (φ SAT)

- Given ${\tt LTL}_f/{\tt LDL}_f$ formula φ 1:
- Compute NFA for φ (exponential) 2.
- 3. Check NFA for nonemptiness (NLOGSPACE)
- Return result of check

${ m LTL}_f/{ m LDL}_f$ validity (arphi VAL)

- 1: Given $\mathtt{LTL}_f/\mathtt{LDL}_f$ formula arphi
- Compute NFA for $\neg \varphi$ (exponential)
- Check NFA for nonemptiness (NLOGSPACE)
- 4: Return complemented result of check

LTL_f/LDL_f logical implication $(\Gamma \models \varphi)$

- 1: Given ${
 m LTL}_f/{
 m LDL}_f$ formulas Γ and arphi
- Compute NFA for $\Gamma \wedge \neg \varphi$ (exponential)
- Check NFA for nonemptiness (NLOGSPACE)
- Return complemented result of check

Thm: [IJCAI13] All above reasoning tasks are PSPACE-complete. (As for infinite traces.)

(Construction of NFA can be done while checking nonemptiness.)

Relationship to Classical Planning

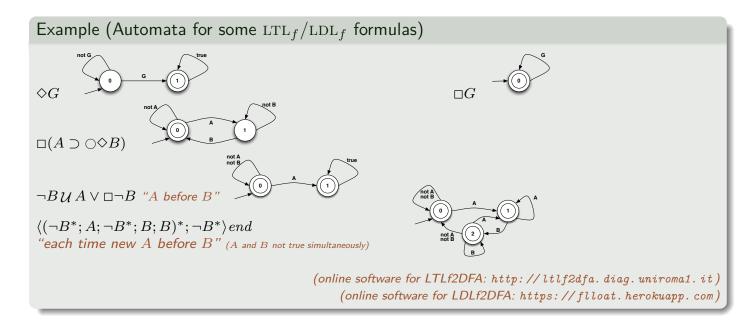
Let Ψ_{domain} describe action domain (LTL $_f$ formula), ϕ_{init} initial state (prop. formula), and G goal (prop. formula). Classical planning amounts to LTL $_f$ satisfiability of:

 $\phi_{init} \wedge \Psi_{domain} \wedge \Diamond G$

Complexity: PSPACE-complete.



Automata for some ${ m LTL}_f/{ m LDL}_f$ formulas





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Planning in deterministic domain

Deterministic domain (including initial state)

 $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$ where:

- F fluents (atomic propositions)
- A actions (atomic symbols)
- $2^{\mathcal{F}}$ set of states
- s_0 initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$ represents action preconditions
- $\delta(s,a)=s'$ with $a\in\alpha(s)$ represents action effects (including frame).

Traces

A **trace** for \mathcal{D} is a finite sequence:

$$s_0, a_1, s_1, \cdots, a_n, s_n$$

where s_0 is the initial state, and $a_i \in \alpha(s_i)$ and $s_{i+1} = \delta(s_i, a_{i+1})$ for each i.

Goals, planning, and plans

Goal = propositional formula G on fluents

Planning = find a trace $s_0, a_1, s_1, \dots, a_n, s_n$ such that $s_n \models G$. (PSPACE-complete)

Plan = project traces on actions, i.e., return a_1, \dots, a_n .

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Deterministic planning domains as automata

Let's transform the planning domain $\mathcal{D}=(2^{\mathcal{F}},\mathcal{A},s_0,\delta,\alpha)$ into a DFA recognizing all its traces.

DFA A_D for $\mathcal D$

 $A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$ where:

- $2^{\mathcal{F} \cup \mathcal{A}}$ alphabet (actions \mathcal{A} include dummy start action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$ set of states
- ullet s_{init} dummy initial state
- $F = 2^{\mathcal{F}}$ (all states of the domain are final)
- $\rho(s,[a,s'])=s'$ with $a\in\alpha(s)$, and $\delta(s,a)=s'$ $\rho(s_{init},[start,s_0])=s_0$

(notation: $[a\,,\,s^{\,\prime}]$ stands for $\{a\}\,\cup\,s^{\,\prime})$

Traces

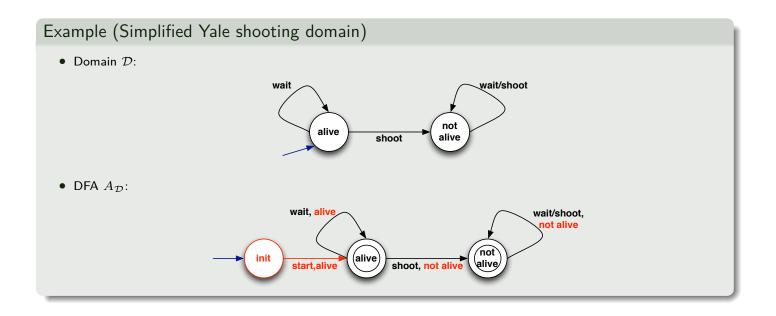
Each trace $s_0, a_1, s_1, \cdots, a_n, s_n$ of the domain \mathcal{D} becomes a finite sequence:

$$[start, s_0], [a_1, s_1], \cdots, [a_n, s_n]$$

recognized by the DFA $A_{\mathcal{D}}$.



Deterministic planning domains as automata





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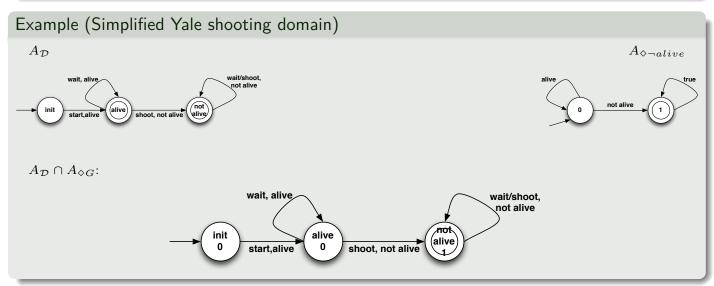
Deterministic planning domains as automata

Planning in deterministic domains

Planning = find a trace of DFA $A_{\mathcal{D}}$ for deterministic domain \mathcal{D} such that is also a trace for the DFA for $\Diamond G$ where G is the goal. That is:

CHECK for nonemptiness $A_{\mathcal{D}} \cap A_{\Diamond G}$: extract plan from witness.

(Computable on-the-fly, PSPACE in \mathcal{D} , constant in G. i.e., optimal)



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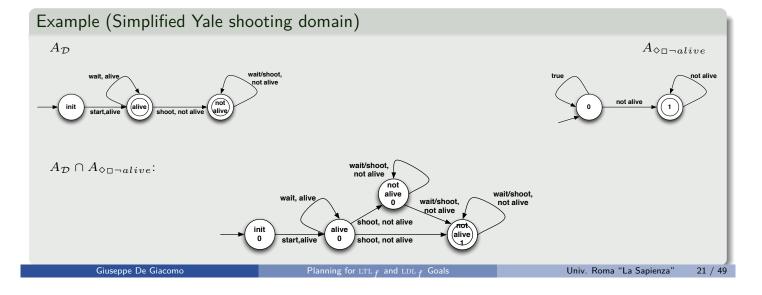
Generalization: planning for LTL_f/LDL_f goals in deterministic domains

Planning in deterministic domains for LTL_f/LDL_f goals

Planning = find a trace of DFA $A_{\mathcal{D}}$ for deterministic domain \mathcal{D} such that is also a accepted by NFA A_{φ} for the LTL $_f$ /LDL $_f$ formula φ . That is:

CHECK for nonemptiness $A_{\mathcal{D}} \cap A_{\varphi}$: extract plan from witness.

(Computable on-the-fly, PSPACE in \mathcal{D} , PSPACE also in φ i.e., optimal) (We can use NFA directly since we are checking for **existence** of a trace satisfying φ)



Generalization: planning for ${ m LTL}_f/{ m LDL}_f$ goals in deterministic domains

Planning for LTL_f/LDL_f goals

Algorithm: Planning for ${ m LDL}_f/{ m LTL}_f$ goals

- 1: Given ${
 m LTL}_f/{
 m LDL}_f$ domain ${\mathcal D}$ and goal ${arphi}$
- 2: Compute corresponding NFA (exponential)
- 3: Compute intersection with DFA of \mathcal{D} (polynomial)
- 5: Check nonemptiness of resulting NFA (NLOGSPACE)
- 6: Return plan

Theorem

Planning for LTL_f/LDL_f goals is:

- PSPACE-complete in the domain;
- PSPACE-complete in the goal.



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FOND_{sp}: strong planning in nondeterministic domains

Nondeterministic domain (including initial state)

 $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$ where:

- F fluents (atomic propositions)
- A actions (atomic symbols)
- 2^F set of states
- ullet s_0 initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$ represents action preconditions
- $\delta(s, a, s')$ with $a \in \alpha(s)$ represents action effects (including frame).

Who controls what?

Fluents controlled by environment

Actions controlled by agent

Observe: $\delta(s, a, s')$

Goals, planning, and plans

Goal = propositional formula G on fluents

Planning = **game** between two players:

agent tries to force eventually reaching G no matter how other environment behave.

Plan = strategy to win the game.

(FOND_{sp} is EXPTIME-complete)

Nondeterministic domains as automata

Let's transform the nondeterministic domain $\mathcal{D}=(2^{\mathcal{F}},\mathcal{A},s_0,\delta,\alpha)$ into an automaton recognizing all its traces.

Automaton A_D for \mathcal{D} is a DFA!!!

 $A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$ where:

- $2^{\mathcal{F} \cup \mathcal{A}}$ alphabet (actions \mathcal{A} include dummy start action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$ set of states
- ullet s_{init} dummy initial state
- $F = 2^{\mathcal{F}}$ (all states of the domain are final)

(notation: [a, s'] stands for $\{a\} \cup s'$)



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Planning for LTL & and LDL & Goals

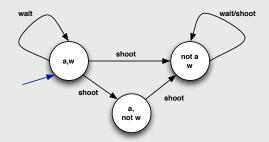
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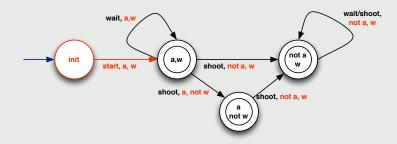
Nondeterministic domains as automata

Example (Simplified Yale shooting domain variant)

• Domain \mathcal{D} :



• DFA $A_{\mathcal{D}}$:



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Planning for LTL $_f$ and LDL $_f$ Goals

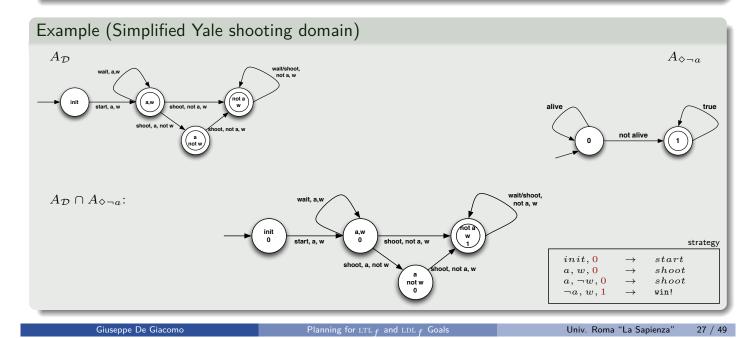
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Nondeterministic domains as automata

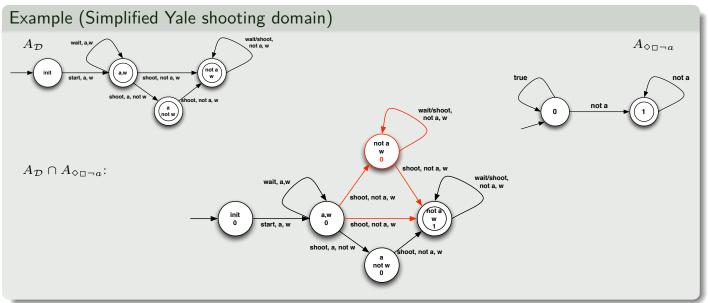
$FOND_{sp}$: strong planning in nondeterministic domains

- Set the arena formed by all traces that satisfy both the DFA $A_{\mathcal{D}}$ for \mathcal{D} and the DFA for $\Diamond G$ where G is the goal.
- Compute a winning strategy.

(EXPTIME-complete in \mathcal{D} , constant in G)



Generalization: $ext{FOND}_{sp}$ for $ext{LTL}_f/ ext{LDL}_f$ goals



Can we use directly NFA's?

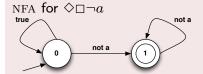
No, because of a basic mismatch

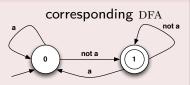
- NFA have perfect foresight, or clairvoyance
- Strategies must be runnable: depend only on past, not future

(angelic nondeterminism)
(devilish nondeterminism)

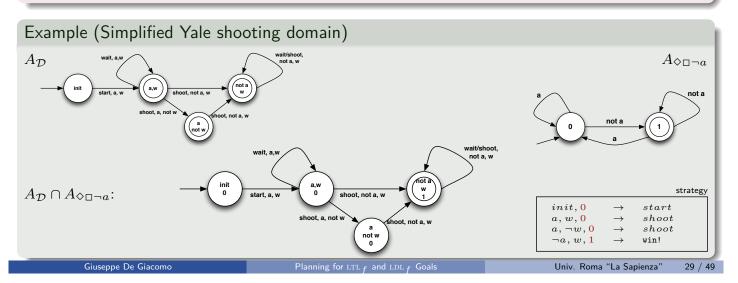
Generalization: FOND_{sp} for LTL_f/LDL_f goals

We need first to determinize the NFA for LTL_f/LDL_f formula





(DFA can be exponential in NFA in general)



Generalization: DFA Games

DFA games

A DFA game $\mathcal{G}=(2^{\mathcal{F}\cup\mathcal{A}},S,s_{init},\varrho,F)$, is such that:

- ullet ${\mathcal F}$ controlled by environment; ${\mathcal A}$ controlled by agent;
- $2^{\mathcal{F} \cup \mathcal{A}}$, alphabet of game;
- ullet S, states of game;
- ullet s_{init} , initial state of game;
- $\varrho: S \times 2^{\mathcal{F} \cup \mathcal{A}} \to S$, transition function of the game: given current state s and a choice of action a and resulting fluents values E the resulting state of game is $\varrho(s, [a, E]) = s'$;
- F, final states of game, where game can be considered terminated.

Winning Strategy:

- A play is winning for the agent if such a play leads from the initial to a final state.
- A strategy for the agent is a function $f:(2^{\mathcal{F}})^* \to \mathcal{A}$ that, given a history of choices from the environment, decides which action \mathcal{A} to do next.
- A winning strategy is a strategy $f:(2^{\mathcal{F}})^* \to \mathcal{A}$ such that for all traces π with $a_i = f(\pi_{\mathcal{F}}|_i)$ we have that π leads to a final state of \mathcal{G} .



Generalization: DFA Games

Winning condition for DFA games

Let

$$PreC(S) = \{ s \in S \mid \exists a \in A. \forall E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S \}$$

Compute the set Win of winning states of a DFA game \mathcal{G} , i.e., states from which the agent can win the game \mathcal{G} , by least-fixpoint:

- $Win_0 = F$ (the final states of \mathcal{G})
- $Win_{i+1} = Win_i \cup PreC(Win_i)$
- $Win = \bigcup_{i} Win_{i}$

(Computing Win is linear in the number of states in \mathcal{G})

Computing the winning strategy

Let's define $\omega: S \to 2^{\mathcal{A}}$ as:

$$\omega(s) = \{a \mid \text{ if } s \in Win_{i+1} - Win_i \text{ then } \forall E.\varrho(s, [a, E]) \in Win_i\}$$

- Every way of restricting $\omega(s)$ to return only one action (chosen arbitrarily) gives a winning strategy for \mathcal{G} .
- Note s is a state of the game! not of the domain only! To phrase ω wrt the domain only, we need to return a stateful transducer with transitions from the game.

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Planning for LTL $_f$ and LDL $_f$ Goal

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Generalization: FOND $_{sp}$ for LTL $_f/{ m LDL}_f$ goals

FOND_{sp} for $\mathrm{LTL}_f/\mathrm{LDL}_f$ goals

Algorithm: FOND $_{sp}$ for LDL $_f/$ LTL $_f$ goals

- 1: Given ${
 m LTL}_f/{
 m LDL}_f$ domain ${\mathcal D}$ and goal ${arphi}$
- 2: Compute NFA for φ (exponential)
- 3: Determinize NFA to DFA (exponential)
- 4: Compute intersection with DFA of \mathcal{D} (polynomial)
- 5: Synthesize winning strategy for DFA game (linear)
- 6: Return strategy

Theorem

FOND_{sp} for LTL_f/LDL_f goals is:

- EXPTIME-complete in the domain;
- 2-EXPTIME-complete in the goal.



Outline

- ① LTL $_f$ /LDL $_f$: LTL/LDL on finite traces
- \bigcirc LTL_f/LDL_f and automata
- 3 Planning for LTL_f/LDL_f goals: deterministic domains
- \P FOND_{sp} for LTL_f/LDL_f goals: nondeteministic domains
- \bullet FOND_{sc} for LTL_f/LDL_f goals: nondeteministic fair domains
- ${ ilde{ t b}}$ POND $_{sp}$ for LTL $_f/$ LDL $_f$ goals: nondeteministic domains with partial observability
- Conclusion



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Planning for LTL f and LDL f Goals

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${ m FOND}_{sc}$: strong cyclic planning in nondeterministic fair domains

Nondeterministic domain (including initial state)

 $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, s_0, \delta, \alpha)$ where:

- F fluents (atomic propositions)
- A actions (atomic symbols)
- $2^{\mathcal{F}}$ set of states
- s_0 initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$ represents action preconditions
- $\delta(s, a, s')$ with $a \in \alpha(s)$ represents action effects (including frame).

Who controls what?

Fluents controlled by environment, though under stochastic fairness:

(i.e., all effects will happen with non-zero (unknown) probability) Observe: $\delta(s,a,s')$

Actions controlled by agent

Goals, planning, and plans

Goal = propositional formula G on fluents

Planning = agent, in spite of the environment, stays in an area from where is possible to reach G

(with the cooperation of environment! it is not a pure adversarial game!)

Plan = **strategy** to stay within the good area.

(FOND $_{sc}$ is EXPTIME-complete)

Nondeterministic fair domains as automata

We transform the nondeterministic fair domain $\mathcal{D}=(2^{\mathcal{F}},\mathcal{A},s_0,\delta,\alpha)$ into an automaton recognizing all its traces exactly as we did for nondeterministic (non-fair) domains. Fairness play no role!

Automaton A_D for \mathcal{D} is a DFA!!!

 $A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$ where:

- $2^{\mathcal{F} \cup \mathcal{A}}$ alphabet (actions \mathcal{A} include dummy start action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$ set of states
- ullet s_{init} dummy initial state
- $F=2^{\mathcal{F}}$ (all states of the domain are final)
- $\quad \bullet \ \ \rho(s,[a,s']) = s' \ \text{with} \ \ a \in \alpha(s) \text{, and} \ \ \delta(s,a,s') \qquad \rho(s_{init},[start,s_0]) = s_0$

(notation: [a,s'] stands for $\{a\} \cup s'$)



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Planning for LTL f and LDL f Goals

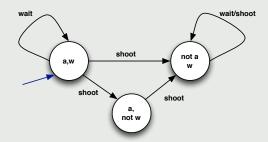
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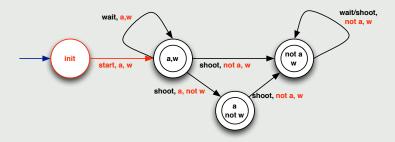
Nondeterministic fair domains as automata

Example (Simplified Yale shooting domain variant)

• Domain \mathcal{D} :



• DFA $A_{\mathcal{D}}$:

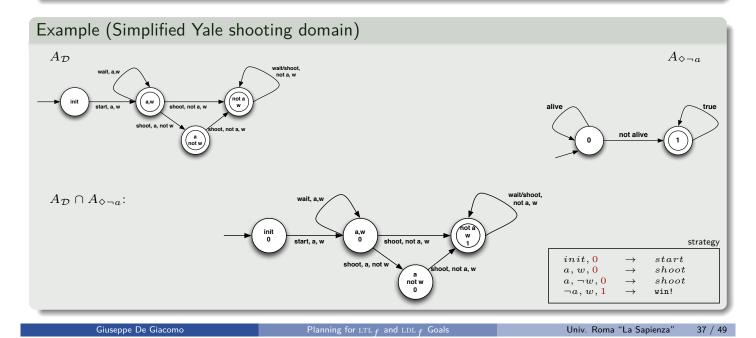


Nondeterministic fair domains as automata

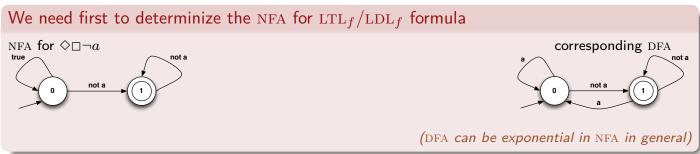
FOND $_{sc}$: strong cyclic planning in nondeterministic domains

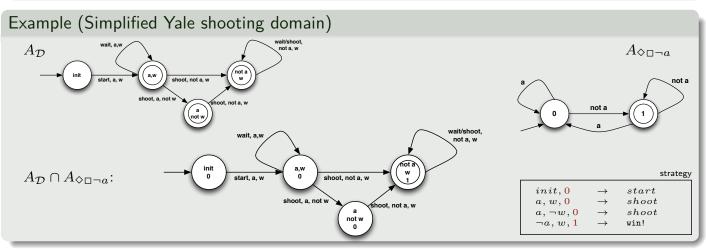
- Set the arena formed by all traces that satisfy both the DFA $A_{\mathcal{D}}$ for \mathcal{D} and the DFA for $\Diamond G$ where G is the goal.
- Compute a winning strategy.

(EXPTIME-complete in \mathcal{D} , constant in G)



Generalization: \mathtt{FOND}_{sc} for $\mathtt{LTL}_f/\mathtt{LDL}_f$ goals





Generalization: Fair DFA Games

Because of stochastic fairness we need to change the game we play over the DFA!

Fair DFA games

A fair DFA game $\mathcal{G}=(2^{\mathcal{F}\cup\mathcal{A}},S,s_{init},\varrho,F)$, is such that:

- ullet ${\mathcal F}$ controlled by environment; ${\mathcal A}$ controlled by agent;
- $2^{\mathcal{F} \cup \mathcal{A}}$, alphabet of game;
- S, states of game;
- s_{init}, initial state of game;
- $\varrho: S \times 2^{\mathcal{F} \cup \mathcal{A}} \to S$, transition relation of the game: given current state s and a choice of action a and resulting fluents values E the resulting state of game is $s' = \varrho(s, [a, E])$;
- F, final states of game, where game can be considered terminated.

Winning Strategy:

- A play is winning for the agent if from the initial the agent can force to remain in a safe area from which is possible to cooperatively reach the final state.
- A strategy for the agent is a function $f:(2^{\mathcal{F}})^* \to \mathcal{A}$ that, given a history of choices from the environment, decides which action \mathcal{A} to do next.
- A winning strategy is a strategy $f:(2^{\mathcal{F}})^* \to \mathcal{A}$ such that all traces π with $a_i = f(\pi_{\mathcal{F}}|_i)$ are winning for the agent.

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Planning for LTL f and LDL f Goals

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Generalization: Fair DFA Games

Existential and universal preimages wrt the environament

$$PreE(a, S) = \{ s \in S \mid a \in A. \exists E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S \}$$

$$PreA(a, S) = \{s \in S \mid a \in A. \forall E \in 2^{\mathcal{F}}. \varrho(s, [a, E]) \in S\}$$

Agent forces that always agent reach a final state if environment cooperates

The winning condition of the game is defined by two nested fixpoints, a **greatest-fixpoint** (for safety) and **least fixpoint** (for reachability):

$$Safe = \nu X.\mu Y.F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X) \cap PreE(a, Y))$$

This gives rise to the following nested fixpoint computation:

- $X_0 = S$ (all states of \mathcal{G})
- $X_{i+1} = Y_{i+1} = \mu Y.F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X_i) \cap PreE(a, Y))$
- $Safe = \bigcap_i X_i$

where $\mu Y.F \cup \bigcup_{a \in \mathcal{A}} (PreA(a,X_i) \cap PreE(a,Y))$ is computed as

- $Y_{i,0} = F$ (the final states of \mathcal{G})
- $Y_{i,j+1} = F \cup \bigcup_{a \in \mathcal{A}} (PreA(a, X_i) \cap PreE(a, Y_{i,j}))$
- $\bullet \ Y_i = \bigcup_i Y_{i,j}$

(Computing each Y_i is linear in the number of states in G, hence computing Safe is quadratic.)

Generalization: Fair DFA Games

Computing the winning strategy

We can stratify Safe according to when a state enters the least fixpoint:

- $Reach_1 = F$,
- $Reach_{j+1} = Reach_j \cup PreA(a, Safe) \cap PreE(a, Reach_j)$.

Note that $Safe = \bigcup_{j \leq |S|} Reach_j$.

Let's define $\omega: S \to 2^{\mathcal{A}}$ as:

$$\omega(s) = \{a \mid \text{ if } s \in Reach_{j+1} - Reach_j \text{ then } \exists E. \varrho(s, [a, E]) \in Reach_j \}$$

- Every way of restricting $\omega(s)$ to return only one action (chosen arbitrarily) gives a winning strategy for \mathcal{G} .
- Note s is a state of the game! not of the domain only! To phrase ω wrt the domain only, we need to return a stateful transducer with transitions from the game.



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Planning for UTL & and LDL & Goals

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Generalization: $ext{FOND}_{sc}$ for $ext{LTL}_f/ ext{LDL}_f$ goals

${ m FOND}_{sc}$ for ${ m LTL}_f/{ m LDL}_f$ goals

Algorithm: FOND $_{sc}$ for LDL $_f/$ LTL $_f$ goals

- 1: Given ${
 m LTL}_f/{
 m LDL}_f$ domain ${\mathcal D}$ and goal ${arphi}$
- 2: Compute NFA for φ (exponential)
- 3: Determinize NFA to DFA (exponential)
- 4: Compute intersection with DFA of \mathcal{D} (polynomial)
- 5: Synthesize winning strategy for resulting fairDFA game (quadratic)
- 6: Return strategy

Theorem

FOND_{sc} for LTL_f/LDL_f goals is:

- EXPTIME-complete in the domain;
- 2-EXPTIME-complete in the goal.



Outline

- ① LTL $_f$ /LDL $_f$: LTL/LDL on finite traces
- \bigcirc LTL_f/LDL_f and automata
- 3 Planning for LTL_f/LDL_f goals: deterministic domains
- \P FOND_{sp} for LTL_f/LDL_f goals: nondeteministic domains
- FOND_{sc} for LTL_f/LDL_f goals: nondeterministic fair domains
- $oldsymbol{6}$ POND $_{sp}$ for LTL $_f/$ LDL $_f$ goals: nondeteministic domains with partial observability
- Conclusion



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Planning for UTL & and LDL & Goals

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$POND_{sn}$: strong planning in nondeter. domain with partial observability

Nondeterministic partially observable domain (including initial state)

 $\mathcal{D} = (2^{\mathcal{F}}, \mathcal{A}, Obs, s_0, \delta, \alpha, obs)$ where:

- F fluents (atomic propositions)
 - $Obs \subseteq \mathcal{F}$ observable fluents
 - A actions (atomic symbols)
 - 2^F set of states
 - ullet s_0 initial state (initial assignment to fluents)
 - $\alpha(s) \subseteq \mathcal{A}$ represents action preconditions
 - $\delta(s, a, s')$ with $a \in \alpha(s)$ represents action effects (including frame)
 - $obs(s) \subseteq Obs$ project s on the observable fluents only.

Goals, planning, and plans

Goal = propositional formula G on fluents **Planning** = **game** between two players:

agent tries to force eventually reaching G no matter how other environment behave and in spite of seeing only observable fluents.

Plan = strategy (depending on observable fluens only) to win the game.

Strategies $f:(2^{Obs})^* \to \mathcal{A}$ such that for all generated traces π with E_i compatible with observations O_i and $a_i = f(\pi_{Obs}|_i)$, we have that π satisfies eventually \mathcal{G} .

(POND, strong plans, 2-EXPTIME-complete)

Does Belief-State Construction for POND_{sp} work for LTL_f/LDL_f? Yes ...

Belief-states DFA game

Given a DFA game $\mathcal{A}=(2^{\mathcal{F}\cup\mathcal{A}},S,s_{init},\varrho,F)$ the associated **belief-states** DFA **game** is the following DFA game: $\mathcal{G}_A^{Obs}=(2^{\mathcal{F}\cup\mathcal{A}},\mathcal{B},B_{init},\partial,\mathcal{F})$, where:

- $2^{\mathcal{F} \cup \mathcal{A}}$ is the alphabet which is the same of the original game;
- $\mathcal{B} = 2^S$ are the **belief states**, corresponding to sets of the states of the original game;
- $B_{init} = \{s_{init}\}$ is the initial belief-state, formed by initial state of the original game;
- $\partial: \mathcal{B} \times 2^{\mathcal{F} \cup \mathcal{A}} \to \mathcal{B}$ is the transition function: given the current belief-state B and a choice of action a and propositions E, respectively for the agent and the environment:

$$\partial(B,[a,E]) = \{s' \mid \exists s,E'.s \in B \land obs(E') = obs(E) \land \delta(s,[a,E]) = s'\}$$

ullet $\mathcal{F}=2^{m{F}}$ are the final belief-states, formed only by final states of the original game.

NB: belief-states DFA game \mathcal{G}_A^{Obs} is itself a DFA game (with full observability) over the \mathcal{F} and \mathcal{A} .



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Planning for LTL f and LDL f Goals

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... Belief-State Construction is 3EXPTIME in the goal!

Belief-State Algorithm for $ext{POND}_{sp}$ for $ext{LDL}_f/ ext{LTL}_f$ goals

- 1: Given domain ${\mathcal D}$ and ${
 m LTL}_f/{
 m LDL}_f$ formula ${arphi}$
- 2: Compute NFA for φ (exponential)
- 3: Determinize NFA to DFA (exponential)
- 4: Compute intersection with DFA of \mathcal{D} (polynomial)
- 5: Compute the belief-state DFA game (exponential)
- 6: Synthesize winning strategy for resulting DFA game (linear)
- 7: Return strategy

Cost is 2-EXPTIME in $\mathcal D$ and 3-EXPTIME in $\varphi!$

Can we do better? YES!



Projection-based Construction

$POND_{sp}$ for LTL_f/LDL_f goals

Projection-based Algorithm for POND_{sp} for LDL_f/LTL_f goals

- 1: Given domain ${\mathcal D}$ and ${
 m LTL}_f/{
 m LDL}_f$ formula ${arphi}$
- 2: Compute NFA for $\neg \varphi$ (exponential)
- 4: Compute NFA for union with complement of DFA for \mathcal{D} (polynomial)
- 3: Project out unobservable props, getting NFA *A* (linear) (behaves existentially on unobservables)
- 4: Complement NFA \overline{A} getting DFA A (exponential) (behaves universally on unobservables)
- 5: Synthesize winning strategy for resulting DFA game A (linear)
- 6: Return strategy

(Inspired by [DeGiacomoVardi-IJCAI16])

Theorem

POND_{sp} for LTL_f/LDL_f goals is:

- 2-EXPTIME-complete in the domain;
- 2-EXPTIME-complete in the goal.

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Planning for LTL $_f$ and LDL $_f$ Goa

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Outline

- 1 LTL $_f$ /LDL $_f$: LTL/LDL on finite traces
- \bigcirc LTL $_f/$ LDL $_f$ and automata
- \bigcirc Planning for $\mathrm{LTL}_f/\mathrm{LDL}_f$ goals: deterministic domains
- \P FOND $_{sp}$ for ${
 m LTL}_f/{
 m LDL}_f$ goals: nondeterministic domains
- ${ ilde{ ilde{5}}}$ FOND $_{sc}$ for LTL $_f/$ LDL $_f$ goals: nondeteministic fair domains
- \bigcirc POND $_{sp}$ for LTL $_f/$ LDL $_f$ goals: nondeteministic domains with partial observability
- Conclusion



Conclusion

Summary

In Planning we separate the domain from the goal (this is not the case in synthesis), for good reasons!

- Domain: it is are presentation of the world in which the agent acts, hence typically large
 - Cost for FOND_{sp}, FOND_{sc} is EXPTIME-complete, ...
 - ightharpoonup ... independently from the goal being classical reachability, LTL $_f$ or LDL $_f$
- Goal: it is an objective the agent wants to obtain, hence typically small
 - ightharpoonup Costs depends on the size of the DFA corresponding the LTL $_f$ /LDL $_f$ expressing the goal
 - Polynomial for reachability, i.e., $\Diamond G$, (G propositional), as well as for many LTL_f/LDL_f formulas that admit a small (bounded) DFA.
 - ightharpoonup Exponential for those ${
 m LTL}_f/{
 m LDL}_f$ that do not require to determinization
 - ≥ 2EXPTIME-complete, in general

Two basic solvers

We used two basic solvers on which the planning community has the best know-how:

- for DFA games ("eventually good"), i.e., a FOND strong planner
- for fair DFA games ("eventually good (under fairness)"), i.e., a FOND strong cyclic planner

See work in progress at: http://fond4ltlfpltl.diag.uniroma1.it

See papers by Alberto Camacho, Christian Muise, Jorge A. Baier, Sheila A. McIlraith at IJCAI18 and ICAPS18

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Planning for LTL $_{m f}$ and LDL $_{m f}$ Goals

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