Regular Decision Processes

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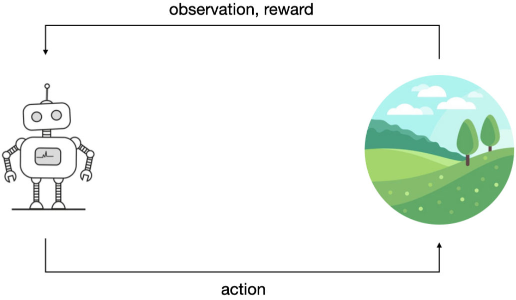
Motivation: Artificial Intelligence

- Optimal agents
  - “Acting rationally” in Russell and Norvig’s AI book
  - Optimal agent for the Turing test

An agent and the environment



A Markov agent and the environment

- Is the state of affairs readily available?
- If not, who is computing states for the agent?
- It is intelligence given an oracle to compute such states
    - Computing states could be the most challenging part

Outline

1. Non-Markov Decision Processes (NMDPs)
2. Regular Decision Processes (RDPs)
3. LDL_f specification of RDPs
4. Learning RDPs via Probabilistic Automata
5. References

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Motivation

Optimal agents in stochastic domains.

Non-Markov Decision Processes (NMDPs)

$$\mathcal{P} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$

- Actions $A = \{a_1, a_2, \ldots, a_n\}$
- Observations $O = \{o_1, o_2, \ldots, o_m\}$
  - An element h of O^* is called a history
- Rewards $R = \{r_1, r_2, \ldots, r_k\} \subseteq \mathbb{R}$
  - An element t of $(AOR)^*$ is called a trace
- Transition function $\mathbf{T} : O^* \times A \times O \to [0, 1]$
  - $\mathbf{T}(\cdot|h, a)$ is a probability distribution on O
- Reward function $\mathbf{R} : O^* \times A \times R \to [0, 1]$
  - $\mathbf{R}(\cdot|h, a)$ is a probability distribution on R
- $\gamma \in (0, 1)$ is the discount factor (can be replaced with a finite horizon)

Dynamics function

- The transition and reward functions can be combined into the dynamics function $\mathbf{D} : O^* \times A \times O \times R \to [0, 1]$
  - Definition: $\mathbf{D}(o, r|h, a) = \mathbf{T}(o|h, a) \cdot \mathbf{R}(r|h, a)$
  - Note that $\mathbf{D}(\cdot|h, a)$ is a probability distribution on $O \times R$

Policy functions

- A policy is a function $\pi : O^* \times A \to [0, 1]$
  - with $\pi(\cdot|h)$ a probability distribution on A
  - It is deterministic if, on every history h, it assigns probability one to an action a (we can write $\pi(h) = a$)

Value functions

$$\mathbf{v}_\pi(h) = \sum_{aor} \pi(a|h) \cdot \mathbf{D}(o, r|h, a) \cdot (r + \gamma \cdot \mathbf{v}_\pi(ho))$$

$$\mathbf{v}_*(h) = \max_\pi \mathbf{v}_\pi(h) = \max_a \left(\sum_{or} \mathbf{D}(o, r|h, a) \cdot (r + \gamma \cdot \mathbf{v}_*(ho)) \right)$$

$$\mathbf{q}_\pi(h, a) = \sum_{or} \mathbf{D}(o, r|h, a) \cdot (r + \gamma \cdot \mathbf{v}_\pi(ho))$$

$$\mathbf{q}_*(h, a) = \max_\pi \mathbf{q}_\pi(h, a) = \sum_{or} \mathbf{D}(o, r|h, a) \cdot (r + \gamma \cdot \mathbf{v}_*(ho))$$

Optimality and Near-optimality

- A policy π is optimal if $\mathbf{v}_\pi(h) = \mathbf{v}_*(h)$ on every history h.
- A policy π is ϵ-optimal if $|\mathbf{v}_\pi(h) - \mathbf{v}_*(h)| \leq \epsilon$ on every history h

- Optimal agent $\rightsquigarrow$ an agent that follows an optimal policy
- Near-optimal agent $\rightsquigarrow$ an agent that follows an ϵ-optimal policy for some ϵ

Comments on NMDPs

- They are very general
  - maybe too general...
- The transition and reward functions are not even required to be computable!
- Optimal policies may not be computable
  - We could require from an agent to be able to solve the halting problem (prove it!)
- There is no hope for computationally-feasible approaches for NMDPs

Markov Decision Processes (MDPs)

$$\mathcal{M} = \langle A, S, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$

- All history-dependent functions depend on the last observation only
  - Since the last observation captures the current state of affairs, it is referred to as a state s, coming from a finite set S (which takes the place of the set of observations).
- The functions become as follows:
  - transition function $\mathbf{T}(s'|s, a)$
  - reward function $\mathbf{R}(r|s, a)$
  - dynamics function $\mathbf{D}(s', r|s, a)$
  - policy function $\pi(a|s)$
  - value functions $\mathbf{v}_\pi(s)$, $\mathbf{q}_\pi(s)$, $\mathbf{v}_*(s)$.

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Motivation

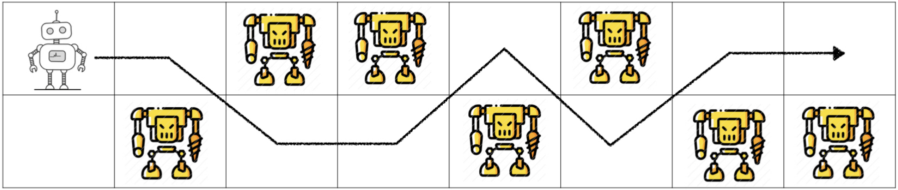
The motivation of NMDPs + Favourable computational properties

Regular Decision Processes (RDPs)

$$\mathcal{R} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$

- $\mathbf{T}$ and $\mathbf{R}$ are 'regular' functions of the history
- They can be characterised/specified in a number of formalisms:
  1. finite-state transducers,
  2. LDL$_f$ formulas,
  3. probabilistic automata (suitable for learning),
  4. other formalisms (e.g., regular expressions)

Example



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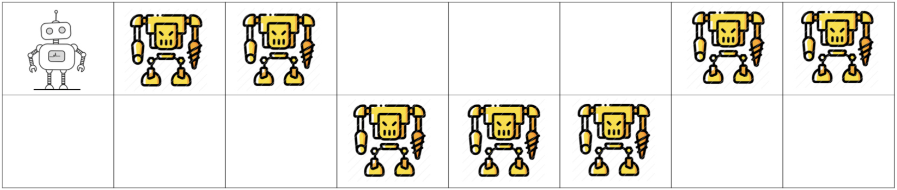

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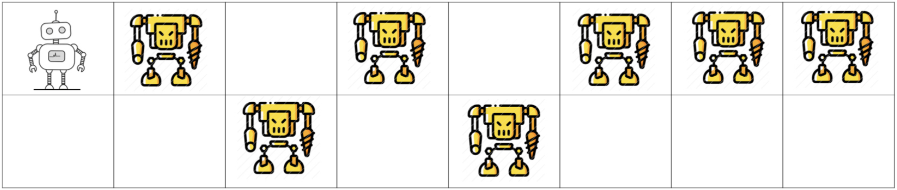

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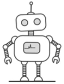	???	???	???	???	???	???	???
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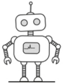
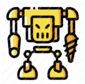
		???	???	???	???	???	???
		???	???	???	???	???	???

Example

Finite Transducers (Moore's variant)

$$T = \langle S, s_0, \Sigma, \tau, \Gamma, \theta \rangle$$

- S is the finite set of states;
- $s_0 \in S$ is the initial state;
- Σ is the finite input alphabet;
- $\tau : S \times \Sigma \to S$ is the deterministic transition function;
- Γ is the finite output alphabet;
- $\theta : S \to \Gamma$ is the output function.

Finite Transducers (Moore's variant) (2)

$$T = \langle S, s_0, \Sigma, \tau, \Gamma, \theta \rangle$$

- The transition function τ and output function θ are extended to strings as follows:

$$\tau(s, \sigma_1\sigma_2 \ldots \sigma_n) = \tau(\tau(s, \sigma_1), \sigma_2 \ldots \sigma_n) \text{ with } \tau(s, \varepsilon) = s$$

$$\theta(s, \sigma_1 \ldots \sigma_n) = \theta(s)\, \theta(\tau(s, \sigma_1), \sigma_2 \ldots \sigma_n) \text{ with } \theta(s, \varepsilon) = \theta(s)$$

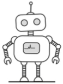
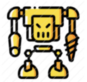
- A transducer T can be seen as a device that maps (or transduces) every string $w \in \Sigma^*$ to the string $\theta(s_0, w)$
- A transducer T can be seen as a representation of the function $T : \Sigma^* \to \Gamma^*$ such that $T(w) = \theta(s_0, w)$

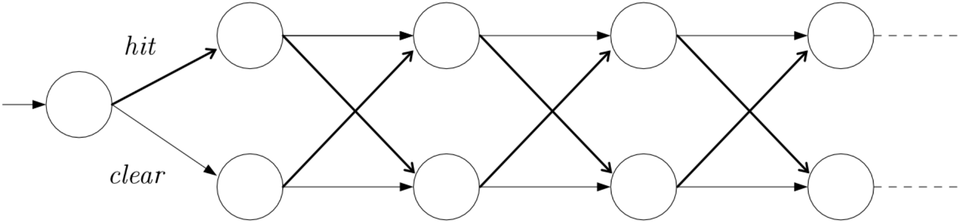
Transducer characterisation of RDPs

$$\mathcal{R} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$

- $\mathcal{R}$ is an RDP if there exist two transducers $T_{\mathbf{T}}$ and $T_{\mathbf{R}}$ such that:
  1. transducer $T_{\mathbf{T}}$ maps every history h to the function $\mathbf{T}_h : A \times O \to [0,1]$ induced by $\mathbf{T}$ when its first argument is h, i.e.,
     - $\mathbf{T}_h(o|a) = \mathbf{T}(o|h,a) \quad \forall a \in A, \forall o \in O.$
  2. transducer $T_{\mathbf{R}}$ maps every history h to the function $\mathbf{R}_h : A \times R \to [0,1]$ induced by $\mathbf{R}$ when its first argument is h, i.e.,
     - $\mathbf{R}_h(r|a) = \mathbf{R}(r|h,a) \quad \forall a \in A, \forall r \in R.$
- Note that $T_{\mathbf{T}}$ and $T_{\mathbf{R}}$ can be combined into a transducer $T_{\mathbf{D}}$ for the dynamics function $\mathbf{D}$.

Example: Transitions

		???	???	???	???	???	???
		???	???	???	???	???	???



Example: Full specification

- $\mathcal{R} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$
  - $A = \{a_1, a_2\}$
  - $O = [0, m-1] \times \{hit, clear\}$
  - p_i^j is the j-th probability of enemy i of being in the upper cell
- $T = \langle S, s_0, O, \tau, \Gamma, \theta \rangle$
  - $S = [0, m-1] \times \{0, 1\}$
  - $s_0 = \langle m-1, 0 \rangle$,
  - the transition function is:
    - $\tau(\langle i, b \rangle, \langle i+1 \bmod m, hit \rangle) = \langle i+1 \bmod m, b+1 \bmod 2 \rangle$,
    - $\tau(\langle i, b \rangle, \langle i+1 \bmod m, clear \rangle) = \langle i+1 \bmod m, b \rangle$,
  - the output function is:
    - $\theta(\langle i, b \rangle)(a_k, \langle j, enemy \rangle, 0) = p_j^b$ where $j = i+1 \bmod m$,
    - $\theta(\langle i, b \rangle)(a_k, \langle j, clear \rangle, 1) = 1 - p_j^b$ where $j = i+1 \bmod m$.

Equivalent MDP

Consider an RDP $\mathcal{R} = \langle A, O, R, \mathbf{D}^{\mathcal{R}}, \gamma \rangle$

Consider a transducer $T = \langle S, s_0, O, \tau, \Gamma, \theta \rangle$ for the dynamics of $\mathcal{R}$.
Namely, $T(h) = \mathbf{D}_h$ and $\mathbf{D}_h(o, r|a) = \mathbf{D}(o, r|h, a)$,
or simply $T(h)(a, o, r) = \mathbf{D}(o, r|h, a)$.

We define the equivalent MDP $\mathcal{M} = \langle A, S, R, \mathbf{D}^{\mathcal{M}}, \gamma \rangle$ where the dynamics function is defined as follows:

$$\mathbf{D}^{\mathcal{M}}(s_2, r|s_1, a) = \sum_{o:\tau(s_1,o)=s_2} \theta(s_1)(a, o, r).$$

It is the MDP perceived by an agent that interacts with $\mathcal{R}$ but reads state $\tau(h)$ instead of history h.

Equivalent MDP: Properties

Consider a history h generated by an agent interacting with $\mathcal{R}$. Say that the agent reads state $s = \tau(h)$ instead of history h, and acts following a Markov policy π on S, hence picks actions according to $\pi(\cdot|s)$. Thus, the agent acts according the the composition $\pi\tau$ of π with the transition function τ of the transducer.

Theorem

For every history h, $\mathbf{v}_{\pi}^{\mathcal{M}}(\tau(h)) = \mathbf{v}_{\pi\tau}^{\mathcal{R}}(h)$.

Therefore, among all policies π on S, it is best to choose a policy π_* that has maximum value in $\mathcal{M}$, since $\mathbf{v}_{\pi_*}^{\mathcal{M}}(\tau(h)) = \mathbf{v}_{\pi_*\tau}^{\mathcal{R}}(h)$.

Is $\pi_*\tau$ optimal for $\mathcal{R}$? Or there are better policies that cannot be expressed as the composition of a Markov policy with the transition function?

Equivalent MDP: Properties and Usage

Theorem

Every RDP $\mathcal{R}$ admits an optimal policy of the form $\pi\tau$ for τ the transition function of its dynamics transducer and π a Markov policy on the states of the transducer.

Corollary

If π_* is an optimal policy for the equivalent MDP, then $\pi_*\tau$ is optimal for the original RDP.

Application: solve the equivalent MDP $\mathcal{M}$ to obtain an optimal Markov policy π_*, and compose it with τ, to obtain an optimal policy $\pi_*\tau$ for $\mathcal{R}$.

Remark: all claims above about optimality hold for ϵ-optimality as well.

Equivalent MDP: Implications

- Planning, i.e., computing an optimal policy:
  - Given the dynamics transducer as input, computing an optimal policy for an RDP amount to computing an optimal policy for an MDP (value iteration, etc.)

Equivalent MDP: Implications (2)

- Reinforcement Learning:
  - A Markov reinforcement learning agent can achieve optimality in an RDP if it is provided with the transducer states in place of observations.
    - This requires an external helper, who has knowledge of the transducer.
    - It is what has been done (more or less explictly) in reinforcement learning so far, since classic algorithms cannot operate directly on observations.
  - Fully-fledged reinforcement learning in RDPs requires to learn the transducer's transition function τ as part of the overall learning process
    - Challenges: to learn a good transition function you need good data, but to have good data you need a good transition function
    - The claim holds regardless of the specific approach.

The importance of τ

- The transition function τ allows an agent to establish that two different histories can be considered equivalent for the purpose of predicting their respective futures.
- Thus, policies can be based on the states returned by τ, since there is no reason to behave differently on two histories h_1, h_2 when $\tau(h_1) = \tau(h_2) = s$.
- For an agent, it is sufficient to see the world through τ.

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Motivation

- High-level specification of RDPs
  - How does a human (e.g., engineer) write down an RDP?
    - Writing the transducer may be inconvenient
  - It could be a convenient language for agents to exchange information about the domain

LDL_f specification of RDPs (syntax)

- Set of propositions (fluents) $P = \{p_1, \dots, p_n\}$.
- Triples specifying transition function:

  $\{\langle \varphi, a, \delta \rangle \mid \varphi$ is an LDL_f formula on P, $a \in A$, δ is a distribution on $O\}$

- Triples specifying reward function:

  $\{\langle \psi, a, \rho \rangle \mid \psi$ is an LDL_f formula on P, $a \in A$, ρ is a distribution on $R\}$

LDL$_f$ specification of RDPs (semantics)

$$\mathcal{R} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$

- $O = 2^P$
- Note that histories O^* are LDL$_f$ interpretations.
- Transition function:
  1. (mutual exclusivity) for every history h and every action a there do not exist two distinct triples $\langle \varphi, a, \delta \rangle$ and $\langle \varphi', a, \delta' \rangle$ such that $h \models \varphi$ and $h \models \varphi'$;
  2. $\mathbf{T}(\cdot|h, a) = \delta(\cdot)$ for some triple $\langle \varphi, a, \delta \rangle$ with $h \models \varphi$.
- Reward function:
  1. (mutual exclusivity) for every history h and every action a there do not exist two distinct triples $\langle \psi, a, \rho \rangle$ and $\langle \psi', a, \rho' \rangle$ such that $h \models \psi$ and $h \models \psi'$;
  2. $\mathbf{R}(\cdot|h, a) = \rho(\cdot)$ for some triple $\langle \psi, a, \rho \rangle$ with $h \models \psi$.

From LDL$_f$ to transducer

- We can compute the transducer version of the specified RDP.
- The transition transducer $T_{\mathbf{T}} = \langle S, s_0, \Sigma, \tau, \Gamma, \theta \rangle$ is built as follows:
  - Consider the formulas $\varphi_1, \ldots, \varphi_n$ for the transition specifications
  - Let $\mathcal{A}_1, \ldots, \mathcal{A}_n$ be the automata for $\varphi_1, \ldots, \varphi_n$
  - Let $\mathcal{A}$ be the cross product of such automata
  - S, s_0, Σ, τ are the ones of $\mathcal{A}$
  - For each state $s = \langle s_1, \ldots, s_n \rangle$ of $\mathcal{A}$, there is at most one s_i that is a final state in $\mathcal{A}_i$, due to mutual exclusivity.
    - The output $\theta(s)$ of state s is the function that maps each action a to the distribution δ as specified by some triple $\langle \varphi_i, a, \delta \rangle$.
- The reward transducer $T_{\mathbf{R}}$ is built similarly.

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Motivation

- Recall the importance of the transition function τ
- Learning τ means learning the states S and how to transitions among them
- The semantics of states is probabilistic: a state means the probability it determines over its possible futures
- Learning τ as the transition function of a Probabilistic-Deterministic Finite Automaton (PDFA)

Probabilistic-Deterministic Finite Automata (PDFA)

$$\mathcal{A} = \langle S, \Sigma, \tau, \lambda, \zeta, s_0 \rangle$$

- S is a finite set of states;
- Σ is a finite input alphabet;
- $\tau : S \times \Sigma \to S$ is the deterministic transition function;
- $\lambda : S \times (\Sigma \cup \{\zeta\}) \to [0, 1]$ is the probability of emitting the next symbol or stopping;

Extension of λ to strings:

$$\lambda(s, \sigma_1\sigma_2 \dots \sigma_n) = \lambda(s, \sigma_1) \cdot \lambda(\tau(s, \sigma_1), \sigma_2 \dots \sigma_n) \quad \text{with} \quad \lambda(s, \varepsilon) = 1$$

Automaton $\mathcal{A}$ represents the following probability distribution on Σ^*:

$$\mathcal{A}(w) = \lambda(s_0, w\zeta)$$

RDPs seen as PDFA

$$\mathcal{R} = \langle A, O, R, \mathbf{T}, \mathbf{R}, \gamma \rangle$$
$$T = \langle S, s_0, O, \tau, \Gamma, \theta \rangle$$
$$\mathcal{A} = \langle S, \Sigma, \tau', \lambda, \zeta, s_0 \rangle$$

- Agent explores stopping with probability $p > 0$ and choosing an action uniformly at random otherwise.
- alphabet $\Sigma = AOR$,
- transitions $\tau'(s, aor) = \tau(s, o)$,
- probability function:
  - $\lambda(s, aor) = \frac{1-p}{|A|} \cdot \theta(s)(a, o, r)$,
  - $\lambda(s, \zeta) = p$.

Learning RDPs

- PDFA can be learned
- Thus, we can learn the PDFA encoding the RDP and extract the transition function (which is what we are interested in)
- In particular, the transducer's transition function τ can be obtained from the PDFA's transition function τ' by dropping actions and rewards, which have no effect on transitions.
- Application: reinforcement learning

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