Estimating The Entangling Power of a Two-Qubit Gate from Measurement Data:

artificial neural networks and randomized measurements versus standard tomography methods

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Motivation and Objectives

The goal of this project is to develop efficient techniques to quantify the entangling ability of two-qubit gates directly from measurement data with less measurement settings than full quantum tomography.

(A) The first method is to develop a neural network trained and tested with data of 48 measurement settings only. The measurement data are imperfect such that they simulate real experimental data.

(B) The second is to consider statistical methods based on a few samples of locally randomized state preparations and measurements.

Entangling Power

Entangling power is a quantity that indicates how much entanglement can a certain quantum gate ***U*** produces, applied to a two-qubit product state and averaged over all input product states [1]. It is defined according to some entanglement measure ***E*** as:

$$e_p(U) = \langle E\left(U\,|\psi_A\rangle \otimes |\psi_B\rangle\right)\rangle_{|\psi_A\rangle\otimes|\psi_B\rangle}$$

For *linear entropy* entanglement measure:

- ***ep*** Ranges between **0** for non-entanglers and **2/9** for perfect entanglers.
- ***ep*** is invariant under local unitary operations.
- A gate and its inverse have equal ***ep***.

U	$\frac{9}{2}e_p(U)$
$U_1 \otimes U_2$	0
CNOT	1
iSWAP	1
B	1
SWAP	0
$\sqrt{SWAP}$	3/4
$SWAP^{\theta}$	$\frac{3}{4}\sin^2(\pi\theta)$

Table 1: Entangling power of basic two-qubit quantum gates.

Method (A): Neural Networks

A *neural network* is a computational system consisting of a group of connected nodes called *neurons*. The basic structure is an input layer, hidden layers, and an output layer of neurons. The output neuron's value **y** is the weighted sum of the input neuron values with a non-linear function **f** applied to it [2]:

$$f(y) = f\left(\sum_i w_i x_i\right)$$

- 130,000 training data samples.
- 20,000 test data samples.
- 48 measurement settings and 1000 shots per measurement setting.

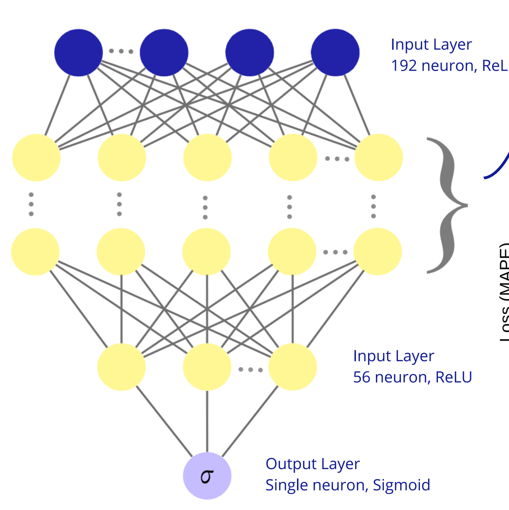

Figure 1: My thesis' neural network structure trained by supervised learning.

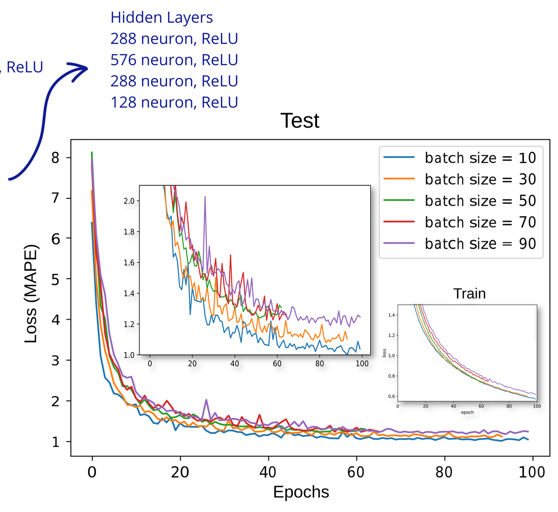

Figure 2: Training of the neural network with mean absolute percentage loss function (MAPE).

Quantum Tomography

Quantum tomography is the task to reconstruct a full description of a possibly unknown quantum state or gate from measurements [3]. The most common tools are: Maximum likelihood estimation, Least-squares fitting and compressed sensing.

Compressed sensing can reconstruct sparse data (e.g. images, signals, and matrices) using a small measurement set [4]. The recovered matrix ***X*** is the solution to the optimization problem:

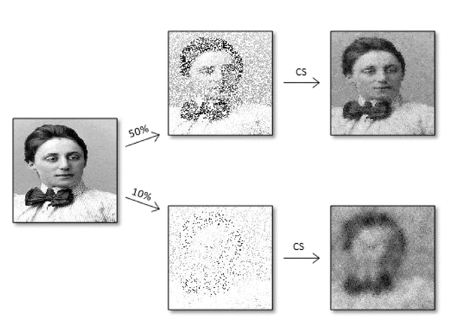

Figure 3: 50% and 10% of Emmy Noether's image reconstructed by compressed sensing.

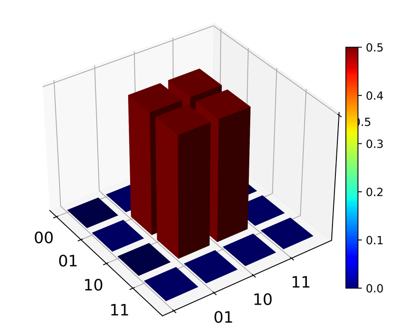

Figure 4: The Choi-matrix of a bit-flip gate reconstructed via compressed sensing.

NP-hard

$$\text{minimize}: \ ||\mathbf{X}||_0 \qquad \text{subject to: } \mathbf{A}\vec{X} = \vec{b}$$

feasible

$$\text{minimize}: \ ||\mathbf{X}||_1 \qquad \text{subject to: } ||\mathbf{A}\vec{X} - \vec{b}||_2 \leq \epsilon$$

Method (B): Randomized Measurements

The *randomized measurements* approach is to apply many local unitaries ***V*** to the input states and ***W*** to some fixed local measurements (e.g. Pauli) to obtain the probability distribution of the correlation functions [5]. This method avoids sharing a common classical reference frame between parties and is not restricted to measuring only fixed local observables.

For two-qubit systems, with a single randomized measurement we get the correlation function:

$$\mathbb{E} = \mathrm{tr}\left[U(V_1 \otimes V_2)\rho_0(V_1^\dagger \otimes V_2^\dagger)U^\dagger(W_1^\dagger \sigma_z^1 W_1 \otimes \mathbb{I})\right]$$

which can be characterized using statistical moments:

$$\mathcal{R}^{(t)} = \int_{Haar} d_{W_1} d_{V_1} d_{V_2} \mathbb{E}^t.$$

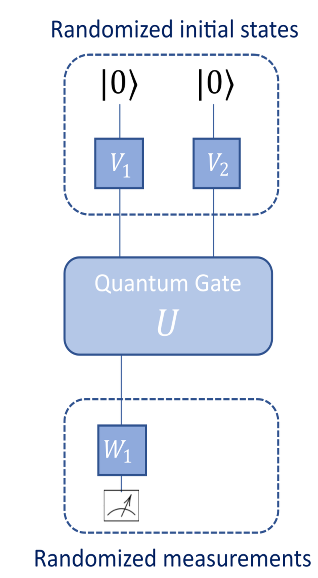

Figure 5: Entangling power estimation via randomized measurments.

RESULT (A):

The neural network is compared to compressed sensing in estimating the entangling power of two-qubit gates directly from reduced and imperfect measurement data:

- Estimations via neural network show a **great match** with the true values.
- The pattern seems **not to have outliers** but the left end of the pattern is shifted upwards. This could be due to bias in the training dataset.
- Compressed sensing shows a very weak fit to the true values with plentiful outliers. This is due to systematic errors imposed by constraints (e.g positive semi-definiteness) [6].

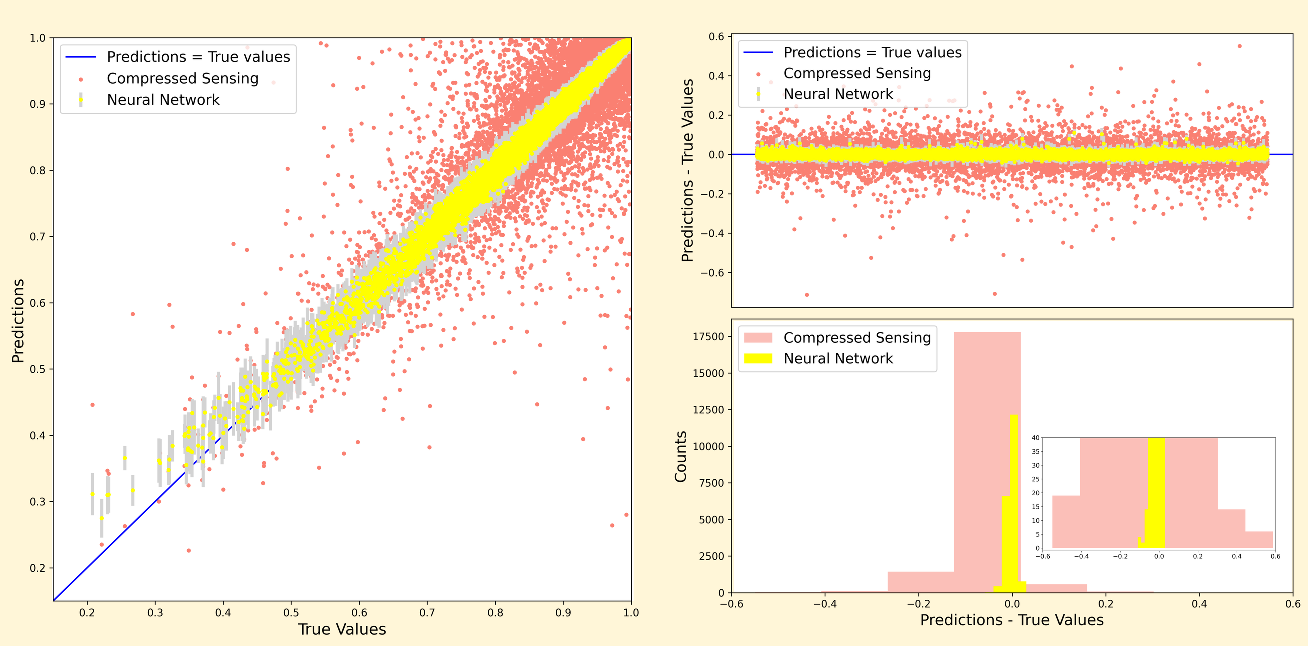

Figure 6: Right: neural networks **outperform compressed sensing** in estimating the entangling power of two-qubit gates from reduced and imperfect measurement data. Left: the differences between predicted and true values are concentrated around zero for a neural network while they are spread around small values near zero for compressed sensing as shown also by the histograms.

U	$e_p^{est}(U)$
$U_1 \otimes U_2$	0.221156 ± 0.035962
CNOT	0.997445 ± 0.0018249
iSWAP	0.998397 ± 0.00069416
B	0.995628 ± 0.0012779
SWAP	0.230064 ± 0.02031265
$\sqrt{SWAP}$	0.775614 ± 0.0130337

Table 2: Entangling power estimation via a neural network for basic two-qubit gates. Despite the **high accuracy and precision** in estimating high and intermediate entangling power, it loses its accuracy in estimating low entangling power values. These gates were not part of the training dataset.

RESULT (B):

We derived an equation that relates the second moment of the distribution of correlations to the entangling power:

$$e_p = \frac{3}{2}\mathcal{R}^{(2)} + \frac{1}{2}$$

- The determination of the entangling power of a unitary gate using randomized measurements requires measuring only a single qubit of the output two-qubit pair which **reduces the effort of measurements**.
- The correlation statistics form a histogram that can **classify** quantum gates into entanglers (peaked) or non-entanglers (flat) and can **quantify** the entangling power with a reasonable number of random measurement settings.

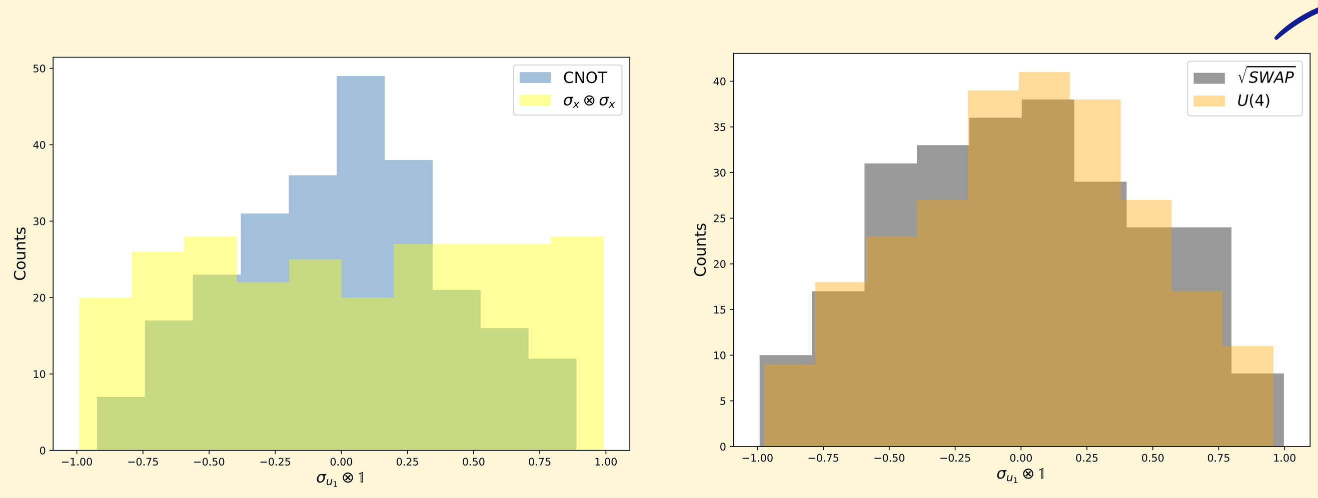

Figure 7: Determination of entangling power of different two-qubit quantum gates using the statistics of random measurements. For each gate, the expectation values (counts) are obtained by measuring random input states of a single qubit in randomly chosen local directions. The four histograms are of the 250 counts observed for CNOT (blue), local spin-flips (yellow), square root SWAP (gray), and a non-local Haar random unitary gate (orange). The entangling power of each gate is obtained from the variance of the statistical data. Estimated values are in Table 3.

U	$e_p^{est}(U)$
CNOT	1.017 ± 0.009360
$\sigma_x^1 \otimes \sigma_x^2$	-0.01746 ± 0.01203
$\sqrt{SWAP}$	0.7651 ± 0.009736
$U(4)$	0.8962 ± 0.007855

Table 3: Entangling power estimation via randomized measurements for three well-known quantum gates and a haar random unitary U(4) with an actual entangling power of 0.9039. This technique is accurate but might yield non-physical results (i.e. negative and above 1) of entanling power.

Conclusion and Future work

In this project, I developed two new and efficient approaches for estimating the entangling power of two-qubit gates directly from measurement data with less effort and resources than full quantum tomography. The first method is using a neural network and it shows a very accurate performance due to its ability in recognizing patterns and fine details in raw data. The second method is randomized measurements which does not require careful alignment of measurement apparatus and it turned out to require measuring a single qubit only.

Future work could be to develope an algorithm that in addition to the determination of the entangling power, it finds the input quantum states that produce the maximum entanglement. Moreover, I aim to test the performance of the neural network on real experimental data. Another idea is to extend these methods to multi-qubit and non-unitary quantum gates.

Acknowledgement

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