Estimation Project

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2. The likelihood function given observed data y=6 and sample size n=10.

Ans:

$f(y|n,w) = \frac{n!}{y!(n-y)!} w^y (1-w)^{n-y}$, so

$L(w|n=10, y=6) = f(y=6|n=10, w\ ) = \frac{10!}{6!4!} w^6 (1-w)^4, 0 \leq w \leq 1$

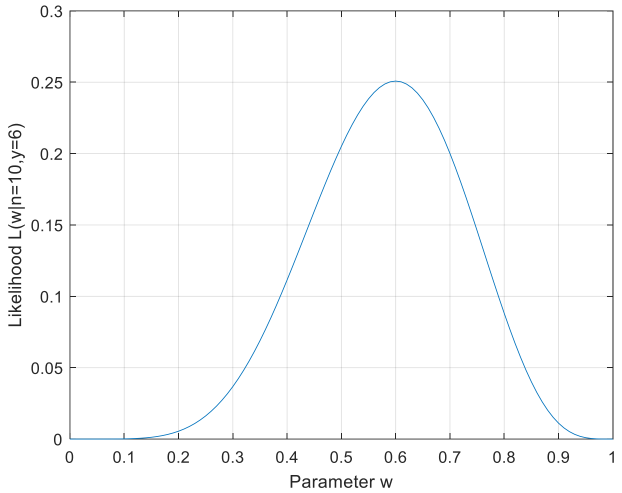

Fig1

3. Determine and plot the PMF of y with a given w=0.6.

Ans:

$f(y|n=10, w=0.6) = \frac{10!}{y!(10-y)!} (0.6)^y (0.4)^{10-y}, y = 0,1, \ldots ,10$

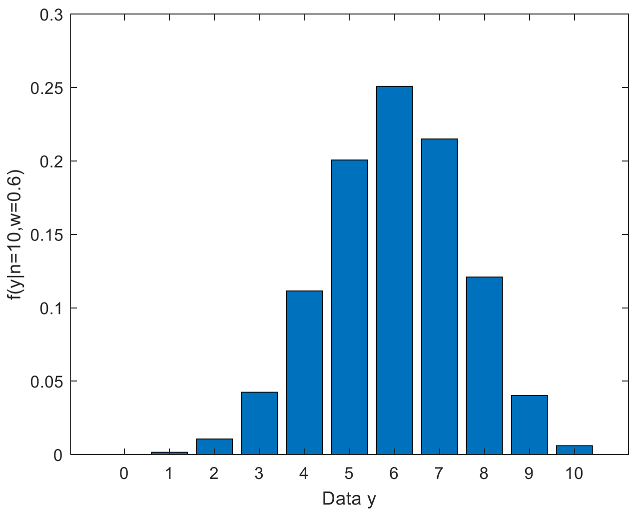

Fig2

What are the differences between the PMF and the associated likelihood function?
Ans:

Likelihood function: $L(w|n=10, y=6) = \frac{10!}{6!4!} w^6(1-w)^4, 0 \leq w \leq 1$

PMF: $f(y|n=10, w=0.6) = \frac{10!}{y!(10-y)!}(0.6)^y(0.4)^{10-y}, y = 0,1,\ldots,10$

Likelihood function is a function of the parameter w given a particular set of observed data, defined on the **parameter scale** $0 \leq w \leq 1$, which is **continous**.

PMF is a function that specifies the probability of observing data given a particular set of parameter values. In this case, PMF is a function of observing data y, which is **discrete**, with a particular set of parameters w and n.

4. What is the maximum likelihood (ML) estimate of w for this observed data (y) and sample size (n)?
Ans:

$$\frac{dlnL(w|n=10, y=6)}{dw} = \frac{6}{w} - \frac{4}{1-w} = \frac{6-10w}{w(1-w)} = 0 \Rightarrow w = 0.6$$

$$\frac{d^2lnL(w|n=10, y=6)}{dw^2}|_{w=0.6} = -\frac{6}{w^2} - \frac{4}{(1-w)^2} = -41.67 < 0$$

$w = 0.6$ is the answer, we can also check the answer from the **Fig1**, where the maximum point is at $w = 0.6$.

5. For this estimated value of w, what is the expected value of y for a sample size of n=10? Use this answer to explain why the ML estimate in part 3 is intuitively plausible.
Ans:

The expected value: $E[y] = \sum_i y_i p_i = \sum_{y=0}^{10} y * \frac{10!}{y!(10-y)!}(0.6)^y(0.4)^{10-y} = 6$

The expected value of y is **6** with sample size n=10, it means that the average value of the number of successes in this sequence of 10 Bernoulli trials is 6.
This result implies that the average probability of success in each trial is 0.6, which has the same value with our estimate w=0.6.