- For AWGN channel, $\underline{h} = \delta[n] = [1]$, $\underline{Q} = \underline{I}_{64\times 64}$ (assume 64 subcarriers)

$\underline{y} = \underline{Q}\cdot\underline{x} + \underline{w}$, $\underline{x}$: An OFDM symbol, $\underline{y}$: received signal, $\underline{w} \sim CN(0, 2\sigma^2\underline{I})$

After FFT: ($\underline{\tilde{F}}$: Unitary FFT matrix)

$$\underline{Y} = \underline{\tilde{F}}\,\underline{y} = \underline{\tilde{F}}(\underline{Q}\,\underline{x} + \underline{w}) = \underline{\tilde{F}}(\underline{\tilde{F}}^H \underline{H}\,\underline{\tilde{F}}\,\underline{x} + \underline{w}) = \underline{H}\,\underline{X} + \underline{\overline{W}}$$

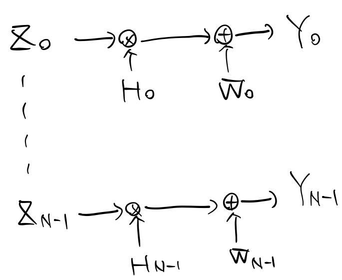
where $\underline{H}$ is a diagonal matrix, where $\underline{H} = \mathrm{diag}(\mathrm{DFT}\{\underline{h}\})$

Find $\underline{\overline{W}}$'s mean and covariance matrix:

$\underline{w} \sim CN(0, 2\sigma^2 I)$, $\underline{\overline{W}} = \underline{\tilde{F}}\,\underline{w}$, then mean : $E[\underline{\overline{W}}] = E[\underline{\tilde{F}}\,\underline{w}] = \underline{0}$,

Covariance matrix: $K_{\overline{W}} = E[\underline{\overline{W}}\,\underline{\overline{W}}^H] = E[\underline{\tilde{F}}\,\underline{w}\,\underline{w}^H\underline{\tilde{F}}^H] = \underline{\tilde{F}}\,2\sigma^2\underline{I}\,\underline{\tilde{F}}^H = \underline{\tilde{F}}\,\underline{\tilde{F}}^H 2\sigma^2\underline{I} = 2\sigma^2\underline{I}$

Since $\underline{H}$ is always diagonal and $\underline{\overline{W}}$ is independent between each channel, every channel can detect independent:



For BPSK, $X_i = \pm 1$, by ML:

$$\frac{Y_i}{H_i} \begin{cases} > 0 \longrightarrow +1 \\ < 0 \longrightarrow -1 \end{cases}$$

One-tap equalizer

Since we only care about real part, define $\mathrm{SNR} \equiv \dfrac{E[|X_i|^2]}{E[|\mathrm{Real}\{\overline{W}_i\}|^2]} = \dfrac{1}{\sigma^2}$

In AWGN, $H_i = 1$, every subcarrier has same BER: $\dfrac{Y_i}{H_i} = X_i + \overline{W}_i$

$$P[\mathcal{E}_i] = \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\rho-1)^2/2\sigma^2}\,d\rho + \frac{1}{2}\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\rho+1)^2/2\sigma^2}\,d\rho$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\rho-1)^2/2\sigma^2}\,d\rho = \int_{\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}}\,d\alpha = Q\left(\frac{1}{\sigma}\right)$$

$= Q(\sqrt{\mathrm{SNR}})$ = Average SNR : Same as BPSK

- In Multipath channel, H_i is different in different subcarrier :

$$\frac{Y_i}{H_i} = X_i + \frac{\overline{W}_i}{H_i}$$, result in different SNR and BER

If define $SNR_i = \dfrac{E[|X_i|^2]}{E[|Real\{\overline{W}_i/H_i\}|^2]} = \dfrac{1}{\frac{1}{|H_i|^2} E[|Real\{\overline{W}_i\}|^2]} = |H_i|^2 \frac{1}{\sigma^2}$

then BER of ith channel $= Q(\sqrt{SNR_i}) = Q\left(\frac{|H_i|}{\sigma}\right)$

Average BER $= \frac{1}{N} \sum_{i=0}^{N-1} Q\left(\frac{|H_i|}{\sigma}\right)$

- If we let $\overline{W} = A + Bj$, where $A = B \sim N(0, \sigma^2)$, and let $H = a+bj$

$$\frac{\overline{W}}{H} = \frac{(A+Bj)(a-bj)}{(a+bj)(a-bj)} = \frac{Aa+Bb}{a^2+b^2} + \frac{-Ab+Ba}{a^2+b^2} j \quad , Real\left\{\frac{\overline{W}}{H}\right\} = \frac{Aa+Bb}{a^2+b^2}$$

$$Var\left(\frac{Aa+Bb}{a^2+b^2}\right) = \frac{a^2}{(a^2+b^2)^2} Var(A) + \frac{b^2}{(a^2+b^2)^2} Var(B) \underset{\substack{\uparrow \\ \text{if A and B} \\ \text{are i.i.d.}}}{=} \frac{a^2+b^2}{(a^2+b^2)^2} Var(A)$$

$$= \frac{1}{a^2+b^2} Var(A) = \frac{1}{|H|^2} Var(A)$$

$$\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 1 \\ \\ \end{bmatrix} \quad [\ 1 \quad 2 \quad 3 \quad 4 \quad 10 \quad \cdots\ ]$$

$$\begin{bmatrix} 1\times & 1\ 2\ 3\ 4 \cdots 10 \\ 1.5\times & 1\ 2\ 3\ 4 \cdots 10 \\ & \end{bmatrix}$$

64×25