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3/8/21
ECE 523

HW #3 note: $\xi = \mathcal{L}$

#1) $\arg\min \{ \frac{1}{2}\|w\|^2 + \frac{C}{2}\sum_{i=1}^{n} \mathcal{L}_i^2$

S.T $y_i(w^T x_i + b) = 1 - \mathcal{L}_i \quad \forall i \in [n]$

form L

$$L(\alpha, \mathcal{L}, w, b) = \frac{1}{2}\|w\|^2 + \frac{C}{2}\sum_{i=1}^{n} \mathcal{L}_i^2 - \sum_{i=1}^{n} \alpha_i [y_i(w^T x + b) - 1 + \mathcal{L}_i]$$

1 $\frac{\partial L}{\partial w} = w - \sum \alpha_i y_i x_i \xrightarrow{=\emptyset} \underline{w = \sum_{i=1}^{n} \alpha_i y_i x_i}$

2 $\frac{\partial L}{\partial b} = -\underline{\sum \alpha_i y_i = \emptyset}$

3 $\frac{\partial L}{\partial \mathcal{L}} = C\mathcal{L} - \alpha_i = \emptyset \rightarrow C\mathcal{L} = \alpha_i \rightarrow C^2 \mathcal{L}^2 = \alpha^2$

$\mathcal{L}^2 = \frac{\alpha^2}{C^2}$

dual form

$$\arg\min \{ \underset{1}{\frac{1}{2}\|w\|^2} + \frac{C}{2}\sum_{i}^{n} \underset{3}{\mathcal{L}^2} - \sum \alpha_i [y_i(w^T x_i + b) - 1 + \underset{1 \,\&\, 3}{\mathcal{L}}] \}$$

$$= \frac{1}{2}\sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i^T x_j + \frac{C}{2}\sum_i^n \frac{\alpha^2}{C^2}\mathcal{L}$$

$$- \sum_i \alpha_i \left[y_i \left(\left(\sum_j \alpha_j y_j x_j \right)^T x_i + b \right) - 1 + \mathcal{L} \right]$$

$$= -\frac{1}{2}\sum^{n}\sum^{m} \alpha_i \alpha_j y_i y_j x_i^T x_j + \frac{1}{2}\sum \alpha_i \mathcal{L}_i - b\left(\cancel{\sum \alpha_i y_i}\right) + \sum \alpha_i - \sum \alpha$$

$$= \frac{1}{2} \qquad - \underbrace{\frac{1}{2}\sum^{n} \alpha_i \mathcal{L}_i}_{\mathcal{L} = \alpha_i/C} + \sum_i \alpha_i$$

$$= \arg\min \{ -\sum_{i=1}^{n} \alpha_i + \frac{1}{2}\sum\sum \alpha_i \alpha_j y_i y_j x_i^T x + \frac{1}{2C}\sum_{i=1}^{n} \alpha_i^2 \}$$

transform

$$\arg\max \{ \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} \alpha_i \alpha_j y_i y_j x_i^T x - \frac{1}{2C}\sum_{i=1}^{n} \alpha_i^2 \}$$

S.T $\alpha_i \geq 0, \forall i = [n], \sum \alpha_i y_i = \emptyset$

sol: $\overline{\arg\max_{\alpha_i}} \{ \sum \alpha_i (1-\beta y_i w_s^T x_i) - \frac{1}{2} \sum_i^n \sum_j^n \alpha_i \alpha_j y_i y_j x_i^T x_j$

#2

$$\arg\min_{w_T \mathcal{L}} \{ \frac{1}{2} \|w_T\|^2 + C \sum_{i=1}^n \mathcal{L}_i - B w_T^T w_s \}$$

$$\text{S.T. } y_i(w_T^T x_i + b) \geq 1 - \mathcal{L}_i \quad \& \quad \mathcal{L}_i \geq 0$$

$$L = \frac{1}{2}\|w_T\|^2 + C \sum_i^n \mathcal{L} - B w_T^T w_s - \sum \alpha_i [y_i(w_T^T x_i + b) - 1 + \mathcal{L}_i] - \sum \mu_i \mathcal{L}_i$$

$$\frac{\partial L}{\partial \mathcal{L}} = C - \sum \alpha_i - \sum \mu_i = 0 \rightarrow C = \alpha_i + \mu_i$$

$$\frac{\partial L}{\partial w_T} = w_T - B w_s - \sum \alpha_i y_i x_i = 0 \rightarrow w_T = B w_s + \sum_{i=1}^n \alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} = -\sum \alpha_i y_i = 0 \rightarrow \boxed{\sum_i^n \alpha_i y_i = 0}$$

$$\mathcal{L}_i \mu_i = \emptyset$$

$$\frac{\partial L}{\partial w_s} = -B w_T = \emptyset$$

$$\frac{1}{2}\|w_T\|^2 = \frac{1}{2} \| B w_s + \sum \alpha_i y_i x_i \|^2 = \frac{1}{2} B w_s^2 + \frac{1}{2} \sum \sum \alpha \alpha y y x^T x$$

$$\sum \alpha_i [y_i(w_T^T x_i + b) - 1 + \mathcal{L}_i] \rightarrow \sum \alpha_i [y_i (B w_s^T x_i + \sum \alpha y_i x_i^2 + b) - 1 + \mathcal{L}_i]$$

$$\rightarrow \sum \alpha_i y_i B w_s^T x_i + \sum \sum \alpha_i \alpha_j y_i y_j x_i^T x_j + \sum \alpha_i y_i b - \sum \alpha_i + \sum \alpha$$

$$L = \rightarrow -\sum \alpha_i (1 - \beta w_s^T x_i y_i) + \frac{1}{2} \sum \sum \alpha_i \alpha_j y_i y_j x_i^T x_j + C \sum \mathcal{L}$$

$\rightarrow B w_s^T w_s$ & $\frac{1}{2} B w_s^2$ / go away b/c co

$$\sum \sum \mathcal{L} \alpha^{\emptyset} + \sum \cancel{\mathcal{L} \mu}^{\emptyset}$$

domain shift (-)

$$\overline{\arg\min} \{ -\sum \alpha (1 - \beta w_s^T x_i y_i) + \frac{1}{2} \sum \sum \alpha_i \alpha_j y_i y_j x_i^T x_j \}$$

$$\arg\max \{ \sum \alpha_i (1 - \beta y_i w_s^T x_i) - \frac{1}{2} \sum \sum \alpha_i \alpha_j y_i y_j x_i^T x_j \}$$

$$\sum \alpha_i y_i = 0$$

$$0 \leq \alpha_i \leq C$$

```python
# Samuel Freitas
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# problem #3

import numpy as np
import sklearn
import matplotlib.pyplot as plt
from numpy import genfromtxt
from sklearn import svm
from numpy.random import rand
import cvxopt
from cvxopt.solvers import qp
from cvxopt import matrix

def plot_2D_labeled_data(X,y,fig_number,fig_title):
    # put plt.ioff() and plt.show() at end
    plt.ion()
    f = plt.figure(fig_number)
    plt.scatter(X[:,0],X[:,1],c=y)
    plt.axis('equal')
    plt.title(fig_title)
    f.show()

# read in data from csv, split into data and labels
source_csv = genfromtxt('source_train.csv', delimiter=',')
source_labels = source_csv[:,2]
source_data = source_csv[:,0:2]
target_csv = genfromtxt('target_train.csv', delimiter=',')
target_labels = target_csv[:,2]
target_data = target_csv[:,0:2]

# compute the first svm ws
C = 10
B = 1
n = len(target_labels)
svc_source = svm.SVC(kernel='linear', C=C).fit(source_data, source_labels)
Ws = svc_source.coef_[0]

WsT = np.full((50,2),Ws)
q = matrix((target_labels.dot(WsT)).dot(target_data.T))
```
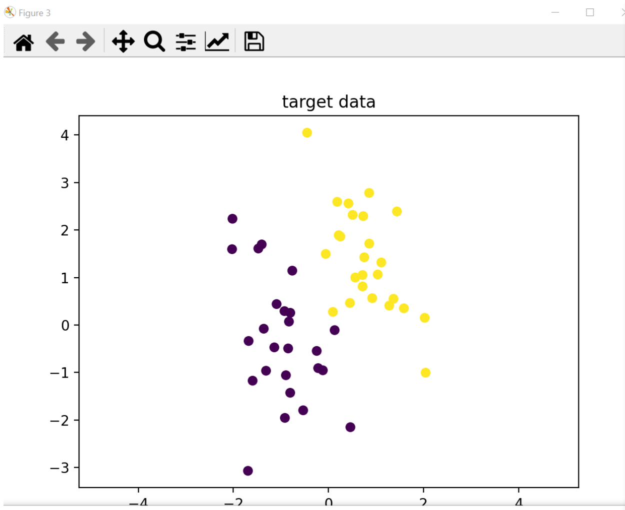
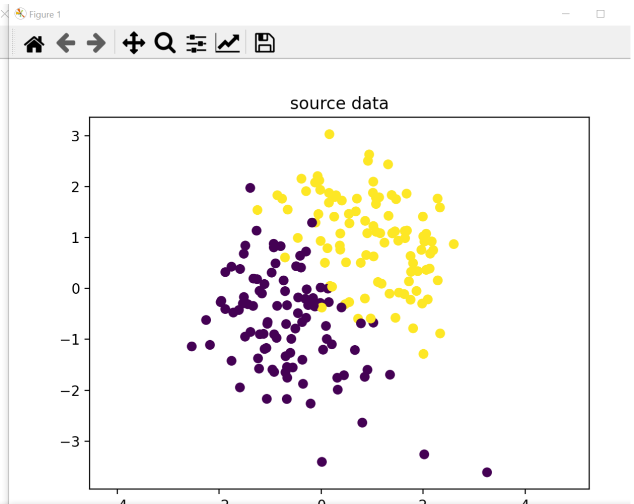
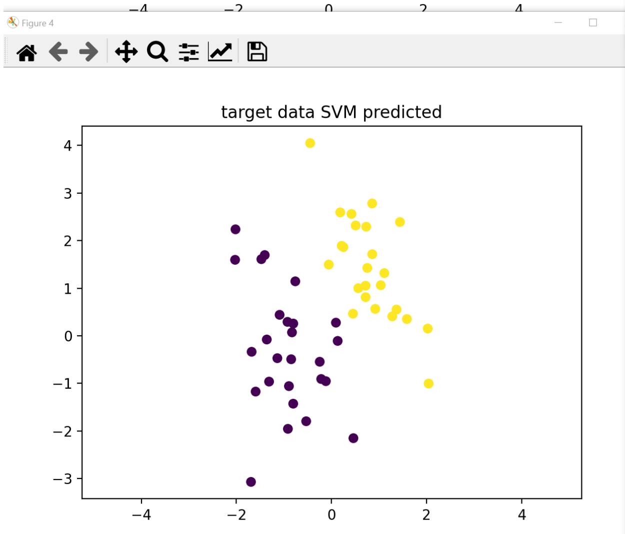
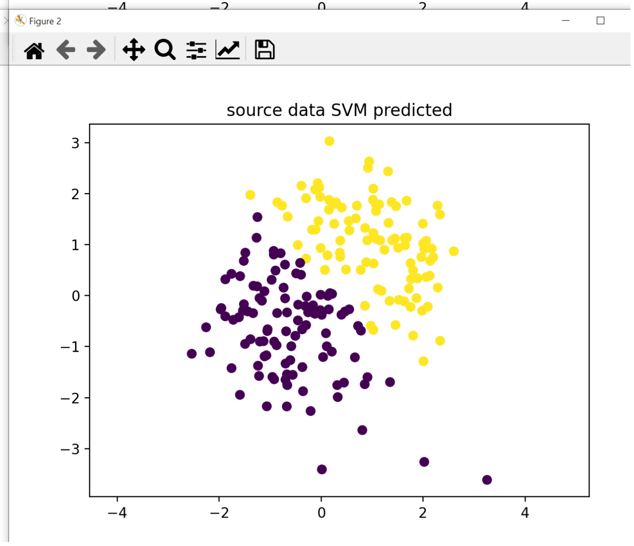

```
G = matrix(0.0, (n,n))
G[::n+1] = -1.0
h = matrix(0.0, (n,1))
A = matrix(1.0, (1,n))
b = matrix(1.0)

# get values for the
P = matrix(np.zeros((len(target_data),len(target_data))))
for i in range(len(target_data)):
    for j in range(len(target_data)):
        a1 = target_labels[i]*target_labels[j]
        a2 = (target_data[i].T).dot(target_data[j])
        P[i,j] = a1*a2
# solve for wt
solv = qp(P,q,G,h)

ai = np.asarray(solv['x'])

np.argmin(np.asarray(solv['x']))

svc_target = svm.SVC(kernel='linear', C=C).fit(target_data, target_labels)
Wt = svc_target.coef_[0]


# after solving for the wt

plot_2D_labeled_data(source_data,source_labels,1,'source data')
plot_2D_labeled_data(source_data,svc_source.predict(source_data),2,'source data S
VM predicted')

plot_2D_labeled_data(target_data,target_labels,3,'target data')
plot_2D_labeled_data(target_data,svc_target.predict(target_data),4,'target data S
VM predicted')

# print the weights ws and wt
print('Ws:',Ws)
print('Wt:',Wt)

plt.ioff()
plt.show()
```










```
     pcost       dcost       gap    pres   dres
 0: -2.0512e-28 -1.4211e-14  5e+01  7e+00  7e+00
 1:  2.8358e-01 -9.4925e-01  8e+00  1e+00  1e+00
 2:  2.9933e-01 -5.7936e-01  4e+00  4e-01  4e-01
 3:  2.3067e-01 -1.0106e+00  3e+00  3e-01  3e-01
 4:  7.4446e-02 -1.1538e+00  2e+00  1e-01  1e-01
 5:  4.1956e-05 -7.8778e-02  8e-02  6e-03  6e-03
 6:  4.2686e-09 -7.9624e-04  8e-04  6e-05  6e-05
 7:  4.5774e-13 -7.9629e-06  8e-06  6e-07  6e-07
 8:  5.0795e-15 -7.9629e-08  8e-08  6e-09  6e-09
Optimal solution found.
Ws: [2.3138911  2.14154075]
Wt: [2.74506314 1.95314389]
```