4)

$$mg\sin\theta = k\alpha\left(1 - \frac{L}{\sqrt{x^2+a^2}}\right)$$

To reduce to form:

$$1 - \frac{h}{u} = \frac{R}{\sqrt{1+u^2}}$$

$$\Rightarrow \quad 1 - \frac{mg\sin\theta}{k}\left(\frac{1}{x}\right) = \frac{L}{a\sqrt{1+\left(\frac{x}{a}\right)^2}}$$

$$\Rightarrow \quad 1 - \frac{mg\sin\theta}{ak}\left(\frac{a}{x}\right) = \frac{(L/a)}{\sqrt{1+\left(\frac{x}{a}\right)^2}}$$

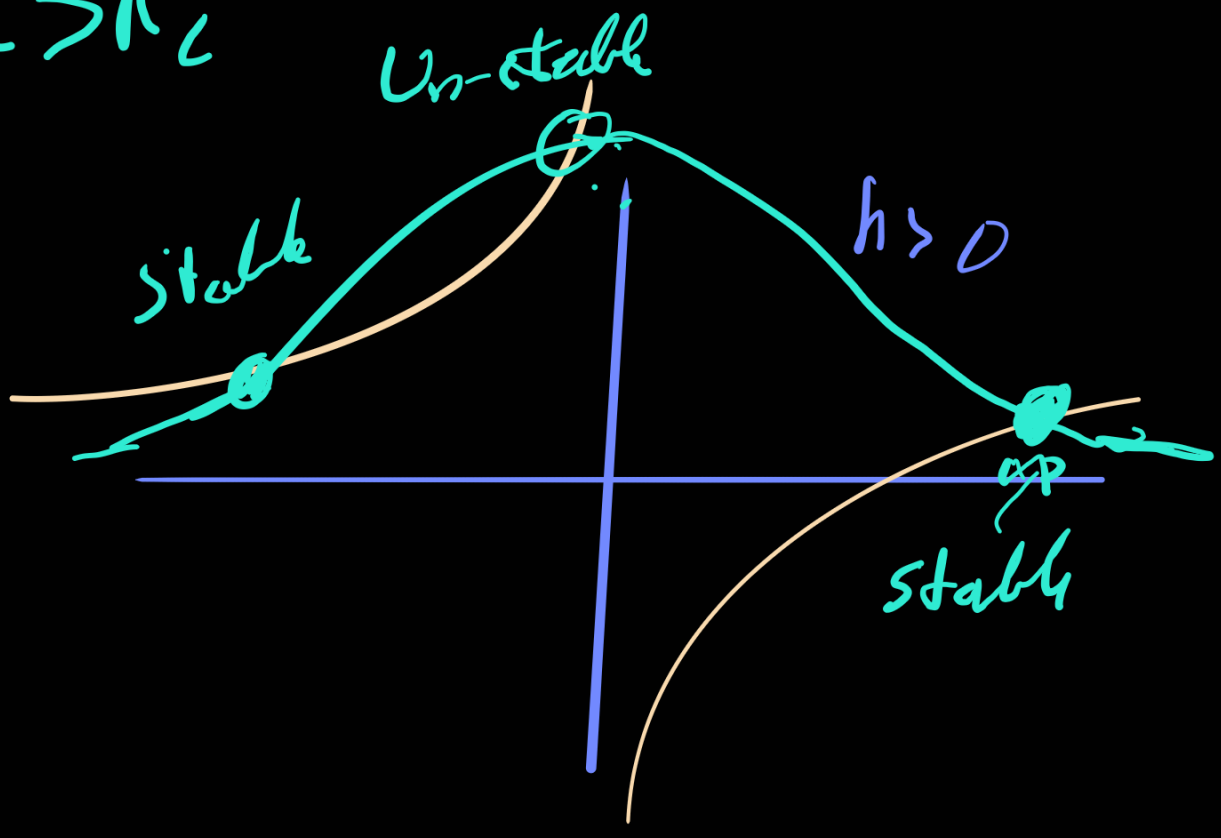
Let $\frac{x}{a} = v$, $\frac{mg\sin\theta}{ak} = h$, $\frac{L}{a} = R$

$$\Rightarrow 1 - \frac{h}{U} = \frac{R}{\sqrt{1+U^2}}$$

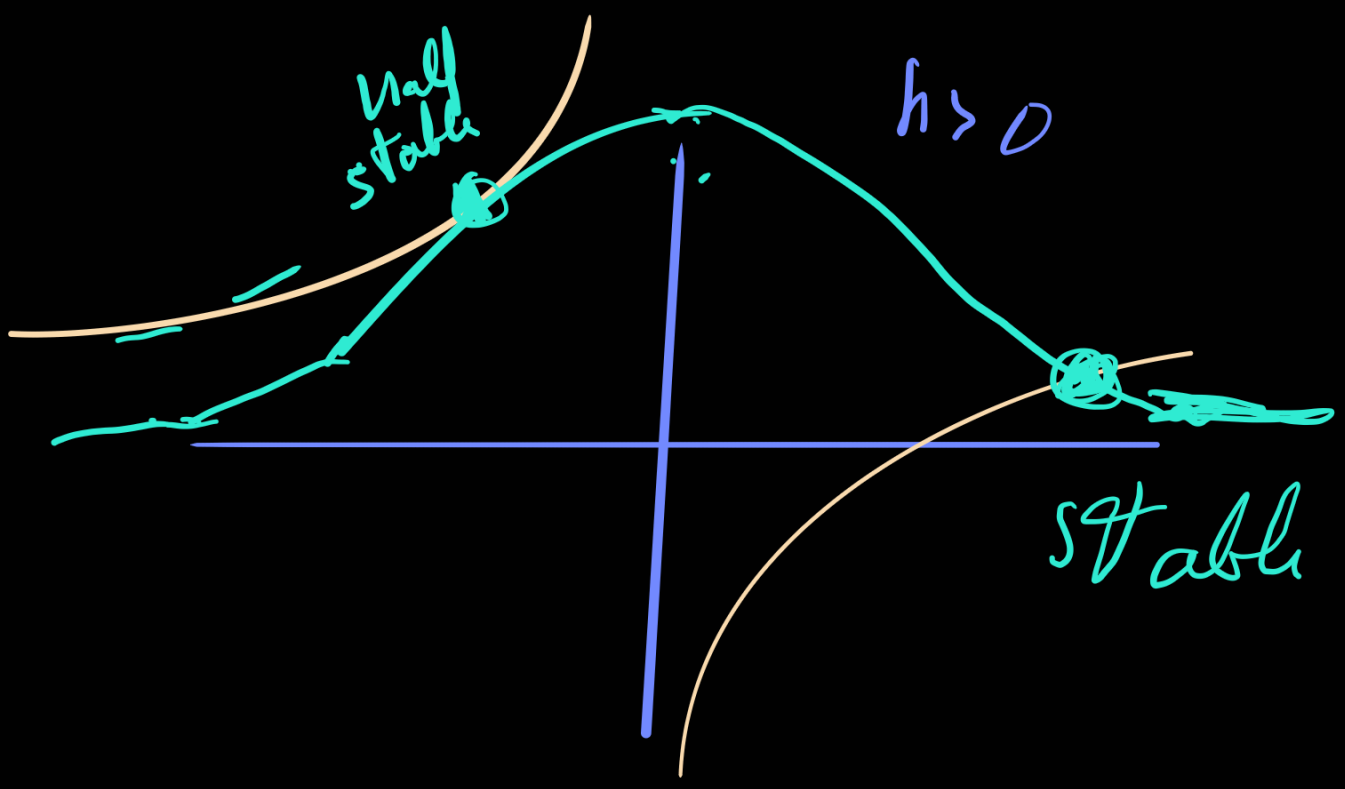
b)

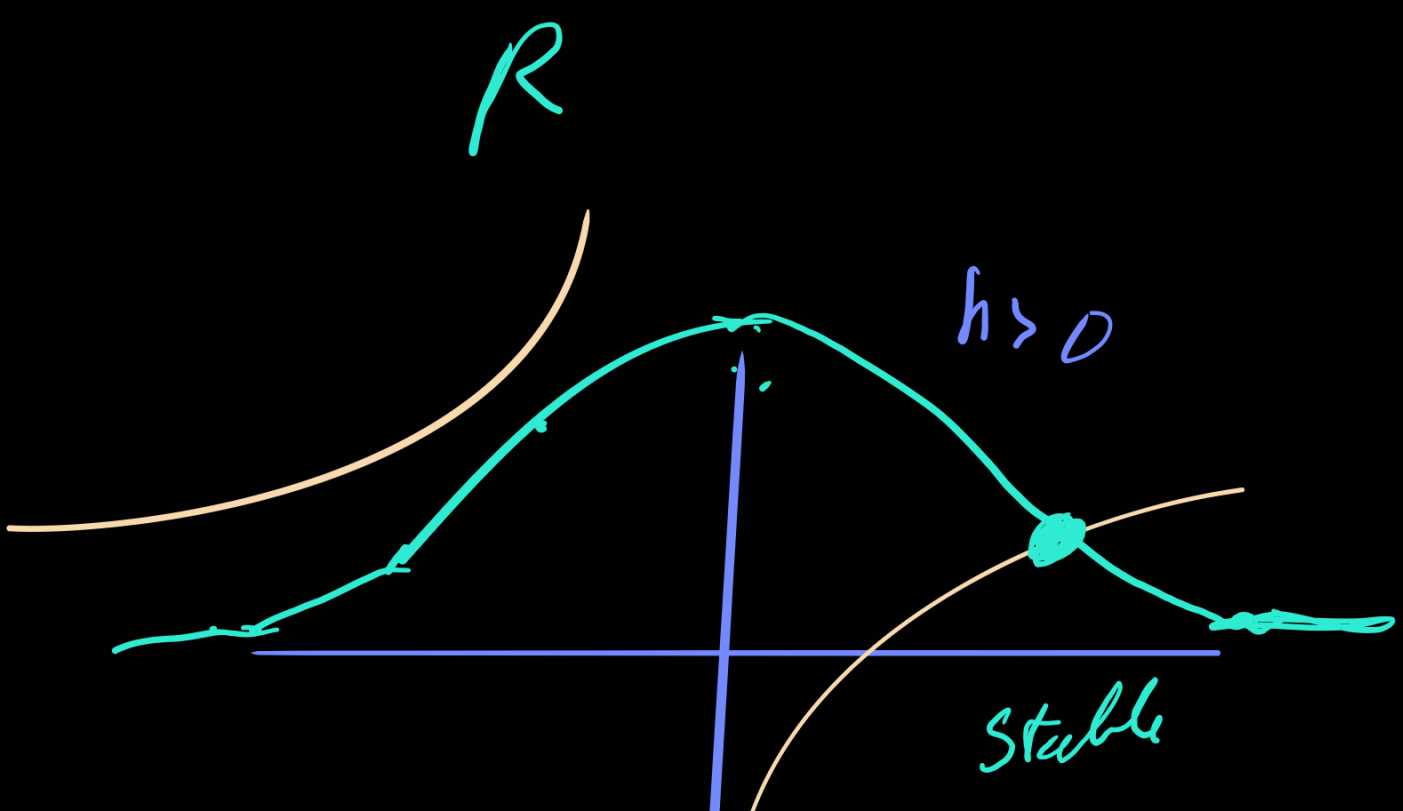
Graph of $1 - \frac{h}{U}$ vs $\frac{R}{\sqrt{1+U^2}}$

$R > R_c$



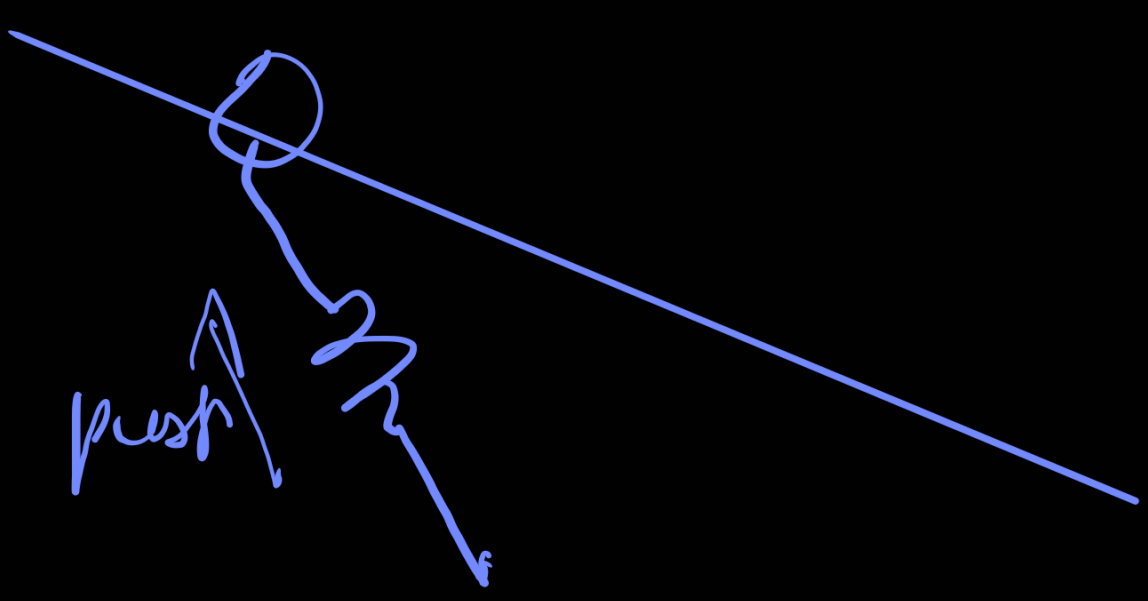
$R = R_c$

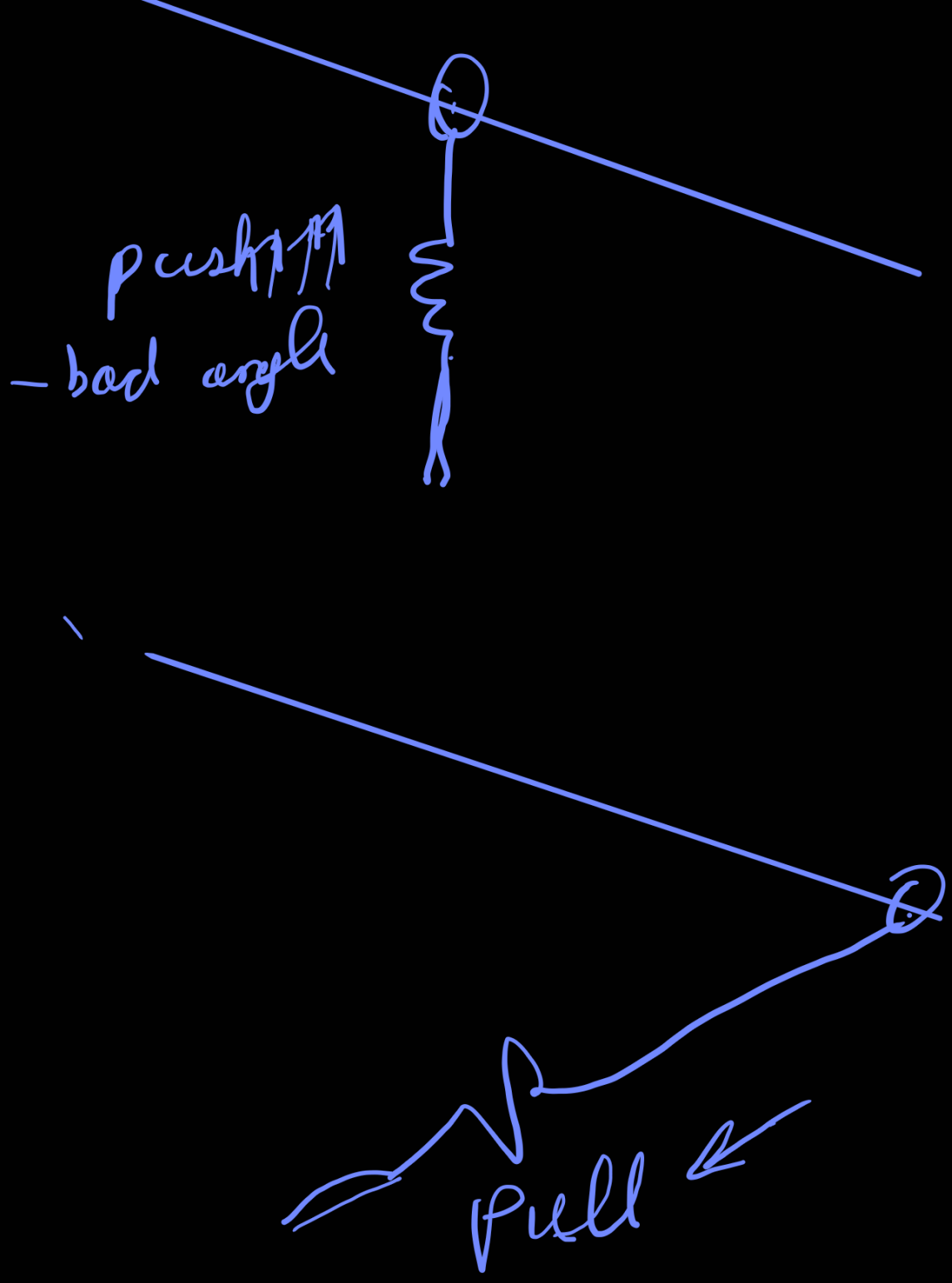

Half stable
h > 0
stable


R
h > 0
Stable

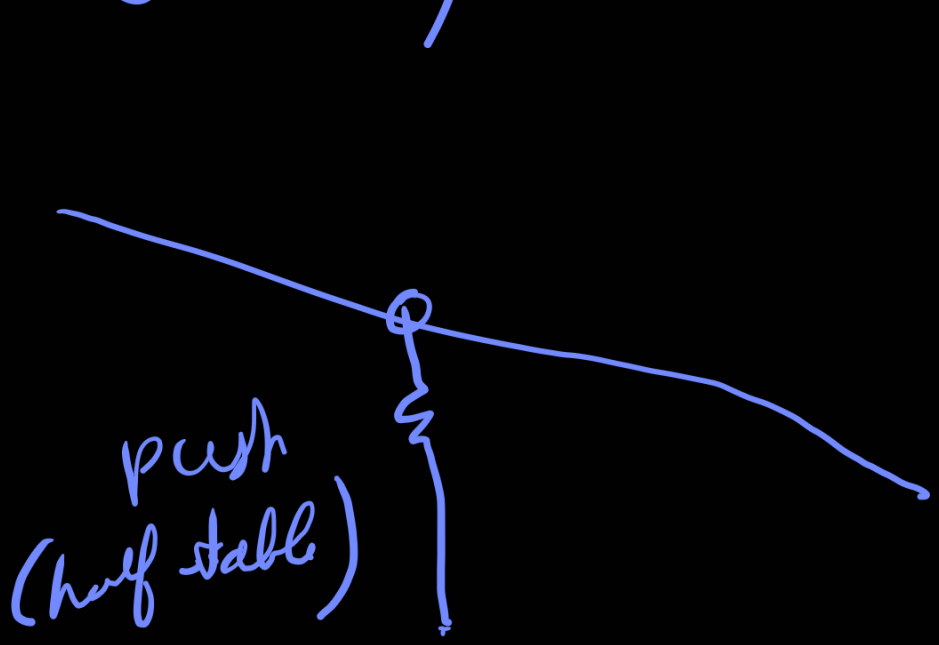
7) Graphs attached

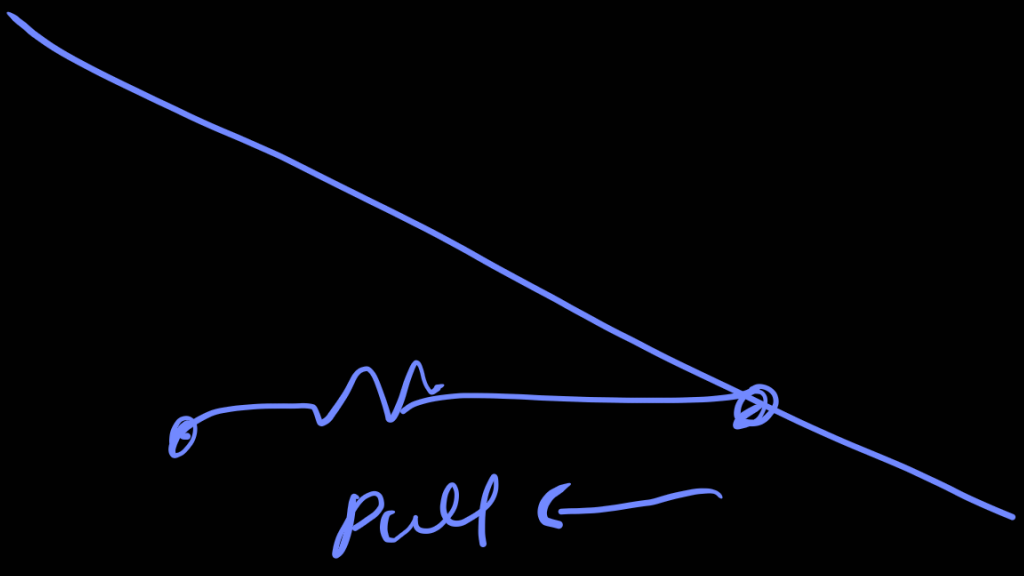
Case 1) Three f.b



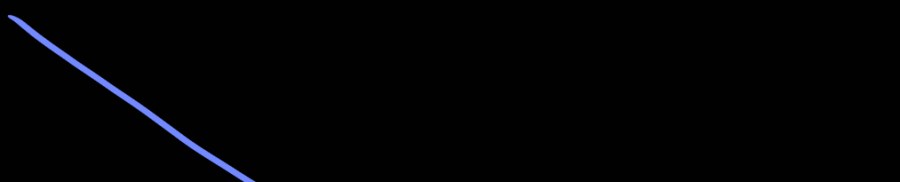


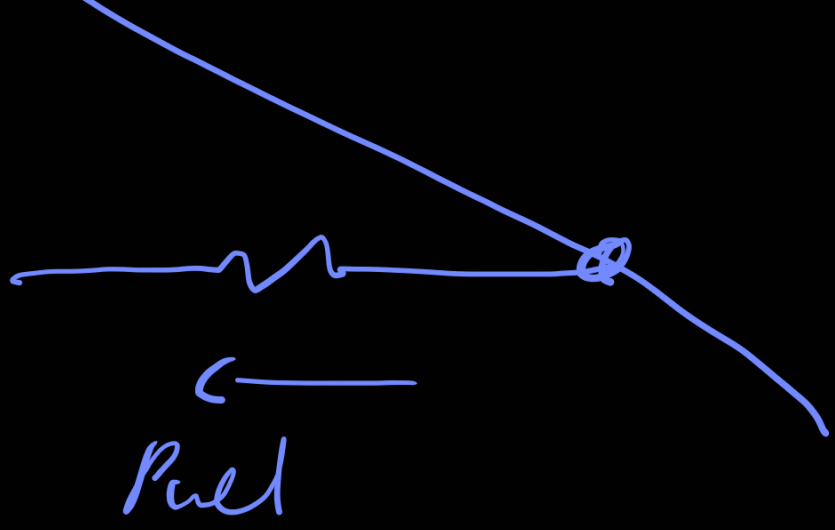
(case 2)





(case 3)




Pull

(a) Equations:

$$b\dot{\theta} + mgL\sin\theta = \Gamma$$

To reduce to:

$$\theta' = \gamma - \sin\theta$$

$$\Rightarrow \frac{b}{mgL}\dot{\theta} + \sin\theta = \frac{\Gamma}{mgL}$$

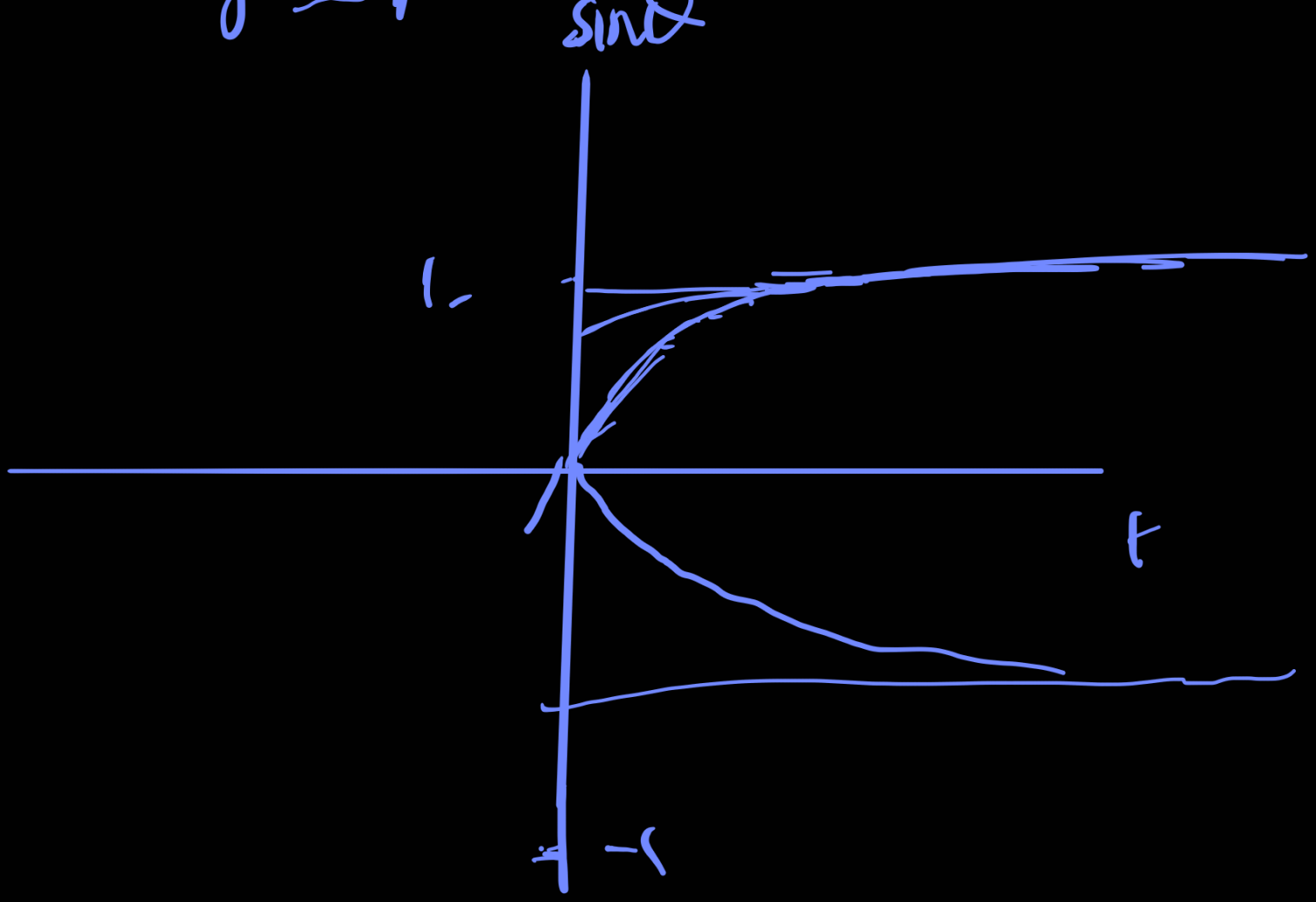
$$\frac{b}{mgL}\frac{d\theta}{dt} = \frac{\Gamma}{mgL} - \sin\theta$$

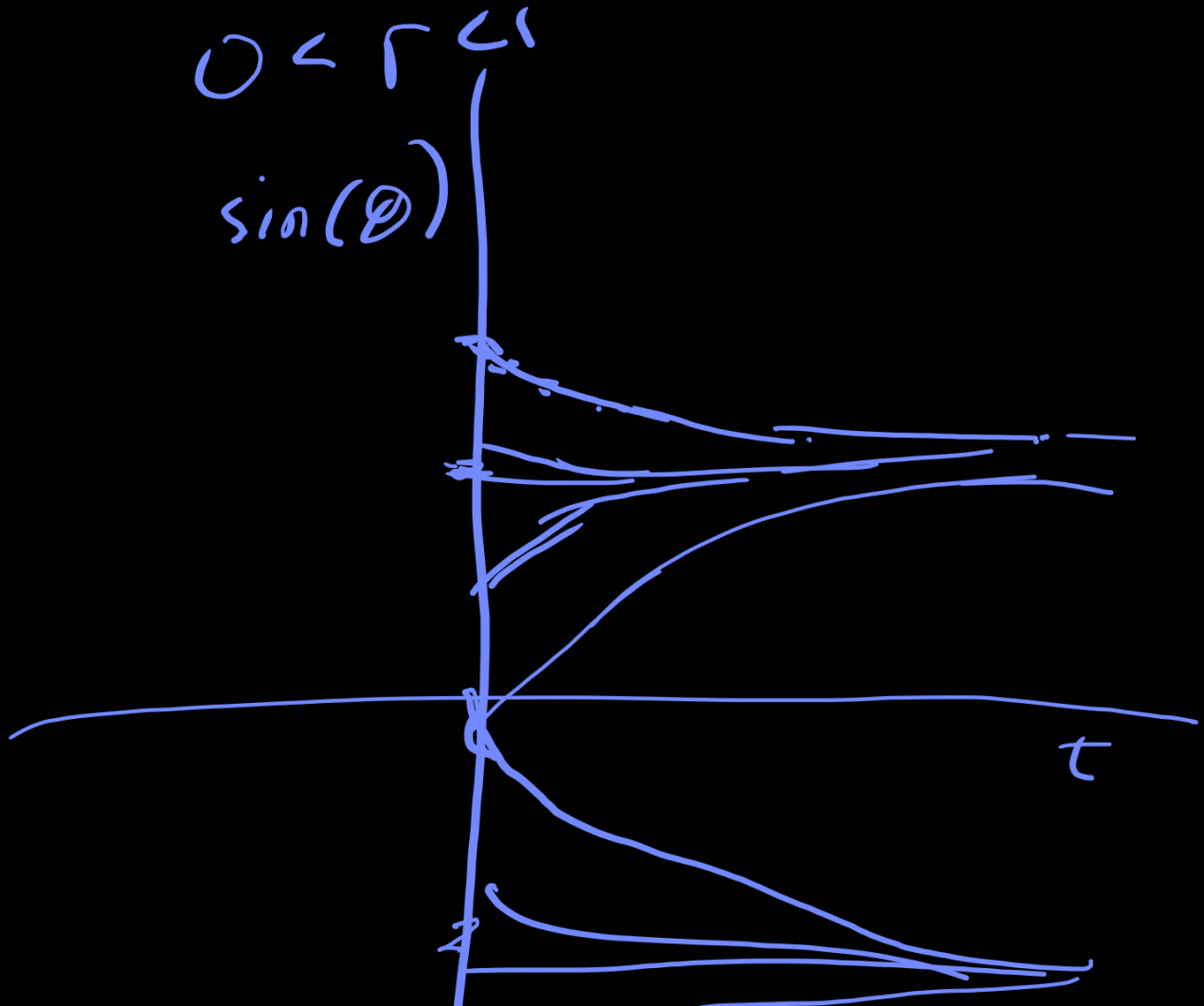
Take: $\tau = \frac{mgLt}{b}$, $\gamma = \frac{\Gamma}{mgL}$

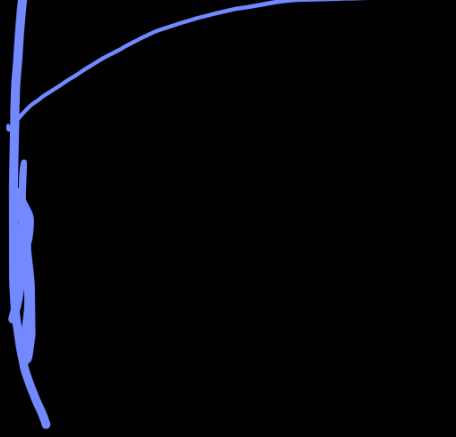
$$\Rightarrow \frac{d\theta}{d\tau} = \gamma - \sin\theta$$

(b)

$\tilde{x}$

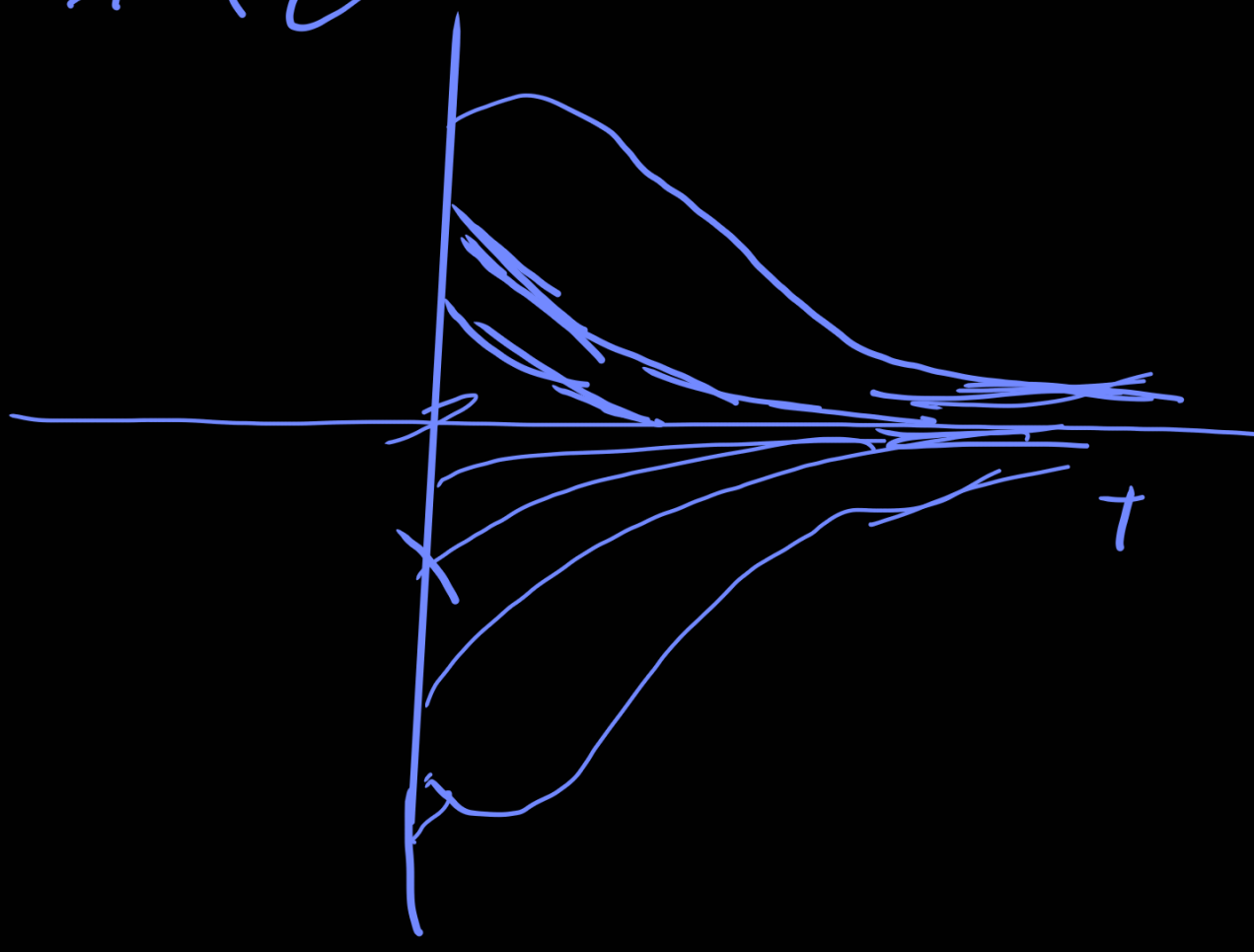

sinθ
1
t
-1


0 < r < 1
sin(θ)
t



$\gamma f << 1$

$\sin(\theta)$

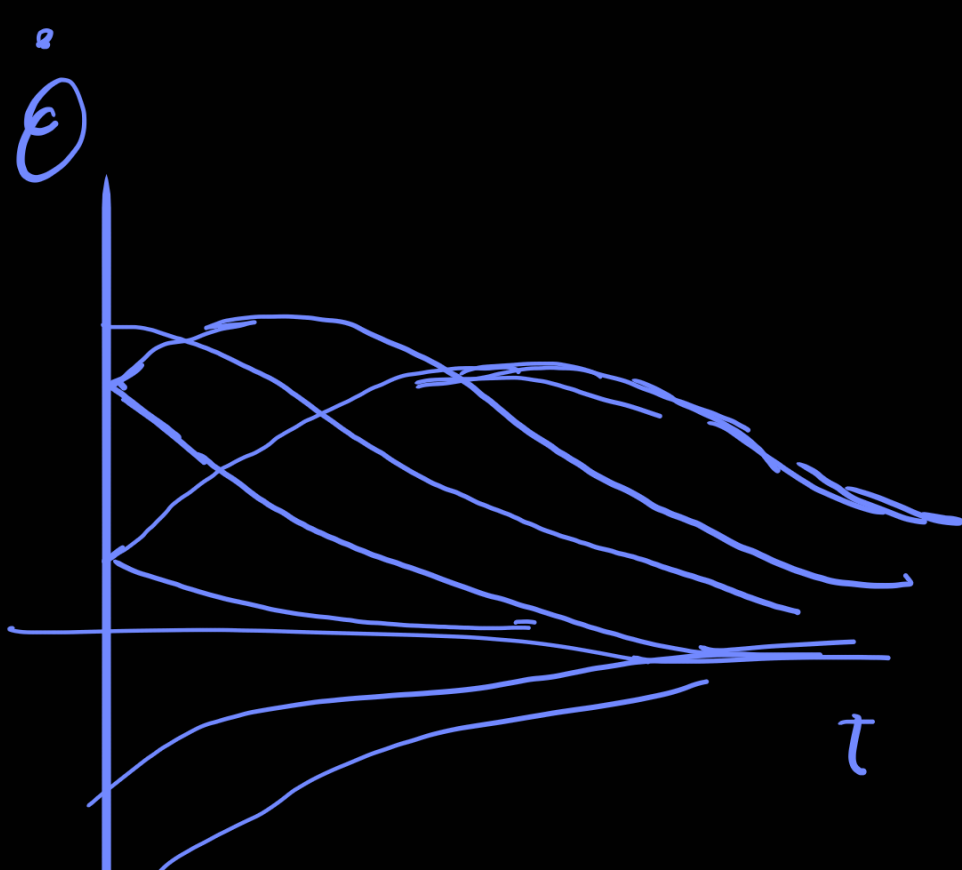


(c) Torque due to gravity is proportional to $\sin\theta$

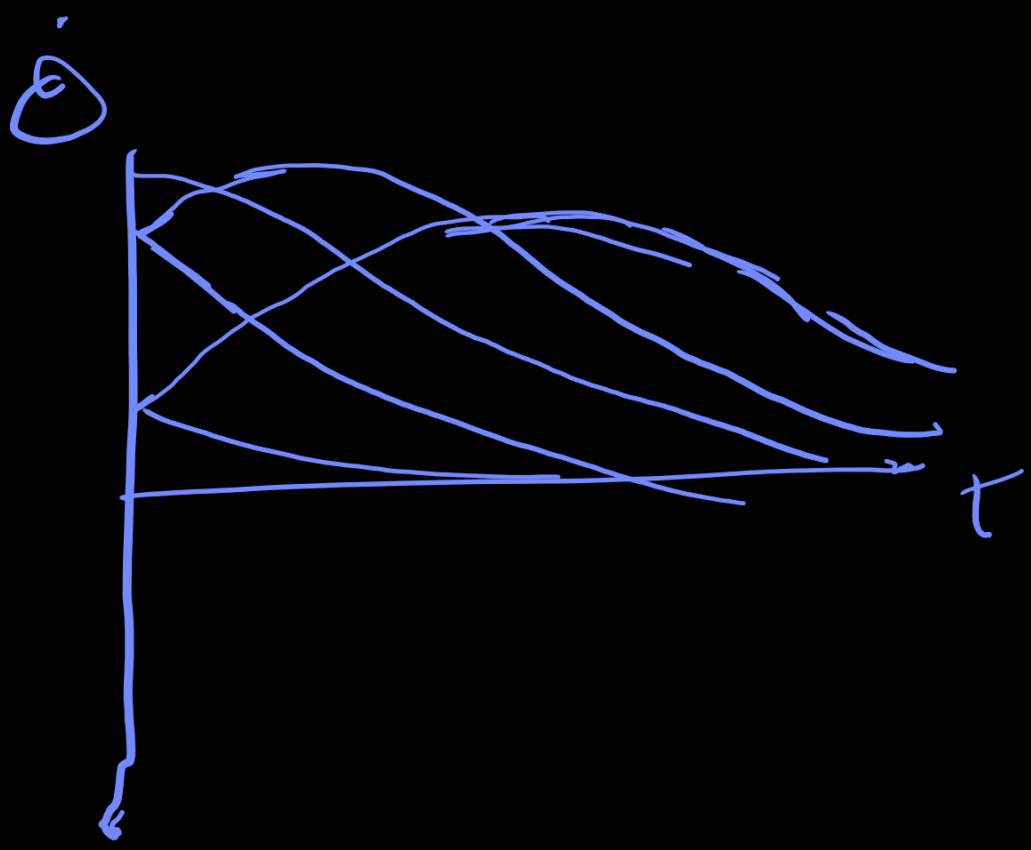
(d)

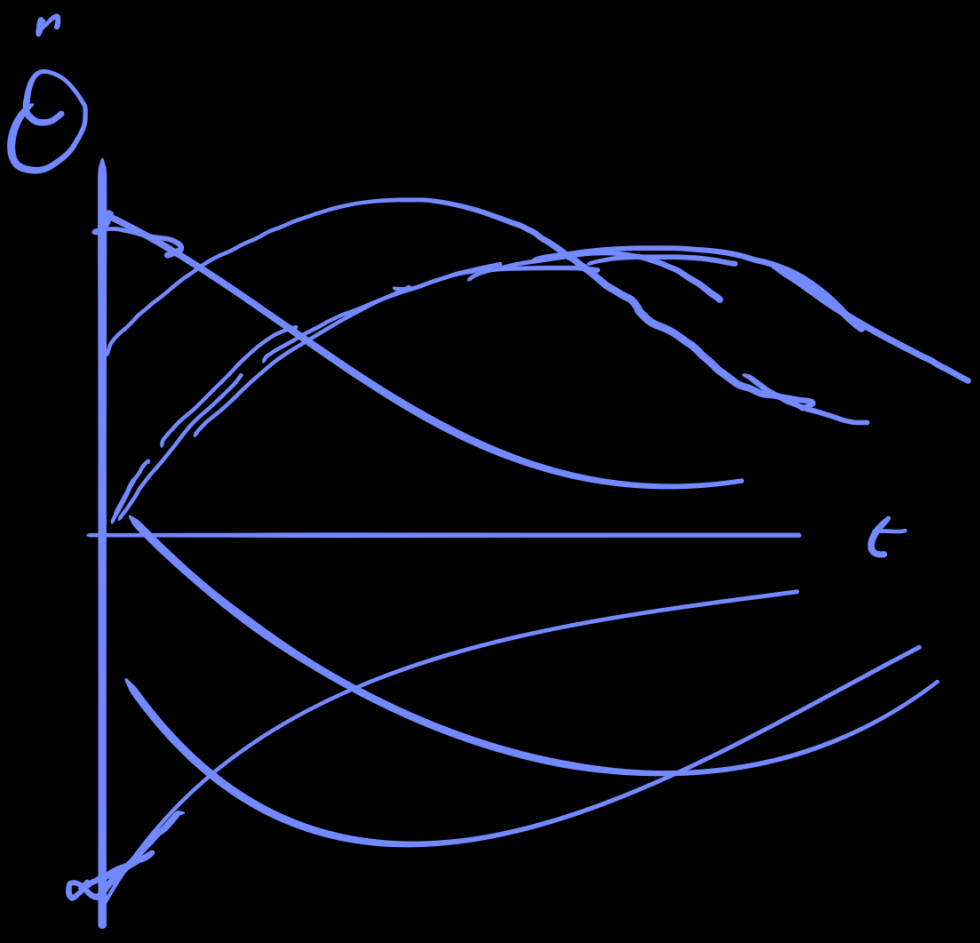
For $\dot{\theta}$ vs t

$1 > \gamma > 0$



$\gamma \approx 1$



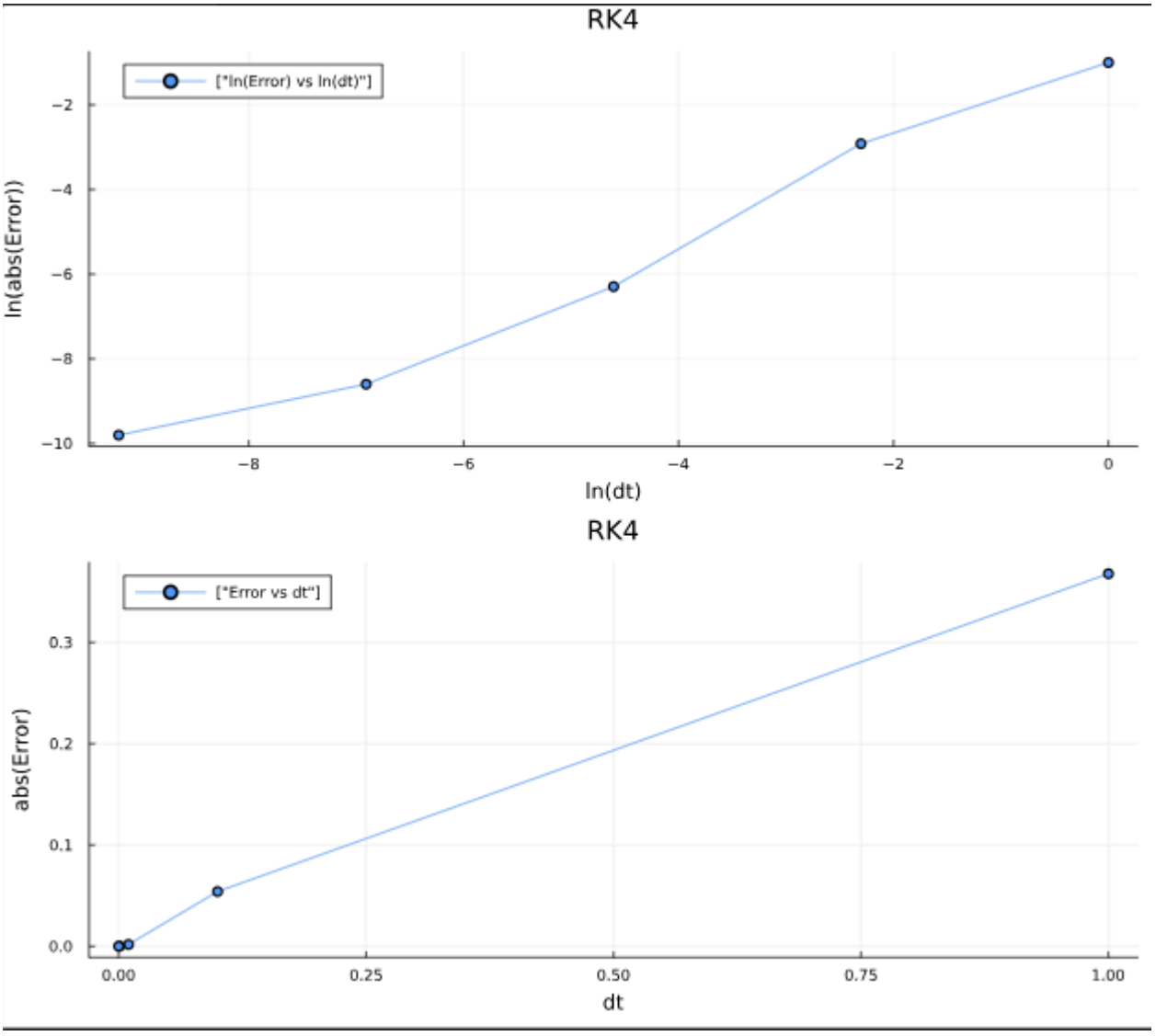

r
θ
t

Tut1_Pictures

1

Exact value is exp(-1)

Simple Euler

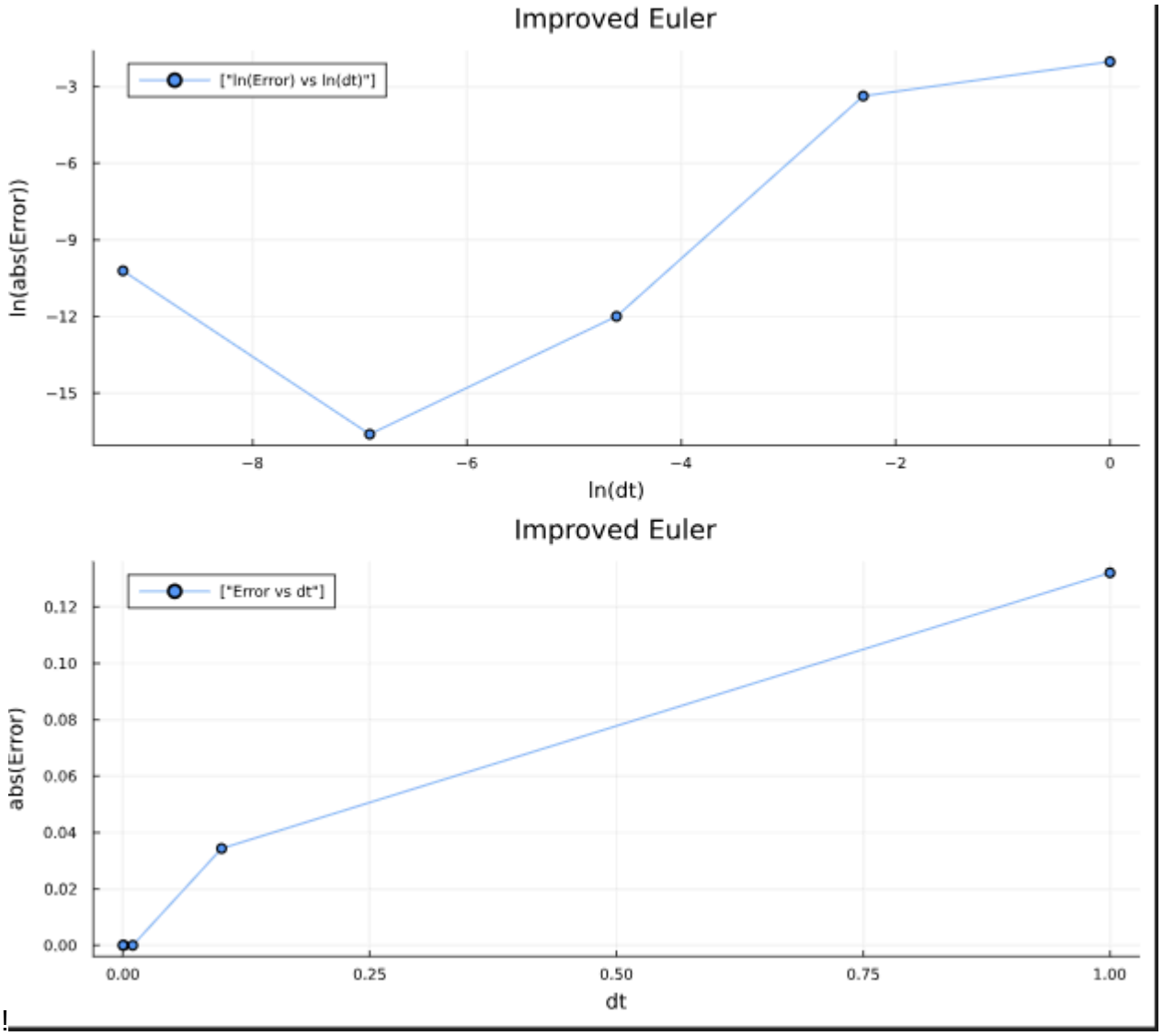



```
For time: 1.0   x(1): 0.0
For time: 0.1   x(1): 0.31381059609
For time: 0.01   x(1): 0.3660323412732295
For time: 0.001   x(1): 0.36769542477096373
For time: 0.0001   x(1): 0.36782426032828575
Actual answer is: 0.36787944117144233

julia>
```

For time: 1.0 x(1): 0.0
For time: 0.1 x(1): 0.31381059609
For time: 0.01 x(1): 0.3660323412732296
For time: 0.001 x(1): 0.36769542477096373
For time: 0.0001 x(1): 0.36782426032828575
Actual answer is: 0.36787944117144233

Improved Euler



!

```
For time: 1.0   x(1): 0.5
For time: 0.1   x(1): 0.33352959127436443
For time: 0.01   x(1): 0.367885618716192
For time: 0.001   x(1): 0.36787950253069096
For time: 0.0001   x(1): 0.3678426556798375
Actual answer is: 0.36787944117144233

julia>
```

For time: 1.0 x(1): 0.5

For time: 0.1 x(1): 0.33352959127436443

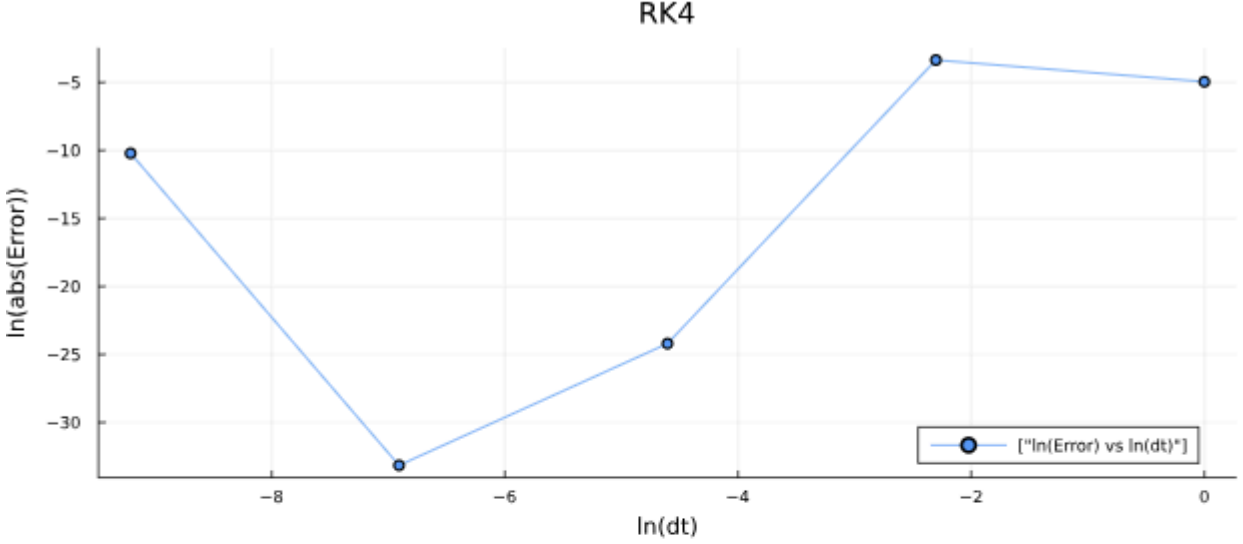
For time: 0.01 x(1): 0.367885618716192

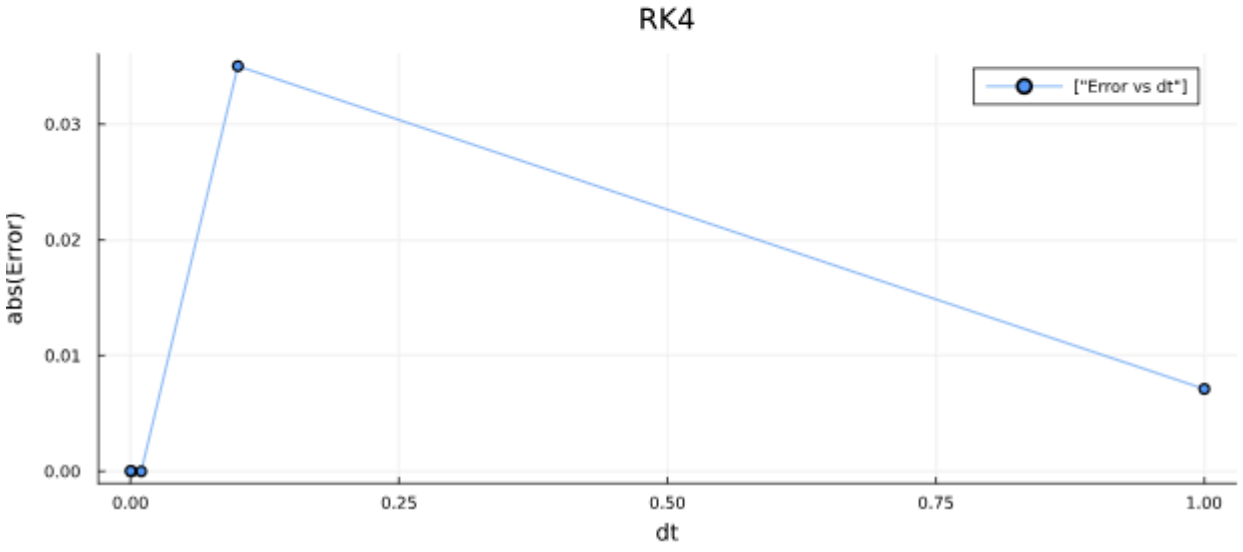
For time: 0.001 x(1): 0.36787950253069096

For time: 0.0001 x(1): 0.3678426556798375

Actual answer is: 0.36787944117144233

Rk4






```
For time: 1.0   x(1): 0.375
For time: 0.1   x(1): 0.332871415379969
For time: 0.01   x(1): 0.3678794412023554
For time: 0.001   x(1): 0.36787944117144633
For time: 0.0001   x(1): 0.36784265506666375
Actual answer is: 0.36787944117144233

julia>
```

For time: 1.0 x(1): 0.375

For time: 0.1 x(1): 0.332871415379969

For time: 0.01 x(1): 0.3678794412023554

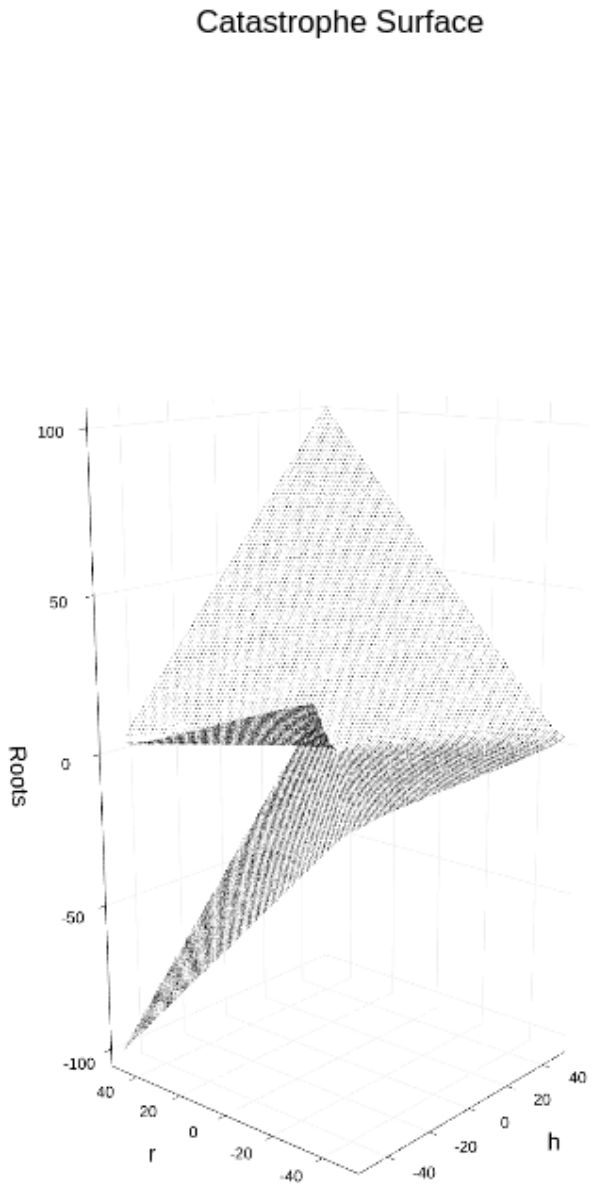
For time: 0.001 x(1): 0.36787944117144633

For time: 0.0001 x(1): 0.36784265506666375

Actual answer is: 0.36787944117144233

2

Below is the Catastrophe surface



The interactive HTML can be found in the github repo

Heatmap corresponds to the number of fixed points