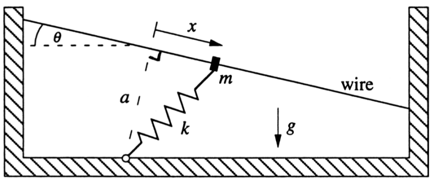
**AE454/777: Dynamics and Bifurcations**
**Tutorial 1**

1. Consider the initial value problem $\dot{x} = -x$, $x(0) = 1$.
(a) Solve the problem analytically. What is the exact value of $x(1)$?
(b) Using the Euler method with step size $\Delta t = 1$, estimate $x(1)$ numerically—call the result $\hat{x}(1)$. Then repeat using $\Delta t = 10^{-n}$, for $n = 1, 2, 3, 4$.
(c)Plot the error $E = |\hat{x}(1) - x(1)|$ as a function of $\Delta t$. Then plot $\ln E$ vs. $\ln \Delta t$. Explain the results.
(d) Redo the exercise using the improved Euler method.
(e) Redo the exercise using the Runge-Kutta method.

2. Consider the following mechanical system. A bead of mass $m$ is constrained to slide along a straight wire inclined at an angle $\theta$ with respect to the horizontal. The mass is attached to a spring of stiffness $k$ and relaxed length $L_0$, and is also acted on by gravity. We choose coordinates along the wire so that $x = 0$ occurs at the point closest to the support point of the spring (see figure). The equilibrium positions of the head satisfy



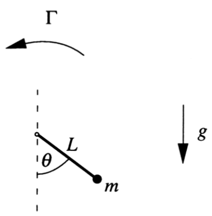

$$mg \sin\theta = kx \left(1 - \frac{L_0}{\sqrt{x^2 + a^2}}\right)$$

(a) Show that this equation can be written in dimensionless form as

$$1 - \frac{h}{u} = \frac{R}{\sqrt{1 + u^2}}$$

for appropriate choices of $R$, $h$, and $u$.
(b) Give a graphical analysis of the dimensionless equation for the cases $R < 1$ and $R > 1$. How many equilibria can exist in each case?
(c) Let $r = R - 1$. Give a numerically accurate plot of the bifurcation curves in the $(r, h)$ plane. Interpret your results physically.

3. The equation of an overdamped pendulum is given by $b\dot{\theta} + mgL \sin\theta = \Gamma$, where $b$ is the viscous damping constant and $\Gamma$ is a constant applied torque (see figure).
(a) Reduce this equation to the form of a nonuniform oscillator $\theta' = \gamma - \sin\theta$ with appropriate values for $t$ and $\gamma$.
(b) Sketch $\sin\theta(t)$ vs. $t$ for different $\gamma$ , including the limiting cases $\gamma \approx 1$ and $\gamma \ll 1$.




(c)What physical quantity is proportional to $\sin\theta(t)$?
(d) Redo the exercise for $\dot{\theta}(t)$ instead of $\sin\theta(t)$.