AE-6104: Home work-5 [XXX points]

Use any symbolic software and mention it during submission. Use simplify option if available.

1. A curve in a spherical coordinate u^i is given by

$$u^1 = t, \quad u^2 = \sin^{-1}\left(\frac{1}{t}\right), \quad u^3 = 2\sqrt{t^2 - 1} \quad t \in \mathbb{R}.$$

   Find the arclength $1 \leq t \leq 2$.

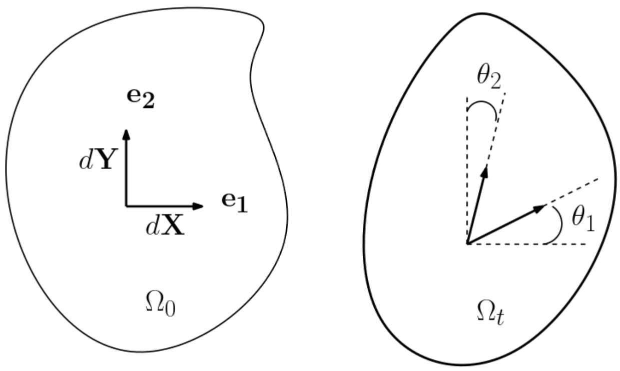

Figure 1: Ω_0 is the reference configuration and Ω_t is the current configuration. Deviation from the orthogonality is given by $\gamma_{12} = \theta_1 + \theta_2$.

2. The shear strain of two material line elements $d\boldsymbol{X} = |d\boldsymbol{X}|\boldsymbol{e}_1$ and $d\boldsymbol{Y} = |d\boldsymbol{Y}|\boldsymbol{e}_2$, which is orthogonal in the reference configuration (i.e. $\boldsymbol{e}_1 \cdot \boldsymbol{e}_2 = 0$), is defined by the deviation from the orthogonality γ_{12} (see Fig. 1). Show that

$$\sin\gamma_{12} = \frac{2\boldsymbol{e}_1 \cdot \boldsymbol{E}\boldsymbol{e}_2}{\sqrt{1 + 2\boldsymbol{e}_1 \cdot \boldsymbol{E}\boldsymbol{e}_1}\sqrt{1 + 2\boldsymbol{e}_2 \cdot \boldsymbol{E}\boldsymbol{e}_2}}$$

   where $\boldsymbol{E}$ is the Green strain tensor.

3. Consider the polar decomposition of the deformation gradient $\boldsymbol{F} = \boldsymbol{R}\boldsymbol{U} = \boldsymbol{V}\boldsymbol{R}$. Let $\{\boldsymbol{N}_a\}$ be the (right) eigenvectors of $\boldsymbol{C}$ and $\{\boldsymbol{n}_a\}$ be the (right) eigenvectors of $\boldsymbol{B}$ for $a = 1, 2, 3$. Then show that

(a) Show that $\boldsymbol{C}$ and $\boldsymbol{U}$ has the same eigenvectors.

(b) Show that $\boldsymbol{B}$ and $\boldsymbol{V}$ has the same eigenvectors.

(c) Show that $\boldsymbol{n}_a = \boldsymbol{R}\boldsymbol{N}_a$ and $\boldsymbol{F}\boldsymbol{N}_a = \lambda_a \boldsymbol{n}_a$, where λ_a^2's are the eigenvalues of $\boldsymbol{C}$.

4. Consider a circular cylindrical tube which in the initial reference configuration is of length l, and has external and internal radii R_1 and R_2 respectively. The tube is now deformed as

$$r = f(R), \quad \theta = \Theta + \psi X_3, \quad x_3 = \lambda X_3. \tag{1}$$

where (R, Θ, X^3) and (r, θ, x^3) are the cylindrical coordinates in the reference and deformed configuration respectively (see HW5). Comment on the nature of the mappings, i.e., what type of deformation they are initiating. Compute the Green strain tensor $\boldsymbol{E}$ and its components with respect to the appropriate basis. Compute its physical components. Write a symbolic code to compute its actual and physical components.

5. Consider a spherical shell which in the initial reference configuration has external and internal radii R_1 and R_2 respectively. The sphere is now deformed as

$$r = f(R), \quad \theta = \Theta, \quad \phi = \Phi. \tag{2}$$

where (R, Θ, Φ) and (r, θ, ϕ) are the spherical coordinates in the reference and deformed configuration respectively (see HW5). Comment on the nature of the mappings, i.e., what type of deformation they are initiating. Compute the Green strain tensor $\boldsymbol{E}$ and its components with respect to the appropriate basis. Compute its physical components. Write a symbolic code to compute its actual and physical components.

6. Consider a toroid tube which in the initial reference configuration has external and internal radii R_1 and R_2 respectively. The tube is now deformed as

$$r = f(R, \Theta), \quad \phi = \Phi, \quad \theta = g(R, \Theta). \tag{3}$$

where (R, Θ, Φ) and (r, θ, ϕ) are the local coordinates in the reference and deformed configuration respectively. The distance OA (see HW5, which was named R (not the same R here)), say C remains same for both the reference and current configuration. Comment on the nature of the mappings, i.e., what type of deformation they are initiating. Compute the Green strain tensor $\boldsymbol{E}$ and its components with respect to the appropriate basis. Compute its physical components. Write a symbolic code to compute its actual and physical components.