1)

$$u^1 = t \longrightarrow r$$
$$u^2 = \sin^{-1}\left(\frac{1}{t}\right) \longrightarrow \phi$$
$$u^3 = 2\sqrt{t^2-1} \longrightarrow \theta$$

$$du^1 = dt$$

$$du^2 = \frac{1}{\sqrt{1-\frac{1}{t^2}}} \times \left(\frac{-1}{t^2}\right) dt = \frac{-dt}{t\sqrt{t^2-1}}$$

$$du^3 = \frac{2t}{\sqrt{t^2-1}} dt$$

$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cancel{\tfrac{1}{r}} r^2 & 0 \\ 0 & 0 & \cancel{r^2\sin} \; r^2\sin^2\phi \end{bmatrix}$$

$$ds^2 = (du^1)^2 + r^2(du^2)^2 + r^2\sin^2\phi\,(du^3)^2$$

$$= \cancel{(dt)^2}\,(dt)^2 + t^2 \cancel{\phi} \left(\frac{(-dt)}{t\sqrt{t^2-1}}\right)^2 + t^2 \times \frac{1}{t^2} \times \left(\frac{2t\,dt}{\sqrt{t^2-1}}\right)^2$$

$$= dt^2 + \frac{dt^2}{t^2-1} + \cancel{4} \frac{4t^2}{t^2-1} dt^2$$

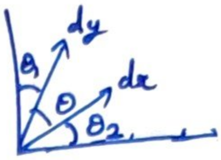
$$ds^2 = \left(1 + \frac{1}{t^2-1} + \frac{4t^2}{t^2-1}\right) dt^2$$

$$ds = \sqrt{\frac{5t^2}{t^2-1} dt^2}$$

$$ds = \frac{\sqrt{5}\,t\,dt}{\sqrt{t^2-1}}$$

$$\text{Length} = \int_1^2 \frac{\sqrt{5}\,t}{\sqrt{t^2-1}} dt = \Big|_1^2 \frac{\sqrt{5}}{2}\sqrt{t^2-1} = \frac{\sqrt{15}}{2}$$

$$= \cancel{\frac{\sqrt{5}}{2}\log(t^2)}$$



2) $\sin(\gamma_{12})$

$(\theta + \theta_1 + \theta_2 = 90^\circ)$

$= \sin(90 - \theta)$

$(a.b = |a||b| \cos(\theta_{ab}))$

$$= \cos(\theta) = \frac{dy . dx}{\sqrt{dy.dy} \times \sqrt{dx.dx}}$$

$$= \frac{Fe_2 . Fe_1}{\sqrt{Fe_2.Fe_2} \times \sqrt{Fe_1 \times Fe_1}}$$

$$= \frac{e_2.F^TF.e_1}{\sqrt{e_2 F^TFe_2}\sqrt{e_1 F^TFe_1}}$$

But we know: $E = \frac{1}{2}(C - G) = \frac{1}{2}(F^TF - G)$

$\Rightarrow F^TF = 2E + G$

$$\Rightarrow \cos(\theta) = \sin(90 - \theta) = \sin(\gamma_{12}) = \frac{e_2(2E+G)e_1}{\sqrt{e_2(2E+G)e_2}\sqrt{e_1(2E+G)e_1}}$$

$$= \frac{e_2Ge_1 + 2e_2Ee_1}{\sqrt{e_2Ge_2 + 2e_2Ee_2}\sqrt{e_1Ge_1 + 2e_1Ee_1}}$$

$$= \frac{2e_2Ee_1}{\sqrt{1 + 2e_2Ee_2}\sqrt{1 + 2e_1Ee_1}} = \frac{2e_1Ee_2}{\sqrt{1+2e_1Ee_1}\sqrt{1+2e_2Ee_2}}$$

$(E^T = E)$

3 $F = RU = VR$

a) ~~R~~ $C = F^T F = U^T R^T R U = U^T U = U^2 \quad (U^T = U)$

$\therefore C = U^2$

~~Since $U \in Psym$~~,

~~Let $U \in ($~~

Let the rank of C be n. Then the rank of U will also be n. Also, both are full-rank ~~mode~~ tensors.

Let $\underline{v}$ be an eigenvector of $\underline{\underline{U}}$ with eigenvalue λ,

Then $\underline{\underline{C}}\underline{v} = \underline{\underline{U}}^2 \underline{v} = \underline{\underline{U}}(\underline{\underline{U}}(\underline{v})) = \lambda \underline{\underline{U}}(\underline{v}) = \lambda^2 \underline{v}$

$\therefore \underline{v}$ is also an eigenvector of C with eigenvalue λ^2.

$\therefore$ Every eigenvector of U is an ~~eigenvalue~~ eigenvector of C.

~~B~~ Both C & U have n eigenvectors.

Possible iff ~~every~~ both have same eigenvectors

b) ~~For~~ $B = FF^T = VRR^TV^T = VV^T = V^2 \quad (V^T = V)$

Same arguement as (a)

c) We know $CN_a = \lambda_c N_a$

$$C = F^T F$$

$$\Rightarrow F^T F N_a = \lambda_a N_a$$

Applying F on both sides,

$$\underbrace{F F^T}_{B} F N_a = \lambda_a F N_a$$

$$B \, F N_a = \lambda_a F N_a$$

$\therefore$ FN_a is an eigenvector of B with eigenvalue λ_a

$$\therefore FN_a = \lambda_a n_a$$

$$BRN_a = FF^TRN_a = RR^TFF^TRN_a$$

$$= RR^TRUU^TR^TRN_a$$

$$= RUU^TN_a$$

$$= RCN_a$$

$$= R\lambda_a N_a \quad (N_a \text{ is eigenvector of } C)$$

$$= \lambda_a RN_a$$

$$\Rightarrow \quad BRN_a = \lambda_a RN_a$$

$\therefore$ RN_a is an eigenvector of B with eigen value λ_a

$$\therefore RN_a = n_a$$