- ## Intro
  - Let's look at Euler's Method. Euler's Method is for graphing differential equations. This one, the derivative of y with respect to x is equal to y. meaning that the slope is always equal to y on the curve.
- ## SimpleAttempt
  - If you tried to graph this function as is, you would quickly notice that there isn't even a place to start, the given information doesn't help at all. There's nothing restricting it vertically, all we know is the shape of the graph based on where it is.
  - The only way you could graph this function is by filling in the number plane, as each combination of x and y follows this rule, which is extremely unhelpful. To make any progress of understanding what this equation means, we'll need an initial point, or somewhere that we know the exact value on the curve. This is usually given with the function, but could also be the solution. Here we'll say that when $x=0$, $y=1$, or that $f(0) = 1$.
- ## SimpleAttemptLines
  - To continue to the next point, we'll need the line tangent to our graph at the initial point. Thankfully, this is very easy to find, as we have an equation for the slope at any point. With the slope of a line, and the point that it lies on, its as trivial point slope form to graph. Before we move on, we'll make a small table to organize the x positions, y positions, and slopes at every point we graph, even though in this

problem the y positions and slopes will be equal, it’s important for more complex problems.

- Now, for this next step, we have to keep in mind that this is just an approximation. Unless the differential function is extremely simple, or we get extremely lucky, we won't get a correct answer. What makes it incorrect is that we are going to be saying that a point on this tangent line we’ve graphed is going to be on the actual function. In reality, we know that tangent lines pass through a curve exactly once, excluding when the graph turns around. Outside, these situations, we can actually be confident that any given point on our tangent line is not on the actual curve. But we can also be confident that it’s reasonably close, which is what we’re running with.
- So, we’ll make a second point somewhere on this line. Using our derivative, we can see what the slope would be if this point was on the curve, optimally this would be only very slightly different to our existing slope, as the further from the original point it is, the less accurate our approximation will tend to be. Where we do decide to put it depends, but for now, we’ll put it at (1,2), so that the jump from the first point’s x value to our second’s x value, or our step size, is 1.
- And then we just repeat the process, essentially making a new problem but with our second point at the initial point. Meaning there’s a new tangent line through it. When finding the location of the third point, we could technically choose anywhere along this tangent line to put it, but we’ll stick with increasing x by 1 so the step size is constant. making the next point go to (2,4). New we do it again, we’ll adjust our line to the new slope of 4, and make more space. Then add in a

point at (3,8). We'll stop here as it's enough to get the idea of the curve, but you could of course go much further with the same logic.

➢ SimpleAttemptExplained

- Looking at this graph and chart, you may notice that the points we got perfectly follow f(x)=2^x. It won't always be easy or even possible to make a function for your estimation, but here we can. Usually, the answer would just be a value at a given x, something like f(3) where here we would say 8, but here we can actually graph the exact function and compare the overall shapes. Usually this is very difficult, but with something as simple as this, the actual answer is just e^x.
- We can already see that this isn't a great approximation, by x=3 the real solution is way far away from what we got, and if we needed something much larger, there wouldn't even be close. Just within this range, the area of the estimation is only 53% of the correct function, which is bad even with the BC curve. Thankfully, this is fine for a few reasons.
- First, in some problems, such as the limit as x approaches infinity for f(x) divided by x, the two answers are extremely similar. Just by knowing that the function is exponential, we can safely say that both of these functions approach infinity, and at similar rates too, and sometimes that's enough. Although this will probably come up less. The real reason this is fine is because we can try again with some slight changes.

- SecondAttempt
  - Right now our step size is 1 because on the x axis, we're going from 0 to 1 to 2 and so on, but, if we lowered this, say to 0.5, so we went from 0 to 0.5 to 1, and so on, it would help our approximation. This is because, at the end of every step, we update the slope of the graph. By shortening the steps, we update it more frequently, thus, it spends less time being out of date, and more time being accurate. We'll update this second point, as well as the rest of our chart, and then extend it so x reaches 3 again.
  - This new setup is more accurate than the previous, equal to 2.25^x. While this still isn't correct, if we continue decreasing the step size, our answer will be more and more accurate. So, the question is, what step size should we use?
  - Well, it depends on what you have and what you need. if we want to calculate something by hand quickly, usually you would stick to something large, like what we did here. But with a computer, you can really go as low as you'd like to get an extremely correct answer. There are also plenty of examples as we talked about, where something as inaccurate as a step of 1 is perfectly fine and will give a correct answer.
  - You might also ask, isn't there some way to use an integral to get an exact answer? Unfortunately no, this is because this process is recursive, meaning that every step depends on the previous one. Integrals rely on the ability to find the value at any point independently. So it would require a very large amount of effort. Let's look at a different example.

- Problem2

  - Now, we'll say that the derivative of y with respect to x is equal to 1 over x+y, our initial point will be (0.5,0.5), and our goal will be f(5), for the sake of getting the same, wrong answer, we'll also say the step size is 0.5. If everything is done correctly the answer will be roughly 2.06, and the graph will start steep, then level out over time.
  - Even though the differential equation was significantly more complex than the original example, it shouldn't have made this much harder. This is one of the benefits of this method, it's generally quite simple, or at worst as simple as the given equation.

- Problem3

  - Let's do another simpler problem, that's very similar to the first equation. The derivative of y with respect to x is equal to negative 2.3 y, with the initial point of (0,1), and the goal of f(5), by using a step size of 1. Looks pretty straightforward. Let's graph it.
  - Interesting. If you did this correctly, you might be thinking you've done something wrong, but the extreme spikes are expected. if you tried to guess what the actual graph was, you might say something like x*sin(x), however, the real function is just e^-2.3x. So, why did this happen?
  - Basically, our step size was too big, from this first point, it ended up way below where it was supposed to be, then overcorrected to being way above, back and

forth, forever getting less accurate. To fix this, we can simply lower the step size. While the shape is still misleading, it correctly trends to y=0.

- ## Problem4

    - For this last problem, we'll go backward, starting with an equation, but not getting the initial point, instead getting the final point and the step size. While this may seem like the same thing we've been doing with the goal to the left of the point. An important distinction is when the slope gets updated. If we were to treat this problem that way we would end up getting the slope of the segment based on where it ends instead of where it starts.
    - It's helpful for these problems to make a chart as opposed to a graph. We know all of our x values, we start at 0, go up by 1s, and end up at 3. For the y values, we end at 2 but start at some mystery number.
    - We can immediately remove the last slope, as it only affects the graph outside of our range. then we can find the slope of the first point in terms of a, here it's also a. Then we can get the second y value. We know that no matter what a is equal to, we're increasing y by that value over this interval. So it would be the previous value + a or just 2 a.
    - The slope of the line at point 2 is also easy to find in terms of a, just 3+2a. And the 3rd point doesn't give too much difficulty either. Finally, we can find the last point in terms of a, except we know that whatever we get is equal to the given 2. Some simple algebra tells us the a = -2 so the initial point is (0,-2).

➢ Outro

- Overall, to use Euler's method, you need to have an equation for the slope of a curve a a given point and a given point to start at or end at. The step size you use is proportional to the error, usually, you want both to be as low as possible, and occasionally a high step size will cause massive errors in the shape and values of a graph.