Time Series Models: From Statistics to AI

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Exercise Sheet 1

Regulations

Please submit your solutions via Moodle in teams of **2 students**, before the exercise group on **Wednesday, April 30th, 2025**. Each submission must include **exactly** two files:

- A `.pdf` file containing both your Jupyter notebook and solutions to analytical exercises. The Jupyter notebook can be exported to pdf by selecting **File → Download as → pdf** in JupyterLab. If this method does not work, you may print the notebook as a pdf instead. Your analytical solutions can be either scanned handwritten solutions or created using LaTeX.
- A `.ipynb` file containing your code as Jupyter notebook.

Both files must follow the naming convention:
`Lastname1-Lastname2-sheet01.pdf`
`Lastname1-Lastname2-sheet01.ipynb`

Task 1. Weather
The file `dailyweather.csv` contains daily measurements of the temperature, humidity, and air pressure on the balcony of an esteemed colleague's parents during the last five years.

1. Plot the time series in a sensible way.
2. Visually inspect the plotted time series and describe whether you think any of them are correlated with each other. If you do, explain which ones and why, and how strong you think the correlation might be. Then calculate the actual correlations and compare them with your guesses. Were your predictions correct?
3. Now we want to look at return-maps. Are you expecting a trend in the first return-map? How do you think the return-maps will differ for longer intervals like a month, 90 days, or a year? Discuss your expectations first, then create the plots to check.
4. Do you notice a trend in the temperature, e.g. can you observe the climate changing over time? What kind of analysis could help you confirm whether such a trend exists?

Task 2. Detrending and autocorrelation
The file `investment.xls` contains scaled quarterly United States private investment per capita rates over the years 1948–1989.

1. First, we want to remove the overall trend from the time series. Should you rather use the model

$$\text{investment}(t_n) = m \cdot t_n, \qquad m \in \mathbf{R} \tag{I}$$

or

$$\text{investment}(t_n) = m \cdot t_n + b, \qquad m, b \in \mathbf{R} \tag{II}$$

for the expected linear trend? Explain.
Now remove the trend from the dataset using linear regression.
(Hint: There are in-built functions/packages in Python and Julia that can do this for you.)

2. Stationarity

   (a) Write down the definitions for weak and strong stationarity.

   (b) Examine (loosely) whether the time series with the linear trend removed is stationary (the implementation does not have to be perfect)?
   Is the time series (of the original series before regression) of first differences stationary (i.e. $\{x_{t+1} - x_t\}$)? How about the time series of second-order differences? (i.e. $\{x_{t+2} - x_t\}$)?

3. The autocorrelation function is given by

$$\text{acorr}(x_t, x_{t+\Delta t}) = \frac{E[(x_t - \mu_t)(x_{t+\Delta t} - \mu_{t+\Delta t})]}{\sigma_t \sigma_{t+\Delta t}} \tag{III}$$

   (a) Implement a function to compute the autocorrelation of a time series from scratch yourself. While doing so, think about which parts of the theoretical formula are challenging to implement in practice. What assumptions do you need to make in order for your implementation to be valid?

   (b) Compute the autocorrelation function of the detrended time series. Can you find periodic business cycles (corresponding to peaks in the autocorrelation function)?

Task 3. AR models

1. Create your own AR time series of length $T = 200$ and order $p = 4$, with the following coefficients:

$$a_0 = 0, \quad a_1 = -0.8, \quad a_2 = 0, \quad a_3 = 0, \quad a_4 = 0.4,$$

   with $\epsilon_t \sim \mathcal{N}(0, 1)$, i.e., the noise process drawn from a standard normal distribution, and with the initial value of the time series being $(x_{-3}, x_{-2}, x_{-1}, x_0) = (0, 0, 0, 0)$.

2. Plot the time series in time as well as the first return-map. What do you notice?