PART5

第五部分

Chapter 7
Momentum

In this chapter you will learn how to:

- calculate the momentum of a moving body or a system of bodies
- use the principle of conservation of momentum to solve problems involving the direct impact of two bodies that separate after impact
- use the principle of conservation of momentum to solve problems involving the direct impact of two bodies that coalesce on impact.

7.1 Momentum

PREREQUISITE KNOWLEDGE

Where it comes from	What you should be able to do	Check your skills
Chapter 1	Calculate velocity when the acceleration is constant.	**1** A car is travelling at $15\,\mathrm{m\,s^{-1}}$ when the brakes are applied. It takes $6\,\mathrm{s}$ for the car to come to rest. Assume that the braking force is constant (and hence the acceleration is constant, but negative). **a** Show that car travels $45\,\mathrm{m}$ under braking before coming to rest. **b** Calculate speed of the car when it has been braking for $3\,\mathrm{s}$. **c** Calculate the speed of the car when it has travelled $22.5\,\mathrm{m}$ under braking.
Chapter 6	Calculate velocity using calculus.	**2** A car is travelling at $15\,\mathrm{m\,s^{-1}}$ when the brakes are applied. It takes $6\,\mathrm{s}$ for the car to come to rest. Assume that the acceleration under braking is given by $\frac{5t(t-6)}{12}$ where t is the time from when braking starts. **a** Calculate the speed of the car when it has been braking for $3\,\mathrm{s}$. **b** Find the speed of the car when it has travelled $22.5\,\mathrm{m}$ under braking.

7.1 Momentum

In mechanics, momentum measures the impetus possessed by a moving object. By considering the transfer of momentum between objects you can calculate what happens when objects interact.

DID YOU KNOW?

The philosopher René Descartes (1596–1650) introduced the concept of momentum. Descartes' built on ideas first written down by Jean Buridan (1295–1363) who defined the 'amount of motion' as the product of the mass of a body and its speed. Using these ideas, Descartes formulated his three laws of motion, which then became the basis for Newton's laws of motion.

KEY POINT 7.1

A body of mass m kg moving with speed v m s^{-1} has momentum given by mv.

Momentum is a vector quantity, having the same direction as the velocity. For one-dimensional motion along a line you only need to work out whether the momentum is positive or negative. The units of momentum are N s.

7.1 Momentum

Find the momentum of a body of mass 3 kg moving at $5\,\mathrm{m\,s^{-1}}$.

$$\text{Momentum} = mv = 3 \times 5$$

$$= 15\,\mathrm{N\,s}$$

A ball of mass 50 g hits the ground with speed $10\,\mathrm{m\,s^{-1}}$ and rebounds with speed $6\,\mathrm{m\,s^{-1}}$. Find the change in momentum that occurs in the bounce.

$$50\,\mathrm{g} = 0.050\,\mathrm{kg}$$

$$\begin{aligned}\text{Momentum before} &= 0.050 \times 10\\ &= 0.5\,\mathrm{N\,s}\end{aligned}$$

$$\begin{aligned}\text{Momentum after} &= 0.050 \times -6\\ &= -0.3\,\mathrm{N\,s}\end{aligned}$$

$$\begin{aligned}\text{So change in momentum} &= -0.3 - 0.5\\ &= -0.8\,\mathrm{N\,s}\end{aligned}$$

7.1 Momentum

A ball bearing of mass 25g is thrown vertically upwards, and is caught on the way back down. The ball bearing has an initial speed of 3 m/s upwards and is travelling at 2 m/s when it is caught. Find the change in its momentum.

0.125 Ns

7.1 Momentum

› The magnitude of the momentum of an object is often thought of as its resistance to being stopped. Compare the momentum and kinetic energy of a cricket ball of mass 0.15 kg bowled very fast at $40\,\mathrm{m\,s^{-1}}$ and a 20 tonne railway truck moving at the very slow speed of 1 cm per second. Which would you rather be hit by, an object with high momentum and low energy, or one with high energy and low momentum?

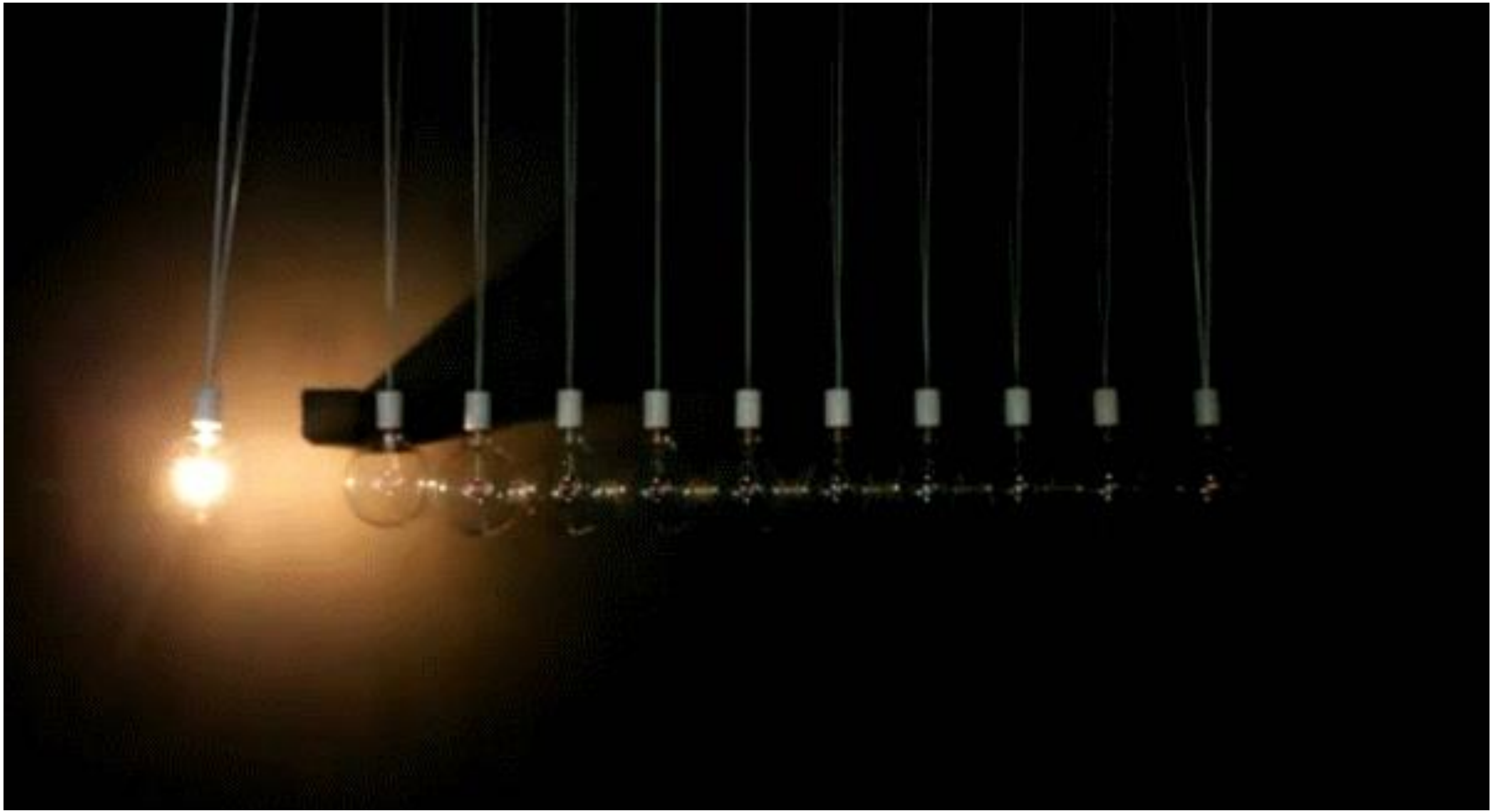
Ball: momentum 6 N s; K.E. 120 J

Truck: momentum 200 N s; K.E. 1 J

Better to be hit by an object with low momentum.

7.2 Collisions and conservation of momentum

During an impact when two bodies collide, there is a transfer of momentum between them.
Newton’s cradle is used to demonstrate both moment and kinetic energies are conserved.

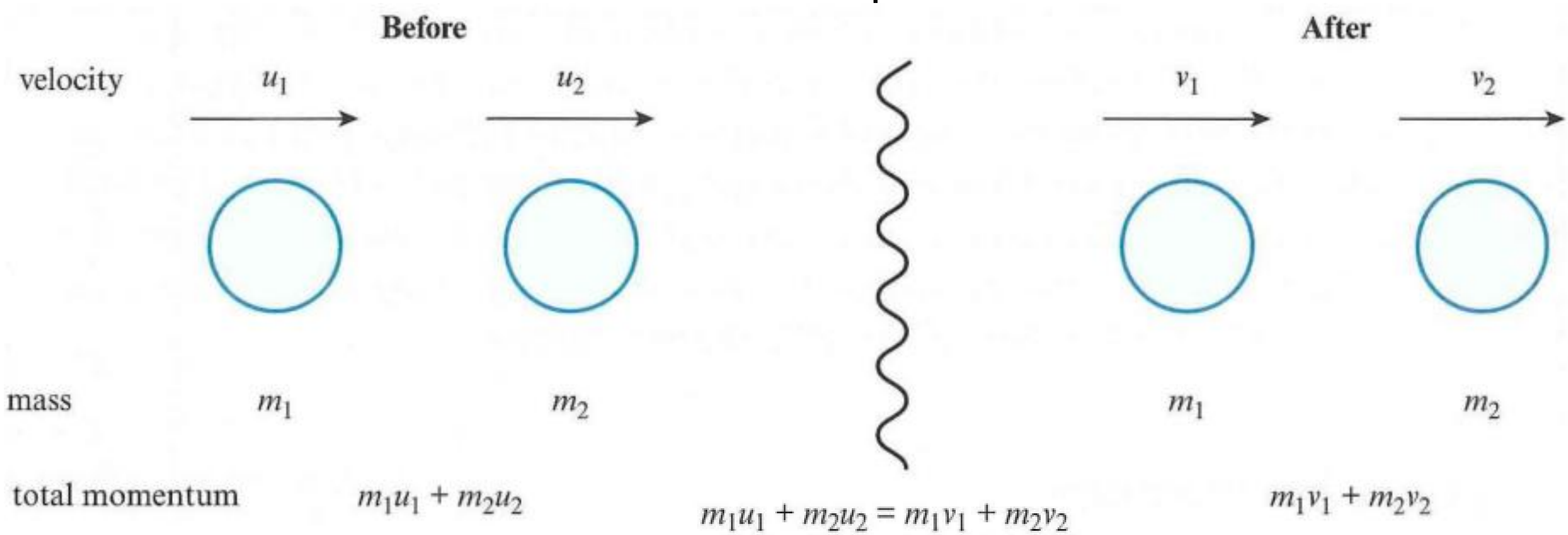


7.2 Collisions and conservation of momentum

MODELLING ASSUMPTIONS

In reality a snooker player would not usually want a direct, one-dimensional impact and would probably prefer to use an oblique, two-dimensional impact, where the motion is not all in the same straight line

- One-dimensional instantaneous impact.

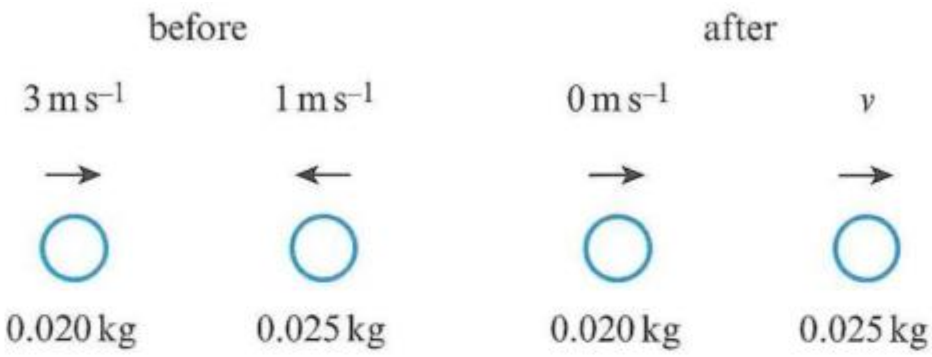


KEY POINT 7.2

Momentum is conserved in impacts. The total momentum is constant.

7.2 Collisions and conservation of momentum

Two ball bearings are moving directly towards one another. The first ball bearing has mass 20g and is moving at 3 m/s. The second ball bearing has mass 25g and is moving at 1 m/s. After the collision the first ball bearing is stationary. What is the speed of the second ball bearing after the collision?



Draw a diagram to summarise the information.

Total momentum before collision

$= (0.020 \times 3) + (0.025 \times -1)$

$= 0.035\ \text{N s}$

Total momentum after collision

$= 0 + 0.025\,v$

$= 0.025v\ \text{N s}$

$0.035 = 0.025\,v$

$v = 1.4\ \text{m s}^{-1}$

7.2 Collisions and conservation of momentum

This example illustrates the **law of conservation of momentum**.

» The law of conservation of momentum states that when there are no external influences on a system, the total momentum of the system is constant.

Since momentum is a vector quantity, this applies to the magnitude of the momentum in any direction.

For a collision you can say

Total momentum before collision = total momentum after collision

Note

It is important to remember that although momentum is conserved in a collision, mechanical energy is not conserved. Some of the work done by the forces is converted into heat and sound.

7.2 Collisions and conservation of momentum

Sometimes, instead of moving apart after an impact, the objects may **coalesce**. This means that they collide and then move off together as a single object. The objects can be thought of as having merged into a single object with a mass equal to the sum of the individual masses.

Examples of coalescence include a railway truck being pushed up to an engine and coupling with it, a person jumping onto a moving vehicle or two ice skaters meeting up and holding hands to continue as one.

The opposite of coalescence is called an **explosion**. This would happen, for example, when the engine and truck become decoupled, when the person jumps off the moving vehicle or when the ice skaters stop holding hands and drift apart.

CONSERVATION OF MOMENTUM

The law of conservation of linear momentum states that 'the total linear momentum of a system of interacting particles remains constant **provided there is no resultant external force'**.

To see why, we start by imagining two isolated particles A and B that collide with one another.

- The force from A onto B, F_{AB} will cause B's momentum to change by a certain amount.
- If the time taken was Δt, then the momentum change (the impulse) given to B will be given by $\Delta p_B = F_{AB}\,\Delta t$
- By Newton's third law, the force from B onto A, F_{BA} will be equal and opposite to the force from A onto B, $F_{AB} = -F_{BA}$.
- Since the time of contact for A and B is the same, then the momentum change for A is equal and opposite to the momentum change for B, $\Delta p_A = -F_{AB}\,\Delta t$.
- This means that the total momentum (momentum of A plus the momentum of B) will remain the same. Total momentum is conserved.

This argument can be extended up to any number of interacting particles so long as the system of particles is still isolated. If this is the case, the momentum is still conserved.

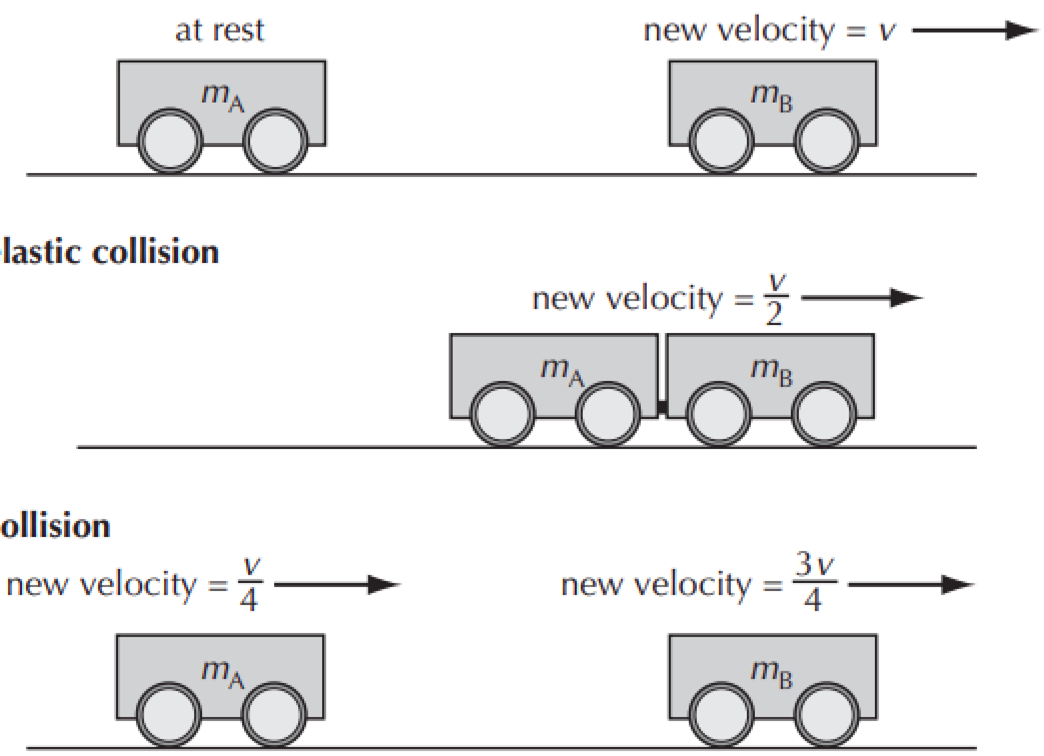
ELASTIC AND INELASTIC COLLISIONS

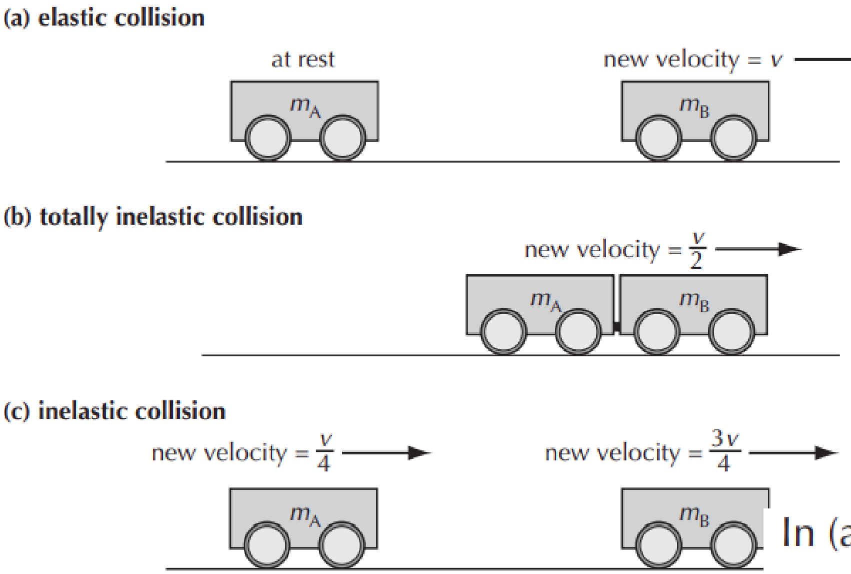
The law of conservation of linear momentum is not enough to always predict the outcome after a collision (or an explosion). This depends on the nature of the colliding bodies. For example, a moving railway truck, m_A, velocity v, collides with an identical stationary truck m_B. Possible outcomes are:

(a) elastic collision

(b) totally inelastic collision

(c) inelastic collision





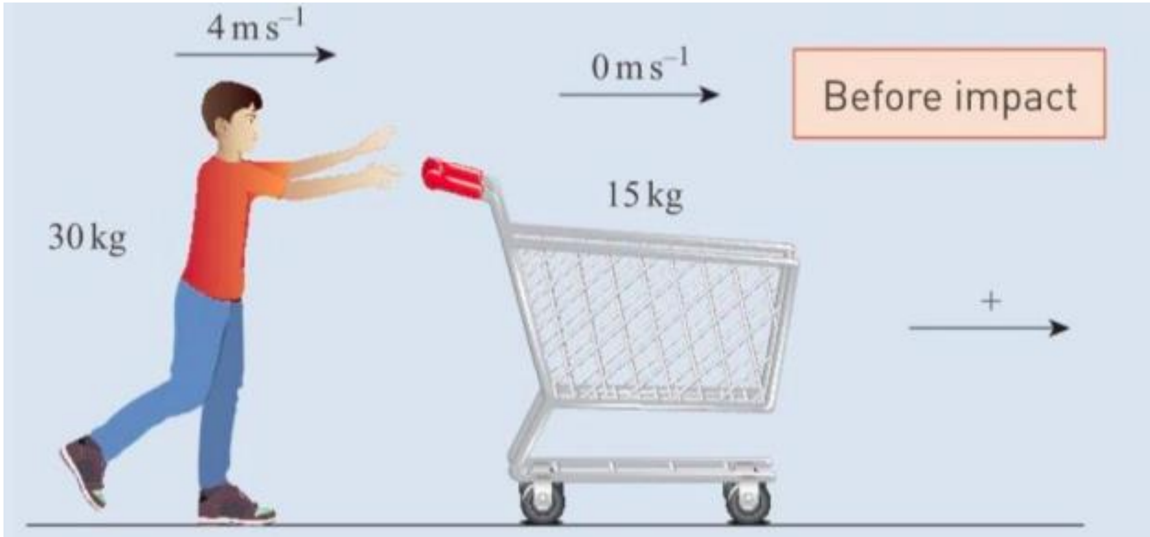
In energy terms, (c) is somewhere between (a) and (b). Some energy is lost, but the railway trucks do not join together. This is an example of an **inelastic collision**. Once again the total momentum is conserved.

In (a), the trucks would have to have elastic bumpers. If this were the case then no mechanical energy at all would be lost in the collision. A collision in which no mechanical energy is lost is called an **elastic collision**. In reality, collisions between everyday objects always lose some energy – the only real example of elastic collisions is the collision between molecules. For an elastic collision, the relative velocity of approach always equals the relative velocity of separation.

In (b), the railway trucks stick together during the collision (the relative velocity of separation is zero). This collision is what is known as a **totally inelastic collision**. A large amount of mechanical energy is lost (as heat and sound), but the total momentum is still conserved.

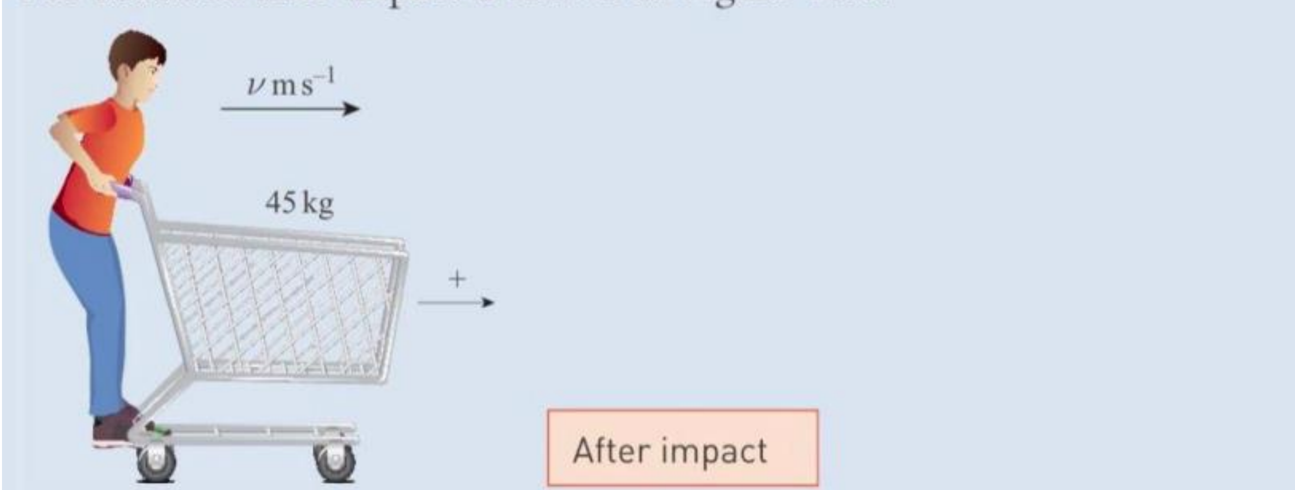
In energy terms, (c) is somewhere between (a) and (b). Some energy is lost, but the railway trucks do not join together. This is an example of an **inelastic collision**. Once again the total momentum is conserved.

A child of mass 30 kg running through a supermarket at $4\,m\,s^{-1}$ leaps on to a stationary shopping trolley of mass 15 kg. Find the speed of the child and trolley together, assuming that the trolley is free to move easily.



Taking the direction of the child's velocity as positive, the total momentum before impact is equal to $4 \times 30 + 0 \times 15 = 120\,N\,s$.

The situation after impact is shown in Figure 10.6.



The total mass of child and trolley is 45 kg, so the total momentum after is $45v$ N s.

Conservation of momentum gives:

$$45v = 120$$
$$v = 2\frac{2}{3}$$

The child and the trolley together move at $2\frac{2}{3}\ \mathrm{m\,s^{-1}}$.

Example 10.4

Two smooth spheres, A of mass 2 kg and B of mass 3 kg, of equal radii are moving towards each other along the same straight line on a smooth, horizontal plane. Sphere A has velocity $7\,\mathrm{m\,s^{-1}}$ and sphere B has a velocity of $2\,\mathrm{m\,s^{-1}}$. After the collision, sphere B moves in the opposite direction with a speed of $2\,\mathrm{m\,s^{-1}}$.

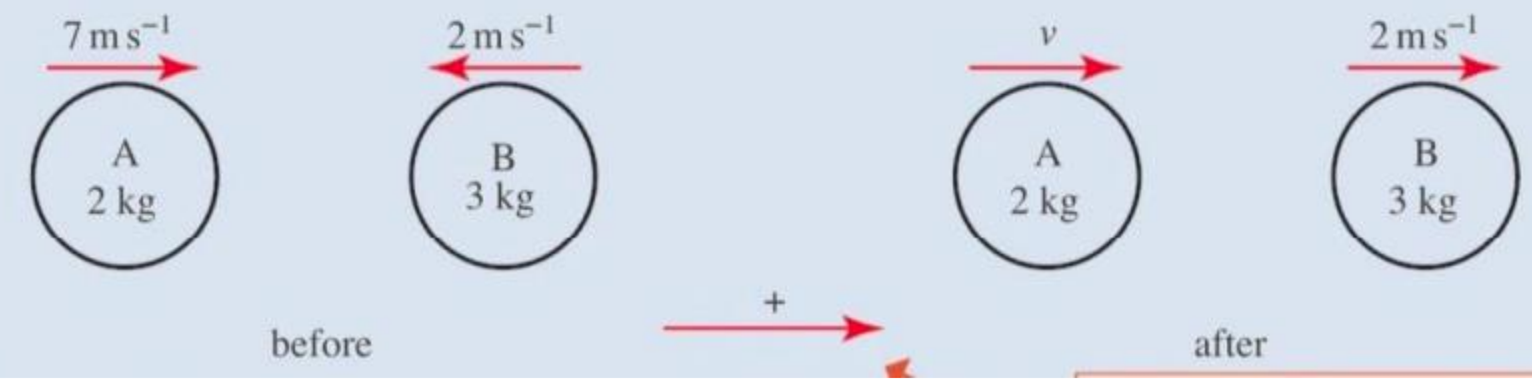
(i) Find the velocity of sphere A and the loss in kinetic energy of the system.

After the collision with sphere A, sphere B collides with sphere C of mass 1 kg which is moving in the same direction as B with velocity $0.5\,\mathrm{m\,s^{-1}}$. The spheres coalesce to form an object D.

Coalesce means stick together.

(ii) Find the velocity of D after this collision.

(i)



$$\text{total momentum before} = \text{total momentum after}$$

$$2 \times 7 + 3 \times (-2) = 2v + 3 \times 2$$

$$8 = 2v + 6$$

$$2v = 2$$

$$v = 1$$

A common mistake is to try to use *kinetic energy before* = *kinetic energy after* to solve problems on collisions. This is wrong because energy is usually lost as heat and sound.

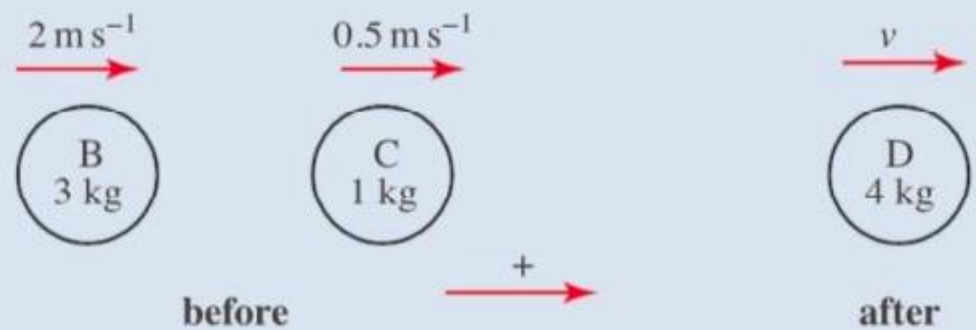
So the final velocity of A is $1\,\mathrm{m\,s^{-1}}$.

Total initial K.E. $= \frac{1}{2} \times 2 \times 7^2 + \frac{1}{2} \times 3 \times 2^2 = 55\,\mathrm{J}$

Total final K.E. $= \frac{1}{2} \times 2 \times 1^2 + \frac{1}{2} \times 3 \times 2^2 = 7\,\mathrm{J}$

Loss in K.E. $= 55\,\mathrm{J} - 7\,\mathrm{J} = 48\,\mathrm{J}$

(ii)



▲ **Figure 10.8**

Using the law of conservation of momentum:

$$\text{total momentum before} = \text{total momentum after}$$
$$3 \times 2 + 1 \times 0.5 = 4v$$
$$6.5 = 4v$$

The velocity of D is $1.625\,\mathrm{m\,s^{-1}}$

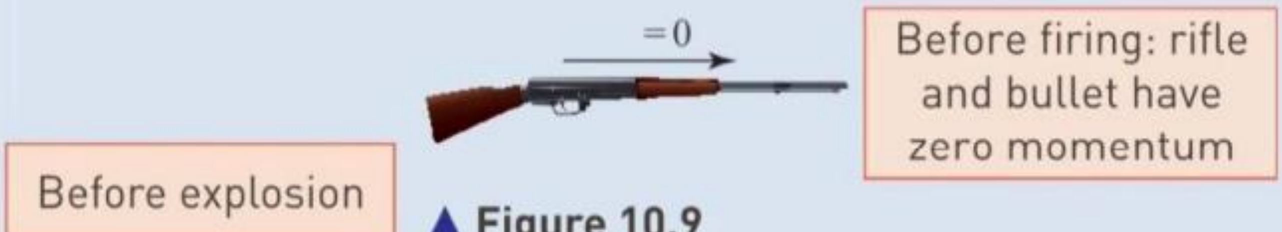
Explosions

Conservation of momentum also applies when explosions take place provided there are no external forces. For example when a bullet is fired from a rifle, or a rocket is launched.

A rifle of mass 8 kg is used to fire a bullet of mass 80 g at a speed of $200\,\mathrm{m\,s^{-1}}$. Calculate the initial recoil speed of the rifle.

Solution

Before the bullet is fired, the total momentum of the system is zero (Figure 10.9).



▲ **Figure 10.9**

After firing, the situation is as shown in Figure 10.10.

▲ **Figure 10.10**

The total momentum in the positive direction after the firing is

$$8v + 0.08 \times 200$$

For momentum to be conserved,

$$8v + 0.08 \times 200 = 0$$

so that

$$v = \frac{-0.08 \times 200}{8} = -2$$

The recoil speed of the rifle is $2\,\mathrm{m\,s^{-1}}$.

You have probably realised that v would turn out to be negative.