

B. TECH
(SEM-III) THEORY EXAMINATION 2019-20
MATHEMATICS-IV

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

| Q no. | Question | Marks | CO | | | | | | | | |
|-------|--|---------------|---------------|---|---|------|---------------|---------------|---------------|---|---|
| a. | Solve the following partial differential equation $yq - xp = z$. | 2 | 1 | | | | | | | | |
| b. | Solve the Cauchy's problem $u_x - u_y = 0$. $u(x, 0) = x$ | 2 | 1 | | | | | | | | |
| c. | Classify the following equation. $x^2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u$ | 2 | 2 | | | | | | | | |
| d. | Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = 0$. | 2 | 2 | | | | | | | | |
| e. | Find the median of 6, 8, 9, 10, 11, 12, 13. | 2 | 3 | | | | | | | | |
| f. | The first three central moments of a distribution are 0, 15, -31. Find the moment of coefficient of skewness. | 2 | 3 | | | | | | | | |
| g. | If the p.m.f of a discrete random variable X is <table border="1" style="margin: 10px auto;"> <tr> <td>X</td><td>1</td><td>2</td><td>3</td></tr> <tr> <td>f(x)</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td></tr> </table> Determine E(X) and V(X). | X | 1 | 2 | 3 | f(x) | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | 2 | 4 |
| X | 1 | 2 | 3 | | | | | | | | |
| f(x) | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | | | | | | | | |
| h. | The probability density function f(x) of a continuous random variable X is defined by $f(x) = \begin{cases} \frac{A}{x^2}, & 5 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$ Find the value of A. | 2 | 4 | | | | | | | | |
| i. | Find the mean of the Binomial Distribution $B\left(4, \frac{1}{3}\right)$. | 2 | 4 | | | | | | | | |
| j. | A machine which produces mica insulating washers for use in electric device to turn out washers having a thickness of 10 mm. A sample of 10 washers has an average thickness 9.52 mm with a standard deviation of 0.6 mm. Find out t. | 2 | 5 | | | | | | | | |

SECTION B

2. Attempt any three of the following:

3 x 10 = 30

| Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$ | 10 | 1 |
| b. | A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge $y=0$ is given by: $u(x, 0) = \begin{cases} 20x & 0 \leq x \leq 5 \\ 20(10-x) & 5 \leq x \leq 10 \end{cases}$ While the two edges $x=0$ and $x=10$ as well as the other short edge are kept at 0°C . Find the steady state temperature at any point (x, y) of the plate. | 10 | 2 |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------------|--|--------------|----------|--------------|-------|------------|----|-----|-----|----------------|-----|-----|-----|-------|-----|-----|-----|----|---|---|---|---|---|---|---|--|--|
| c. | Find an exponential curve $PV^x = k$ for the data: | 10 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"><tr><td>V</td><td>50</td><td>100</td><td>150</td><td>200</td></tr><tr><td>P</td><td>135</td><td>48</td><td>26</td><td>17</td></tr></table> | V | 50 | 100 | 150 | 200 | P | 135 | 48 | 26 | 17 | | | | | | | | | | | | | | | | |
| V | 50 | 100 | 150 | 200 | | | | | | | | | | | | | | | | | | | | | | | |
| P | 135 | 48 | 26 | 17 | | | | | | | | | | | | | | | | | | | | | | | |
| d. | Fit a Poisson distribution to the following data which give the number of yeast cells per square for 400 squares | 10 | 4 | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>F</td><td>103</td><td>143</td><td>98</td><td>42</td><td>8</td><td>4</td><td>2</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table> <p>It is given that $e^{-1.52} = 0.2674$.</p> | X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | F | 103 | 143 | 98 | 42 | 8 | 4 | 2 | 0 | 0 | 0 | 0 | | |
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | | | | | | | | | | | | | | |
| F | 103 | 143 | 98 | 42 | 8 | 4 | 2 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | |
| e. | To test the effectiveness of inoculation against cholera, the following table was obtained | 10 | 5 | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"><tr><td></td><td>Attached</td><td>Not attached</td><td>Total</td></tr><tr><td>Inoculated</td><td>30</td><td>160</td><td>190</td></tr><tr><td>Not inoculated</td><td>140</td><td>460</td><td>600</td></tr><tr><td>Total</td><td>170</td><td>620</td><td>790</td></tr></table> <p>(The figure represents the number of persons)</p> <p>Use Chi square test to defend or refute the statement. The inoculation prevents attack from cholera. The value of χ^2 for 1 degree of freedom at 5% level is 3.841.</p> | | Attached | Not attached | Total | Inoculated | 30 | 160 | 190 | Not inoculated | 140 | 460 | 600 | Total | 170 | 620 | 790 | | | | | | | | | | |
| | Attached | Not attached | Total | | | | | | | | | | | | | | | | | | | | | | | | |
| Inoculated | 30 | 160 | 190 | | | | | | | | | | | | | | | | | | | | | | | | |
| Not inoculated | 140 | 460 | 600 | | | | | | | | | | | | | | | | | | | | | | | | |
| Total | 170 | 620 | 790 | | | | | | | | | | | | | | | | | | | | | | | | |

3. Attempt any one part of the following:

1 x 10 = 10

| Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | Solve $(D+1)(D+D'-1)z = \sin(2x+3y)$ | 10 | 1 |
| b. | In a partially destroyed laboratory record of an analysis of correlation data, the following result only are legible: Variance of $x = 9$ Regression equation: $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. What were (a) the mean value of x and y (b) the standard deviation of y and the coefficient of correlation between x and y ? | 10 | 3 |

4. Attempt any one part of the following:

1 x 10 = 10

| Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | Solve $x^2 \frac{\partial^2 z}{\partial x^2} - 4y^2 \frac{\partial^2 z}{\partial y^2} - 4y \frac{\partial z}{\partial y} - z = x^2 y^2 \log y$ | 10 | 1 |
| b. | A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x,t)$. | 10 | 2 |

5. Attempt any one part of the following:

1 x 10 = 10

| Q no. | Question | Marks | CO |
|-------|---|-------|----|
| a. | An insulated rod of length l its ends A and B maintained at 0°C and 100°C respectively until the steady state condition prevails. If B is suddenly reduced to 0°C and maintained at 0°C , Find the temperature at a distance x from A at time t . | 10 | 2 |

| | | | | | | | | | | | | | | | | | | | | | | | | |
|-------|--|-------|----|----|----|----|----|----|-------|----|----|---|---|---|---|-------|----|----|----|----|----|---|--|--|
| b. | Find the multiple regression equation of X_1 on X_2 and X_3 from the data Given below: | 10 | 3 | | | | | | | | | | | | | | | | | | | | | |
| | <table><tr><td>X_1</td><td>3</td><td>5</td><td>6</td><td>8</td><td>12</td><td>10</td></tr><tr><td>X_2</td><td>10</td><td>10</td><td>5</td><td>7</td><td>5</td><td>2</td></tr><tr><td>X_3</td><td>20</td><td>25</td><td>15</td><td>16</td><td>15</td><td>2</td></tr></table> | X_1 | 3 | 5 | 6 | 8 | 12 | 10 | X_2 | 10 | 10 | 5 | 7 | 5 | 2 | X_3 | 20 | 25 | 15 | 16 | 15 | 2 | | |
| X_1 | 3 | 5 | 6 | 8 | 12 | 10 | | | | | | | | | | | | | | | | | | |
| X_2 | 10 | 10 | 5 | 7 | 5 | 2 | | | | | | | | | | | | | | | | | | |
| X_3 | 20 | 25 | 15 | 16 | 15 | 2 | | | | | | | | | | | | | | | | | | |

6. Attempt any *one* part of the following: 1 x 10 = 10

| Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | State the Bayes' theorem. The probability that a civilian can hit a target is $\frac{2}{5}$ and the probability that an army officer can hit the same target is $\frac{3}{5}$. While the civilian can fire 8 shots in the time, the army officer fires 10 shots. If they fire together, then what is the probability that army officer shoots the target? | 10 | 4 |
| b. | Define the Normal distribution. The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wage will be (i) between Rs. 140 and Rs. 144, (ii) less than Rs. 126 (iii) more than Rs. 160. | 10 | 4 |

7. Attempt any *one* part of the following: 1 x 10 = 10

| Q no. | Question | Marks | GO | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------------------|--|--------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-------------------|----|----|----|----|----|----|----|----|----|----|----|---|----|----|----|----|----|----|---|
| a. | An IT company wants to appoint an effective trainer to improve the performance of their engineers. Four group of 7, 8, 10 and 11 Engineers from total 36 engineers were given 5 days training by the 4 trainers. Scores were awarded to the engineers at the end of the training on their Skills. Let us examine the preference of one engineer of one trainer over other three trainers. Given that $\alpha=0.05$ i.e. at 5% level of significance the value of $F(3,32)=3.29$. | 10 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| b. | Distinguish between p chart and C chart. The number of defectives in 17 samples of size 500 each from 17 lots is shown below: <table border="1" style="margin: 10px auto;"><thead><tr><th>Sample</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>13</th><th>14</th><th>15</th><th>16</th><th>17</th></tr></thead><tbody><tr><td>No. of defectives</td><td>20</td><td>25</td><td>35</td><td>45</td><td>15</td><td>65</td><td>15</td><td>20</td><td>35</td><td>23</td><td>12</td><td>9</td><td>21</td><td>22</td><td>32</td><td>35</td><td>38</td></tr></tbody></table> <p>Find out the control limits for the number of defective units and also check whether the process is under control or not.</p> | Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | No. of defectives | 20 | 25 | 35 | 45 | 15 | 65 | 15 | 20 | 35 | 23 | 12 | 9 | 21 | 22 | 32 | 35 | 38 | 10 | 5 |
| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | | | | | | | | | | | | | | | | | | | | | | |
| No. of defectives | 20 | 25 | 35 | 45 | 15 | 65 | 15 | 20 | 35 | 23 | 12 | 9 | 21 | 22 | 32 | 35 | 38 | | | | | | | | | | | | | | | | | | | | | | |