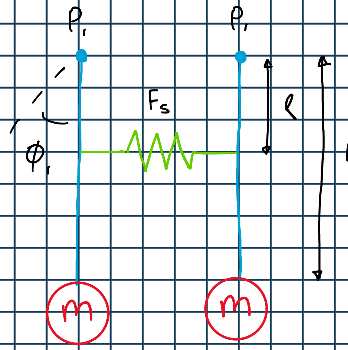
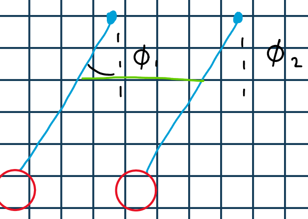
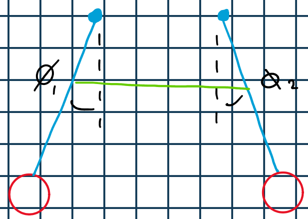
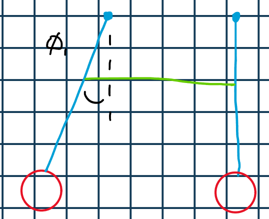
Preliminaries / Foundational knowledge

Tuesday, 31 May 2022 12:15 pm



The coupled Pendulum:

- For us, $m = 1kg$, assume strings are massless, frictionless etc
- Maximum angular displacement is $|\phi| = |\phi_1| = |\phi_2|$
- The system is symmetric, i.e the solutions for P_1 and P_2 are interchangeable

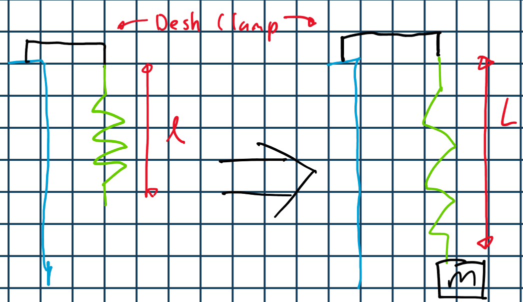
IN PHASE	OUT of PHASE	BEAT
		
$\phi_{1\,max} = \phi_{2\,max}$	$\phi_{1\,max} = -\phi_{1\,max}$	$\phi_{max} = C,\ \phi_{2\,max} = 0$
Pendulums TOGETHER	Pendulums APPART	One starts at Res-

SPRING CONSTANTS

- IOT make predictions we need to know k to a good precision

Recall $F = kx$, $V = \frac{1}{2}kx^2$

- We can subject the spring to a force and measure deformation



$$\Delta F = mg,\ \Delta x = L - \ell,\ \therefore mg = k(L - \ell),\quad k = mg/(L-\ell)$$

Also, spring is conductive so must be insulated from metal!
This is to avoid shorting current and ruining potentiometer results

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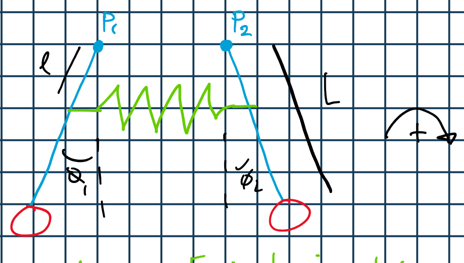
Let us consider this general system
- $m = 1kg$, therefore implied



$$\ell_{y_1} = \ell\cos(\phi_1),\quad L_{x_1} = L\sin(\phi_1)$$

$$|\tau_1| = \sum \tau_1 = F_s \ell\cos(\phi_1)^* + gL\sin(\phi_1)$$

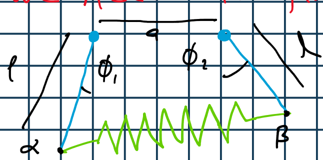
$$\therefore |\tau_2| = \sum \tau_2 = F_s\ell\cos(\phi_2) + gL\sin(\phi_2)$$

* Assumes F_s is horizontal
will come back later!

Recall $\tau = I\ddot{r}^2$, and $L = mr^2$ for a pendulum

$$\therefore L^2\ddot{\phi}_1 = F_s\ell\cos(\phi_1) + gL\sin(\phi_1),\quad L^2\ddot{\phi}_2 = F_s\ell\cos(\phi_2) + gL\sin(\phi_2)$$

We need to find an expression for F_s



Consider this as its own coordinate system

Having defined these points, $\Delta x = |\vec{\alpha\beta}|^2 - a$

$$\vec{\alpha} = \begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}\ell\sin(\phi_1)\\ \ell\cos(\phi_1)\end{pmatrix},\quad \vec{\beta} = \begin{pmatrix}a\\0\end{pmatrix} + \begin{pmatrix}\ell\sin(\phi_2)\\ \ell\cos(\phi_2)\end{pmatrix}$$

$$\therefore \vec{\alpha\beta} = \vec{\beta} - \vec{\alpha} = \begin{pmatrix} a + \ell\sin(\phi_2) - \ell\sin(\phi_1) \\ \ell\cos(\phi_2) - \ell\cos(\phi_1) \end{pmatrix}$$

With small angle approximations $\sin(x) = x$, $\cos(x) = 1$

$$\vec{\alpha\beta} = \begin{pmatrix} a + \ell(\phi_2 - \phi_1) \\ 0 \end{pmatrix}$$ ← This is our horizontal F_s assumption validated!

$$|\vec{\alpha\beta}| = a + \ell(\phi_2 - \phi_1)$$ $\because$ From the physical system, $a >> \ell(\phi_2 - \phi_1)$

$$\therefore \Delta x = |\vec{\alpha\beta}| - a = \ell(\phi_2 - \phi_1)$$

$$\therefore F_s = -\ell(\phi_1 - \phi_2)k = k\ell(\phi_2 - \phi_1)$$

$$\therefore L^2\ddot{\phi}_1 = Lg\sin(\phi_1) - \ell^2 k(\phi_1 - \phi_2)\cos(\phi_1)$$

Again, applying $\cos(x) = 1$, $\sin(x) = x$

$$L^2\ddot{\phi}_1 = Lg\phi_1 - \ell^2 k(\phi_1 - \phi_2)$$

Spring acts in opposite direction!

$$\therefore \ddot{\phi}_1 = \frac{g}{L}\phi_1 + \frac{\ell^2 k}{L^2}(\phi_2 - \phi_1), \quad \ddot{\phi}_2 = \frac{g}{L} + \frac{\ell^2 k}{L^2}(\phi_1 - \phi_2)$$

We will now switch to $(-)$ from $(+)^*$, use $\omega^2 = g/L$

$$\ddot{\phi}_1 + \omega^2\phi_1 = -\frac{\ell^2 k}{L^2}(\phi_2 - \phi_1)$$

$$\ddot{\phi}_2 + \omega^2\phi_2 = \frac{\ell^2 k}{L^2}(\phi_2 - \phi_1)$$

* Should have done from start $\because$ F_s is positive

Lets make the substitution $B = \frac{\ell}{L}\sqrt{k}$

Now we can solve these using the provided general solutions!

In phase: $\phi_1(t) = \phi_2(t) = \phi_{max}\cos(\omega t)$

Out of phase: $\phi_1(t) = -\phi_2(t) = \phi_{max}\cos(\sqrt{\omega^2 + 2B^2}\, t)$

Beat: $\phi_1(t) = \phi_{max}\cos\left(\frac{\sqrt{\omega^2 + 2B^2} - \omega}{2} t\right)\cos\left(\frac{\sqrt{\omega^2 + 2B^2} + \omega}{2} t\right)$

(ϕ_2 at rest)

$$\phi_2(t) = -\phi_{max}\sin\left(\frac{\sqrt{\omega^2 + 2B^2} - \omega}{2} t\right)\sin\left(\frac{\sqrt{\omega^2 + 2B^2} + \omega}{2} t\right)$$

We can fourier transform these!

Done with Mathematica

In Phase: ω (duh)

Out of Phase: $\sqrt{2B^2 + \omega^2}$ → One negative and can ignore?

Beat: Big, complex expressions, 4 'nodes'

Recall: $\omega^2 = g/L$, $B = \frac{\ell}{L}\sqrt{k}$

$$\omega \pm \sqrt{2\frac{\ell^2}{L^2}k + \omega^2} \qquad \omega^2 = \sqrt{\frac{g}{L}}$$

$$\omega^4 = \frac{g}{L}$$

$$\therefore \omega \pm \sqrt{2\ell^2\left(\frac{\omega^4}{g^2}\right) + \omega^2} \qquad L^2 = \frac{g^2}{\omega^4}$$

$$= \omega \pm \omega\sqrt{\frac{2\ell^2}{g^2}\omega^2 + 1}$$

Week 2

Friday, 24 June 2022 2:15 pm

Objective: Run 10min beat experiments
1x for $\ell = 200, 400, 600$ mm

Redo linear regression for spring constants

Attempt non-SAA Approach

• Have confirmed $L = 1.0$m exactly

$$Ae^{-Rt}(\sin(\omega t + \phi)) + K$$

Curve fit 2 DATA

USE LORENTZIAN TO GET ω

M	0	10	20	30	40	50	60	g
L	184	233	265	294	328	355	386	mm
dx	0	49	81	110	144	171	202	mm

Spring Constant Determination

First Lab Experimental Notebook

Friday, 10 June 2022 2:08 pm

Initial Determination of Spring constant

(g) m	0	10	20	30	40	50	60
(mm) ℓ	119	159	182	216	248	285	306
Δx	0	40	63	97	129	166	187

$mg = kx$

$m = \frac{k}{g}x$

$m\frac{g}{k} = x$

First Experiment: Verify Pendulums are independent when uncoupled.

· Expect sinusoidal motion with $\omega = \sqrt{l/g}$ ☆

3 × 2 minute runs on each pendulum

Left Pendulum | Right Pendulum

Notes:

· Right sensor has significantly less noise

· Overtones Apparent on F.T. Benzene fingers

- Only apparent on left sensor

· Set right Pendulum to Ch1 for better F.T. Data

· Controlled lateral modes as well as practacable

Second Experiment: Out of phase

· 5 min experiment time

· 3 runs per ℓ (mm)

Use right as Ch1 for data

$\ell = 400$ | $\ell = 600$ | $\ell = 200$

* Start pendulum TOGETHER to avoid crash

Experiment 3: Beat

· Just gathering data for now

Experiment 3: Beat

- Just gathering data for now
- 5 minute run time
- Fixed $\ell = 200$, one of F.T starts stationary + moving

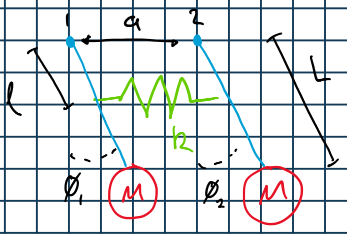
Experiment 4: In Phase

- 5 minute runtime
- 200, 400, 600 mm ℓ, one trial each

time constraints prevented more trials

Better Derivation

Friday, 3 June 2022 6:02 pm



For us:
$m = L = 1$
∴ they are implied

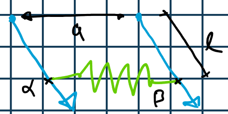
$\tau_a = I\alpha = MR^2\ddot{\phi} = L\ddot{\phi}_1$ (keep like this for later!)

$L\ddot{\phi}_1 = \Sigma\tau_1 = L \cdot F_g \sin(\phi_1) + \ell \cdot F_s \cos(\phi_1)$

$F_g = mg = 1 \cdot g = g.$ $F_s = -k\Delta x$

$\therefore \quad L\ddot{\phi} = -g\sin(\phi_1) - \ell\cos(\phi_1) k \Delta x$

Now what is Δx?



$x_0 = a, \quad \Delta x = |\vec{\alpha\beta}| - a$

$\vec{\alpha} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \ell\sin(\phi_1) \\ \ell\cos(\phi_1) \end{pmatrix}, \quad \vec{\beta} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} \ell\sin(\phi_2) \\ \ell\cos(\phi_2) \end{pmatrix}$

$\therefore \vec{\alpha\beta} = \vec{\beta} - \vec{\alpha} = \begin{pmatrix} \ell\sin\phi_2 + a - \ell\sin\phi_1 \\ \ell\cos\phi_2 - \ell\cos\phi_1 \end{pmatrix}$

Taylor small angle approximation,

$\therefore \quad \vec{\alpha\beta} = \begin{pmatrix} \ell\phi_2 - \ell\phi_1 + a \\ \ell - \ell \end{pmatrix} = \begin{pmatrix} a + \ell(\phi_2 - \phi_1) \\ 0 \end{pmatrix}$

$\therefore \Delta x = |\vec{\alpha\beta}| - a = a + \ell(\phi_2 - \phi_1) - a = \ell(\phi_2 - \phi_1)$

Now sub back in and apply small angle

$L\ddot{\phi}_1 = -g\phi_1 - \ell k(\ell(\phi_2 - \phi_1))$

$\ddot{\phi}_1 = -g/L - \ell^2 k(\phi_2 - \phi_1)$

For a pendulum, natural frequency $\omega = \sqrt{g/L}$

$\therefore \quad \ddot{\phi}_1 + \omega^2 = -(\ell\sqrt{k})^2(\phi_2 - \phi_1)$

And it follows that (Fs acts in opposite direction

$\ddot{\phi}_2 + \omega^2 = (\ell\sqrt{k})^2(\phi_2 - \phi_1)$