IBDP Mathematics

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Pure mathematics is, in its way, the poetry of logical ideas. — Albert Einstein, German theoretical physicist. This revision book is to the students of IBDP and if there are something to replenish please contact me by My email (click on "My email")

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1 Sequence and Series

1.1 Arithmetic Sequence

Definition 1.1. *Arithmetic Sequence/Arithmetic Progression (AP) is defined as:*

$$u_n = u_1 + (n-1)d \tag{1}$$

u_n is the general term
u_1 is the first term
n is the number of the term
d is is the common difference.

Note that the u_n in the sum equation of Arithmetic Sequence can be written as $u_1 + (n-1)d$ Thus that the equation can also be written as$S_n = \frac{1}{2}[2u_1 + (n-1)d]$

Theroem 1.1. *The sum of Arithmetic Sequence is :*

$$S_n = \frac{n(u_n + u_1)}{2} \tag{2}$$

Please notice till now we do not learn the Infinite sum equation for Arithmetic Sequence yet.

1.2 Geometric Sequence

Definition 1.2. *Geometric Sequence is defined as :*

$$u_n = u_1 \times (r)^{n-1} \tag{3}$$

where r is the common ratio of this sequence.

In Geometric Sequence there are some theorem which I will list below.

Corollary 1.1. *For the common ratio r it follows the rule:*

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3} = ... \tag{4}$$

Theroem 1.2. *The sum of finite Geometric Sequence is :*

$$S_n = \frac{u_1(1-r^n)}{1-r} \tag{5}$$

If r > 1, the sequence is an exponential growth.
If 0 < r < 1, the sequence has an exponential decay.

For the limitation of r , -1 < r < 1 can also be written as r<∣1∣

Theroem 1.3. *When the n = ∞ the sum of Geometric sequence is:*

$$S_\infty = \frac{u_1}{1-r} \ (-1 < r < 1) \tag{6}$$

2 Counting and Binomial Theorem

2.1 Binomial Theorem

In algebra, a binomial is a polynomial that is the sum of two terms, each of which is a monomial. It is the simplest kind of sparse polynomial after the monomials.[1]

[1]

Definition 2.1. *A Binomial is a polynomial which satisfy :*

$$(a+b)^n \tag{7}$$

The method we use to expand the binomial is using the binomial theorem.

The meaning of $\binom{n}{r}$ is the combination.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

.

Theroem 2.1.

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + b^n \quad n \in \mathbb{N} \tag{8}$$

The independent term: The term do not involve x in it since the independent term does not vary as x varies. (constant term)

Using Theorem 2.2 in the case of $(a+x)^n$ which a is not equal to 1

Theroem 2.2.

$$(a+b)^n = a^n(1+\frac{b}{a})^n \ , \ |\frac{b}{a}| < 1 \ , \ n \in \mathbb{Q} \tag{9}$$

Please notice that when $n \in \mathbb{N}$ (n is a natural number) $(a+b)^n$ have n terms. But in the case which $n \in \mathbb{Q}$ (n is a rational number) the binomial $(a+b)^n$ would have infinite terms.

Problem 1. *Write the first three terms in the expansion of* $(2+x)^{-3}$

Solution:

Using the theorem 2.2 to change $(2+x)^{-3}$ into another form.

$$(2+x)^{-3} = 2^{-3}(1+\frac{x}{x})^{-3}$$

$$= (\frac{1}{8})(1+(-3)\frac{x}{2} + \binom{-3}{2}(\frac{x}{2})^2 + ...)$$

To calculate the $\binom{-3}{r}$ you can use GDC, or I will discuss this later in the section of Counting theorem.

$$= \frac{1}{8}(1 - \frac{3}{2}x + \frac{12}{4}x^2 + ...)$$

$$= \frac{1}{8} - \frac{3}{16}x + \frac{3}{8}x^2$$

2.2 *Counting: permutation and combination*

In mathematics, a combination is a selection of items from a set that has distinct members, such that the order of selection does not matter (unlike permutations).[2]

[2]

Definition 2.2. *Combination, choose r from n is :*

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \tag{10}$$

$$n! = n(n-1)(n-2)(n-3)...3 \cdot 2 \cdot 1 \tag{11}$$

The number of ways of arranging n distinct objects in a row is $n!$.

A permutation of a set is, an arrangement of its members into a sequence or linear order, or if the set is already ordered, a rearrangement of its elements.

In permutations, the order matters.In combinations, the order does not matter.

Definition 2.3. *The permutation(Arrangement) of r objects out of n objects is:*

$$_nP_r = \frac{n!}{(n-r)!} \tag{12}$$

3 Exponents and Logarithms

3.1 Basic concepts

The formula of exponents are listed below. In the exam these are the most important formula, therefore you should not forget them.

Theroem 3.1.

$$a^n \cdot a^m = a^{n+m}$$

$$a^n \div a^m = a^{n-m}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$a^{-m} = (\frac{1}{a})^m = \frac{1}{a^m}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Tip: When solving exponential equations, convert them to the same base.

The combine of square root and exp are like this $a^{\frac{m}{n}=(\sqrt[n]{a})^m}$.

3.2 Logarithms

Logarithms are often defined as the inverse function of exponents, and in IBDP program logarithms are a very common topic. Hence you should remember all the concepts below.

Definition 3.1. *The function $a = b^x$ can be written as $x = \log_b a$*

The rules of logarithms:

Theroem 3.2.

$$\log_a x^n = n \log_a x$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

Proof. To proof formula (13) Let $\log_a x = p$, $\log_a y = q$, as a result $a^p = x$, $a^q = y$, hence $a^p \times a^q = a^{p+q} = xy$. Therefore $\log_a xy = p + q = \log_a x + \log_a y$ □

Proof. To proof the formula (14)Let $\log_a x = p$, $\log_a y = q$, as a result $a^p = x$, $a^q = y$, hence $a^p \div a^q = a^{p-q} = \frac{x}{y}$. Therefore $\log_a \frac{x}{y} = p - q = \log_a x - \log_a y$ □

$$-\log_a x = \log_a \frac{1}{x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_a b = \frac{1}{\log_b a}$$

$$\log_a x + \log_a y = \log_a xy \tag{13}$$

$$\log_a x - \log_a y = \log_a \frac{x}{y} \tag{14}$$