Elena Maria Varghese
59 - S5 - CSE - A

FLAT
Assignment

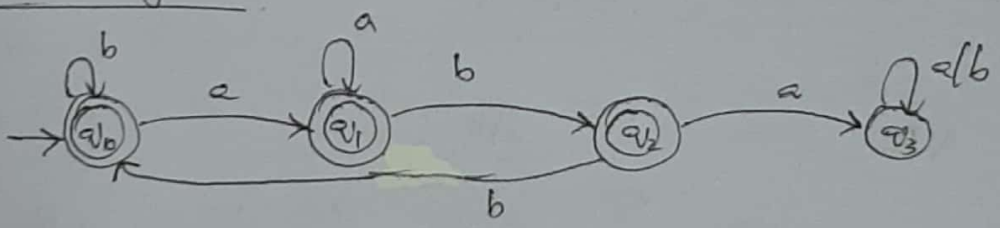
Sample Question Paper

PART-A

1- Design a DFA for the language

$L = \{x \in \{a,b\}^* \mid aba \text{ is not a substring in } x\}$

Ans:- State Diagram



Formal Definition

$D = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 = q_0$.

$A = \{q_0, q_1, q_2\}$

$\delta : \{1.\ \delta(q_0, a) = q_1\ \ 2.\ \delta(q_0, b) = q_0,\ \ 3.\ \delta(q_1, a) = q_1,$

$4.\ \delta(q_1, b) = q_2\ \ 5.\ \delta(q_2, a) = q_3\ \ 6.\ \delta(q_2, b) = q_0,$

$7.\ \delta(q_3, a) = q_3\ ,\ 8.\ \delta(q_3, b) = q_3\}$.

State Transition Table :-

$Q \backslash \Sigma$	a	b
$\rightarrow * q_0$	q_1	q_0
$* q_1$	q_1	q_2
$* q_2$	q_3	q_0
q_3	q_3	q_3

2→ Write a regular grammar for the language
$L = \{axb \mid x \in \{a,b\}^*\}$

Ans:- NFA

$M = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$A = \{q_2\}$

$\delta : \{ \delta(q_0, a) = \{q_1\}$

$\delta(q_0, b) = \{ \}$

$\delta(q_1, a) = \{q_1\}$

$\delta(q_1, b) = \{q_1, q_2\}$

$\delta(q_2, a) = \{\}, \ \delta(q_2, b) = \{\} \}$

Regular Grammar

$G = (N, T, P, S)$

$N = \{Q\} = \{q_0, q_1, q_2\}$

$T = \Sigma = \{a, b\}$

$S = q_0.$

$P : \{ P1 : q_0 \rightarrow aq_1,$

$P2 : q_1 \rightarrow aq_1,$

$P3 : q_1 \rightarrow bq_1,$

$P4 : q_1 \rightarrow bq_2$

$P5 : q_2 \rightarrow \epsilon \}$

Q3 → Write a Regular Expression for the language $L = \{x \in \{0,1\}^* \mid$ there are no consecutive 1's in $x\}$

Ans:- let the regular expression be r

$$r = (0 + 10)^* . (1 + \epsilon)$$

Q4 → Prove that the language $L_1 = \{a^{n!} \mid n \in N\}$ is not regular.

Ans:- $L_1 = \{a, a^{2!}, a^{3!} \ldots a^{n!}, a^{(n+1)!} \ldots\}$

Step 1

Assume that L_1 is regular, then by pumping lemma, there exists a positive integer n such that $\forall$ strings $x \in L$ & $|x| \geq n$, it is possible to find three strings u, v, w such that

$x = uvw$, $|uv| \leq n$, $|v| > 0$

Then we can say $\forall t \geq 0,\ uv^t w \in L$

Step 2

Let

$x = a^{n!}$

$|x| = |a^{n!}| = n! \geq n$

By pumping lemma there exists strings of the form u, v, w, such that $|uv| \leq n$ & $|v| > 0$

Possible length of v ranges from 1 to n

$1 \leq |v| \leq n$

Suppose $t = 2$ and $|v| = n$

$uv^2w = uvvw$

$|uv^2w| = |uvw| + |v| = n! + n$

New string's length is $n! + n$

$n! \leq n! + n \leq (n+1)!$

This is a contradiction

strings of length $n! + n$ is not part of the language L_1

5 → list out the applications of Myhill Nerode Theorem.

Ans:- Myhill Nerode Theorem is used to prove that a certain language is regular or not.
It can be also used to find the minimal no. of states in a deterministic Finite Automata (DFA)

6 → Write a Context - Free grammar for the language
$L = \{ x \in \{a,b\}^* \mid \#_a(x) = \#_a(x) \}$.
Here the notation $\#_1(\omega)$ represents the no. of occurences of the symbol 1 in the string ω.

Ans:- $G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b\}$

$P = \{ 1.\ S \rightarrow \varepsilon,\ 2.\ S \rightarrow aSb,\ 3.\ S \rightarrow bSa,$
$4.\ S \rightarrow SS \}$

$S = S$

Eg:- abba

$S \xRightarrow{4.} SS \xRightarrow{2} aSbS \xRightarrow{1} abS$
$\xRightarrow{3} abbSa \xRightarrow{1} abba$

7 → Design a PDA for the language of odd length binary palindromes

Let the PDA be

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, A)$$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, \$, 1\}$

$q_0 = q_0$

$z_0 = \$$

$A = \{q_2\}$

$\delta : \{ \delta(q_0, 0, 1) = \{(q_0, 01), (q_1, 1)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10), (q_1, 0)\}$

$\delta(q_0, 1, \$) = \{(q_0, 1\$), (q_1, \$)\}$

$\delta(q_0, 0, \$) = \{(q_0, 0\$), (q_1, \$)\}$

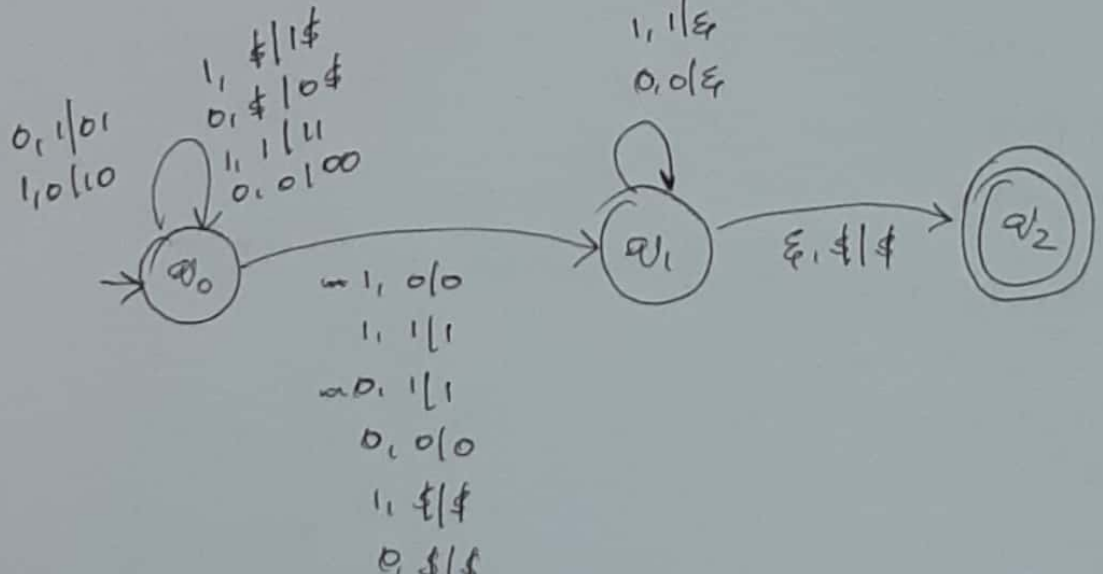
$\delta(q_0, 1, 1) = \{(q_0, 11), (q_1, 1)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00), (q_1, 0)\}$

$\delta(q_1, 0, 0) = \{(q_1, \epsilon)\}$

$\delta(q_1, 1, 1) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, \$) = \{(q_2, \$)\}$



8- Prove that Context Free languages are closed under set union.

Ans:- Let L_1 and L_2 are two CFL with grammars G_1 and G_2 respectively.

Then if a new language $L = L_1 \cup L_2$ exists, we can say that L is also context free language.

L is generated by a new grammar 'G' created by combining all the symbols and rules of G_1 and G_2.

The union property ~~of~~ for grammar G is achieved by having an additional set of rules of the form $S \rightarrow S_1 | S_2$ where S_1 and S_2 are start symbols of G_1 and G_2 respectively.

Eg:- Let $L = \{a, b\}$, $S(a) = L_1$ and

$S(b) = L_2$

$\therefore S(L) = L_1 \cup L_2$

$L_1 = \{a^n b^n \mid n \geq 0\}$, $L_2 = \{b^n a^n \mid n \geq 0\}$

Rules of $G_1 = \{S_1 \longrightarrow a S_1 b \mid \epsilon\}$

Rules of $G_2 = \{S_2 \longrightarrow b S_2 a \mid b\epsilon\}$

$\therefore$ $L_1 \cup L_2$ is generated by grammar G
with rules $G = (\{S_1, S_2, S\}, \{a, b\}, P, S)$

Rules are
$P = \{P_1 \cup P_2 \cup \{S \rightarrow S_1 | S_2\}\}$ where rules ~~the~~

P_1 & P_2 corresponds to rules of G_1 and

G_2 respectively.
Then we can have

$P = \{ S \rightarrow S_1 | S_2,$
$S_1 \rightarrow a S_1 b | \epsilon,$
$S_2 \rightarrow b S_2 a | ba \}$

9→ Write a context sensitive grammar for the
language $L = \{ a^n b^n c^n \mid n \geq 0 \}$

$G = (N, T, P, S)$

$N = \{S, A, B, C\}$

$T = \{a, b, c\}$

$S = S$

$P: \{$ 1. $S \rightarrow aSBC$
2. $S \rightarrow aBC$
3. $CB \rightarrow BC$
4. $aB \rightarrow ab$
5. $bB \rightarrow bb$
6. $bC \rightarrow bc$
~~7. cC → cc~~
7. $cC \rightarrow cc$

10 → Differentiate between Recursive and Recursively Enumerable Languages.

Ans → RE languages or type-0 languages are generated by type-0 grammars. An RE language can be accepted or recognized by Turing machine which means it will enter into final state for the strings of language and may or may not ~~enter~~ enter into rejecting state for the strings which are not part of the language.

RE languages are also called as Turing recognizable languages.

REC

Recursive language (subset of RE) can be decided by Turing machine which means it will enter into final state for the strings of language and rejecting state for the strings which are not part of the language.

e.g., $L = \{a^n b^n c^n \mid n >= 1\}$ is recursive because we can construct a turing machine which will move to final state if the string is of the form $a^n b^n c^n$ else move to non-final state. So the TM will always halt in this case. REC languages are called as Turing decidable languages.

PART-B

11 (a) Draw the state-transition diagram showing an NFA N for the following language L. Obtain the DFA D equivalent to N by applying the subset construction algorithm

$L = \{ x \in \{a,b\}^* \mid$ the second last symbol in x is $b \}$

Ans:- **NFA**

State Diagram :-

$N = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\delta : \{ 1.\ \delta(q_0, a) = \{q_0\}.$

$2.\ \delta(q_0, b) = \{q_0, q_1\}$

$3.\ \delta(q_1, a) = \{q_2\}$

$4.\ \delta(q_1, b) = \{q_2\} \}$

$q_0 = q_0$

$A = \{q_2\}$

Q \ Σ	a	b
$\rightarrow q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$
$*q_2$	$\{\}$	$\{\}$

Conversion of NFA to DFA using subset construction

δ \ Σ	a	b
$\rightarrow [q_0]$	$[q_0]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_2]$	$[q_0, q_1, q_2]$
$*[q_0, q_2]$	$[q_0]$	$[q_0, q_1]$
$*[q_0, q_1, q_2]$	$[q_0, q_2]$	$[q_0, q_1, q_2]$

$P_0 = [q_0]$

$P_1 = [q_0, q_1]$

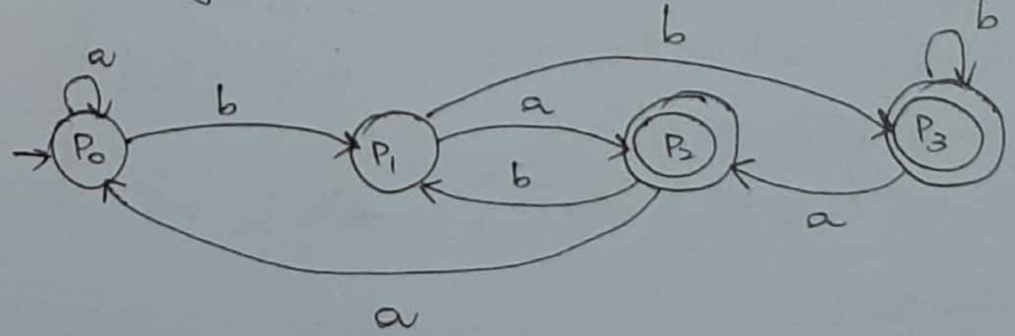
$P_2 = [q_0, q_2]$

$P_3 = [q_0, q_1, q_2]$

State ~~Diagram &~~ Transition Table :-

δ \ Σ	a	b
$\rightarrow P_0$	P_0	P_1
P_1	P_2	P_3
$* P_2$	P_0	P_1
$* P_3$	P_2	P_3

State Diagram:-



Formal Definition

$D = (Q_D, \Sigma, \delta', q_0', A')$

$Q_D = \{P_0, P_1, P_2, P_3\}$

$\Sigma = \{a, b\}$

$\delta' = \{$ 1. $\delta'(P_0, a) = P_0$

2. $\delta'(P_0, b) = P_1$

3. $\delta'(P_1, a) = P_2$

4. $\delta'(P_1, b) = P_3$

5. $\delta'(P_2, a) = P_0$

6. $\delta'(P_2, b) = P_1$

7. $\delta'(P_3, a) = P_2$

8. $\delta'(P_3, b) = P_3 \}$

$q_0 = P_0$

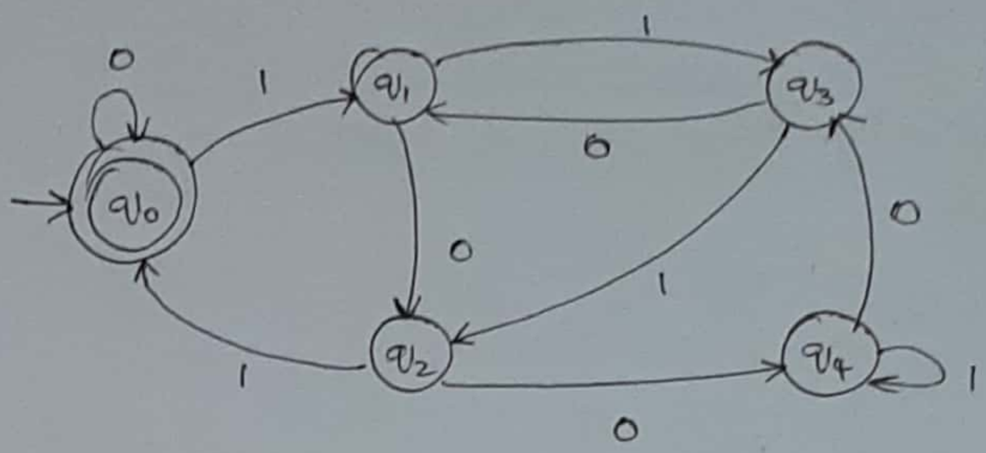
$A' = \{P_2, P_3\}$

11

(b) Draw the state-transition diagram showing a DFA for recognizing the following language

$L = \{ x \in \{0,1\}^* \mid x$ is a binary representation of a natural number which is a multiple of $5\}$

State diagram:-



Formal Definition

$$D = (Q, \Sigma, \delta, q_0, A)$$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$; $q_0 = q_0$; $A = \{q_0\}$

$\delta : \{$1. $\delta(q_0, 0) = q_0$, 2. $\delta(q_0, 1) = q_1$

3. $\delta(q_1, 0) = q_2$ 4. $\delta(q_1, 1) = q_3$

5. $\delta(q_2, 0) = q_4$ 6. $\delta(q_2, 1) = q_0$

7. $\delta(q_3, 0) = q_1$ 8. $\delta(q_3, 1) = q_2$

9. $\delta(q_4, 0) = q_3$ 10. $\delta(q_4, 1) = q_4 \}$

State Transition Table

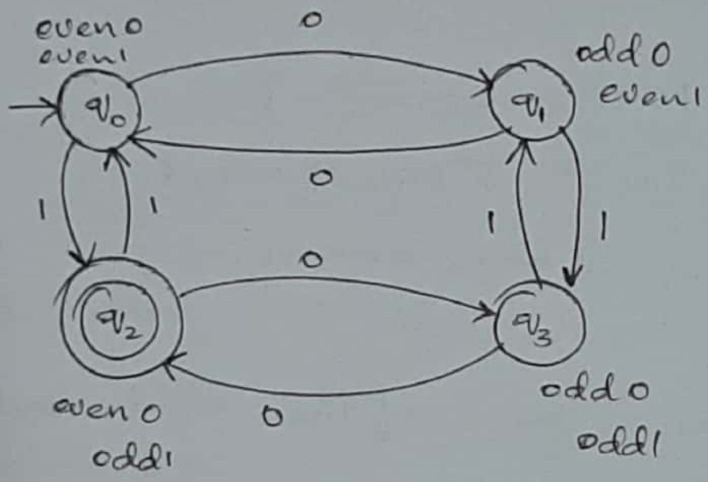
$Q \backslash \Sigma$	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

15

(a) Show ~~that~~ the equivalence classes of the canonical Myhill nerode ~~theorem~~ relation for the language of binary strings with odd no. of 1's and even number of 0's.

Ans →

DFA for odd no. of 1's and even no. of 0's.



$D = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$\delta : \{ \delta(q_0, 0) = q_1 \; ; \; \delta(q_0, 1) = q_2$

$\delta(q_1, 0) = q_0 \; ; \; \delta(q_1, 1) = q_3$

$\delta(q_2, 0) = q_3 \; ; \; \delta(q_2, 1) = q_0$

$\delta(q_3, 0) = q_2 \; ; \; \delta(q_3, 1) = q_1 \}$

$q_0 = q_0$

$A = \{q_2\}$.

let the equivalence classes be as $[J_0]$, $[J_1]$, $[J_2]$ and $[J_3]$

$$[J_0] = \{ x \mid x \in \{0,1\}^*,\ \delta^*(q_0, x) = q_0 \}$$

$$[J_0] = \{ \epsilon, 0011, 00001111, \ldots \}$$

$$[J_1] = \{ x \mid x \in \{0,1\}^*,\ \delta^*(q_0, x) = q_1 \}$$

$$[J_1] = \{ 0, 000, 011, 0001111, \ldots \}$$

$$[J_2] = \{ x \mid x \in \{0,1\}^*,\ \delta^*(q_0, x) = q_2 \}$$

$$[J_2] = \{ 1, 111, 100, 1110000, \ldots \}$$

$$[J_3] = \{ x \mid x \in \{0,1\}^*,\ \delta^*(q_0, x) = q_3 \}$$

$$[J_3] = \{ 01, 10, 111000, 000111, \ldots \}$$

L is the union of some of the equivalent classes

here $L = [J_2]$

15 (b) With an example, explain ambiguity in Context Free Grammar.

Ans- A context free grammar (CFG) is said to be ambiguous if there exists atleast one terminal string for which the grammar has more than one leftmost or rightmost derivation, or more than one parse tree.

<u>Example:-</u>

$P: \{ S \rightarrow S+S \mid S*S \mid a \}$

for the string $a+a*a$

I

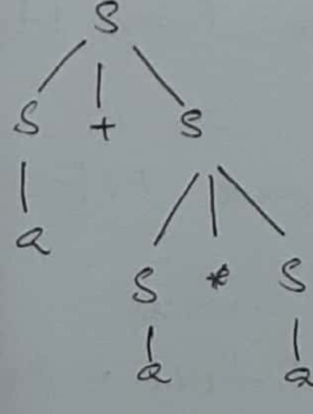
$S \xRightarrow{1} S+S$

$\xRightarrow{3} a+S$

$\xRightarrow{2} a+S*S$

$\xRightarrow{3} a+a*S$

$\xRightarrow{3} a+a*a$



II

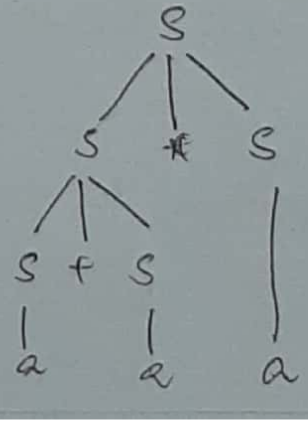
$S \xRightarrow{2} S*S$

$\xRightarrow{1} S+S*S$

$\xRightarrow{3} a+S*S$

$\xRightarrow{3} a+a*S$

$\xRightarrow{3} a+a*a$



Scanned with CamScanner

14

(b) Using ultimate periodicity for regular languages, prove that the language

$L = \{a^{n^2} \mid n \geq 0\}$ is not regular.

Step 1

Assume that L is regular, then by pumping lemma, there exists a positive integer n such that $\forall$ strings $x \in L$ & $|x| \geq n$, it is possible to find 3 strings u, v, w such that $x = uvw$

$|uv| \leq n$, $|v| > 0$

Then we can say $\forall t \geq 0$, $uv^t w \in L$

Step 2

Let $x = a^{n^2}$

$|x| = |a^{n^2}| = n^2 \geq n$

By pumping lemma, there exists strings of the form u, v, w, such that $|uv| \leq n$ and $|v| > 0$

Possible length of v ranges from 1 to n

$1 \leq |v| \leq n$

Suppose, t value is 2 and $|v| = n$

$uv^t w = uv^2 w = uvvw$

$|uv^2 w| = |uvw| + |v| = n^2 + n$

New string is having a length n^2+n

$a^{1^2}, a^{2^2}, \ldots a^{i^2} \ldots a^{n^2}, \ldots a^{(n+1)^2}$

$|a^{n^2}| = n^2$

$|a^{(n+1)^2}| = (n+1)^2 = n^2+2n+1$

n^2+n lies between n^2 and n^2+2n+1
which is a contradiction

$\therefore$ The language is not regular.

13-
(b) Using pumping lemma for regular languages,
prove that the language
$L = \{a^n b^n \mid n \geq 0\}$ is not regular.

Ans →
We assume that language 'L' is regular
then by pumping lemma there exists an integer
constant 'n' such that for all strings $x \in L$
which satisfies the condition $|x| \geq n$
It is possible to find 3 strings of the form
u, v, w with the condition

$x = uvw$

$|uv| \leq n$

$|v| > 0$ Then $\forall t \geq 0$, $uv^t w \in L$

Step 2

take $x = a^n b^n$

$|x| = |a^n b^n| = 2n \geq n$

Then we can split ~~the~~ x into uvw

under certain conditions

$|uv| = |a_1 a_2 \ldots a_i| = i < n$

$|v| = |a_4 a_5 \ldots a_i| > 0$

$u = a^p$

$v = a^q$

$w = a^r b^n$

where $p+q+r = n$

$x = a^n b^n = a^p a^q a^r b^n$

$|uv| = |a^p a^q| = p+q \leq n$

$|v| = |a^q| = q > 0$

Step 3

$t=0$; $uv^t w = uv^0 w = a^p (a^q)^0 a^r b^n$

$= a^{p+r} b^n \notin L$

$(\because p+r \neq n)$

$t=2$, $uv^2 w = a^{p+2q+r} b^n \notin L$

$(\because p+2q+r \neq n)$

This violates the pumping lemma conditions &

hence the language is not regular.

16-

(a) Convert the Context-Free Grammar with productions:

$\{S \rightarrow aSb \mid \varepsilon\}$ into Greibach Normal form

Since grammar generates empty strings, empty production is part of the language Such production cannot be converted to GNF

$P: \{S \rightarrow aSb\}$

Step 1 :- Substitute non terminals for terminals

$a \leftrightarrow A$

$b \leftrightarrow B$

$P: \{S \rightarrow ASB,\ A \rightarrow a,\ B \rightarrow b\}$

Step 2 :- Introduce an ordering among non terminals by renaming them.

let $S = A_1$

$A = A_2$

$B = A_3$

$P: \{A_1 \rightarrow A_2 A_1 A_3,\ A_2 \rightarrow a,\ A_3 \rightarrow b\}$

Step 3 :- use lemma-1 or lemma-2 if necessary so that after step 3 the rules will be of the form.

① GNF

② $A_i \rightarrow A_j\alpha \ ; j > i$

③ Z-rules

Step 4:- Convert all Ai rules to GNF

1. $A_1 \rightarrow a A_1 A_3$

2. $A_2 \rightarrow a$

3. $A_3 \rightarrow b$

Step 5:- Convert all Z-rules to GNF — No Z-rules.

∴ final grammar is

$G' = (N', T, P', S')$

$N' = \{A_1, A_2, A_3\}$

$T = \{a, b\}$

$P' = \{1.\ A_1 \rightarrow aA_1A_3,\ 2.\ A_2 \rightarrow a,\ 3.\ A_3 \rightarrow b,\ 4.\ A_1 \rightarrow \epsilon\}$

$S' = S.$

16 (b) Convert the CFG with ~~proper~~ productions $\{S \rightarrow aSa \mid bSb \mid SS \mid \epsilon\}$ in Chomsky Normal Form.

This language generates empty strings

Empty production is part of the language

Such productions cannot be converted to CNF

$\{S \rightarrow aSa \mid bSb \mid SS\}$

Step 1

$a \leftrightarrow A \quad ; \quad b \leftrightarrow B$

$\{ S \rightarrow ASA \mid BSB \mid SS, \quad A \rightarrow a, \quad B \rightarrow b \}$

$S \rightarrow ASA$ can be rewritten as

$S \rightarrow AD$

$D \rightarrow SA$

$S \rightarrow BSB$ can be rewritten as

$S \rightarrow BX$

$X \rightarrow SB$

$\therefore$ Final Grammar is

$G = (N, T, P, S)$

$N = \{A, S, B, X, D\}$

$T = \{a, b\}$

$P : \{$ 1. $S \rightarrow AD$,

2. $D \rightarrow SA$,
3. $S \rightarrow BX$
4. $X \rightarrow SB$.
5. $S \rightarrow SS$,
6. $A \rightarrow a$
7. $B \rightarrow b$
8. $S \rightarrow \epsilon \}$

$S = S$

Scanned with CamScanner

17.

(a) Design a PDA for the language

$L = \{a^m b^n c^{m+n} \mid n \geq 0, m \geq 0\}$

Also illustrate the computation of the PDA on a string in the language

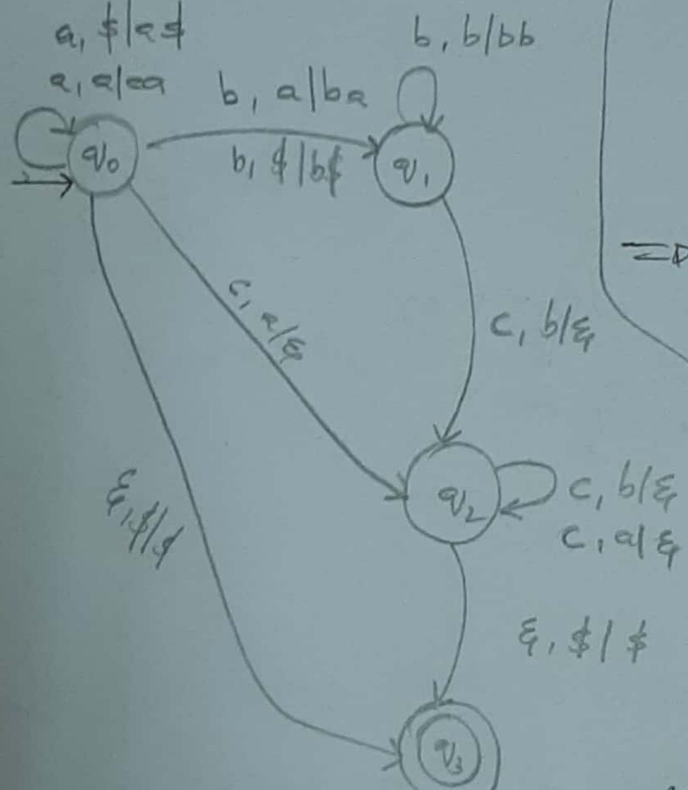
$a^m b^n c^{m+n}$

$a^m b^n c^{n+m}$

$a^m \underline{b^n c^n} c^m$

$\Rightarrow$ no. of a's + no. of b's = no. of c's.

State diagram:-



Formal definition

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, A)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{\$, a, b, c\}$

$q_0 = q_0$.

$z_0 = \$$

$A = \{q_3$.

$\delta(q_0, c, a) = \{(q_2, \epsilon)\}$

$\delta(q_0, b, \$) = \{(q_1, b\$)\}$

$\delta: \{\delta(q_0, a, \$) = \{(q_0, a\$)\},$

$\delta(q_0, a, a) = \{(q_0, aa)\},$

$\delta(q_0, b, a) = \{(q_1, ba)\},$

$\delta(q_1, b, b) = \{(q_1, bb)\},$

$\delta(q_1, c, b) = \{(q_2, \epsilon)\},$

$\delta(q_2, c, b) = \{(q_2, \epsilon)\}$

$\delta(q_2, \epsilon, \$) = \{(q_3, \$)\}$

$\delta(q_2, c, a) = \{(q_2, \epsilon)\}$ ACCEPT.

$\delta(q_0, \epsilon, \$) = \{(q_3, \$)\}$ ACCEPT.

Instantaneous description for the string $a^2b^2c^4$

aabbcccc

$(q_0, aabbcccc, \$) \vdash (q_0, abbcccc, a\$) \vdash (q_0, bbcccc, aa\$)$

$\vdash (q_1, bcccc, baa\$) \vdash (q_1, cccc, bbaa\$) \vdash (q_2, ccc, baa\$)$

$\vdash (q_2, cc, aa\$) \vdash (q_2, c, a\$) \vdash (q_2, \varepsilon, \$)$

$\vdash (q_3, \varepsilon, \$)$ ACCEPT.

14)

b) $L = \{a^{n^2} \mid n \geq 0\}$ is not regular

$a^0, a^1, a^4, a^9, \ldots a^{n^2}$

$u = \{0, 1, 4, 9, 16, 25, 36, 49, 64, \ldots\}$

Consider $n=16$, $p=9$, $m=25$

$n+p = 25+9$

$= 34 \notin U$

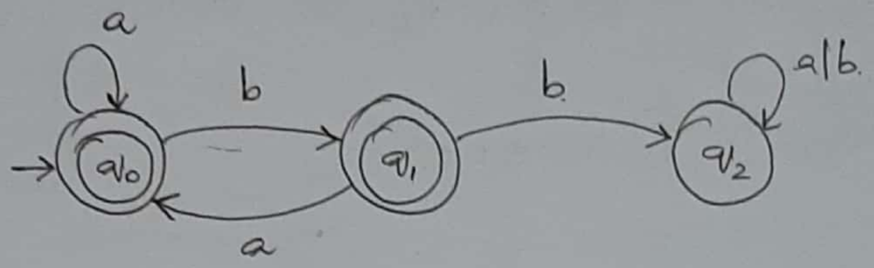
$\therefore$ The language is not regular.

12-

(a) Write a Regular Grammar G for the following language L defined as

$L = \{ x \in \{a,b\}^* \;\; x \text{ does not contain consecutive b's} \}$

DFA



$M = (Q, \Sigma, \delta, q_0, A)$

$Q = \{ q_0, q_1, q_2 \}$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$A = \{ q_0, q_1 \}$

$\delta : \{ \delta(q_0, a) = q_0,$
$\delta(q_0, b) = q_1,$
$\delta(q_1, a) = q_0$
$\delta(q_1, b) = q_2$
$\delta(q_2, a) = q_2$
$\delta(q_2, b) = q_2 \}.$

Regular Grammar is,

$G = (N, T, P, S)$

$N = \{Q\} = \{q_0, q_1, q_2\}$

$T = \Sigma = \{a, b\}$

$S = q_0$

$P : \{ P_1 : q_0 \rightarrow a q_0,$
$P_2 : q_0 \rightarrow b q_1$
$P_3 : q_1 \rightarrow a q_0,$
$P_4 : q_1 \rightarrow b q_2$
$P_5 : q_2 \rightarrow a q_2$
$P_6 : q_2 \rightarrow b q_2$
$P_7 : q_1 \rightarrow \epsilon,$
$P_8 : q_0 \rightarrow \epsilon \}$

Scanned with CamScanner

12

(b) Obtain the DFA A_G over the alphabet set $\Sigma = \{a,b\}$, equivalent to the regular grammar G with start symbol S and production : $S \rightarrow aA$ and $A \rightarrow aA | bA | b$

Ans:- $G = (N, T, P, S)$

$N = \{S, A\}$

$T = \{a, b\}$

$P : \{S \rightarrow aA \,,\; A \rightarrow aA | bA | b\}$

Equivalent NFA →

$M = (Q, \Sigma, \delta, q_0, A)$

$Q = N \cup \{q_f\} = \{S, A, q_f\}$

$\Sigma = T = \{a, b\}$

$q_0 = S$

$A = \{q_f\}$

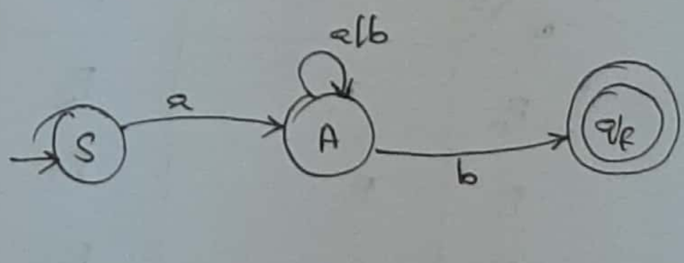
State diagram (NFA)

$\delta : \{ \delta_1 (S, a) = \{A\},$

$\delta_2 (A, a) = \{A\},$

$\delta_3 (A, b) = \{A\},$

$\delta_4 (A, b) = \{q_f\} \}$



To find the equivalent DFA using subset construction

NFA's state transition table

$Q \backslash \Sigma$	a	b
S	$\{A\}$	ϕ
A	$\{A\}$	$\{A, q_f\}$
q_f	ϕ	ϕ

DFA state transition table

$Q \backslash \Sigma$	a	b
$\rightarrow [S]$	$[A]$	$[\phi]$
$[A]$	$[A]$	$[A, q_f]$
$[\phi]$	$[\phi]$	$[\phi]$
$*[A, q_f]$	$[A]$	$[A, q_f]$

$P_0 \rightarrow [S]$

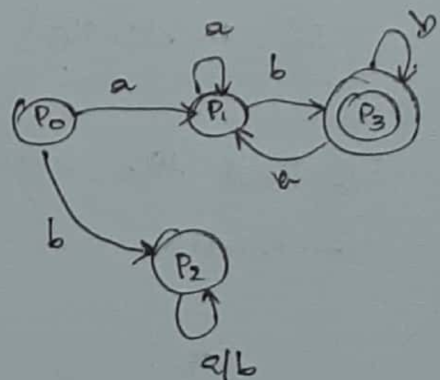
$P_1 \rightarrow [A]$

$P_2 \rightarrow [\phi]$

$P_3 \rightarrow [A, q_f]$

$Q \backslash \Sigma$	a	b
$\rightarrow P_0$	P_1	P_2
P_1	P_1	P_3
P_2	P_2	P_2
$*P_3$	P_1	P_3

State Diagram



Formal Definition for the equivalent DFA

$D = (S', \Sigma, \delta', q_0', A')$

$S' = \{P_0, P_1, P_2, P_3\}$

$\Sigma = \{a, b\}$

$\delta' : \{ \delta_1'(P_0, a) = P_1, \quad \delta_2'(P_0, b) = P_2,$

$\delta_3'(P_1, a) = P_1, \quad \delta_4'(P_1, b) = P_3,$

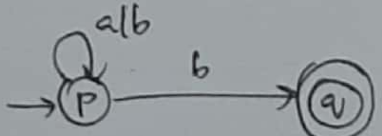
$\delta_5'(P_2, a) = P_2, \quad \delta_6'(P_2, b) = P_2,$

$\delta_7'(P_3, a) = P_1, \quad \delta_8'(P_3, b) = P_3 \}$

$q_0' = P_0$

$A' = \{P_3\}$

13-(a) Using Kleen's constuction, obtain the regular expression for the language represented by the



Ans→ Step 1

Step 2

Eliminate the states → $q_{rip} = 1$

$q_i = S \qquad q_j = 2$

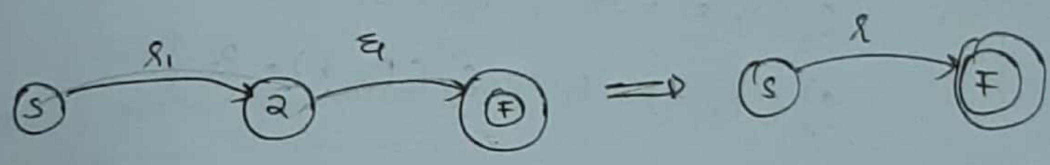
$R_1 = \delta(q_i, q_{rip}) = \delta(s, 1) = \epsilon$

$R_2 = \delta(q_{rip}, q_{rip}) = \delta(1,1) = a+b$

$R_3 = \delta(q_{rip}, q_j) = \delta(1,2) = b$

$R_4 = \delta(q_i, q_j) = \delta(s,2) = \phi$

$R_1 R_2^* R_3 + R_4 = (a+b)^* b \longrightarrow$ let this be r_1



$R_1 = r_1$

$R_2 = \phi$

$R_3 = \epsilon$

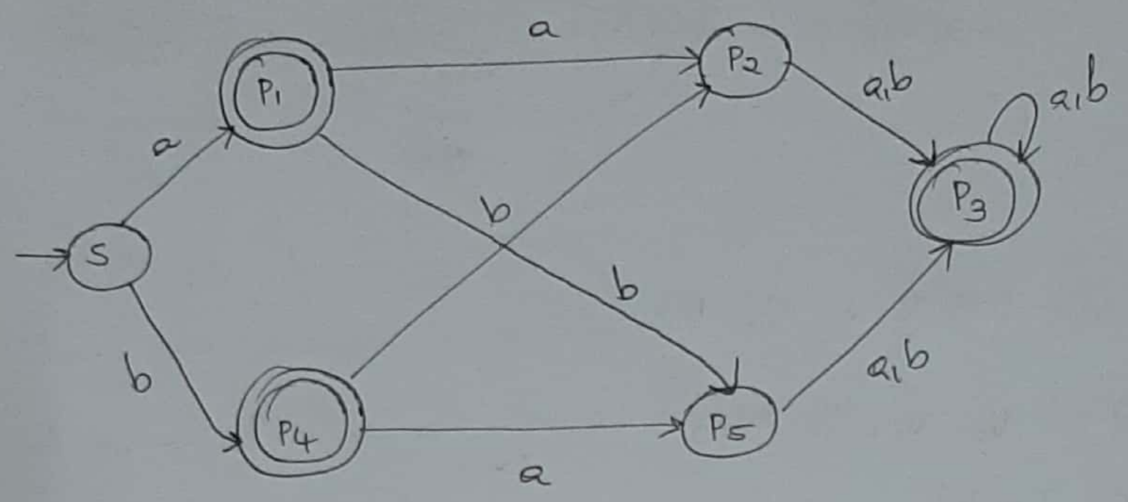
$R_4 = \phi$; Regular expression r will be $\longrightarrow$

$\therefore \; r = (a+b)^* b$

14

(a) Obtain the minimum state DFA from the following DFA

Ans:-



Ans:- Step 1

A　　　　　　Q − A

$\underbrace{P_1, P_3, P_4}$　　　　$\underbrace{S, P_2, P_5}$

$\underbrace{S, P_5, P_2}$

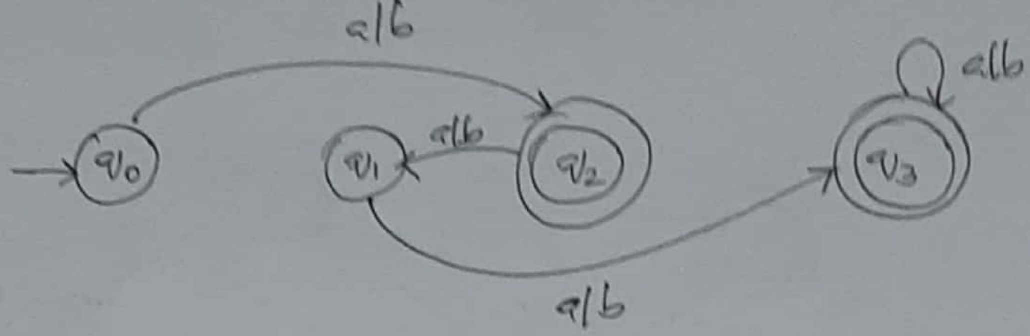
a-successors

$\underbrace{P_1\, P_4}$　　$\underbrace{P_3}$

a and b successors

$\underbrace{P_1, P_4}$　　$\underbrace{P_3}$　　$\underbrace{S}$　　$\underbrace{P_2, P_5}$

q_2　　q_3　　q_0　　q_1



$M = (Q', \Sigma, \delta', q_0', A)$

$Q' = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\delta' : \{ \delta'_1(q_0, a) = q_2, \quad \delta'_2(q_0, b) = q_2,$

$\delta'_3(q_1, a) = q_3, \quad \delta'_4(q_1, b) = q_3,$

$\delta'_5(q_2, a) = q_1, \quad \delta'_6(q_2, b) = q_1,$

$\delta'_7(q_3, a) = q_3, \quad \delta'_8(q_3, b) = q_3 \}$

$A = \{q_2, q_3\}.$

18.

(a) Using pumping lemma for context-free languages, prove that the language:

$L = \{ww \mid w \in \{a,b\}^*\}$ is not a context free language.

Ans:- Assume L is a CFL

Then we have a constant n

let $x = a^n b^n a^n b^n$ be a string

$x \in L(G)$

$|x| = 4n > n$

↳ Then we can have x splitted into

uvwyz with constants $|vwy| \leq n$

$|vy| > 0$

case 1

$$x = \underset{u}{a^n}\ \underset{v}{b^p}\ \underset{w}{b^q}\ \underset{y}{b^r}\ \underset{z}{0^n 1^n}$$

$p+q+r = n$

$p = r$

The split satisfies the constraints and can have

$x = uv^t wy^t z \quad t \geq 0$

Then the string will be of the form

$x = a^n b^m a^n b^n \notin L(G)$ $m \neq n$

$\therefore \{ww \mid w \in \{a,b\}^*\}$ is not a CFL as per pumping lemma.

20

(b) Write a context sensitive grammar for the language $L = \{a^n b^n c^n \mid n \geq 0\}$.

Also illustrate how the string $a^2 b^2 c^2$ can be derived from the start symbol of the proposed grammar.

Ans→

$G = (N, T, P, S)$

$N = \{S, A, B, C\}$

$T = \{a, b, c\}$

$S = S$

P: { 1. $S \longrightarrow aSBC$

2. $S \longrightarrow aBC$

3. $CB \longrightarrow BC$

4. $aB \longrightarrow ab$

5. $bB \longrightarrow bb$

6. $bC \longrightarrow bc$

7. $cC \longrightarrow cc$ }

Deriving $a^2 b^2 c^2 = aabbcc$

$S \xRightarrow{1} aSBC \xRightarrow{2} aaBCBC \xRightarrow{4} aabCBC$

$\xRightarrow{3} aabBCC \xRightarrow{5} aabbCC \xRightarrow{6} aabbcC$

$\xRightarrow{7} aabbcc$

Sample Course level Assessment Questions

Course Outcome 1 CO1 :-

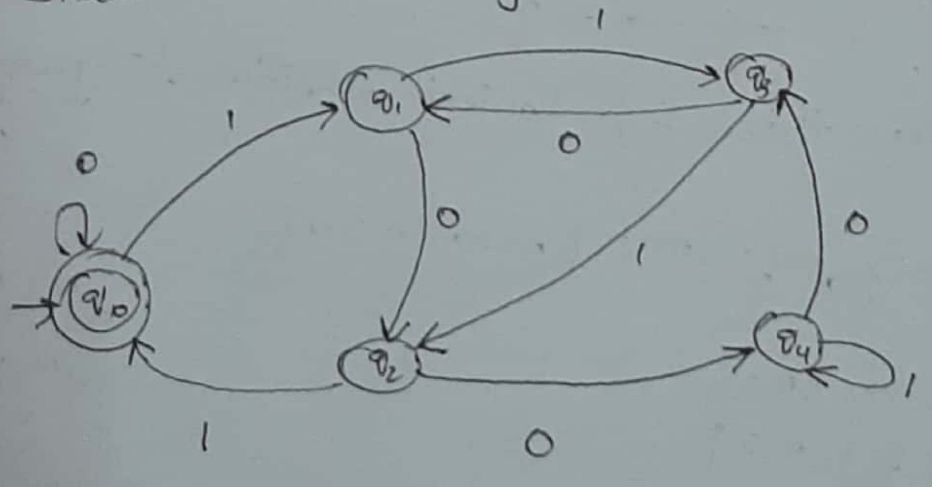
$L_2 = \{ x \ \{0,1\}^* \mid x$ is the binary representation of a decimal number which is a multiple of 5 $\}$

Ans:- We can construct a DFA that accepts L_2

So L_2 is a regular ~~lag~~. language generated by regular grammars

DFA

State Transition diagram.



$L_3 = \{a^n b^n c^n \mid n \geq 0\}$

This language is generated by context sensitive grammers.

$G = (N, T, P, S)$

$N = \{$

$T = \{a, b, c\}$

$S = S.$

$P: \{$ 1. $S \rightarrow aSBC$

2. $S \rightarrow aBC$
3. $CB \rightarrow BC$
4. $aB \rightarrow ab$
5. $bB \rightarrow bb$
6. $bC \rightarrow bb$
7. $cC \rightarrow cc\}$

P: is of the form

$u \rightarrow v$

$|u| \leq |v|$

Hence these are Context Sensitive languages.

$L_4 = \{a^m b^n c^{m+n} \mid m \geq 0, n \geq 0\}$

$a^m b^n c^{n+m}$

$a^m b^n c^n c^M$

$G = (N, T, P, S)$

$N = \{S, A, B\}$

$T = \{a, b, c\}$

$P = \{$ 1. $S \rightarrow aAc$

2. $A \rightarrow aAc$
3. $A \rightarrow bBc$
4. $B \rightarrow bBc$
5. $B \rightarrow \epsilon\}$

P: Is of the form

$A \rightarrow \alpha$

$A \in N$ and

$\alpha \in (N \cup T)^*$

$\therefore$ L_4 is a context free ~~sensitive~~ language.

$L_1 = \{a^p \mid p \text{ is a prime number}\}$

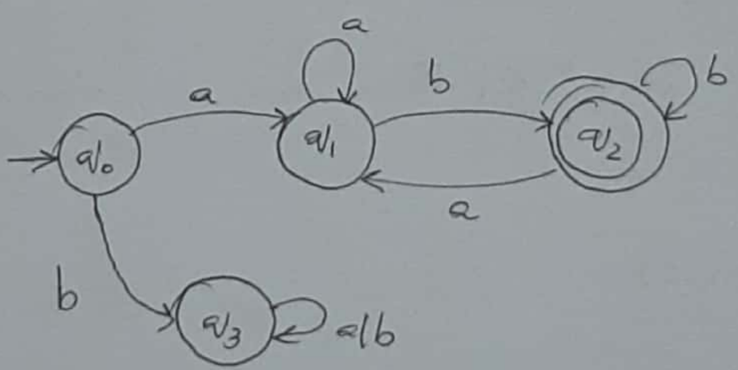
Using pumping lemma and ultimate periodicity, we cannot design a PDA or a FSA. We cannot design an LBA as the tape is limited.

Then this language will be generated by unrestricted grammar accepted by turing machine.

Course Outcome 2 (CO2):-

3) Design a DFA for the language $L = \{axb \mid x \in \{a,b\}^*\}$

State diagram



$D = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$; $q_0 = q_0$; $A = \{q_2\}$

$\delta = \{\delta_1(q_0, a) = q_1,\ \delta_2(q_0, b) = q_3,\ \delta_3(q_1, a) = q_1,\ \delta_4(q_1, b) = q_2,$
$\delta_5(q_2, a) = q_1,\ \delta_6(q_2, b) = q_2,\ \delta_7(q_3, a) = q_3,\ \delta_8(q_3, b) = q_3\}$

State Table

$Q \setminus \Sigma$	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
$* q_2$	q_1	q_2
q_3	q_3	q_3.

(ii) Write a Regular Expression for the language:

$L = \{ x \in \{a, b\}^* \mid$ third last symbol in x is b

$$r = (a+b)^* b (a+b)(a+b)$$

(iii) Write a regular grammar for the languages

$L = \{ x \in \{0,1\}^* \mid$ there are no consecutive zeros in $x \}$

Ans → $G = (N, T, P, S)$

$N = \{S\}$

$T = \{0, 1\}$

$S = S$

$P: \{ S \rightarrow 01S$

$S \rightarrow 1S$

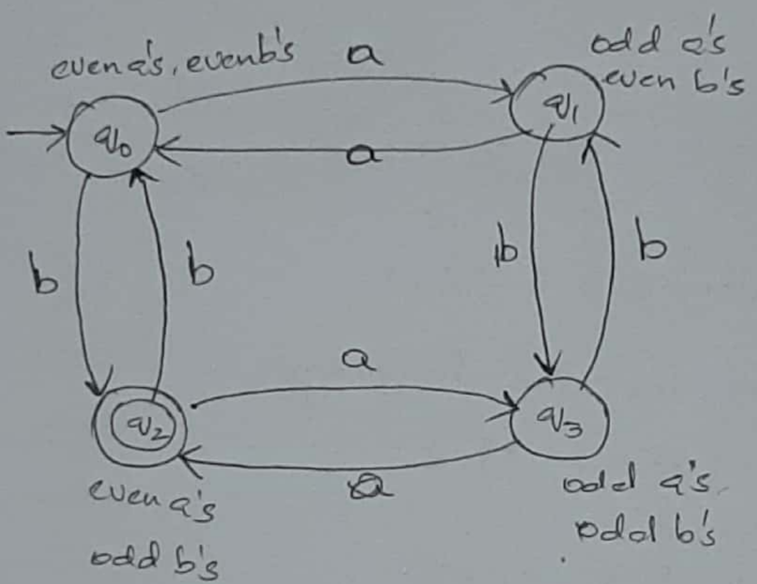
$S \rightarrow 0$

$S \rightarrow \epsilon \}$

(iv) Show the equivalence classes of canonical Myhill-Nerode relation induced by the language:

$L = \{ x \in \{a,b\}^* : x$ contains even no. of a's and odd number of b's

DFA



$D = (Q, \Sigma, \delta, q_0, A)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\delta = \{$ 1. $\delta(q_0, a) = q_1$, 2. $\delta(q_0, b) = q_2$

3. $\delta(q_1, a) = q_0$, 4. $\delta(q_1, b) = q_3$

5. $\delta(q_2, a) = q_3$, 6. $\delta(q_2, b) = q_0$

7. $\delta(q_3, a) = q_2$, 8. $\delta(q_3, b) = q_1$ $\}$

let the equivalent classes be $[J_0]$, $[J_1]$, $[J_2]$ and $[J_3]$ and the language be L

$[J_0] = \{x \mid x \in \{a,b\}^*, \delta^*(q_0, x) = q_0\}$

$[J_0] = \{\epsilon, aabb, aaaabbbb, \ldots\}$.

Scanned with CamScanner

$[J_1] = \{x \mid x \in \{a,b\}^*, \delta^*(q_0, x) = q_1\}$

$[J_1] = \{a, abb, aaabb, \ldots\}$

$[J_2] = \{x \mid x \in \{a,b\}^*, \delta^*(q_0, x) = q_2\}$

$[J_2] = \{b, aab, aaaabbb, \ldots\}$

$[J_3] = \{x \mid x \in \{a,b\}^*, \delta^*(q_0, x) = q_3\}$

$[J_3]$

$L = [J_2]$

Course Outcome 3 (CO3):

i) Design a PDA for the language
$L = \{ww^R \mid w \in \{a,b\}^*\}$.

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, A)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{X, Y, \$\}$

$q_0 = q_0$

$z_0 = \$$

$A = \{q_2\}$

$\delta : \{$ 1. $\delta(q_0, a, \$) = \{(q_0, X\$)\}$

2. $\delta(q_0, b, \$) = \{(q_0, Y\$)\}$

3. $\delta(q_0, a, X) = \{(q_0, XX), (q_1, \epsilon)\}$

4. $\delta(q_0, b, Y) = \{(q_0, YY), (q_1, \epsilon)\}$

5. $\delta(q_0, b, X) = \{(q_0, YX)\}$

6. $\delta(q_0, a, Y) = \{(q_0, XY)\}$

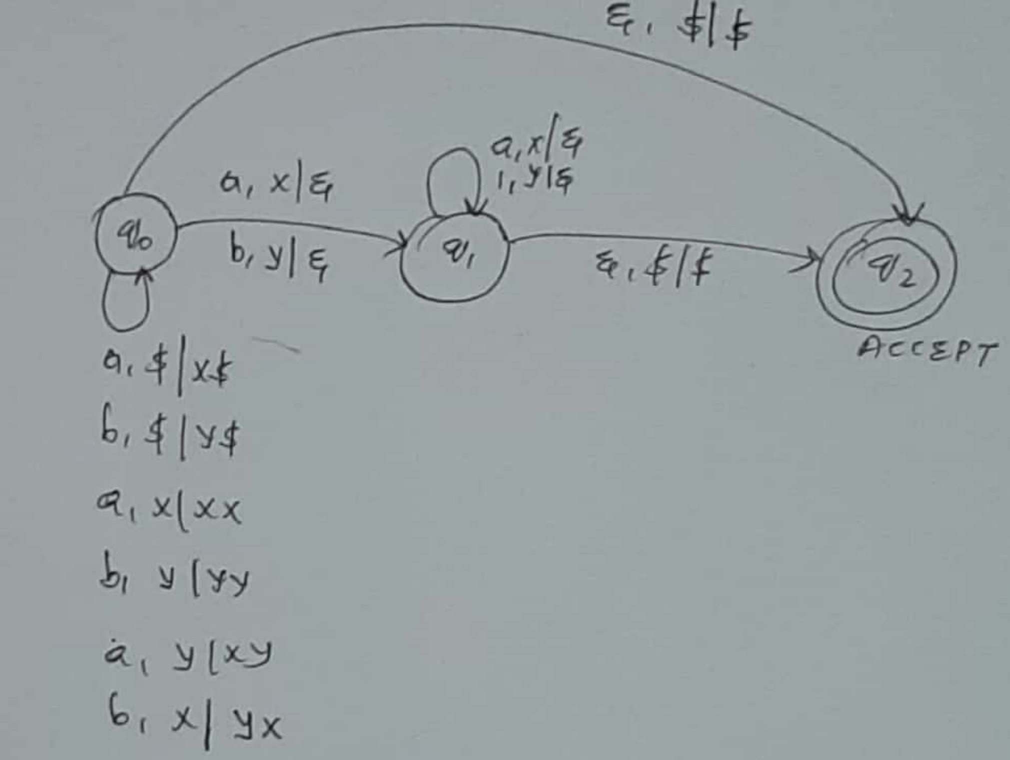
7. $\delta(q_1, a, x) = \{(q_1, \varepsilon)\}$

8. $\delta(q_1, b, y) = \{(q_1, \varepsilon)\}$

9. $\delta(q_1, \varepsilon, \$) = \{(q_2, \varepsilon)\}$ ACCEPT }

10. $\delta(q_0, \varepsilon, \$) = \{(q_2, \varepsilon)\}$ ACCEPT }

State Diagram



(ii) Write a Context-Free Grammar for the language

$L = \{a^n b^{2n} \mid n \geq 0\}$

$G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b\}$

$P = \{1.\ S \longrightarrow \varepsilon \mid aSbb\}$

$S = S.$