# SVD and its Application in Statistics - Question 1

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**You may be knowing about a typesetting language called LaTeX. By default LaTeX places the things in the regular horizontal way, it can also rotate and scale (possibly different scaling factors in x- and y-directions): While writing my math books, I needed a way to place a writing on a slant surface drawn on the page:**

**LaTeX has no direct way to achieve this. But I could get the effect by first doing a little SVD computation. How?**

**Solution:**

## 1 Algorithm:

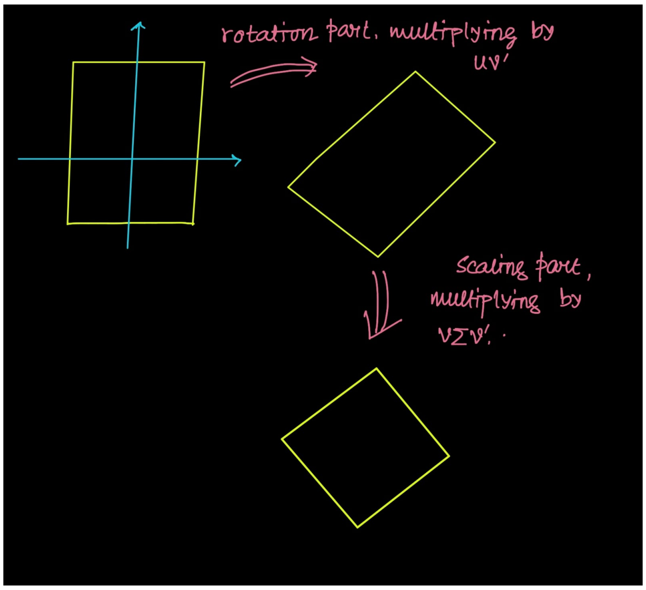
We will use **Polar Decomposition** which is a special form of SVD.

We first ignore the content of the page and we concentrate on the rectangular sheet only. At first we rotate it by a specific angle $\theta$. We name the rotation matrix as $Q = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

Now we scale the matrix by multiplying scaling positive definite matrix S. Here $S = BRB^T$ , where $R$ is the diagonal matrix which is used for scaling and $B$ is used for second time rotation. So in this linear transformation the matrix related to the basis $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is $A = SQ$. We transform all the points of the plane :

$$x \longrightarrow Ax.$$

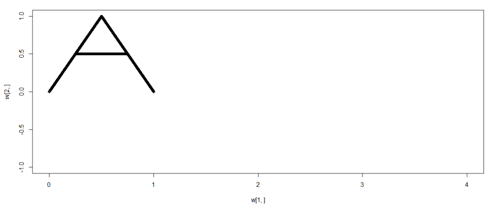
Now $Q$ will be of the form $BG^T$, Then $A$ becomes of the form $BRG^T$, which is the SVD decomposition of $A$.

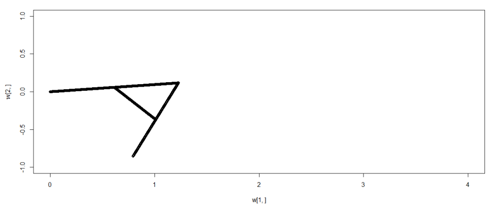



## 2 R code regarding this algorithm:

```
SVD = function(t,u,a,b)
{
x=c(seq(0,1,by=0.001),seq(0.25,0.75,by=0.001))
y=c(2*x[1:500],2-2*x[501:1001],rep(0.5,501))
plot(x,y,xlim = c(0,4),ylim = c(-1,1))

A=matrix(c(cos(t),sin(t),-sin(t),cos(t)),2)
v=as.matrix(rbind(x,y))
z=A%*%v
B=matrix(c(cos(u),sin(u),-sin(u),cos(u)),2)
R=diag(c(a,b))
Q=B%*%R%*%t(B)
w=Q%*%z
plot(w[1,],w[2,],xlim = c(0,4),ylim = c(-1,1))

}
SVD(-0.9,-0.5,1.2,0.95)
SVD(0,0,1,1)
```

The output : We are showing the tranformation wrt the original A using t=-0.9 , u= -0.5 , a=1.2 , b=0.95.The new figure of 'A' becomes:





## 3 Reshaping a square to a parallelogram using R:

```
f=function(x1,y1,x2,y2)
{
a1=(x2-x1)/2
a2=-(x1+x2)/2
```

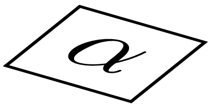
```
a3=(y2-y1)/2
a4=-(y1+y2)/2

A=matrix(c(a1,a2,a3,a4),2,2,byrow=TRUE)
b=svd(A)
U=b$u
V=b$v

if (abs(U[1,1]+U[2,2])<10^(-10))
{
U[,1]=U[,1]*(-1)
}

if (abs(V[1,1]+V[2,2])<10^(-10))
{
V[,1]=V[,1]*(-1)
}

sig=diag(b$d,2)

temp1=acos(V[1,1])
temp2=acos(U[1,1])
c=180/pi
theta1=(-c)*temp1
theta2=c*temp2

if (abs(sin(temp1)-V[1,2])<10^(-10))
{
theta1=-theta1
}

if (abs(sin(temp2)-U[1,2])<10^(-10))
{
theta2=-theta2
}

x=sig[1,1]
y=sig[2,2]

cat('\n\nFirst angle of rotation (in degrees) is',theta1,'\n')
cat('\n\nScale factor along x-axis is',x,'\n')
cat('\n\nScale factor along y-axis is',y,'\n')
cat('\n\nSecond angle of rotation (in degrees) is',theta2,'\n')

}

f(-6,-1,1,-3)   #For modifying the letter alpha
f(-4,-3,2,-3)   #For Italics Font
##The Output:
First angle of rotation (in degrees) is -38.7356


Scale factor along x-axis is 4.320261


Scale factor along y-axis is 2.198941


Second angle of rotation (in degrees) is 6.264404
> f(-4,-3,2,-3)   #For Italics Font


First angle of rotation (in degrees) is -49.73116
```

```
Scale factor along x-axis is 3.541381

Scale factor along y-axis is 2.541381

Second angle of rotation (in degrees) is 40.26884
```

1. The letter alpha (boxed) looks like: $\boxed{\alpha}$.

   We now assume that the coordinates of the vertices of the square containing $\alpha$ are $(-1,-1),(1,-1),(1,1),(-1,1)$ in a counterclockwise direction. We also assume that all rotations are performed about the centre of the letter $\alpha$. We will transform this into a parallelogram having $(x_1, y_1),(x_2, y_2),(x_3, y_3),(x_4, y_4)$ as the coordinates of the vertices.

   To do this, we compute the matrix of the linear transformation and find out its SVD. Composing rotatebox and scalebox and then again rotatebox with the angles found from SVD and the singular values of $\mathbf{A}$ in place of the xfactors and yfactors gives the following picture:

2. Writing in italics form using SVD: *We can also write text in italics using SVD!*