HAPSRAV – Kinematic Aircraft 6DOF Model Description

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Description

Kinematic Aircraft 6DOF

- Aircraft simulation using ideal response models from the commanded inputs
- Kinematic relationships used to simulated the vehicle’s inertial behavior
- Drag polar and sum of forces used to compute wind angles, lift, drag, and thrust
- Solar model computes sun’s position
- Battery model for energy storage/consumption
- Propulsion model for power consumption

Modes

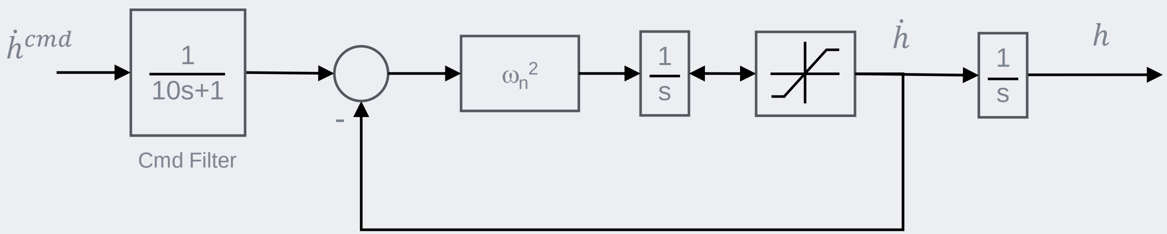
- Trajectory Optimization
  - Uses rate of climb for vertical control and bank angle for lateral control
- Waypoint Following
  - Uses altitude for vertical control and course angle for lateral control
- Both modes use equivalent airspeed for longitudinal control
- Both modes take wind vector as input with individual components for North-East-Down directions, $W^T = [w_n \;\; w_e \;\; w_d]$

Commanded Inputs

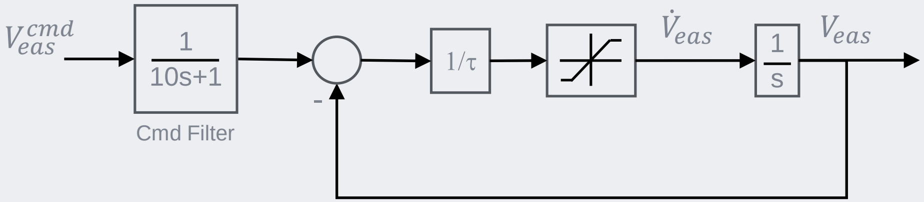
Trajectory Optimization Mode

Vertical

Altitude Rate

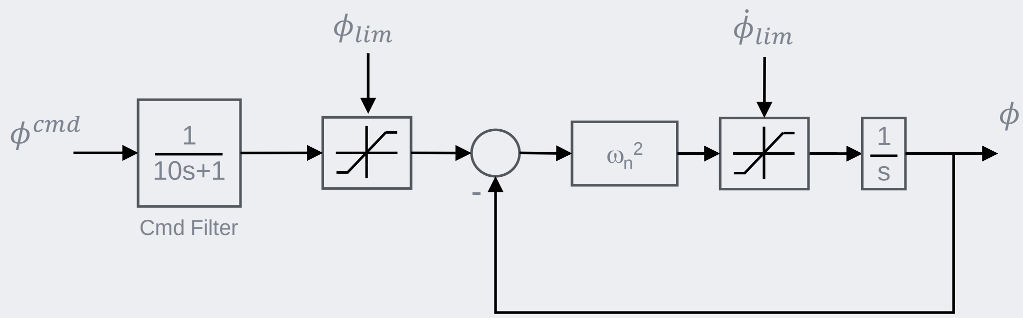


Airspeed



- True airspeed and rate computed using $\sqrt{\rho/\rho_{SL}}$

Lateral
Bank Angle



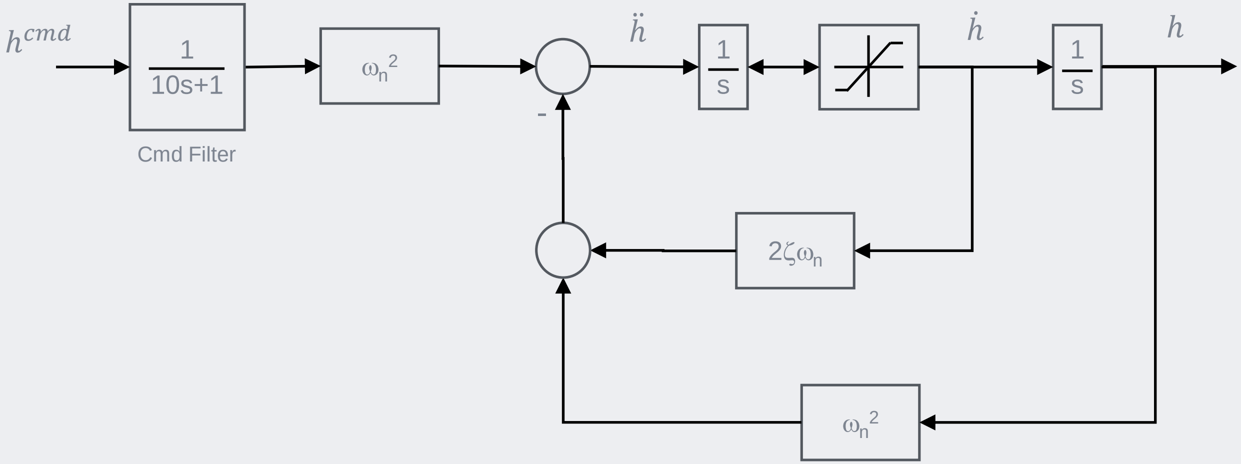
- ϕ_{lim} = 5° at SL up to 10° at 23 km
- $\dot{\phi}_{lim}$= 1°/sec at SL up to 3°/sec at 23 km

Commanded Inputs

Waypoint Following Mode

Vertical

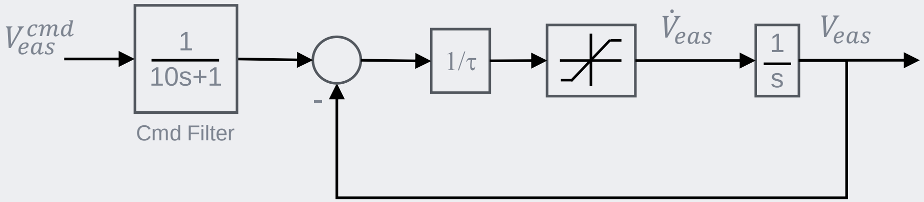
Altitude



Altitude Rate Limits

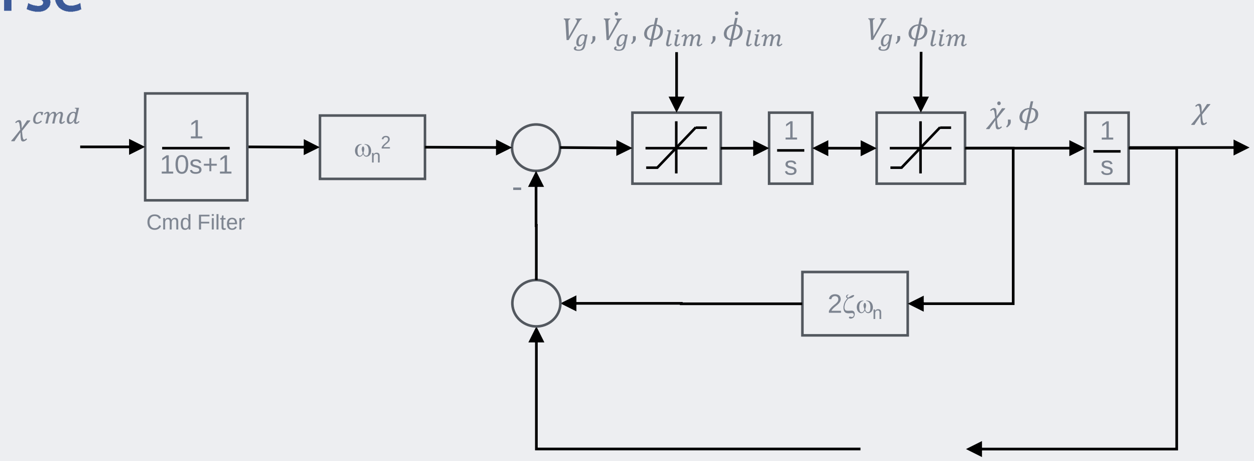
- Descent Rate Limit
  - -3.0 deg glideslope
  - Descent rate not to exceed -0.8 m/sec
- Climb Rate Limit
  - Climb rate not to exceed 0.8 m/sec

Airspeed



Lateral

Course



Course Limits

- Course Rate Limit, $\dot{\chi}_{lim}$
  - $\dot{\chi}_{lim} = \frac{g}{V_g} \tan \phi_{lim}$
- Course Error Limit,
  - $\ddot{\chi}_{lim} = \frac{g}{V_g} \dot{\phi}_{lim} \sec^2 \phi_{lim} - \frac{g}{V_g^2} \dot{V}_g \tan \phi_{lim}$

Kinematic Models

Flight Path Angles

- Inertial Flight Path, γ_i
  - $\gamma_i = \sin^{-1}(\dot{h}/V_g)$
- Wind Flight Path, γ
  - $\gamma = \sin^{-1}((\dot{h} + w_d)/V_{tas})$

Euler Angles

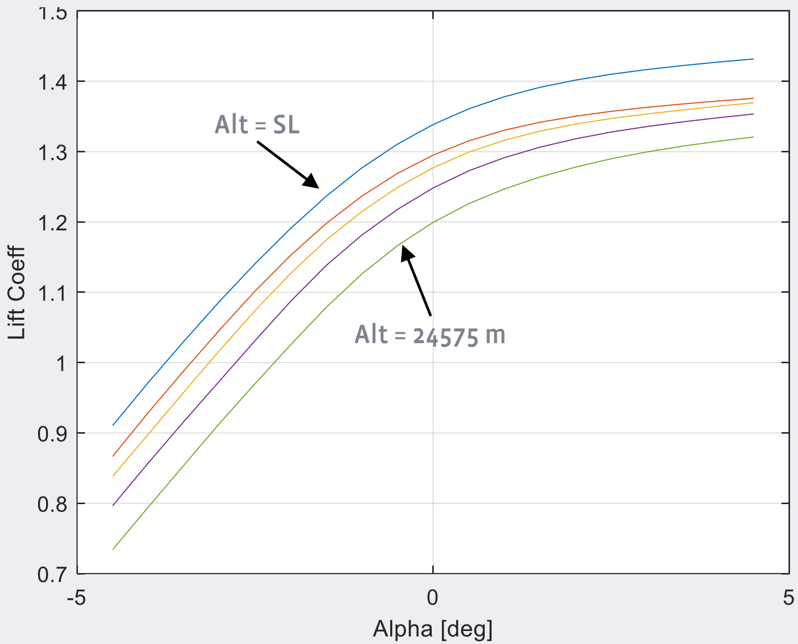
- Roll, ϕ
  - $\phi = \tan^{-1}(\dot{\chi} V_g / g)$
- Pitch, θ
  - $\theta = \gamma + \alpha$
- Heading, ψ
  - $\psi = \chi - \sin^{-1}\left\{\frac{w_e \cos\chi - w_n \sin\chi}{V_{tas}\cos\gamma}\right\}$
- Euler rates computed from band limited differentiator
  - $\frac{s}{\left(\frac{1}{2\pi f_d}\right)s+1}$

Ground Speed

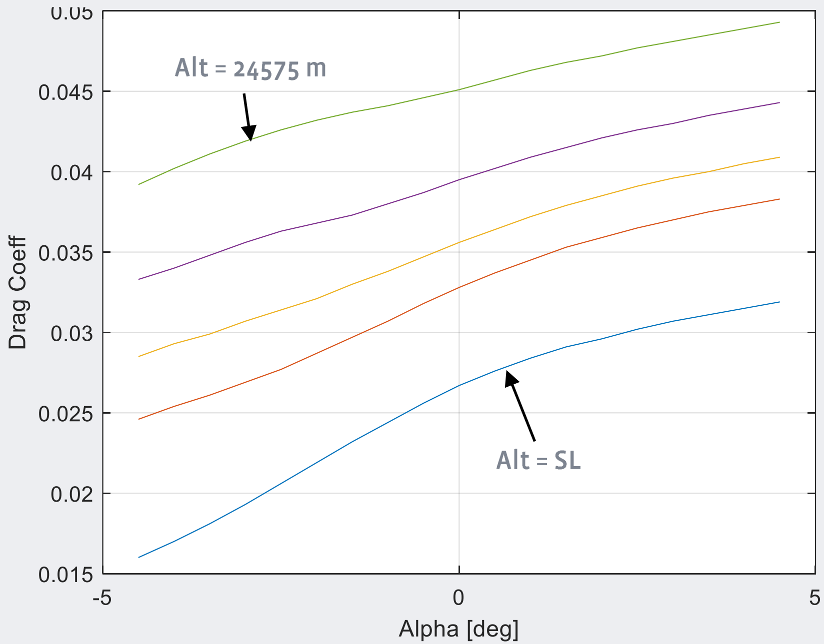
- V_g, solve from wind triangle
  - $V_g^2 = V_{tas}^2 + V_W^2 + 2V_{tas}\begin{bmatrix}\cos\psi\cos\gamma \\ \sin\psi\cos\gamma \\ -\sin\gamma\end{bmatrix}^T \begin{bmatrix}w_n \\ w_e \\ w_d\end{bmatrix}$
- $\dot{V}_g$ also computed from band limited differentiator

Aerodynamic Models

Lift & Drag



Lift Coeff
Alpha [deg]
Alt = SL
Alt = 24575 m
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
-5
0
5



Drag Coeff
Alpha [deg]
Alt = 24575 m
Alt = SL
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
-5
0
5

Aerodynamics

- Dynamic Pressure, $\bar{q}$
  - $\bar{q} = \rho V_{tas}^2/2$
- Unbanked Lift Coefficient, $C_{L,ub}$
  - Sum of forces along lift axis
  - $C_{L,ub}\bar{q}S - mg\cos\gamma = m\frac{V_{tas}^2}{(r_{earth}+h)}$
- Lift Coefficient, C_L
  - $C_L = C_{L,ub}/\cos\phi$
- Angle of Attack, α
  - Reverse lookup from $C_{L,ub}$ vs α curve

Aerodynamics

- Angle of Attack, α
  - Reverse lookup from $C_{L,ub}$ vs α curve
- Drag Coefficient, C_D
  - Table lookup from C_D vs α curve
- Note
  - α and C_D are only exact in level unaccelerated flight

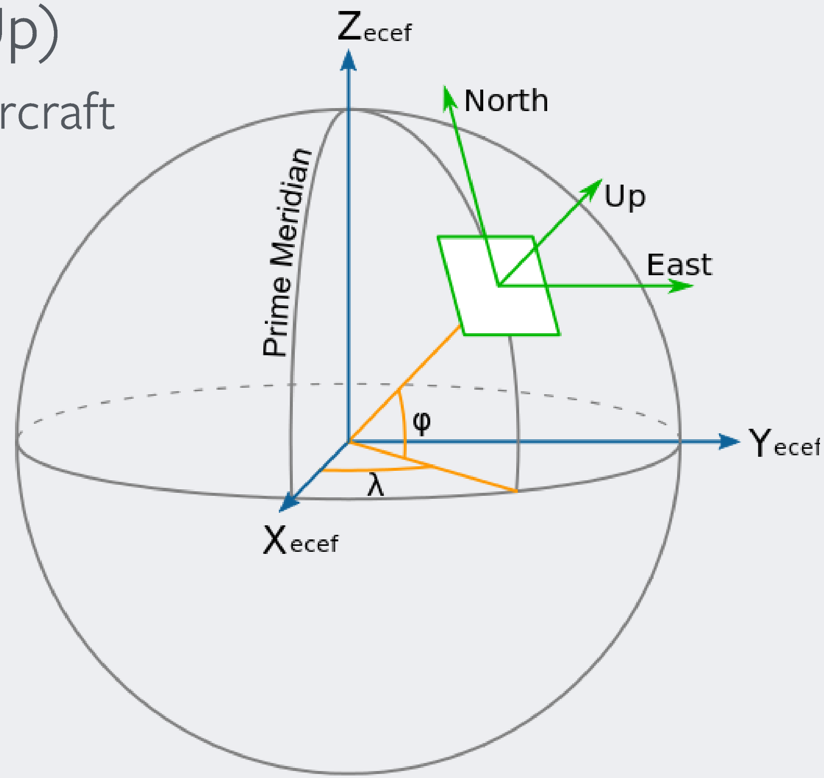
Forces

- Lift & Drag Forces
  - D = $C_D\bar{q}S$
  - L = $C_L\bar{q}S$
- Centerline Thrust, T
  - Sum of forces along speed axis
  - $T - C_D\bar{q}S - mg\sin\gamma = m\dot{V}_{tas}$

Flat Earth Model

NED (ENU) Reference Frame

- North-East-Down (East-North-Up)
  - Origin at aircraft CG, moves with aircraft
  - North axis points to true North
  - East axis points to East
  - Down axis points to the ground



Flat Earth Position

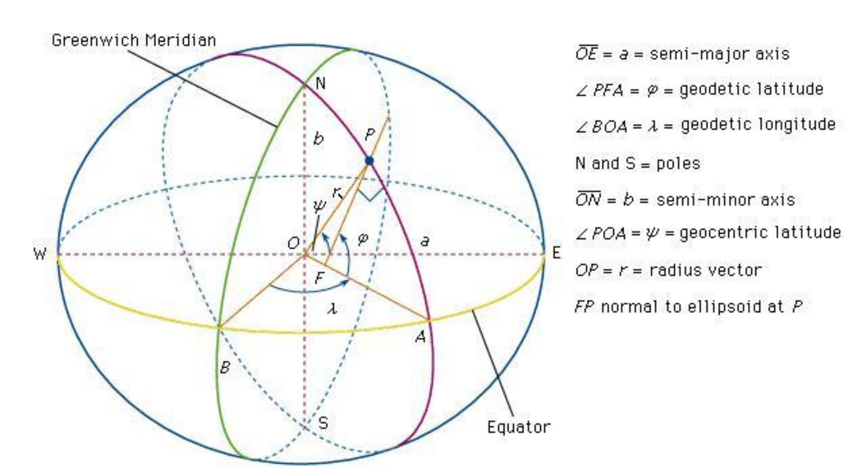
- North Position, p_n
  - $\dot{p}_n = V_{tas} \cos\psi + w_n$
  - $p_n = \int \dot{p}_n + p_{n,init}$
- East Position, p_e
  - $\dot{p}_e = V_{tas} \sin\psi + w_e$
  - $p_e = \int \dot{p}_e + p_{e,init}$

LLA Coordinate System

Reference Ellipsoid

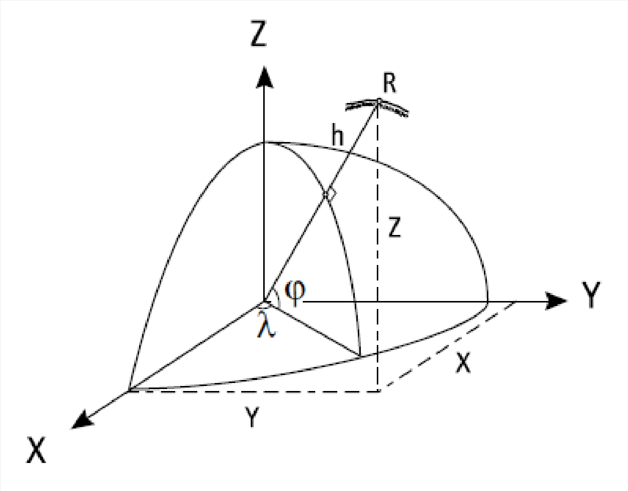
WGS-84

- f – flattening
  - $f = 1 - b/a$
- e – eccentricity
  - $e^2 = (a^2 - b^2)/a^2$
- M – Meridian radius of curvature
  - $M = a(1 - e^2)/(1 - e^2 \sin^2 \phi)^{3/2}$
- N – Prime vertical radius of curvature
  - $N = a/(1 - e^2 \sin^2 \phi)^{1/2}$



LLA

- Polar coordinate system relative to WGS-84, origin at Earth's center of mass
- Frame rotates with earth, coordinates of a fixed point along the surface do not change
- Longitude, λ
- Latitude, ϕ
- Height above reference ellipsoid, h
- Convention is to first rotate by λ



Flat Earth to LLA Transformation

- Latitude
  - $\phi = \int(\dot{p}_n/(M+h)) + \phi_{init}$
- Longitude
  - $\lambda = \int(\dot{p}_e/((N+h)\cos\phi)) + \lambda_{init}$
- Implementation uses the Aerospace Blockset

Moments

Body Rotational Rates

- Roll Rate, p
  - $p = \dot{\phi} - \dot{\psi} \sin \theta$
- Pitch Rate, q
  - $q = \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi$
- Yaw Rate, r
  - $r = \dot{\psi} \cos \theta \cos \phi$- $\dot{\theta} \sin \phi$

Solar Model

Solar Model

- Adjust solar irradiance for annual variation
  - $G_1 = G_0\left(1 + 0.034\cos\frac{2\pi\, JD}{365.25}\right)$
- Adjust for atmospheric absorption: $G_2 = G_1 \cdot f(h, \theta_s)$
  - Rays travel through more atmosphere when the sun is lower
- Solar angle calculations (azimuth ϕ_s, zenith θ_s) from NREL
- Solar panel distribution with mean pitch offset approx. 9º from vehicle body axes

Power Collection		
η_{panel}	=	23%
η_{MPPT}	=	97%
A_{solar}	=	143.30 m^2
G_0	=	1367 W/m^2

Battery Model

Batteries

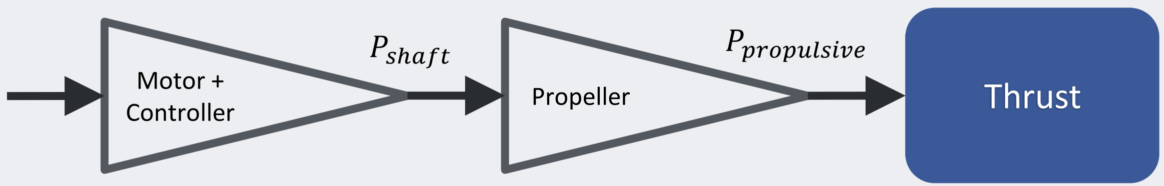
- Battery modeled as energy storage device with fixed energy capacity
- Battery has charge and discharge efficiencies $\eta_{B,in}$ and $\eta_{B,out}$
- Max. charge/discharge rates, voltage vs. SOC not considered

Power Storage		
$\eta_{B,in}$	=	93%
$\eta_{B,out}$	=	97%
m_{batt}	=	200 kg
e_{batt}	=	320 Wh/kg
E_{batt}	=	64.0 kWh

Propulsion Model

Propulsion

- Propulsion system consists of motor controller(s), motors, propellers
- Propulsive power $P_p = TV_{tas}$, and electric power to produce this propulsive power is $P_{el,p} = TV_{tas} / \eta_{motor}\eta_{prop}\eta_{ctrl}$
- Propeller efficiency modeled as $\eta_{prop} = \dfrac{2\,\eta_{prop,0}}{1+\sqrt{\dfrac{2T}{A_{disk}V_{tas}^2\rho}+1}}$

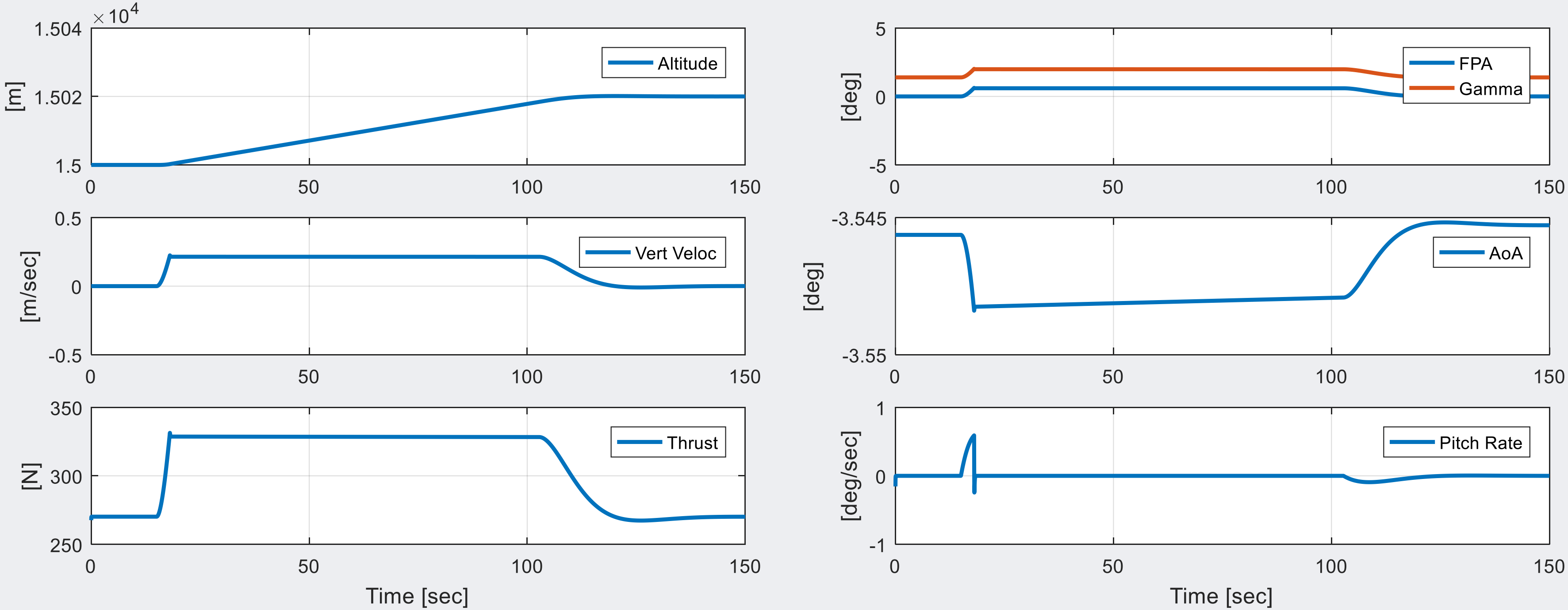


Power Consumption		
η_{motor}	=	95%
η_{ctrlr}	=	97%
$\eta_{prop,0}$	=	90%
R_{prop}	=	1.5 m
n_{motors}	=	4
$P_{el,acc}$	=	360 W

Sample Simulations

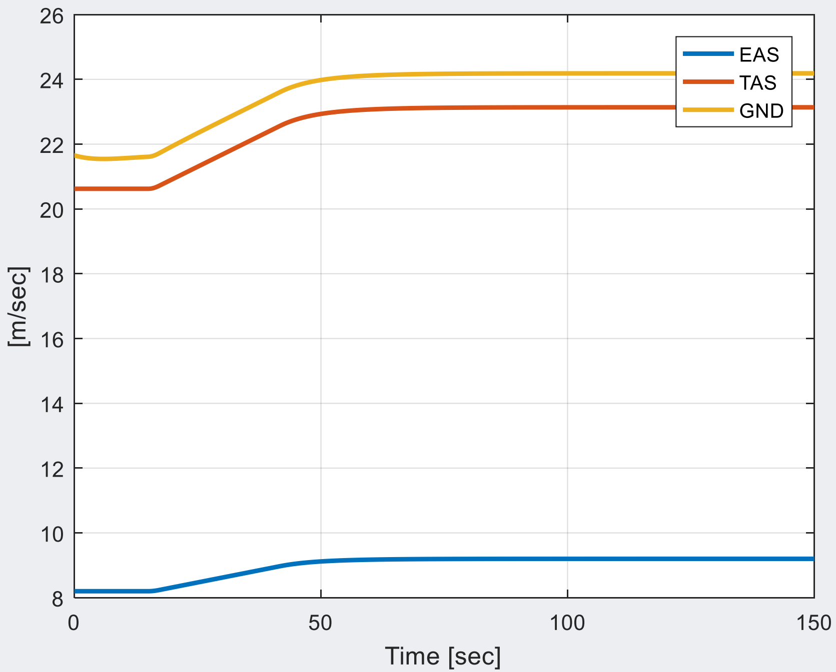
20 m Altitude Step at 8.2 m/sec

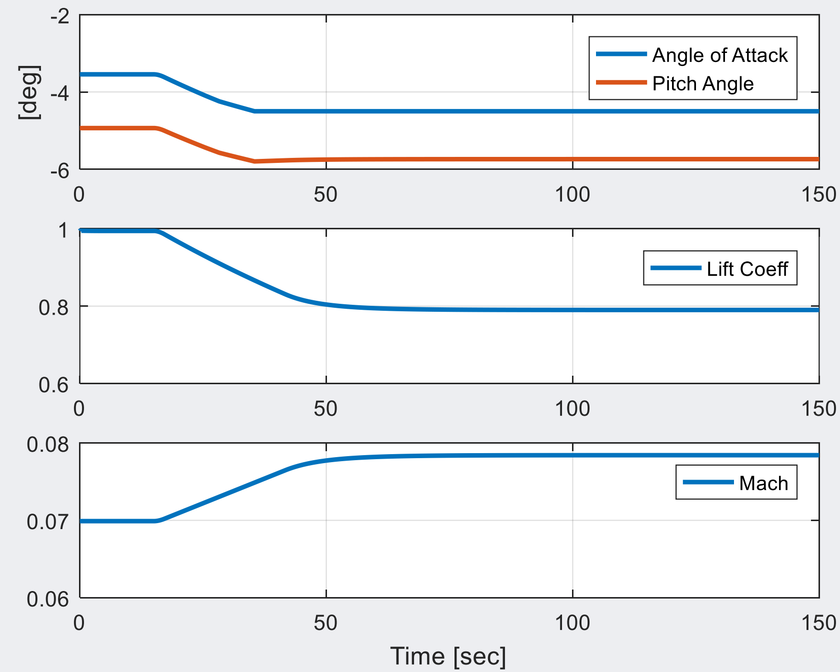
$w_n = 0$, $w_e = 0$, $w_d = 0.5$ m/sec



1 m/sec EAS Step at 1500 m

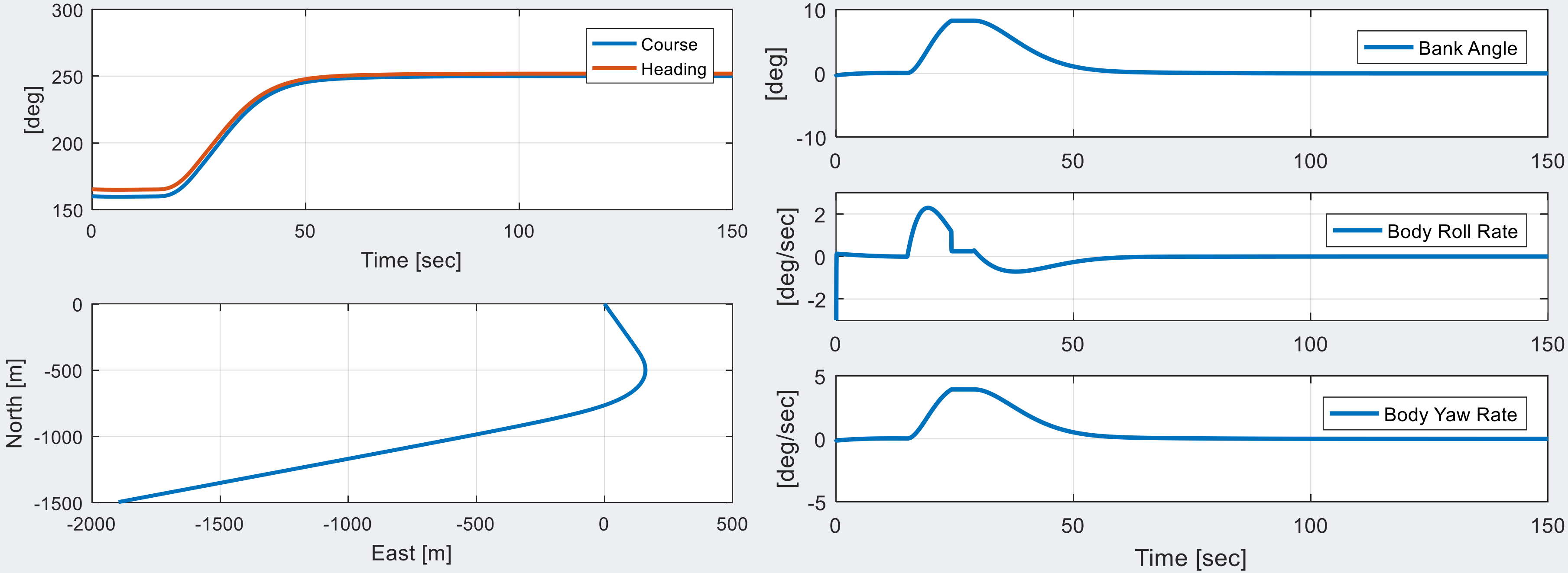
$w_n = -2$, $w_e = -2$, $w_d = -0.5$ m/sec





90 deg Course Step

$w_n = 0, w_e = 2, w_d = 0$ m/sec



Interface

Input Signals

Trajectory Optimization Mode (default)

SW Variable	Symbol	Signal	Units
kenny_U.u_vert_cmd	$\dot{h}^{cmd}$	Commanded vertical rate of climb	m/sec
kenny_U.u_eas_cmd	V_{eas}^{cmd}	Commanded equivalent airspeed	m/sec
kenny_U.u_lat_cmd	ϕ^{cmd}	Commanded bank (roll) angle	rad
kenny_U.u_winds	W	Wind components along North, East, Down directions	m/sec

Input Signals
Waypoint Following Mode

Symbol	Signal	Units
h^{cmd}	Commanded altitude	m
V_{eas}^{cmd}	Commanded equivalent airspeed	m/sec
χ^{cmd}	Commanded course (ground track) angle	rad
W	Wind components along North, East, Down directions	m/sec

Output Signals

Vehicle State

Symbol	Signal	Units
u	Forward axis body velocity	m/sec
v	Right wing axis body velocity Note: always zero	m/sec
w	Down axis body velocity	m/sec
p	Angular rate about forward body axis	rad/sec
q	Angular rate about right wing body axis	rad/sec
r	Angular rate about body down axis	rad/sec
ϕ	Euler roll angle	rad
θ	Euler pitch angle	rad
ψ	Euler yaw angle	rad

Output Signals

Vehicle State

Symbol	Signal	Units
p_N	Position along North axis	m
p_E	Position along East axis	m
h	Geopotential altitude above sea level	m
V_{eas}	Equivalent airspeed	m/sec
V_{tas}	True airspeed	m/sec
V_g	Ground speed	m/sec
V_v	Vertical velocity	m/sec
α	Angle of attack	rad
β	Angle of sideslip Note: always zero	rad

Output Signals

Vehicle State

Symbol	Signal	Units
γ	Flight path angle	rad
γ_i	Inertial flight path angle	rad
χ	Course	Rad
C_L	Lift coefficient	non-dim
C_D	Drag Coefficient	non-dim
T	Centerline Thrust	N
$\bar{q}$	Dynamic pressure	N/m^2
M	Mach number	non-dim
ϕ	Latitude	deg
λ	Longitude	deg

Output Signals

Power State

Symbol	Signal	Units
Az	Sun azimuth angle: deviation from North	deg
El	Sun elevation angle: deviation from horizon	deg
JD	Julian day	non-dim
P_{elec}^{in}	Electric power in: power being stored as electricity	kW
P_{elec}^{out}	Electric power out: power being consumed from batteries	kW
P_{batt}	Available battery power	kW
E_{elec}^{in}	Electric energy in: electric energy being stored	kW-hr
E_{elec}^{out}	Electric energy out: electric energy being spent	kW-hr

Output Signals

Power State

Symbol	Signal	Units
E_{batt}	Available battery energy	kW-hr
E_{grav}	Stored gravitational energy: above minimum altitude limit	kW-hr
E_{panel}	Integrated solar flux in aircraft's horizontal plane	kW-hr
A_{proj}	Projected air vehicle area facing sun	m^2
Q	Solar Intensity	W/m^2
η	Propeller efficiency: ratio of propulsion being converted into thrust	non-dim
$\dot{h}^{max}$	Rate of climb at available power: climbing faster than this rate drains the batteries	m/sec

Output Signals

Power State

Name	Description
Night Mode	1 – Sun is below horizon 0 – Sun is above horizon
Climb Mode	-2 – Recommend descent at idle power -1 – Recommend powered descent 0 – Recommend holding altitude 1 – Recommend climbing
Charge Mode	-1 – Batteries discharging 0 – No power flow 1 – Batteries charging

References

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