$ident, x, y, y_p, y_f, -$, abbrev, r subscripts: p for pointers, f for functions

n, i, j index variables

 $impl_const$ implementation-defined constant member C struct/union member name

Ott-hack, ignore (annotations)

nat OCaml arbitrary-width natural number

 mem_ptr abstract pointer value mem_val abstract memory value

Ott-hack, ignore (locations)

mem_iv_c OCaml type for memory constraints on integer values

 UB_name undefined behaviour

string OCaml string

Ott-hack, ignore (OCaml type variable TY) Ott-hack, ignore (OCaml Symbol.prefix)

mem_order, _ OCaml type for memory order

linux_mem_order OCaml type for Linux memory order

Ott-hack, ignore (OCaml type variable bt)

```
Sctypes_{-}t, \tau
                                                 C type
                                                    pointer to type \tau
tag
                                                 OCaml type for struct/union tag
                     ::=
                           ident
β, _
                                                 base types
                     ::=
                                                    unit
                           unit
                           bool
                                                    boolean
                                                    integer
                           integer
                                                    rational numbers?
                           real
                                                   location
                           loc
                           \operatorname{array} \beta
                                                    array
                           \mathtt{list}\, eta
                                                    list
                                                    tuple
                           \mathtt{struct}\,tag
                                                    struct
                           \operatorname{\mathfrak{set}} \beta
                                                    \operatorname{set}
                           opt(\beta)
                                                    option
                                                   parameter types
                           \beta \to \beta'
                           \beta_{\tau}
                                           Μ
                                                    of a C type
binop
                                                 binary operators
                                                    addition
                                                    subtraction
                                                    multiplication
                                                    division
                                                    modulus
                                                    remainder
                           rem_f
                                                    exponentiation
                                                    equality, defined both for integer and C types
```

	!= > < >= <= /\	inequality, similiarly defined greater than, similarly defined less than, similarly defined greater than or equal to, similarly defined less than or equal to, similarly defined conjunction disjunction
$binop_{arith}$::=	arithmentic binary operators
$binop_{rel}$::=	relational binary operators
$binop_{bool}$::= 	boolean binary operators
mem_int	::=	memory integer value

		1 0	M M	
$object_value$::=	$\begin{array}{l} mem_int \\ mem_ptr \\ \operatorname{array}\left(\overline{loaded_value_i}^i\right) \\ (\operatorname{struct} ident)\{\overline{.member_i:\tau_i = mem_val_i}^i\} \\ (\operatorname{union} ident)\{.member = mem_val\} \end{array}$		C object values (inhabitants of object types), which can be read/stored integer value pointer value C array value C struct value C union value
$loaded_value$::= 	$\verb specified object_value $		potentially unspecified C object values specified loaded value
value	::=	$object_value \ loaded_value \ Unit \ True \ False \ eta[\overline{value_i}^i] \ (\overline{value_i}^i)$		Core values C object value loaded C object value unit boolean true boolean false list tuple
$bool_value$::= 	True False		Core booleans boolean true boolean false
$ctor_val$::=	$\begin{array}{c} \operatorname{Nil}\beta\\ \operatorname{Cons}\\ \operatorname{Tuple} \end{array}$		data constructors empty list list cons tuple

		Array Specified	C array non-unspecified loaded value
	ı	Specifica	-
$ctor_expr$::=		data constructors
		Ivmax	max integer value
		Ivmin	min integer value
		Ivsizeof	sizeof value
		Ivalignof	alignof value
		IvCOMPL	bitwise complement
		IvAND	bitwise AND
		IvOR	bitwise OR
		IvXOR	bitwise XOR
		Fvfromint	cast integer to floating value
		Ivfromfloat	cast floating to integer value
name	::=		
name	—	ident	Core identifier
		$impl_const$	implementation-defined constant
	'	1	•
pval	::=		pure values
		ident	Core identifier
		$impl_const$	implementation-defined constant
		value	Core values
		$\mathtt{constrained}(\overline{mem_iv_c_i,pval_i}^{i})$	constrained value
		$\mathtt{error}\left(string, pval ight)$	impl-defined static error
		$ctor_val(\overline{pval_i}^i)$	data constructor application
		$(\mathtt{struct}ident)\{\overline{.member_i=pval_i}^{i}\}$	C struct expression
		$(\verb"union" ident") \{ .member = pval \}$	C union expression
tpval	::=		top-level pure values
cpout			top tevel pure variets

		$\begin{array}{l} {\tt undef} \ \ UB_name \\ {\tt done} \ pval \end{array}$		undefined behaviour pure done
$ident_opt_eta$::= 	$_{::}eta \ ident:eta$	$binders = \{\}$ $binders = ident$	type annotated optional identifier
pattern	::= 	$ident_opt_eta \ ctor_val(\overline{pattern_i}^i)$	$\begin{aligned} & \text{binders} = \text{binders}(ident_opt_\beta) \\ & \text{binders} = \text{binders}(\overline{pattern}_i^{\ i}) \end{aligned}$	
z	::=	$i \\ mem_int \\ size_of(au) \\ offset_of_{tag}(member) \\ ptr_size \\ max_int_{ au} \\ min_int_{ au}$	M M M M M M	OCaml arbitrary-width integer literal integer size of a C type offset of a struct member size of a pointer maximum value of int of type τ minimum value of int of type τ
$\mathbb{Q},\ q,\ _{-}$::=	$rac{int_1}{int_2}$		OCaml type for rational numbers
lit	::=	$ident$ unit $bool$ z \mathbb{Q}		

```
ident\_or\_pattern
                                   ident
                                                                              binders = ident
                                                                               binders = binders(pattern)
                                   pattern
bool\_op
                                   \neg term
                                  term_1 = term_2

    \bigwedge(\overline{term_i}^i) \\
    \bigvee(\overline{term_i}^i)

                                  term_1 \ binop_{bool} \ term_2
                                                                              Μ
                                  if term_1 then term_2 else term_3
arith\_op
                                  term_1 + term_2
                                  term_1 - term_2
                                  term_1 \times term_2
                                  term_1/term_2
                                  term_1 \, {\tt rem\_t} \, term_2
                                  term_1 \, {\tt rem\_f} \, term_2
                                  term_1 \hat{} term_2
                                  term_1 \ binop_{arith} \ term_2
                                                                              Μ
cmp\_op
                                  term_1 < term_2
                                                                                                                       less than
                                  term_1 \le term_2
                                                                                                                       less than or equal
                                  term_1 binop_{rel} term_2
                                                                              Μ
list\_op
                                   nil
                                  {\tt tl}\, term
```

```
term^{(int)}
tuple\_op
                  ::=
                         (\overline{term_i}^i)
                        term^{(int)}
pointer\_op
                  ::=
                        mem\_ptr
                        term_1 +_{ptr} term_2
                        {\tt cast\_int\_to\_ptr}\, term
                         {\tt cast\_ptr\_to\_int}\, term
array\_op
                        term_1[term_2]
param\_op
                  ::=
                        ident:\beta.\ term
                        term(term_1, ..., term_n)
struct\_op
                  ::=
                        term.member
ct\_pred
                  ::=
                        \texttt{representable}\left(\tau, term\right)
                        aligned(\tau, term)
                         alignedI(term_1, term_2)
term, \ \_
                        lit
                        arith\_op
```

```
bool\_op
                   cmp\_op
                   tuple\_op
                   struct\_op
                   pointer\_op
                   list\_op
                   array\_op
                   ct\_pred
                   param\_op
                                                                S
                                                                        parentheses
                   (term)
                                                                Μ
                   \sigma(term)
                                                                        simul-sub \sigma in term
                   pval
                                                                Μ
                                                                     pure expressions
pexpr
             ::=
                  pval
                                                                        pure values
                   ctor\_expr(\overline{pval_i}^i)
                                                                        data constructor application
                   array\_shift(pval_1, \tau, pval_2)
                                                                        pointer array shift
                                                                        pointer struct/union member shift
                  member\_shift(pval, ident, member)
                                                                        boolean not
                  not(pval)
                  pval_1 \ binop \ pval_2
                                                                        binary operations
                  memberof(ident, member, pval)
                                                                        C struct/union member access
                   name(\overline{pval_i}^i)
                                                                        pure function call
                   assert_undef (pval, UB_name)
                   {\tt bool\_to\_integer}\,(pval)
                   conv_int(\tau, pval)
                   wrapI(\tau, pval)
                                                                     top-level pure expressions
tpexpr
             ::=
                                                                        top-level pure values
                   tpval
                   case pval of trac{|trace_branch_i|^i}{|trace_branch_i|^i} end
                                                                        pattern matching
```

		$\begin{split} & \texttt{let} ident_or_pattern = pexpr \texttt{in} tpexpr\\ & \texttt{let} ident_or_pattern : (y_1 : \beta_1. term_1) = tpexpr_1 \texttt{in} tpexpr_2\\ & \texttt{if} pval \texttt{then} tpexpr_1 \texttt{else} tpexpr_2\\ & \sigma(tpexpr) \end{split}$	bind binders $(ident_or_pattern)$ in $tpexpr$ bind binders $(ident_or_pattern)$ in $tpexpr_2$ bind y_1 in $term_1$	pure let pure let
$tpexpr_case_branch$::=	$pattern \Rightarrow tpexpr$	bind $binders(pattern)$ in $tpexpr$	pure top-level case expression top-level case expression br
m_kill_kind	::= 	$\begin{array}{l} {\tt dynamic} \\ {\tt static}\tau \end{array}$		
bool, _	::= 	true false		OCaml booleans
$int,$ _	::=	i		OCaml fixed-width integer literal integer
res_term	::= 	$\begin{array}{l} \texttt{emp} \\ points_to \\ ident \\ \langle res_term_1, res_term_2 \rangle \\ \texttt{pack} \left(pval, res_term \right) \end{array}$		resource terms empty heap single-cell heap variable seperating-conjunction pair packing for existentials
mem_action	::= 	$\mathtt{create}\left(pval, au ight) \ \mathtt{create_readonly}\left(pval_1, au,pval_2 ight)$		memory actions

```
alloc(pval_1, pval_2)
                             kill(m_kill_kind, pval, pt)
                             store(bool, \tau, pval_1, pval_2, mem\_order, pt)
                                                                                                               true means store is locking
                             load(\tau, pval, mem\_order, pt)
                             rmw(\tau, pval_1, pval_2, pval_3, mem\_order_1, mem\_order_2)
                             fence (mem\_order)
                              cmp\_exch\_strong(\tau, pval_1, pval_2, pval_3, mem\_order_1, mem\_order_2)
                              cmp\_exch\_weak(\tau, pval_1, pval_2, pval_3, mem\_order_1, mem\_order_2)
                             linux_fence(linux_mem_order)
                             linux\_load(\tau, pval, linux\_mem\_order)
                             linux\_store(\tau, pval_1, pval_2, linux\_mem\_order)
                             linux_rmw(\tau, pval_1, pval_2, linux_mem\_order)
                                                                                                            polarities for memory actions
polarity
                       ::=
                                                                                                               (pos) sequenced by let weak and let strong
                                                                                                               only sequenced by let strong
                              neg
pol\_mem\_action
                                                                                                            memory actions with polarity
                             polarity\ mem\_action
                                                                                                            operations involving the memory state
mem\_op
                             pval_1 \ binop_{rel} \ pval_2
                                                                                                               pointer relational binary operations
                             pval_1 -_{\tau} pval_2
                                                                                                               pointer subtraction
                             \mathtt{intFromPtr}\left(	au_{1},	au_{2},pval
ight)
                                                                                                               cast of pointer value to integer value
                             \mathtt{ptrFromInt}\left(\tau_{1},\tau_{2},pval\right)
                                                                                                               cast of integer value to pointer value
                             ptrValidForDeref(	au, pval, pt)
                                                                                                               dereferencing validity predicate
                             ptrWellAligned(\tau, pval)
                             ptrArrayShift (pval_1, \tau, pval_2)
                             memcpy(pval_1, pval_2, pval_3)
                             memcmp(pval_1, pval_2, pval_3)
```

```
realloc(pval_1, pval_2, pval_3)
                       va_start(pval_1, pval_2)
                       va\_copy(pval)
                       va\_arg(pval, \tau)
                       va_{end}(pval)
                                                                                                                      spine element
spine\_elem
                 ::=
                       pval
                                                                                                                         pure or logical value
                                                                                                                         resource value
                       res\_term
spine
                                                                                                                      spine
                 ::=
                       spine\_elem_i
                                                                                                                      (effectful) top-level values
tval
                 ::=
                                                                                                                         end of top-level expression
                       {\tt done}\, spine
                       undef UB\_name
                                                                                                                         undefined behaviour
res\_pattern
                                                                                                                      resource terms
                 ::=
                                                        binders = \{\}
                                                                                                                         empty heap
                       emp
                                                         binders = \{\}
                                                                                                                         single-cell heap
                       pt
                                                        binders = ident
                                                                                                                         variable
                       ident
                                                        binders = binders(res\_pattern_1) \cup binders(res\_pattern_2)
                       \langle res\_pattern_1, res\_pattern_2 \rangle
                                                                                                                         seperating-conjunction pair
                       pack (ident, res_pattern)
                                                         binders = ident \cup binders(res\_pattern)
                                                                                                                         packing for existentials
ret\_pattern
                                                                                                                      return pattern
                       comp ident\_or\_pattern
                                                         binders = binders(ident\_or\_pattern)
                                                                                                                         computational variable
                       \log ident
                                                         binders = ident
                                                                                                                         logical variable
                                                        binders = binders(res\_pattern)
                                                                                                                         resource variable
                       {\tt res}\ res\_pattern
init,
                                                                                                                      initialisation status
                 ::=
```

		✓ ×		initialised uninitalised
$points_to, pt$::=	$term_1 \stackrel{init}{\mapsto}_{\tau} term_2$		points-to separation logic predicate
res	::= 	emp $points_to$ $res_1 * res_2$ $\exists ident: \beta. res$ $term \land res$ $\langle res \rangle$ $[\sigma] res$	S M	resources empty heap points-top heap pred. seperating conjunction existential logical conjuction parentheses simul-sub σ in res
$ret,$ _	::=	$\Sigma ident:\beta. \ ret$ $\exists ident:\beta. \ ret$ $res \otimes ret$ $term \wedge ret$ I $\sigma(ret)$	M	return types return a computational value return a logical value return a resource value return a predicate (post-condition) end return list simul-sub σ in ret
seq_expr	::= 	$\begin{array}{c} \texttt{ccall}\left(\tau, pval, spine\right) \\ \texttt{pcall}\left(name, spine\right) \end{array}$		sequential (effectful) expressions C function call procedure call
seq_texpr	::= 	$tval \ { m run} ident \overline{pval_i}^{i}$		sequential top-level (effectful) expressions (effectful) top-level values run from label

		$\begin{array}{l} \texttt{let} ident_or_pattern = pexpr \texttt{in} texpr\\ \texttt{let} ident_or_pattern: (y_1:\beta_1. term_1) = tpexpr \texttt{in} texpr \end{array}$	bind binders($ident_or_pattern$) in $texpr$ bind binders($ident_or_pattern$) in $texpr$ bind y_1 in $term_1$	pure let pure let
		$\begin{array}{l} \operatorname{let}\overline{ret_pattern_i}^i = seq_expr\operatorname{in}texpr \\ \operatorname{let}\overline{ret_pattern_i}^i : ret = texpr_1\operatorname{in}texpr_2 \\ \operatorname{case}pval\operatorname{of}\overline{\mid texpr_case_branch_i}^i \operatorname{end} \\ \operatorname{if}pval\operatorname{then}texpr_1\operatorname{else}texpr_2 \\ \operatorname{bound}[int](is_texpr) \end{array}$	bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$ bind binders $(\overline{ret_pattern_i}^i)$ in $texpr_2$	bind return patterns annotated bind return patterns pattern matching conditional limit scope of indet seq behaviour
$texpr_case_branch$::= 	$pattern \Rightarrow texpr$	bind $binders(pattern)$ in $texpr$	top-level case expression branch top-level case expression branch
is_expr	::= 	$tval \\ exttt{memop}\left(mem_op\right) \\ exttt{pol_mem_action}$		indet seq (effectful) expressions (effectful) top-level values pointer op involving memory memory action
is_texpr	::= 	$\begin{array}{l} {\tt let weak } \overline{ret_pattern_i}^{i} = is_expr {\tt in } texpr \\ {\tt let strong } \overline{ret_pattern_i}^{i} = is_expr {\tt in } texpr \end{array}$	bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$ bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$	indet seq top-level (effectful) express weak sequencing strong sequencing
texpr	::= 	seq_texpr is_texpr $\sigma(texpr)$	M	top-level (effectful) expressions sequential (effectful) expressions indet seq (effectful) expressions simul-sub σ in $texpr$
arg	::=	$\Pi ident:\beta. \ arg \ orall ident:\beta. \ arg$		argument/function types

 $res \multimap arg$

```
term \supset arg
                          \sigma(arg)
                                                                 simul-sub \sigma in arg
                                                              pure argument/function types
pure\_arg
                          \Pi ident:\beta. pure\_arg
                          term \supset pure\_arg
                          pure\_ret
pure\_ret
                                                              pure return types
                          \Sigma ident:\beta. pure\_ret
                          term \land pure\_ret
\mathcal{C}
                                                              computational var env
                          C, ident: \beta
\mathcal{L}
                                                              logical var env
                          \overline{\mathcal{L}_{i}}^{i}
\mathcal{L}, ident: \beta
Φ
                                                              constraints env
                          \Phi, term
\mathcal{R}
```

resources env

::=

```
\mathcal{R}, ident:res
                                                                                                                                                       substitutions
\sigma
                                        spine\_elem/ident, \sigma
 typing
                                        \mathtt{smt}\left(\Phi\Rightarrow term\right)
                             ident: eta \in \mathcal{C} ident: eta \in \mathcal{L} ident: \operatorname{struct} tag \& \overline{member_i: 	au_i}^i \in \operatorname{Globals} \overline{\mathcal{C}_i; \mathcal{L}_i; \Phi_i \vdash mem\_val_i \Rightarrow \operatorname{mem} eta_i}^i
                                                                                                                                                             dependent on memory object model
 opsem
                                      \forall i < j. \ \mathtt{not} \left( pattern_i = pval \leadsto \sigma_i \right)
                                        \mathtt{fresh}\left( \overset{\cdot}{mem\_ptr} \right)
                                        term
                                        pval:\beta
formula
                                        judgement
                                         typing
                                         opsem
                                        term \equiv term'
                                        name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in \texttt{Globals} pval:arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals}
```

```
heap, h
                                                                                                                                      heaps
                                                  h + \{points\_to\}
object\_value\_jtype
                                                 C; \mathcal{L}; \Phi \vdash object\_value \Rightarrow \mathsf{obj}\,\beta
pval\_jtype
                                                 C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta
res\_jtype
                                                 \Phi \vdash res \equiv res'
                                                 C; \mathcal{L}; \Phi; \mathcal{R} \vdash res\_term \Leftarrow res
spine\_jtype
                                         ::=
                                                 C; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret
pexpr\_jtype
                                        ::=
                                                 C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term
comp\_pattern\_jtype
                                        ::=
                                                  term as pattern: \beta \leadsto \mathcal{C}; \Phi
                                                  term as ident\_or\_pattern:\beta \leadsto \mathcal{C};\Phi
res\_pattern\_jtype
                                         ::=
                                                  res\_pattern:res \leadsto \mathcal{L}; \Phi; \mathcal{R}
ret\_pattern\_jtype
                                        ::=
```

 $\overline{ret_pattern_i}^i: ret \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}$

```
tpval\_jtype
                                                      C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term
tpexpr\_jtype
                                            ::=
                                                     C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term
action\_jtype
                                                      C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_action \Rightarrow ret
tval\_jtype
                                                      \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathit{tval} \Leftarrow \mathit{ret}
texpr\_jtype
                                            ::=
                                                     C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq\_expr \Rightarrow ret
                                                     C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_op \Rightarrow ret
                                                     C; \mathcal{L}; \Phi; \mathcal{R} \vdash is\_expr \Rightarrow ret
                                                     C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq\_texpr \Leftarrow ret
                                                     C; \mathcal{L}; \Phi; \mathcal{R} \vdash is\_texpr \Leftarrow ret
                                                      C; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret
subs\_jtype
                                                      pattern = pval \leadsto \sigma
                                                      ident\_or\_pattern = pval \leadsto \sigma
                                                      res\_pattern = res\_term \leadsto \sigma
                                                      ret\_pattern = spine\_elem \leadsto \sigma
                                                      \overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret
pure\_opsem\_jtype
                                                      \langle pexpr \rangle \longrightarrow \langle pexpr' \rangle
                                                      \langle pexpr \rangle \longrightarrow \langle tpexpr:(y:\beta. term) \rangle
```

```
\langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle
   opsem\_jtype
                                                             \begin{array}{c|c} ... \\ & \langle seq\_expr \rangle \longrightarrow \langle texpr:ret \rangle \\ & \langle h; seq\_texpr \rangle \longrightarrow \langle h'; texpr \rangle \\ & \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle \\ & \langle h; mem\_action \rangle \longrightarrow \langle h'; tval \rangle \\ & \langle h; is\_expr \rangle \longrightarrow \langle h'; is\_expr' \rangle \\ & \langle h; is\_texpr \rangle \longrightarrow \langle h'; texpr \rangle \\ & \langle h; texpr \rangle \longrightarrow \langle h'; texpr' \rangle \\ \end{array} 
  lemma\_jtype
\boxed{\mathcal{C};\mathcal{L};\Phi \vdash object\_value} \Rightarrow \mathtt{obj}\,eta
                                                                                                                                                    \frac{}{\mathcal{C};\mathcal{L};\Phi \vdash mem\_int \Rightarrow \mathtt{objinteger}} \quad \text{TY\_PVAL\_OBJ\_INT}
                                                                                                                                                          C; \mathcal{L}; \Phi \vdash mem\_ptr \Rightarrow objloc TY_PVAL_OBJ_PTR
                                                                                                                           \frac{\overline{\mathcal{C};\mathcal{L};\Phi \vdash loaded\_value_i \Rightarrow \beta}^i}{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{array}\left(\overline{loaded\_value_i}^i\right) \Rightarrow \mathtt{obj}\,\mathtt{array}\,\beta} \quad \mathsf{TY\_PVAL\_OBJ\_ARR}
                                                                                                                             ident : \mathtt{struct} \ tag \ \& \ \overline{member_i {:} \tau_i}^i \in \mathtt{Globals}
                                                                                                                             \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash mem\_val_i} \Rightarrow mem \beta_{\tau_i}^{i}
                                                                                                                                                                                                                                                                                   Ty_Pval_Obj_Struct
                                                                                   \overline{\mathcal{C};\mathcal{L};\Phi \vdash (\mathtt{struct}\, tag)\{\overline{.member_i:\tau_i = mem\_val_i}^i\}} \Rightarrow \mathtt{obj}\,\mathtt{struct}\, tag
```

$$C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$$

$$\frac{x : \beta \in \mathcal{C}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Comp}$$

$$\frac{x : \beta \in \mathcal{L}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow obj \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \beta} \quad \text{Ty_Pval_Obj}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow obj \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash specified object_value \Rightarrow \beta} \quad \text{Ty_Pval_Loaded}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Unit} \Rightarrow \text{unit}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Unit} \Rightarrow \text{unit}} \quad \text{Ty_Pval_Unit}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{True} \Rightarrow \text{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{False} \Rightarrow \text{bool}} \quad \text{Ty_Pval_False}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{value}_i \Rightarrow \beta^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash \beta[\overline{value}_i^i] \Rightarrow \text{list} \beta} \quad \text{Ty_Pval_List}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{value}_i \Rightarrow \beta^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash (\overline{value}_i^i) \Rightarrow \beta^i_i} \quad \text{Ty_Pval_Tuple}$$

$$\frac{\operatorname{smt}(\Phi \Rightarrow \operatorname{false})}{C;\mathcal{L};\Phi \vdash \operatorname{error}(\operatorname{string},\operatorname{pval}) \Rightarrow \beta} \quad \operatorname{Ty_Pval_Error}$$

$$\overline{C;\mathcal{L};\Phi \vdash \operatorname{Nil}\beta() \Rightarrow \operatorname{list}\beta} \quad \operatorname{Ty_Pval_Ctor_Nil}$$

$$\frac{C;\mathcal{L};\Phi \vdash \operatorname{pval}_1 \Rightarrow \beta}{C;\mathcal{L};\Phi \vdash \operatorname{pval}_2 \Rightarrow \operatorname{list}\beta} \quad \operatorname{Ty_Pval_Ctor_Cons}$$

$$\overline{C;\mathcal{L};\Phi \vdash \operatorname{cons}(\operatorname{pval}_1,\operatorname{pval}_2) \Rightarrow \operatorname{list}\beta} \quad \operatorname{Ty_Pval_Ctor_Cons}$$

$$\frac{\overline{C};\mathcal{L};\Phi \vdash \operatorname{pval}_i \Rightarrow \overline{\beta_i}^i}{C;\mathcal{L};\Phi \vdash \operatorname{Tuple}(\overline{\operatorname{pval}_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \operatorname{Ty_Pval_Ctor_Tuple}$$

$$\frac{\overline{C};\mathcal{L};\Phi \vdash \operatorname{pval}_i \Rightarrow \overline{\beta}^i}{C;\mathcal{L};\Phi \vdash \operatorname{Array}(\overline{\operatorname{pval}_i}^i) \Rightarrow \operatorname{array}\beta} \quad \operatorname{Ty_Pval_Ctor_Array}$$

$$\frac{C;\mathcal{L};\Phi \vdash \operatorname{pval} \Rightarrow \beta}{C;\mathcal{L};\Phi \vdash \operatorname{Specified}(\operatorname{pval}) \Rightarrow \beta} \quad \operatorname{Ty_Pval_Ctor_Specified}$$

$$\frac{\operatorname{ident:} \operatorname{struct} \operatorname{tag} \& \overline{\operatorname{member}_i : \tau_i}^i \in \operatorname{Globals}$$

$$\overline{C;\mathcal{L};\Phi \vdash \operatorname{pval}_i \Rightarrow \beta_{\tau_i}^i}$$

$$\overline{C;\mathcal{L};\Phi \vdash (\operatorname{struct} \operatorname{tag}) \{\overline{\cdot \operatorname{member}_i = \operatorname{pval}_i^i}^i\} \Rightarrow \operatorname{struct} \operatorname{tag}} \quad \operatorname{Ty_Pval_Struct}$$

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 $\overline{\Phi \vdash \mathtt{emp} \ \equiv \ \mathtt{emp}} \quad \mathrm{TYRES_EQ_EMP}$

 $\Phi \vdash res \equiv res'$

$$\begin{split} \operatorname{smt} \left(\Phi \Rightarrow (term_1 = term_1') \wedge (term_2 = term_2') \right) \\ \Phi \vdash term_1 & \stackrel{init}{\mapsto_{\tau}} term_2 \equiv term_1' & \stackrel{init}{\mapsto_{\tau}} term_2' \\ \\ \frac{\Phi \vdash res_1 \equiv res_1'}{\Phi \vdash res_2 \equiv res_2'} \\ \hline \Phi \vdash res_1 * res_2 \equiv res_1' * res_2' \end{split}$$
 Tyres_Eq_SepConj

$$\frac{\Phi \vdash \mathit{res} \; \equiv \; \mathit{res'}}{\Phi \vdash \exists \, \mathit{ident:} \beta. \; \mathit{res} \; \equiv \; \exists \, \mathit{ident:} \beta. \; \mathit{res'}} \quad \mathsf{TyRes_Eq_Exists}$$

$$\begin{array}{l} \operatorname{smt}\left(\Phi, term \Rightarrow term'\right) \\ \operatorname{smt}\left(\Phi, term' \Rightarrow term\right) \\ \Phi \vdash res \equiv res' \\ \hline \Phi \vdash term \land res \equiv term' \land res' \end{array} \quad \text{Tyres_Eq_Term} \end{array}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash\mathtt{emp}\Leftarrow\mathtt{emp}}\quad \mathsf{TyRes_EMP}$$

$$\frac{\Phi \vdash points_to \equiv points_to'}{\mathcal{C}; \mathcal{L}; \Phi; \cdot, .:points_to \vdash points_to' \Leftarrow points_to'} \quad \text{Tyres_PointsTo}$$

$$\frac{\Phi \vdash res \equiv res'}{\mathcal{C}; \mathcal{L}; \Phi; \cdot, r : res \vdash r \Leftarrow res'} \quad \text{TyRes_Var}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term_1 \Leftarrow res_1 \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash res_term_2 \Leftarrow res_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash \langle res_term_1, res_term_2 \rangle \Leftarrow res_1 * res_2 \end{array} \quad \text{TyRes_SepConj}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term_2 \Leftarrow [pval/y, \cdot] res \\ & \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \operatorname{pack}(pval, res_term_2) \Leftarrow \exists \, y \text{:} \beta. \, res} \end{split} \quad \text{TyRes_Pack} \end{split}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash :: ret \gg \cdot; ret} \quad \text{TY_Spine_Empty}$$

$$\begin{array}{c}
\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\
\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\
\overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \Pi x:\beta. arg \gg pval/x, \sigma; ret}
\end{array}$$
TY_SPINE_COMP

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \forall x:\beta. \ arg \gg pval/x, \sigma; ret \end{array} \quad \text{Ty_Spine_Log}$$

$$\begin{aligned} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash \underbrace{\mathit{res_term} \Leftarrow \mathit{res}}_{i} \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \underbrace{\overline{x_i = \mathit{spine_elem}_i}^i :: \mathit{arg} \gg \sigma; \mathit{ret}}_{i} \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash x = \mathit{res_term}, \underbrace{\overline{x_i = \mathit{spine_elem}_i}^i :: \mathit{res} \multimap \mathit{arg} \gg \mathit{res_term}/x, \sigma; \mathit{ret}}_{i} \end{aligned}$$
 TY_SPINE_RES

$$\frac{\mathsf{smt}\;(\Phi\Rightarrow term)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_i=spine_elem_i}^i::arg\gg\sigma;ret} \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_i=spine_elem_i}^i::arg\gg\sigma;ret}{\overline{x_i=spine_elem_i}^i::term\supset arg\gg\sigma;ret} \\ \text{TY_Spine_Phi}$$

 $C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow y: \beta. \ y = pval} \quad \text{TY_PE_VAL}$$

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\mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathsf{loc}
                                                                             \mathcal{C}: \mathcal{L}: \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                                                                                                                                                                                                                      TY_PE_ARRAY_SHIFT
           \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{array\_shift}(pval_1, \tau, pval_2)} \Rightarrow y : \mathtt{loc}. \ y = pval_1 +_{\mathtt{ptr}} (pval_2 \times \mathtt{size\_of}(\tau))
                                                           C; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathsf{loc}
                                                           \exists: struct tag \ \& \ \overline{member_i : 	au_i}^i \in \texttt{Globals}
                                                                                                                                                                                                                            TY_PE_MEMBER_SHIFT
\overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{member\_shift}\left(pval,tag,member_j\right)} \Rightarrow y\mathtt{:loc}.\ y = pval +_{\mathtt{ptr}} \mathtt{offset\_of}_{tag}(member_j)
                                                                       \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{not} (pval) \Rightarrow y \texttt{:bool}. \ y = \neg pval} \quad \text{TY\_PE\_NOT}
                                                                             \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{integer}
                                                                             \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                                                                                                                                                                                                           TY_PE_ARITH_BINOP
                      \overline{\mathcal{C};\mathcal{L};\Phi \vdash pval_1 \ binop_{arith} \ pval_2 \Rightarrow y \text{:integer.} \ y = (pval_1 \ binop_{arith} \ pval_2)}
                                                                                \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathtt{integer}
                                                                                \mathcal{C}: \mathcal{L}: \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                                 \frac{\mathcal{C}, \mathcal{L}, \text{ } \vdash pval_2 \rightarrow \text{ Integer}}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{rel} \ pval_2 \Rightarrow y \text{:bool.} \ y = (pval_1 \ binop_{rel} \ pval_2)}
                                                                                                                                                                                                      TY_PE_REL_BINOP
                                                                                   \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow bool
                                                                                  \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow bool
                              \frac{\mathcal{C}, \mathcal{L}, 1 + pear_2 \rightarrow bool}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{bool} \ pval_2 \Rightarrow y : bool. \ y = (pval_1 \ binop_{bool} \ pval_2)}
                                                                                                                                                                                                      TY_PE_BOOL_BINOP
                                                      name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in Globals
                                                     \frac{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma \ y:\beta. \ term \land \mathtt{I}}{\mathcal{C}; \mathcal{L}; \Phi \vdash name(\overline{pval_i}^i) \Rightarrow y:\beta. \ \sigma(term)}
                                                                                                                                                                                               Ty PE CALL
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$$\frac{\overline{term[i]} \text{ as } pattern_i : \beta \leadsto \mathcal{C}_i ; \overline{\Phi_i}^i}{term \text{ as } \operatorname{Array}(\overline{pattern_i}^i) : \operatorname{array} \beta \leadsto \overline{\mathcal{C}_i}^i ; \overline{\Phi_i}^i} \quad \text{TY_PAT_COMP_ARRAY}$$

 $\frac{term \, \text{as} \, pattern: \beta \leadsto \mathcal{C}; \Phi}{term \, \text{as} \, \text{Specified}(pattern): \beta \leadsto \mathcal{C}; \Phi} \quad \text{Ty_Pat_Comp_Specified}$

 $term \ \mathtt{as} \ ident_or_pattern{:}\beta \leadsto \mathcal{C}; \Phi$

 $\overline{term \text{ as } x : \beta \leadsto \cdot, x : \beta; \cdot, x = term} \quad \text{TY_PAT_SYM_OR_PATTERN_SYM}$

 $\frac{term \text{ as } pattern: \beta \leadsto \mathcal{C}; \Phi}{term \text{ as } pattern: \beta \leadsto \mathcal{C}; \Phi} \quad \text{Ty_Pat_Sym_Or_Pattern_Pattern}$

 $res_pattern:res \leadsto \mathcal{L}; \Phi; \mathcal{R}$

$$\frac{}{\texttt{emp:emp} \leadsto \cdot; \cdot; \cdot} \quad \text{TY_PAT_RES_EMPTY}$$

 $\frac{}{points_to:points_to} \leadsto \cdot; \cdot; \cdot, _:points_to \qquad \text{Ty_Pat_Res_Points} \text{To}$

 $\frac{}{r:res \leadsto \cdot; \cdot; \cdot, r:res} \quad \text{Ty_Pat_Res_Var}$

$$\frac{\mathit{res_pattern}_1:\mathit{res}_1 \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1}{\mathit{res_pattern}_2:\mathit{res}_2 \leadsto \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{\mathit{res_pattern}_2:\mathit{res}_2 \leadsto \mathcal{L}_2; \Phi_2; \mathcal{R}_2}{\langle \mathit{res_pattern}_1, \mathit{res_pattern}_2 \rangle :\mathit{res}_1 * \mathit{res}_2 \leadsto \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2} \quad \text{Ty_Pat_Res_SepConj}$$

$$\frac{res_pattern:res \leadsto \mathcal{L}; \Phi; \mathcal{R}}{res_pattern:term \land res \leadsto \mathcal{L}; \Phi, term; \mathcal{R}} \quad \text{Ty_Pat_Res_Conj}$$

$$\frac{res_pattern:[x/y,\cdot]res \leadsto \mathcal{L}; \Phi; \mathcal{R}}{\operatorname{pack}(x, res_pattern): \exists y: \beta. res \leadsto \mathcal{L}, x: \beta; \Phi; \mathcal{R}} \quad \text{TY_PAT_RES_PACK}$$

 $\boxed{\overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}}$

$$\frac{1}{:I \rightsquigarrow :; :; :; :} TY_PAT_RET_EMPTY$$

$$\frac{y \text{ as } ident_or_pattern: \beta \leadsto \mathcal{C}_1; \Phi_1}{\overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{comp \, ident_or_pattern, \, \overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}_1; \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2}{\mathsf{Comp} \, ident_or_pattern, \, \overline{ret_pattern_i}^i : \Sigma \, y : \beta. \, ret \leadsto \mathcal{C}_1, \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \text{TY_PAT_RET_COMP}$$

$$\frac{\overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}}{\log y, \ \overline{ret_pattern_i}^i : \exists \ y : \beta. \ ret \leadsto \mathcal{C}; \mathcal{L}, y : \beta; \Phi; \mathcal{R}} \quad \text{Ty_Pat_Ret_Log}$$

$$\frac{\underset{res_pattern:res \rightarrow \mathcal{L}_{1}; \Phi_{1}; \mathcal{R}_{1}}{\underset{ret_pattern_{i}}{ret_pattern_{i}}^{i} : ret \rightarrow \mathcal{C}_{2}; \mathcal{L}_{2}; \Phi_{2}; \mathcal{R}_{2}}}{\underset{res_pattern, \ \overline{ret_pattern_{i}}^{i} : res \otimes ret \rightarrow \mathcal{C}_{2}; \mathcal{L}_{1}, \mathcal{L}_{2}; \Phi_{1}, \Phi_{2}; \mathcal{R}_{1}, \mathcal{R}_{2}}} \quad \text{Ty_Pat_Ret_Res}$$

$$\frac{\overline{\mathit{ret_pattern_i}}^i : \mathit{ret} \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}}{\overline{\mathit{ret_pattern_i}}^i : \mathit{term} \land \mathit{ret} \leadsto \mathcal{C}; \mathcal{L}; \Phi, \mathit{term}; \mathcal{R}} \quad \mathsf{TY_PAT_RET_PHI}$$

 $C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{undef}\;\mathit{UB_name}\Leftarrow\mathit{y}{:}\beta.\,\mathit{term}}\quad \mathsf{TY_TPVAL_UNDEF}$$

$$\begin{array}{l} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \operatorname{smt} (\Phi \Rightarrow pval/y, \cdot (term)) \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \operatorname{done} pval \Leftarrow y:\beta. \ term \end{array} \quad \text{TY_TPVAL_DONE}$$

 $C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{create}\,(pval, \tau) \Rightarrow \Sigma\,y_p : \mathtt{loc.\,representable}\,(\tau *, y_p) \land \mathtt{alignedI}\,(pval, y_p) \land \exists\,y : \beta_\tau.\,\langle y_p \overset{\times}{\mapsto}_\tau y \rangle \otimes \mathtt{I}} \quad \mathtt{TY_ACTION_CREATE}$$

$$C; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathtt{loc} \\ \mathtt{smt} \ (\Phi \Rightarrow pval_0 = pval_1) \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \overset{\checkmark}{\mapsto}_{\tau} pval_2 \Leftarrow pval_1 \overset{\checkmark}{\mapsto}_{\tau} pval_2 \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{load} \ (\tau, pval_0, \neg, pval_1 \overset{\checkmark}{\mapsto}_{\tau} pval_2) \Rightarrow \Sigma \ y : \beta_{\tau}. \ y = pval_2 \land \langle pval_1 \overset{\checkmark}{\mapsto}_{\tau} pval_2 \rangle \otimes \mathtt{I} \\ C; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathtt{loc} \\ C; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathtt{loc} \\ C; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \beta_{\tau} \\ \mathtt{smt} \ (\Phi \Rightarrow \mathsf{representable} \ (\tau, pval_1)) \\ \mathtt{smt} \ (\Phi \Rightarrow pval_2 = pval_0) \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pval}_2 \mapsto_{\tau} - \Leftarrow pval_2 \mapsto_{\tau} - \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{store} \ (\neg, \tau, pval_0, pval_1, \neg, pval_2 \mapsto_{\tau} -) \Rightarrow \Sigma \ . \mathtt{unit.} \ \langle pval_2 \overset{\checkmark}{\mapsto}_{\tau} pval_1 \rangle \otimes \mathtt{I} \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pval}_0 \Rightarrow \mathtt{loc} \\ \mathtt{smt} \ (\Phi \Rightarrow pval_0 = pval_1) \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pval}_1 \mapsto_{\tau} - \Leftarrow pval_1 \mapsto_{\tau} - \\ C; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{kill} \ (\mathtt{static} \ \tau, pval_0, pval_1 \mapsto_{\tau} -) \Rightarrow \Sigma \ . \mathtt{unit.} \ \mathtt{I} \\ \mathtt{TY_ACTIOn_KILL_STATIC} \\ TY_ACTION_KILL_STATIC}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \, pval, \, \overline{spine_elem_i}^{\; i} \Leftarrow \Sigma \, y : \beta. \, ret} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow \exists \, y : \beta. \, ret} \end{split}$$
 Ty_TVAL_Log

 $\overline{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{done} \ \Leftarrow \mathtt{I}} \quad \mathrm{TY_TVAL_I}$

$$\begin{array}{l} \mathtt{smt} \ (\Phi \Rightarrow term) \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{done} \ spine \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{done} \ spine \Leftarrow term \land ret \end{array} \quad \mathtt{TY_TVAL_PHI} \\$$

$$\begin{aligned} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash \mathit{res_term} \Leftarrow \mathit{res} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \mathsf{done} \, \overline{\mathit{spine_elem}_i}^i \Leftarrow \mathit{ret} }{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash \mathsf{done} \, \mathit{res_term}, \, \overline{\mathit{spine_elem}}^i \Leftarrow \mathit{res} \otimes \mathit{ret} } \end{aligned} \quad \mathsf{TY_TVAL_RES}$$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash\mathtt{undef}\ \mathit{UB_name} \Leftarrow\mathit{ret}}\quad \mathtt{TY_TVAL_UB}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{loc} \\ & pval : arg \equiv \overline{x_i}^i \mapsto texpr \in \mathtt{Globals} \\ & \underline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret} \\ & \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{ccall}\left(\tau, pval, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \end{split} \quad \mathtt{TY_SEQ_E_CCALL} \end{split}$$

$$\begin{array}{l} name: arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals} \\ \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \texttt{pcall}\left(name, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \quad \text{Ty_Seq_E_Proc} \end{array}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathsf{loc} \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathsf{loc} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash pval_1 \ binop_{rel} \ pval_2 \Rightarrow \Sigma \ y \mathsf{:bool.} \ y = (pval_1 \ binop_{rel} \ pval_2) \land \mathsf{I} \end{split} \qquad \mathsf{TY_MEMOP_REL_BINOP} \end{split}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{loc}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{intFromPtr}\left(\tau_1, \tau_2, pval\right) \Rightarrow \Sigma \ y : \mathtt{integer}. \ y = \mathtt{cast_ptr_to_int} \ pval \wedge \mathtt{I}} \quad \mathtt{TY_MEMOP_INTFROMPTR}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrFromInt}\left(\tau_1, \tau_2, pval\right) \Rightarrow \Sigma \, y : \mathtt{loc}. \, y = \mathtt{cast_int_to_ptr} \, pval \wedge \mathtt{I}} \quad \mathtt{TY_MEMOP_PTRFROMINT}$$

$$C; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathsf{loc}$$

$$\mathsf{smt} (\Phi \Rightarrow pval_1 = pval_0)$$

$$C: C: \Phi: \mathcal{R} \vdash pval_1 \xrightarrow{\checkmark} \Leftarrow pval_1 \xrightarrow{\checkmark} \Rightarrow \mathsf{mval}_1 \xrightarrow{} \Rightarrow \mathsf{mval}_1 \xrightarrow{}$$

 $\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \overset{\checkmark}{\mapsto}_{\tau -} \Leftarrow pval_1 \overset{\checkmark}{\mapsto}_{\tau -}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{ptrValidForDeref}} (\tau, pval_0, pval_1 \overset{\checkmark}{\mapsto}_{\tau -}) \Rightarrow \Sigma \ y \text{:bool.} \ y = \text{aligned} \ (\tau, pval_1) \land \langle pval_1 \overset{\checkmark}{\mapsto}_{\tau -} \rangle \otimes \mathbf{I}}$ TY_MEMOP_PTRVALIDFORDEREF

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{loc}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrWellAligned}\left(\tau, pval\right) \Rightarrow \Sigma \ y : \mathtt{bool}. \ y = \mathtt{aligned}\left(\tau, pval\right) \wedge \mathtt{I}} \quad \mathsf{TY_Memop_PtrWellAligned}$$

$$egin{aligned} \mathcal{C}; \mathcal{L}; \Phi dash pval_1 \Rightarrow \mathtt{loc} \ \mathcal{C}; \mathcal{L}; \Phi dash pval_2 \Rightarrow \mathtt{integer} \end{aligned}$$

 $\frac{\mathsf{c}; \mathcal{L}; \Psi \vdash pval_2 \Rightarrow \mathtt{integer}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrArrayShift}\left(pval_1, \tau, pval_2\right) \Rightarrow \Sigma \, y : \mathtt{loc.} \, y = pval_1 +_{\mathtt{ptr}}\left(pval_2 \times \mathtt{size_of}(\tau)\right) \wedge \mathtt{I}}$ Ty_Memop_PtrArrayShift

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash memop \, (mem_op) \Rightarrow ret} \quad \text{Ty_Is_E_Memop}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Action}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash neg \ mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Neg_Action}$$

C; L; Φ ; $R \vdash seg_texpr \Leftarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret} \quad \text{TY_SEQ_TE_TVAL}$$

```
\mathcal{C}:\mathcal{L}:\Phi\vdash pexpr\Rightarrow y:\beta.\ term
                                            y as ident\_or\_pattern:\beta \leadsto \mathcal{C}_1; \Phi_1
                                            \mathcal{C}, \mathcal{C}_1; \mathcal{L}, y:\beta; \Phi, term, \Phi_1; \mathcal{R} \vdash texpr \Leftarrow ret
                     \mathcal{C}; \overline{\mathcal{L}}; \overline{\Phi}; \mathcal{R} \vdash \mathtt{let} \, ident\_or\_pattern = pexpr \, \mathtt{in} \, texpr \Leftarrow ret   \mathsf{TY\_SEQ\_TE\_LETP} 
                                        C; L; \Phi \vdash tpexpr \Leftarrow y:\beta. term
                                         y as ident\_or\_pattern: \beta \leadsto C_1; \Phi_1
                                        \mathcal{C}, \mathcal{C}_1; \mathcal{L}, y:\beta; \Phi, term, \Phi_1; \mathcal{R} \vdash texpr \Leftarrow ret
\overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathtt{let}\,ident\_or\_pattern:(y:\beta.\,\,term) = tpexpr\,\mathtt{in}\,texpr \Leftarrow ret} \quad \mathtt{TY\_SeQ\_TE\_LetPT}
                                                \mathcal{C}: \mathcal{L}: \Phi: \mathcal{R}' \vdash seg\_expr \Rightarrow ret_1
                                                \overline{ret\_vattern_i}^i : ret_1 \leadsto \mathcal{C}_1 : \mathcal{L}_1 : \Phi_1 : \mathcal{R}_1
                 \frac{\mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr \Leftarrow ret_2}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \overline{ret\_pattern_i}^i = seq\_expr \text{ in } texpr \Leftarrow ret_2} \quad \text{TY\_SEQ\_TE\_LET}
                                            \mathcal{C}:\mathcal{L}:\Phi:\mathcal{R}'\vdash texpr_1\Leftarrow ret_1
                                             \overline{ret\_nattern_i}^i: ret_1 \leadsto \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1
           \frac{\mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr_2 \Leftarrow ret_2}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \mathsf{let} \overline{ret\_pattern_i}^i : ret_1 = texpr_1 \mathsf{in} \, texpr_2 \Leftarrow ret_2} \quad \mathsf{TY\_SEQ\_TE\_LETT}
                                 C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_1
                      \frac{\overline{y_1 \text{ as } pattern_i:\beta_1 \leadsto \mathcal{C}_i; \Phi_i}^i}{\overline{\mathcal{C}, \mathcal{C}_i; \mathcal{L}, y_1:\beta_1; \Phi, y_1 = pval, \Phi_i; \mathcal{R} \vdash texpr_i \Leftarrow ret}^i} \frac{\overline{\mathcal{C}, \mathcal{C}_i; \mathcal{L}, y_1:\beta_1; \Phi, y_1 = pval, \Phi_i; \mathcal{R} \vdash texpr_i \Leftarrow ret}^i}{\overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{case } pval \text{ of } | pattern_i \Rightarrow texpr_i}^i \text{ end } \Leftarrow ret}}  TY_SEQ_TE_CASE
                                                    C; \mathcal{L}; \Phi \vdash pval \Rightarrow bool
                                                    C; \mathcal{L}; \Phi, pval = \mathsf{true}; \mathcal{R} \vdash texpr_1 \Leftarrow ret
                                                    C; \mathcal{L}; \Phi, pval = \mathtt{false}; \mathcal{R} \vdash texpr_2 \Leftarrow ret
                                                                                                                                                                                          Ty_Seo_TE_IF
                                     \overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}} \vdash \text{if } pval \text{ then } texpr_1 \text{ else } texpr_2 \Leftarrow ret
```

$$\label{eq:continuous_continuous$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{bound} [int] (is_texpr) \Leftarrow ret} \quad \mathsf{TY_SEQ_TE_BOUND}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret_1 \\ & \overline{ret_pattern_i}^i : ret_1 \leadsto \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ & \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr \Leftarrow ret_2 \\ \hline & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{let} \, \mathtt{strong} \, \overline{ret_pattern_i}^i = is_expr \, \mathtt{in} \, texpr \Leftarrow ret_2 \end{split} \qquad \text{Ty_Is_TE_LetS} \end{split}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret} \quad \text{TY_TE_IS}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret} \quad \text{TY_TE_SeQ}$$

 $pattern = pval \leadsto \sigma$

$$\underline{} := pval \leadsto \cdot$$
 Subs_Decons_Value_No_Sym_Annot

$$\overline{x:_=pval \leadsto pval/x, \cdot} \quad \text{Subs_Decons_Value_Sym_Annot}$$

$$\begin{aligned} pattern_1 &= pval_1 \leadsto \sigma_1 \\ pattern_2 &= pval_2 \leadsto \sigma_2 \\ \overline{\text{Cons}(pattern_1, pattern_2) = \text{Cons}(pval_1, pval_2) \leadsto \sigma_1, \sigma_2} \end{aligned} \text{ Subs_Decons_Value_Cons}$$

$$\frac{\overline{pattern_i = pval_1 \leadsto \sigma_i}^i}{\text{Tuple}(\overline{pattern_i}^i) = \text{Tuple}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Tuple}$$

$$\frac{\overline{pattern_i = pval_1 \leadsto \sigma_i}^i}{\operatorname{Array}(\overline{pattern_i}^i) = \operatorname{Array}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Array}$$

$$\frac{pattern = pval \leadsto \sigma}{\texttt{Specified}(pattern) = pval \leadsto \sigma} \quad \texttt{SUBS_DECONS_VALUE_SPECIFIED}$$

 $ident_or_pattern = pval \leadsto \sigma$

$$\overline{x = pval \leadsto pval/x}$$
. Subs_Decons_Value'_Sym

$$\frac{pattern = pval \leadsto \sigma}{pattern = pval \leadsto \sigma} \quad \text{Subs_Decons_Value'_Pattern}$$

 $res_pattern = res_term \leadsto \sigma$

$$\frac{}{\text{emp} = \text{emp} \leadsto}$$
 Subs_Decons_Res_Emp

$$pt = pt \leadsto \cdot$$
 Subs_Decons_Res_Points_to

 $\overline{ident = \mathit{res_term} \leadsto \mathit{res_term}/ident, \cdot} \quad \text{Subs_Decons_Res_Var}$ $res_pattern_1 = res_term_1 \leadsto \sigma_1$ $res_pattern_2 = res_term_2 \leadsto \sigma_2$ $\frac{cos_{pattern_1} = res_{pattern_2} \sim \sigma_2}{\langle res_pattern_1, res_pattern_2 \rangle = \langle res_term_1, res_term_2 \rangle \leadsto \sigma_1, \sigma_2} \quad \text{Subs_Decons_Res_Pair}$ $\frac{res_pattern = res_term \leadsto \sigma}{\texttt{pack} \, (ident, res_pattern) = \texttt{pack} \, (pval, res_term) \leadsto pval/ident, \sigma} \quad \texttt{Subs_Decons_Res_Pack}$ $ret_pattern = spine_elem \leadsto \sigma$ $\frac{ident_or_pattern = pval \leadsto \sigma}{\mathsf{comp}\,ident_or_pattern = pval \leadsto \sigma} \quad \text{Subs_Decons_Ret_Comp}$ $\frac{1}{\log ident = pval \rightsquigarrow pval/ident}$. SUBS_DECONS_RET_LOG $\frac{\mathit{res_pattern} = \mathit{res_term} \leadsto \sigma}{\mathit{res_pattern} = \mathit{res_term} \leadsto \sigma} \quad \mathit{Subs_Decons_Ret_Res}$ $\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$ $\frac{}{::ret \gg \cdot; ret}$ Subs_Decons_Arg_Empty $\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \overline{x_i = spine_elem_i}^i :: \Pi x:\beta. arg \gg pval/x, \sigma; ret} \quad \text{Subs_Decons_Arg_Comp}$

```
\frac{bool\_value \equiv bool\_value_1 \ binop_{bool} \ bool\_value_2}{\langle bool\_value_1 \ binop_{bool} \ bool\_value_2 \rangle \longrightarrow \langle bool\_value \rangle}
                                                                                                                                                                                                                                              OP_PE_PE_BOOL_BINOP
                                                                                                 \overline{\langle \texttt{assert\_undef} \ (\texttt{True}, \ \mathit{UB\_name}) \rangle \longrightarrow \langle \texttt{Unit} \rangle} \quad \text{Op\_PE\_PE\_Assert\_Undef}
                                                                                                   \frac{}{\langle \texttt{bool\_to\_integer}\left(\texttt{True}\right)\rangle \longrightarrow \langle 1\rangle} \quad \text{OP\_PE\_PE\_BOOL\_TO\_INTEGER\_TRUE}
                                                                                                \frac{}{\langle \texttt{bool\_to\_integer}\left(\texttt{False}\right)\rangle \longrightarrow \langle 0\rangle} \quad \text{Op\_PE\_PE\_Bool\_To\_INTEGER\_False}
                                                                          abbrev_1 \equiv \max_{\cdot} \inf_{\tau} - \min_{\cdot} \inf_{\tau} + 1
                                                                          abbrev_2 \equiv pval \, rem_f \, abbrev_1
                                                                        abbrev_2 \equiv pval \, {\tt rem\_i} \, abbrev_1
mem\_int' \equiv {\tt if} \, abbrev_2 \leq {\tt max\_int}_{\tau} \, {\tt then} \, abbrev_2 \, {\tt else} \, abbrev_2 - abbrev_1
                                                                                                                                                                                                                                                                                      Op_PE_PE_WrapI
                                                                                                                        \langle \mathtt{wrapI} \ (	au, mem\_int) \rangle \longrightarrow \langle mem\_int' \rangle
\langle pexpr\rangle \longrightarrow \langle tpexpr:(y{:}\beta.\,term)\rangle
                                                                                                         \begin{array}{l} name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in \texttt{Globals} \\ \overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma \ y:\beta. \ term \land \texttt{I} \\ \overline{\langle name(\overline{pval_i}^i) \rangle} \longrightarrow \langle \sigma(tpexpr): (y:\beta. \ \sigma(term)) \rangle \end{array} \quad \text{Op\_PE\_TPE\_CALL} \\ \end{array}
\langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle
                                                                                        \frac{pattern_{j} = pval \leadsto \sigma_{j}}{\forall \, i < j. \, \, \text{not} \, (pattern_{i} = pval \leadsto \sigma_{i})} \\ \frac{\forall \, i < j. \, \, \text{not} \, (pattern_{i} = pval \leadsto \sigma_{i})}{\langle \text{case} \, pval \, \text{of} \, \overline{\mid pattern_{i} \Rightarrow tpexpr_{i}}^{i} \, \text{end} \rangle \longrightarrow \langle \sigma_{j}(tpexpr_{j}) \rangle} \quad \text{Op\_TPE\_TPE\_CASE}
                                                                                      \frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle \texttt{let}\, ident\_or\_pattern = pval \, \texttt{in}\, tpexpr \rangle \longrightarrow \langle \sigma(tpexpr) \rangle} \quad \text{Op\_TPE\_TPE\_Let\_Sub}
```

```
\frac{\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle}{\langle \text{let } ident\_or\_pattern = pexpr \text{ in } tpexpr\rangle \longrightarrow \langle \text{let } ident\_or\_pattern = pexpr' \text{ in } tpexpr\rangle}
                                                                                                                                                                                                                                                                           OP_TPE_TPE_LET_LET
                   \frac{\langle pexpr\rangle \longrightarrow \langle tpexpr_1 : (y:\beta.\ term)\rangle}{\langle \text{let}\ ident\_or\_pattern = pexpr\ in} \ tpexpr_2\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern : (y:\beta.\ term) = tpexpr_1\ in} \ tpexpr_2\rangle
                                                                                                                                                                                                                                                                                               OP_TPE_TPE_LET_LETT
                                                        \frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle \texttt{let} \, ident\_or\_pattern: (y:\beta. \, term) = \texttt{done} \, pval \, \texttt{in} \, tpexpr \rangle \longrightarrow \langle \sigma(tpexpr) \rangle} \quad \text{Op\_TPE\_TPE\_Lett\_Sub}
                                                                                                                     \langle tpexpr_1 \rangle \longrightarrow \langle tpexpr_1' \rangle
\frac{\langle \text{let } ident\_or\_pattern:(y:\beta. \ term) = tpexpr_1 \text{ in } tpexpr_2 \rangle}{\langle \text{let } ident\_or\_pattern:(y:\beta. \ term) = tpexpr_1 \text{ in } tpexpr_2 \rangle} \longrightarrow \langle \text{let } ident\_or\_pattern:(y:\beta. \ term) = tpexpr_1 \text{ in } tpexpr_2 \rangle}
                                                                                                                                                                                                                                                                                                              OP_TPE_TPE_LETT_LETT
                                                                                                                                                                                                                             Op_TPE_TPE_IF_True
                                                                                         \overline{\langle \text{if True then } tpexpr_1 \text{ else } tpexpr_2 \rangle \longrightarrow \langle tpexpr_1 \rangle}
                                                                                                                                                                                                                             Op_TPE_TPE_IF_FALSE
                                                                                       \overline{\langle \text{if False then } tpexpr_1 \text{ else } tpexpr_2 \rangle \longrightarrow \langle tpexpr_2 \rangle}
  \langle seq\_expr \rangle \longrightarrow \langle texpr:ret \rangle
                                                                                            \frac{pval:arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals}}{\overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret} \\ \frac{\left\langle \texttt{ccall} \left(\tau, pval, \overline{spine\_elem_i}^i\right) \right\rangle \longrightarrow \left\langle \sigma(texpr) : \sigma(ret) \right\rangle}{\left\langle \texttt{ccall} \left(\tau, pval, \overline{spine\_elem_i}^i\right) \right\rangle \longrightarrow \left\langle \sigma(texpr) : \sigma(ret) \right\rangle}
                                                                                                              name: arg \equiv \overline{x_i}^i \mapsto texpr \in Globals
                                                                                              \frac{\overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret}{\langle \texttt{pcall} \left( name, \overline{spine\_elem_i}^i \right) \rangle \longrightarrow \langle \sigma(texpr) : \sigma(ret) \rangle} \quad \text{Op\_SE\_TE\_PCALL}
```

 $\langle h; seq_texpr \rangle \longrightarrow \langle h'; texpr \rangle$

```
\frac{\langle seq\_expr\rangle \longrightarrow \langle texpr_1 : ret\rangle}{\langle h; \mathtt{let} \ \overline{ret\_pattern_i}^i = seq\_expr \ \mathtt{in} \ texpr_2\rangle \longrightarrow \langle h; \mathtt{let} \ \overline{ret\_pattern_i}^i : ret = texpr_1 \ \mathtt{in} \ texpr_2\rangle} \quad \mathsf{OP\_STE\_TE\_LET\_LETT}
                                                                                                          \langle h; texpr_1 \rangle \longrightarrow \langle h'; texpr_1' \rangle
                               \frac{\langle n; texpr_1 \rangle \longrightarrow \langle n; texpr_1 \rangle}{\langle h; \mathsf{let} \, \overline{ret\_pattern_i}^{\, i} : ret = texpr_1 \, \mathsf{in} \, texpr_2 \rangle \longrightarrow \langle h; \mathsf{let} \, \overline{ret\_pattern_i}^{\, i} : ret = texpr_1' \, \mathsf{in} \, texpr_2 \rangle}
                                                                                                                                                                                                                                                        OP_STE_TE_LETT_LETT
                                                                                                                                                                                                             OP_STE_TE_IF_TRUE
                                                                                    \overline{\langle h; \text{if True then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_1 \rangle}
                                                                                                                                                                                                              OP_STE_TE_IF_FALSE
                                                                                  \overline{\langle h; \text{if False then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_2 \rangle}
                                                                                                                                                                                                    OP_STE_TE_BOUND
                                                                                               \overline{\langle h: \mathtt{bound} [int](is\_texpr) \rangle} \longrightarrow \overline{\langle h: is\_texpr \rangle}
   \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                                \frac{mem\_int \equiv \texttt{cast\_ptr\_to\_int} \, mem\_ptr}{\langle h; \texttt{intFromPtr} \, (\tau_1, \tau_2, mem\_ptr) \rangle \longrightarrow \langle h; \texttt{done} \, mem\_int \rangle}
                                                                                                                                                                                                        Op_Memop_TVal_IntFromPtr
                                                                                    mem\_ptr \equiv \texttt{cast\_ptr\_to\_int} \ mem\_int
                                                                 \frac{1}{\langle h; \mathtt{ptrFromInt}\left(\tau_1, \tau_2, mem\_int\right)\rangle \longrightarrow \langle h; \mathtt{done}\, mem\_ptr\rangle}
                                                                                                                                                                                                        OP_MEMOP_TVAL_PTRFROMINT
                                                                                                           bool\_value \equiv \mathtt{aligned}\left(\tau, mem\_ptr\right)
\frac{bool\_vatae \equiv \mathtt{aligned}(r, mem\_ptr)}{\langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_\}; \mathtt{ptrValidForDeref}(\tau, mem\_ptr, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_) \rangle \longrightarrow \langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_\}; \mathtt{done} \ bool\_vatue, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_\rangle}
                                                                                                                                                                                                                                                                                                               OP_MEMOP_TVAL_PTRVALID
                                                       \frac{bool\_value \equiv \mathtt{aligned} \left(\tau, mem\_ptr\right)}{\langle h; \mathtt{ptrWellAligned} \left(\tau, mem\_ptr\right) \rangle \longrightarrow \langle h; \mathtt{done} \, bool\_value \rangle}
                                                                                                                                                                                                 Op_Memop_TVal_PtrWellAligned
```

$$\frac{mem.ptr' = mem.ptr +_{ptr}(mem.int \times size.of(\tau))}{\langle h; ptrArrayShift (mem.ptr, \tau, mem.int) \rangle \longrightarrow \langle h; done mem.ptr' \rangle} \quad \text{Op Memop_TVal_PtrArrayShift}} \\ \frac{\langle h; mem.action \rangle \longrightarrow \langle h'; tval \rangle}{\text{fresh}(mem.ptr)} \\ \frac{resh(mem.ptr)}{representable (\tau *, mem.ptr)} \\ \frac{pval(\beta)^2}{\langle h; create (mem.int, \tau) \rangle \longrightarrow \langle h + \{mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \}; done mem.ptr, pval, mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \rangle} \\ \langle h; \{mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \}; load (\tau, mem.ptr, \neg, mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval) \rangle \longrightarrow \langle h + \{mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \}; done pval, mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \rangle} \\ \langle h + \{mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} -\}; store (\neg, \tau, mem.ptr, pval, \neg, mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \}; done Unit, mem.ptr \stackrel{\checkmark}{\rightarrow}_{\tau} pval \rangle} \\ \langle h + \{mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle} \\ \langle h; mem.ptr \mapsto_{\tau} -\}; kill (static \tau, mem.ptr \mapsto_{\tau} -) \rangle \longrightarrow \langle h + \{mem.ptr \mapsto_{\tau} pval \}; done Unit, mem.ptr \mapsto_{\tau} pval \rangle}$$

$$\langle h; is_texpr \rangle \longrightarrow \langle h'; texpr \rangle$$

$$\frac{\overline{ret_pattern_i} = spine_elem_i \leadsto \sigma_i^{\ i}}{\langle h; \mathtt{let\,strong}\, \overline{ret_pattern_i}^{\ i} = \mathtt{done}\, \overline{spine_elem_i}^{\ i}\, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \overline{\sigma_i}^{\ i}(texpr) \rangle} \quad \text{Op_IsTe_IsTe_LetS_Sub}$$

$$\frac{\langle h; is_expr\rangle \longrightarrow \langle h'; is_expr'\rangle}{\langle h; \mathsf{let}\,\mathsf{strong}\,\overline{ret_pattern_i}^{\,\,i} = is_expr\,\mathsf{in}\,texpr\rangle \longrightarrow \langle h'; \mathsf{let}\,\mathsf{strong}\,\overline{ret_pattern_i}^{\,\,i} = is_expr'\,\mathsf{in}\,texpr\rangle} \quad \text{Op_ISTE_ISTE_LETS_LETS}$$

 $\langle h; texpr \rangle \longrightarrow \langle h'; texpr' \rangle$

$$\frac{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle}{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle} \quad \text{Op_TE_TE_SEQ}$$

$$\frac{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle}{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle} \quad \text{Op_TE_TE_IS}$$

 $\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret$

$$\frac{}{::ret \leadsto :; :; : | ret} \quad Arg_Env_Ret$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \Pi \, x : \beta. \, arg \leadsto \mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Comp}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \forall x : \beta. arg \leadsto \mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Log}$$

$$\frac{\overline{x_i}^{\;i} :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{\overline{x_i}^{\;i} :: term \supset arg \leadsto \mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \mid ret} \quad \text{Arg_Env_Phi}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: res \multimap arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x: res \mid ret} \quad \text{Arg_Env_Res}$$

 $\boxed{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\sqsubseteq\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}'}$

$$\frac{}{\cdot;\cdot;\cdot;\cdot\sqsubseteq\cdot;\cdot;\cdot}\quad \text{Weak_Empty}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x : \beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Cons_Phi}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x : res \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res} \quad \text{Weak_Cons_Res}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x:\beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Skip_Phi}$$

 $\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma): (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}')$

$$\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\cdot) : (\cdot; \cdot; \cdot; \cdot)$$
 TY_SUBS_EMPTY

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \hline \mathcal{C}; \mathcal{L}; \Phi : (pval/x, \sigma) : (\mathcal{C}', x : \beta; \mathcal{L}'; \Phi'; \mathcal{R}') \end{array} \quad \text{TY_SUBS_CONS_COMP} \end{array}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (pval/x, \sigma) : (\mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}')} \quad \text{Ty_Subs_Cons_Log} \end{split}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}')}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}')} \quad \text{Ty_Subs_Cons_Phi}$$

$$\begin{aligned} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res \\ & \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (res_term/x, \sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res)} \end{aligned} \quad \text{Ty_Subs_Cons_Res}$$

Definition rules: 195 good 0 bad Definition rule clauses: 432 good 0 bad