ident, $x, y, y_p, y_f, -$, abbrev, r, α subscripts: p for pointers, f for functions

n, i, j index variables

 $impl_const$ implementation-defined constant member C struct/union member name

Ott-hack, ignore (annotations)

nat OCaml arbitrary-width natural number

mem_ptr abstract pointer value
mem_val abstract memory value

Ott-hack, ignore (locations)

mem_iv_c OCaml type for memory constraints on integer values

 UB_name undefined behaviour

string OCaml string

Ott-hack, ignore (OCaml type variable TY)
Ott-hack, ignore (OCaml Symbol.prefix)

mem_order, _ OCaml type for memory order

linux_mem_order OCaml type for Linux memory order

Ott-hack, ignore (OCaml type variable bt)

```
Sctypes_{-}t, \tau
                                                  C type
                                                     array of length int of element type \tau
                            \operatorname{array} \operatorname{int} \tau
                                                     pointer to type \tau
                            \tau *
int, -
                                                  OCaml fixed-width integer
                                                     literal integer
                                                     literal integer
                            n
                                                  OCaml type for struct/union tag
tag
                            ident
β, _
                                                  base types
                                                     unit
                            unit
                                                     boolean
                            bool
                                                     integer
                            integer
                            real
                                                     rational numbers?
                                                     location
                            loc
                            \operatorname{array} \beta
                                                     array
                            \operatorname{list} \beta
                                                     list
                                                     tuple
                            \mathtt{struct}\,tag
                                                     struct
                            \mathtt{set}\,eta
                                                     set
                            \mathtt{opt}\left(eta
ight)
                                                     option
                            \beta \to \beta'
                                                     parameter types
                                                     of a C type
                                            Μ
binop
                                                  binary operators
                                                     addition
                                                     subtraction
                                                     multiplication
                                                     division
```

	rem_t rem_f	modulus remainder exponentiation equality, defined both for integer and C types inequality, similiarly defined greater than, similarly defined less than, similarly defined greater than or equal to, similarly defined less than or equal to, similarly defined conjunction disjunction
$binop_{arith}$::=	arithmentic binary operators
$binop_{rel}$::= 	relational binary operators
$binop_{bool}$::= /\	boolean binary operators

		V		
mem_int	::=	1 0	M M	memory integer value
$object_value$::=	$\begin{split} & mem_int \\ & mem_ptr \\ & \texttt{array}\left(\overline{loaded_value_i}^i\right) \\ & (\texttt{struct}ident)\{\overline{.member_i:}\tau_i = mem_val_i^{-i}\} \\ & (\texttt{union}ident)\{.member = mem_val\} \end{split}$		C object values (inhabitants of object types), which can be read/stored integer value pointer value C array value C struct value C union value
$loaded_value$::=	$\verb specified object_value $		potentially unspecified C object values specified loaded value
value	::= 	$object_value \ loaded_value \ $ Unit True False $eta[\overline{value_i}^i] \ (\overline{value_i}^i)$		Core values C object value loaded C object value unit boolean true boolean false list tuple
$bool_value$::=	True False		Core booleans boolean true boolean false
$ctor_val$::=	$\mathtt{Nil}\beta$		data constructors empty list

	Cons	list cons
	Tuple	tuple
	Array	C array
	Specified	non-unspecified loaded value
at an amon		data constructors
$ctor_expr$::= 	data constructors
	Ivmax	max integer value
	Ivmin	min integer value
	Ivsizeof	sizeof value
	Ivalignof	alignof value
	IvCOMPL	bitwise complement
	IVAND	bitwise AND
	IvOR	bitwise OR
	IvXOR	bitwise XOR
	Fvfromint	cast integer to floating value
	Ivfromfloat	cast floating to integer value
name	::=	
	ident	Core identifier
	$impl_const$	$implementation-defined\ constant$
pval	::=	pure values
1	$ident$	Core identifier
	$ impl_const $	implementation-defined constant
	value	Core values
	$ \texttt{constrained} (\overline{\textit{mem_iv_c}_i, \textit{pval}_i}^{ i})$	constrained value
	error $(string, pval)$	impl-defined static error
	$ ctor_val(\overline{pval_i}^i)$	data constructor application
	$ (structident) \{ \overline{.member_i = pval_i}^i \} $	C struct expression
	$ (union ident) \{ .member = pval \}$	C union expression
	(3 32

tpval	::= 	$\begin{array}{l} \texttt{undef} \;\; UB_name \\ \texttt{done} \; pval \end{array}$		top-level pure values undefined behaviour pure done
$ident_opt_eta$::= 	$_{dash}^{dash} \beta \ ident:eta$	$\begin{aligned} & \text{binders} = \{\} \\ & \text{binders} = ident \end{aligned}$	type annotated optional identifier
pattern	::= 	$ident_opt_eta \ ctor_val(\overline{pattern_i}^i)$	$\begin{aligned} & \text{binders} = \text{binders}(ident_opt_\beta) \\ & \text{binders} = \text{binders}(\overline{pattern_i}^i) \end{aligned}$	
z	::=	i mem_int $size_of(\tau)$ $offset_of_{tag}(member)$ ptr_size max_int_{τ} min_int_{τ}	M M M M M M	OCaml arbitrary-width integer literal integer size of a C type offset of a struct member size of a pointer maximum value of int of type τ minimum value of int of type τ
$\mathbb{Q},\ q,\ _{-}$::=	$rac{int_1}{int_2}$		OCaml type for rational numbers
lit	::=	$ident$ unit $bool$ z \mathbb{Q}		

```
ident\_or\_pattern
                                    ident
                                                                                  binders = ident
                                    pattern
                                                                                  binders = binders(pattern)
                                                                                                                         array property formulas
array\_prop
                                                                                  bind \overline{ident}_i^{\ i} in term_1
                                    \forall \overline{ident_i}^i . term_1 \rightarrow term_2
                                                                                  bind \overline{ident_i}^i in term_2
bool\_op
                                    \neg term
                                    term_1 = term_2term_1 \leftrightarrow term_2
                                                                                  Μ
                                    term_1 \rightarrow term_2
                                    \bigwedge(\overline{term_i}^i)
                                    \bigvee (\overline{term_i}^i)
                                    array\_prop
                                    term_1 \ binop_{bool} \ term_2
                                                                                  Μ
                                    if term_1 then term_2 else term_3
arith\_op
                                    term_1 + term_2
                                    term_1 - term_2
                                    term_1 \times term_2
                                    term_1/term_2
                                    term_1 \, {\tt rem\_t} \, term_2
                                    term_1 \, {\tt rem\_f} \, term_2
                                    term_1 ^ term_2
                                    term_1 binop_{arith} term_2
                                                                                  Μ
cmp\_op
                             ::=
                                    term_1 < term_2
                                                                                                                             less than
```

```
term_1 \leq term_2
                                                               less than or equal
                        term_1 \ binop_{rel} \ term_2
                                                       Μ
list\_op
                  ::=
                        nil
                        term_1 :: term_2
                        {\tt tl}\, term
                        term^{(int)}
tuple\_op
                  ::=
                         (\overline{term_i}^i)
                        term^{(int)}
pointer\_op
                        mem\_ptr
                        term_1 +_{ptr} term_2
                        cast_int_to_ptr term
                        {\tt cast\_ptr\_to\_int}\, term
array\_op
                        [|\overline{term_i}^i|]
                        term_1[term_2]
                        {\tt const}\, term
                        term_1[term_2] := term_3
param\_op
                        ident:\beta.\ term
                        term(term_1, .., term_n)
struct\_op
                        term.member \\
```

```
ct\_pred
                           representable (\tau, term)
                           aligned(\tau, term)
                           alignedI(term_1, term_2)
term, _, iguard
                           lit
                           arith\_op
                           bool\_op
                           cmp\_op
                           tuple\_op
                           struct\_op
                           pointer\_op
                           list\_op
                           array\_op
                           ct\_pred
                           param\_op
                           (term)
                                                                   S
                                                                          parentheses
                           \sigma(term)
                                                                   Μ
                                                                          simul-sub \sigma in term
                                                                   Μ
                           pval
                                                                        pure expressions
pexpr
                           pval
                                                                          pure values
                           ctor\_expr(\overline{pval_i}^i)
                                                                          data constructor application
                           array\_shift(pval_1, \tau, pval_2)
                                                                          pointer array shift
                           member\_shift(pval, ident, member)
                                                                          pointer struct/union member shift
                           not(pval)
                                                                          boolean not
                           pval_1 \ binop \ pval_2
                                                                          binary operations
                           memberof(ident, member, pval)
                                                                          C struct/union member access
                           name(\overline{pval_i}^i)
                                                                          pure function call
                           assert_undef (pval, UB_name)
```

	 	$\begin{aligned} &\texttt{bool_to_integer} \ (pval) \\ &\texttt{conv_int} \ (\tau, pval) \\ &\texttt{wrapI} \ (\tau, pval) \end{aligned}$		
tpexpr	::=	$tpval \\ \texttt{case} \ pval \ \texttt{of} \ \overline{\mid tpexpr_case_branch_i}^i \ \texttt{end} \\ \texttt{let} \ ident_or_pattern = pexpr \ \texttt{in} \ tpexpr \\ \texttt{let} \ ident_or_pattern: (y_1:\beta_1. \ term_1) = tpexpr_1 \ \texttt{in} \ tpexpr_2 \\ \texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2 \\ \sigma(tpexpr) \\ \end{cases}$	bind binders $(ident_or_pattern)$ in $tpexpr$ bind binders $(ident_or_pattern)$ in $tpexpr_2$ bind y_1 in $term_1$	top-level pure expressions top-level pure values pattern matching pure let annoted pure let pure if simul-sub σ in $tpexpr$
$tpexpr_case_branch$::=	$pattern \Rightarrow tpexpr$	bind $binders(pattern)$ in $tpexpr$	pure top-level case expression top-level case expression br
m_kill_kind	::= 	$\begin{array}{l} \operatorname{dynamic} \\ \operatorname{static} \tau \end{array}$		
bool, _	::= 	true false		OCaml booleans
$points_to, \ pt$::=	$term_1 \stackrel{init}{\mapsto}_{\tau} term_2$		points-to separation logic prec
$qpoints_to, \ qpt$::=	* $x. iguard; term_1 + x \times \text{size_of}(\tau) \stackrel{init}{\mapsto}_{\tau} term_2$		quantified (integer-indexed) p
res_term	::=			resource terms

	<pre>emp points_to qpoints_to ident $\langle res_term_1, res_term_2 \rangle$ pack (pval, res_term) fold (res_term) explode (res_term:pt) implode (res_term:qpt, int) break (res_term:qpt, int)</pre>		empty heap single-cell heap contiguous-cell heap variable seperating-conjunction pair packing for existentials fold into recursive res. pred. transform points-to-array into quantified points-to transform quantified points-to into points-to-array split a qpt into a qpt and a pt
į	$\mathtt{glue}\left(res_term_1{:}qpt, res_term_2{:}pt\right)$		join a qpt and a pt into a qpt
	$\sigma(res_term)$	M	substitution for resource terms
mem_action ::=	= $\mathtt{create}\left(pval, au ight)$		memory actions
	$ \begin{array}{l} \texttt{create_readonly} \ (pval_1, \tau, pval_2) \\ \texttt{alloc} \ (pval_1, pval_2) \\ \texttt{kill} \ (m_kill_kind, pval, pt) \\ \texttt{store} \ (bool, \tau, pval_1, pval_2, mem_order, pt) \\ \texttt{load} \ (\tau, pval, mem_order, pt) \\ \texttt{rmw} \ (\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_2) \\ \texttt{fence} \ (mem_order) \\ \texttt{cmp_exch_strong} \ (\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_2) \\ \texttt{cmp_exch_weak} \ (\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_2) \\ \texttt{linux_fence} \ (linux_mem_order) \\ \texttt{linux_load} \ (\tau, pval, linux_mem_order) \\ \texttt{linux_store} \ (\tau, pval_1, pval_2, linux_mem_order) \\ \texttt{linux_rmw} \ (\tau, pval_1, pval_2, linux_mem_order) \\ \texttt{linux_rmw} \ (\tau, pval_1, pval_2, linux_mem_order) \\ \end{aligned} $		true means store is locking
polarity ::=	=		polarities for memory actions (pos) sequenced by let weak and let strong

		neg		only sequenced by let strong
pol_mem_action	::=	$polarity\ mem_action$		memory actions with polarity
mem_op	::= 	$\begin{array}{l} pval_1 \; binop_{rel} \; pval_2 \\ pval_1{\tau} \; pval_2 \\ \text{intFromPtr} \; (\tau_1, \tau_2, pval) \\ \text{ptrFromInt} \; (\tau_1, \tau_2, pval) \\ \text{ptrValidForDeref} \; (\tau, pval, pt) \\ \text{ptrWellAligned} \; (\tau, pval) \\ \text{ptrArrayShift} \; (pval_1, \tau, pval_2) \\ \text{memcpy} \; (pval_1, pval_2, pval_3) \\ \text{memcmp} \; (pval_1, pval_2, pval_3) \\ \text{realloc} \; (pval_1, pval_2, pval_3) \\ \text{va_start} \; (pval_1, pval_2) \\ \text{va_copy} \; (pval) \\ \text{va_arg} \; (pval, \tau) \\ \text{va_end} \; (pval) \end{array}$		operations involving the memory state pointer relational binary operations pointer subtraction cast of pointer value to integer value cast of integer value to pointer value dereferencing validity predicate
$spine_elem$::= 	$egin{aligned} pval \ res_term \ \sigma(spine_elem) \end{aligned}$	М	spine element pure or logical value resource value substitution for spine elements / return values
spine	::=	$\overline{spine_elem_i}^{\ i}$		spine
tval	::=	$\mathtt{done}spine$		(effectful) top-level values end of top-level expression

		undef UB_name		undefined behaviour
$res_pattern$::=	$\begin{array}{l} \texttt{emp} \\ ident \\ \texttt{fold} \left(res_pattern\right) \\ \left\langle res_pattern_1, res_pattern_2 \right\rangle \\ \texttt{pack} \left(ident, res_pattern\right) \end{array}$	binders = $\{\}$ binders = $ident$ binders = $\{\}$ binders = binders($res_pattern_1$) \cup binders($res_pattern_2$) binders = $ident \cup binders(res_pattern)$	resource terms empty heap variable unfold (recursive) predicate seperating-conjunction pair packing for existentials
$ret_pattern$::= 	comp $ident_or_pattern$ $log ident$ $res res_pattern$	$\begin{aligned} & \text{binders} = \text{binders}(ident_or_pattern) \\ & \text{binders} = ident \\ & \text{binders} = \text{binders}(res_pattern) \end{aligned}$	return pattern computational variable logical variable resource variable
init,	::= 	✓ ×		initialisation status initialised uninitalised
res		emp $points_to$ $qpoints_to$ $res_1 * res_2$ $\exists ident: \beta. res$ $term \land res$ if $term$ then res_1 else res_2 $\alpha(\overrightarrow{pval_i}^i)$ $\sigma(res)$	M	resources empty heap points-to heap pred. quantified (integer-indexed) points-to heap pred. seperating conjunction existential logical conjuction ordered disjuction predicate simul-sub σ in res
$ret, _$::=	$\Sigma ident: \beta. \ ret$		return types return a computational value

		$\exists ident: \beta. \ ret$ $res \otimes ret$ $term \wedge ret$ I $\sigma(ret)$	M	return a logical value return a resource value return a predicate (post-condition end return list simul-sub σ in ret
seq_expr	::= 	$ exttt{ccall}(au, ident, spine) \\ exttt{pcall}(name, spine)$		sequential (effectful) expressions C function call procedure call
seq_texpr	::=	$tval \\ \operatorname{run} ident \overline{pval_i}^i \\ \operatorname{let} ident_or_pattern = pexpr \operatorname{in} texpr \\ \operatorname{let} ident_or_pattern: (y_1:\beta_1.\ term_1) = tpexpr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i = seq_expr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i : ret = texpr_1 \operatorname{in} texpr_2 \\ \operatorname{case} pval \operatorname{of} \overline{\mid texpr_case_branch_i}^i \operatorname{end} \\ \operatorname{if} pval \operatorname{then} texpr_1 \operatorname{else} texpr_2 \\ \operatorname{bound} [int] (is_texpr) \\ \end{aligned}$	bind binders($ident_or_pattern$) in $texpr$ bind binders($ident_or_pattern$) in $texpr$ bind y_1 in $term_1$ bind binders($\overline{ret_pattern_i}^i$) in $texpr$ bind binders($\overline{ret_pattern_i}^i$) in $texpr_2$	sequential top-level (effectful) expres (effectful) top-level values run from label pure let annotated pure let bind return patterns annotated bind return patterns pattern matching conditional limit scope of indet seq behaviour
$texpr_case_branch$::=	$pattern \Rightarrow texpr$	bind $binders(pattern)$ in $texpr$	top-level case expression branch top-level case expression branch
is_expr	::=	$tval \\ memop (mem_op) \\ pol_mem_action$		indet seq (effectful) expressions (effectful) top-level values pointer op involving memory memory action
is_texpr	::=			indet seq top-level (effectful) express

		$\begin{array}{l} {\tt letweak}\overline{ret_pattern_i}^{i} = is_expr{\tt in}texpr\\ {\tt letstrong}\overline{ret_pattern_i}^{i} = is_expr{\tt in}texpr \end{array}$	bind binders($\overline{ret_pattern_i}^i$) in $texpr$ bind binders($\overline{ret_pattern_i}^i$) in $texpr$	weak sequencing strong sequencing
texpr	::= 	seq_texpr is_texpr $\sigma(texpr)$	M	top-level (effectful) expressions sequential (effectful) expressions indet seq (effectful) expressions simul-sub σ in $texpr$
arg	::=	$\Pi ident:\beta. \ arg$ $\forall ident:\beta. \ arg$ $res \multimap arg$ $term \supset arg$ ret $\sigma(arg)$	M	argument/function types ${\rm simul\text{-}sub}\ \sigma\ {\rm in}\ arg$
$pure_arg$::= 	$\Pi ident:\beta. \ pure_arg$ $term \supset pure_arg$ $pure_ret$		pure argument/function types
$pure_ret$::= 	$\Sigma ident: \beta. \ pure_ret \ term \land pure_ret$ I		pure return types
С	::= 	. $\frac{\mathcal{C}, ident: eta}{\overline{\mathcal{C}_i}^i}$		computational var env

```
logical var env
Φ
                                                                                                                                                                                    constraints env
                                                \begin{array}{l} \cdot \\ \underline{\Phi, term} \\ \overline{\Phi_i}^i \end{array}
\mathcal{R}
                                                                                                                                                                                    resources env
                                  \left| \begin{array}{c} \mathcal{R}, ident:res \\ \overline{\mathcal{R}_i}^i \end{array} \right|
\sigma, \psi
                                                                                                                                                                                    substitutions
                                apply \sigma to all elements in \psi
                                                                                                                                                                       М
typing
                                                 \mathtt{smt}\left(\Phi\Rightarrow term\right)
                                         ident:\beta \in \mathcal{C}
                                  | ident: \beta \in \mathcal{C} 
| ident: \beta \in \mathcal{L} 
| struct tag \& \overline{member_i : \tau_i}^i \in Globals 
| \alpha \equiv \overline{x_i : \beta_i}^i \mapsto res \in Globals 
| \overline{C_i; \mathcal{L}_i; \Phi_i \vdash mem\_val_i \Rightarrow mem \beta_i}^i 
| \overline{C_j; \mathcal{L}_j} \mid \overline{ident_{ij}}^i \vdash guarded(term_j)^j 
| \overline{C_j; \mathcal{L}_j} \mid \overline{ident_{ij}}^i \vdash vconstr(term_j)^j 
                                                                                                                                                                                           recursive resource predicate
                                                                                                                                                                                           dependent on memory object model
```

```
ident \in \mathcal{C}; \mathcal{L}
ident \in \overline{ident_i}^i
opsem
                           \forall i < j. \ \mathsf{not} \left( pattern_i = pval \leadsto \sigma_i \right)
                           fresh(mem\_ptr)
                           term
                           pval:\beta
formula
                           judgement
                            typing
                           opsem
                       res \equiv res'
                      heap, h, f
                                                                                        heaps
                         h + \{points\_to\}
h + f
                                                                                        [O] convenient for the soundness proof
wf_jtyp
                      | \quad \mathcal{C}; \mathcal{L} \vdash \mathsf{guarded\_e}(term)|
                      lemma\_jtype
                    ::= | \overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret
```

```
\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'
                                                             \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}')
res\_jtype
                                                             \Phi \vdash res \equiv res'
                                                             C; \mathcal{L}; \Phi; \mathcal{R} \vdash res\_term \Leftarrow res
                                                              h:\mathcal{R}
object\_value\_jtype
                                                             C; \mathcal{L}; \Phi \vdash object\_value \Rightarrow \mathsf{obj}\,\beta
pval\_jtype
                                                  ::=
                                                             C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta
spine\_jtype
                                                  ::=
                                                             C; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret
pexpr\_jtype
                                                  ::=
                                                             C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term
comp\_pattern\_jtype
                                                  ::=
                                                              pattern: \beta \leadsto \mathcal{C} \text{ with } term
                                                              ident\_or\_pattern: \beta \leadsto \mathcal{C} \text{ with } term
res\_pattern\_jtype
                                                              \Phi \vdash res' = \mathtt{strip\_ifs}(res)
                                                              \Phi \vdash res \text{ as } res\_pattern \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'
                                                              \Phi \vdash res\_pattern:res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'
ret\_pattern\_jtype
                                                              \Phi \vdash \overline{ret\_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'
```

$$tpval_jtype$$
 ::= $| \mathcal{C}; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term$

$$\begin{array}{ll} \textit{tpexpr_jtype} & ::= \\ & | \quad \mathcal{C}; \mathcal{L}; \Phi \vdash \textit{tpexpr} \Leftarrow \textit{ident}: \beta. \textit{term} \end{array}$$

$$\begin{array}{ll} \textit{action_jtype} & ::= \\ & | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \textit{mem_action} \Rightarrow \textit{ret} \end{array}$$

$$\begin{array}{ll} \textit{memop_jtype} & ::= \\ & | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \textit{mem_op} \Rightarrow \textit{ret} \end{array}$$

$$tval_jtype$$
 ::= $| \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$

$$\begin{array}{ll} seq_expr_jtype & ::= \\ & | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret \end{array}$$

$$is_expr_jtype ::= \ | \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$$

$$texpr_jtype \qquad ::= \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C}; \mathcal{C} \vdash texpr \Leftrightarrow ret \\ | \quad \mathcal{C} \vdash texpr \Leftrightarrow ret$$

$$subs_jtype \qquad ::= \\ | pattern = pval \leadsto \sigma \\ | ident_or_pattern = pval \leadsto \sigma \\ | res_pattern = res_term \leadsto \sigma \\ | ret_pattern_i = spine_elem_i^i \leadsto \sigma$$

```
\overline{x_i = spine\_elem_i}^i :: arq \gg \sigma; ret
 pure\_opsem\_jtype
                                                            opsem\_jtype
                                                             \langle h; seq\_expr \rangle \longrightarrow \langle h'; texpr:ret \rangle
                                                            \langle h; seq\_texpr \rangle \longrightarrow \langle h'; texpr \rangle
                                                            \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                           \langle h; mem\_action \rangle \longrightarrow \langle h'; tval \rangle\langle h; is\_expr \rangle \longrightarrow \langle h'; is\_expr' \rangle
                                                            \langle h; is\_texpr \rangle \longrightarrow \langle h'; texpr \rangle
                                                            \langle h; texpr \rangle \longrightarrow \langle h'; texpr' \rangle
\mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}(term)
                                                                                                 \frac{ident \in \mathcal{C}; \mathcal{L}}{\mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(ident\right)} \quad \text{Wf\_GUARDED\_EEXPR\_EVAR}
                                                                                   \frac{ident \in \mathcal{C}; \mathcal{L}}{\mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(z \times ident\right)}
                                                                                                                                                       Wf_Guarded_Eexpr_Scaled_EVar
                                                                                                  \mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}(term_1)
                                                                                                 \mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(term_2\right)
                                                                                                                                                                        WF_GUARDED_EEXPR_PLUS
                                                                                        \overline{\mathcal{C};\mathcal{L}} \vdash \text{guarded\_e}\left(term_1 + term_2\right)
```

$$\boxed{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded}(term)}$$

 $\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded}(term)$

$$\begin{array}{c|c} \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term) \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term') \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term \leqslant term') \end{array} \\ \text{WF_GUARDED_LEQ}$$

$$\begin{array}{c|c} \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term) \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term') \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term = term') \end{array} \\ \end{array} \\ \text{WF_GUARDED_EQ}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{guarded}\left(term_j\right)^j}}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{guarded}\left(\bigvee(\overline{term_j}^j\right))} \quad \text{Wf_GUARDED_OR}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{guarded} (term_j)^j}}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{guarded} (\bigwedge (\overline{term_j}^j))} \quad \text{WF_GUARDED_AND}$$

$$\frac{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (term)}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} \, (\neg \, term)} \quad \text{Wf_Guarded_Neg}$$

 $|\mathcal{C}; \mathcal{L} \vdash \mathtt{well_formed}(array_prop)|$

$$\begin{array}{c|c} \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{guarded} \, (term_1) \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr} \, (term_2) \\ \hline \mathcal{C}; \mathcal{L} \vdash \mathsf{well_formed} \, (\forall \overline{ident_i}^i \, . \, term_1 \rightarrow term_2) \end{array} \quad \text{WF_ARRAY_PROP_BASE}$$

 $\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret$

$$\frac{}{::ret \leadsto :; :; :; \cdot \mid ret}$$
 Arg_Env_Ret

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \Pi \, x : \beta. \, arg \leadsto \mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Comp}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \forall x : \beta. arg \leadsto \mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Log}$$

$$\frac{\overline{x_i}^{\;i} :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{\overline{x_i}^{\;i} :: term \supset arg \leadsto \mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \mid ret} \quad \text{Arg_Env_Phi}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: res \multimap arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x:res \mid ret} \quad \text{Arg_Env_Res}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq C'; \mathcal{L}'; \Phi'; \mathcal{R}'$$

$$\frac{}{\cdot;\cdot;\cdot;\cdot\sqsubseteq\cdot;\cdot;\cdot;}\quad \text{Weak_Empty}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x : \beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}, x:\beta; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x:\beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Cons_Phi}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x : res \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res} \quad \text{Weak_Cons_Res}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x:\beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Skip_Phi}$$

$$\boxed{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma){:}(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')}$$

$$\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash (\cdot) : (\cdot; \cdot; \cdot; \cdot)$$
 TY_SUBS_EMPTY

$$\begin{array}{c} \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta\\ \hline \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}',x:\beta;\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}';\mathcal{L}',x:\beta;\Phi';\mathcal{R}')\\ \hline \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}';\mathcal{L}',x:\beta;\Phi';\mathcal{R}')\\ \hline \frac{smt}{\mathcal{C}};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \frac{smt}{\mathcal{C}};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(r\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(r\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')\\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(res_term/x,\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}',x:res)\\ \hline \hline \mathcal{T}\text{Y_SUBS_CONS_RES}\\ \hline \hline \hline \Phi\vdash \text{emp}\equiv\text{emp} & \text{TY_RES_EQ_EMP} \\ \hline \end{array}$$

 $\Phi \vdash res \equiv res'$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(term_{1}=term_{1}'\right)\wedge\left(term_{2}=term_{2}'\right)\right)}{\Phi\vdash term_{1}(q)\overset{init}{\mapsto}_{\tau}term_{2}\equiv\ term_{1}'(q)\overset{init}{\mapsto}_{\tau}term_{2}'} \quad \text{Ty_Res_Eq_PointsTo}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(iguard\leftrightarrow iguard'\right)\wedge\left(term_{1}=term_{1}'\right)\wedge\left(term_{2}=term_{2}'\right)\right)}{\Phi\vdash *x.\ iguard; term_{1}+x\times\operatorname{size_of}(\tau)\overset{init}{\mapsto_{\tau}}term_{2}\equiv *x.\ iguard'; term_{1}'+x\times\operatorname{size_of}(\tau)\overset{init}{\mapsto_{\tau}}term_{2}'}$$
 TY_RES_EQ_QPOINTSTO

$$\begin{array}{cccc} \Phi \vdash res_1 \equiv res_1' \\ \Phi \vdash res_2 \equiv res_2' \\ \hline \Phi \vdash res_1 * res_2 \equiv res_1' * res_2' \end{array} \text{Ty_Res_Eq_SepConj}$$

$$\frac{\Phi \vdash res \equiv res'}{\Phi \vdash 3ident:\beta. \ res} = \exists ident:\beta. \ res'} \quad \text{Ty.Res.Eq.Exists}$$

$$\frac{\text{smt} (\Phi \Rightarrow term \leftrightarrow term')}{\Phi \vdash res \equiv res'} \quad \frac{\Phi \vdash res \equiv res'}{\Phi \vdash term \land res} \quad \text{Ty.Res.Eq.Term}$$

$$\frac{\text{smt} (\Phi \Rightarrow term_1 \leftrightarrow term_2)}{\Phi \vdash term_1 \Rightarrow res_{21}} \quad \frac{\Phi \vdash res_{11} \equiv res_{21}}{\Phi \vdash res_{21} \equiv res_{22}} \quad \frac{\Phi \vdash res_{21} \equiv res_{22}}{\Phi \vdash \text{if } term_1 \text{ then } res_{11} \text{ else } res_{12} \equiv \text{if } term_2 \text{ then } res_{21} \text{ else } res_{22}} \quad \text{Ty.Res.Eq.OrdDisj}$$

$$\frac{\Phi \vdash \alpha(\overline{pval_i}^i) \equiv \alpha(\overline{pval_i}^i)}{\Phi \vdash \alpha(\overline{pval_i}^i) \equiv \alpha(\overline{pval_i}^i)} \quad \text{Ty.Res.Eq.Pred}$$

$$\frac{\Phi \vdash points.to}{\Phi \vdash points.to' \equiv points.to'} \quad \text{Ty.Res.Emp}$$

$$\frac{\Phi \vdash points.to' \equiv points.to'}{\Phi \vdash points.to' \equiv points.to'} \quad \text{Ty.Res.PointsTo}$$

$$\frac{\Phi \vdash qpoints.to \equiv qpoints.to'}{\Phi \vdash qpoints.to' \equiv qpoints.to'} \quad \text{Ty.Res.QPointsTo}$$

$$\frac{\Phi \vdash res_1' \equiv res_2'}{C:\mathcal{L};\Phi; \cdot, :qpoints.to \vdash qpoints.to' \leftarrow qpoints.to''} \quad \text{Ty.Res.QPointsTo}$$

$$\frac{\Phi \vdash res_1' \equiv strip.ifs(res_1)}{\Phi \vdash res_1' \equiv res_2'} \quad \text{Ty.Res.Var}$$

$$C; \mathcal{L}; \mathfrak{S}; \mathcal{R}_1 \vdash res_term_1 \Leftarrow res_1$$

$$C; \mathcal{L}; \mathfrak{S}; \mathcal{R}_2 \vdash res_term_2 \Leftarrow res_2$$

$$\overline{C; \mathcal{L}; \mathfrak{S}; \mathcal{R}_1, \mathcal{R}_2 \vdash \langle res_term_1, res_term_2 \rangle} = res_1 * res_2$$

$$\operatorname{smt} (\Phi \Rightarrow term)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term \Leftarrow res$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftarrow term \land res$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftarrow term \land res$$

$$C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$$

$$C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$$

$$C; \mathcal{L}; \Phi \vdash \mathcal{R}_1 \vdash res_term \Leftarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$\overline{C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term} \Leftrightarrow pval/y, \cdot (res)$$

$$C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res$$

```
qpt \equiv *x. iguard; term_1 + x \times size\_of(\tau) \xrightarrow{init}_{\tau} term_2
     qpt' \equiv *x. iguard'; term_1 + x \times size\_of(\tau) \stackrel{init}{\mapsto}_{\tau} term_2
    pt \equiv term_1 + i \times size\_of(\tau) \stackrel{init}{\mapsto}_{\tau} i/x, \cdot (term_2)
    \operatorname{smt}(\Phi \Rightarrow (iguard \land (x != i)) \leftrightarrow iguard')
    C; \mathcal{L}; \Phi; \mathcal{R} \vdash res\_term \Leftarrow qpt
           \mathcal{L}; \Phi; \mathcal{R} \vdash res\_term \Leftarrow qpt
\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash break(res\_term:qpt, i) \Leftarrow qpt' * pt
TY\_RES\_BREAK
      qpt' \equiv *x. iguard'; term_1 + x \times size\_of(\tau) \stackrel{init}{\mapsto}_{\tau} term_2'
     pt \equiv term_1'' \stackrel{init}{\mapsto}_{\tau} term_2''
      i \equiv (term_1'' - term_1)/size\_of(\tau)
      qpt \equiv *x. iguard; term_1 + x \times \text{size\_of}(\tau) \stackrel{init}{\mapsto}_{\tau} term_2
       \operatorname{smt}(\Phi \Rightarrow term_2 = (\operatorname{if} x = i \operatorname{then} term_2'' \operatorname{else} term_2'))
       smt(\Phi \Rightarrow iquard \leftrightarrow (iquard' \lor x = i))
      C; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res\_term_1 \Leftarrow qpt'
     C; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash res\_term_2 \Leftarrow pt
\frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1,\mathcal{R}_2 \vdash \mathsf{glue}\left(\mathit{res\_term}_1:\mathit{qpt'},\mathit{res\_term}_2:\mathit{pt}\right) \Leftarrow \mathit{qpt}}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1,\mathcal{R}_2 \vdash \mathsf{glue}\left(\mathit{res\_term}_1:\mathit{qpt'},\mathit{res\_term}_2:\mathit{pt}\right) \Leftarrow \mathit{qpt}}
                                                                                                                                            Ty_Res_Glue
```

 $h:\mathcal{R}$

$$\frac{h:\mathcal{R}}{\frac{\cdot;\cdot;\cdot;\mathcal{R}' \vdash pt \Leftarrow pt}{h + \{pt\}:\mathcal{R},\mathcal{R}'}} \quad \text{TY_HEAP_POINTSTO}$$

 $\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathsf{obj}\,\beta$

 $\overline{\mathcal{C};\mathcal{L};\Phi \vdash mem_int} \Rightarrow \mathtt{obj}\,\mathtt{integer} \quad \mathsf{TY_PVAL_OBJ_INT}$

$$\frac{}{\mathcal{C};\mathcal{L};\Phi \vdash mem_ptr \Rightarrow \mathtt{objloc}} \quad \text{TY_PVAL_OBJ_PTR}$$

$$\frac{\overline{\mathcal{C};\mathcal{L};\Phi \vdash loaded_value_i \Rightarrow \beta}^i}{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{array}\left(\overline{loaded_value_i}^i\right) \Rightarrow \mathtt{obj}\,\mathtt{array}\,\beta} \quad \mathtt{TY_PVAL_OBJ_ARR}$$

$$\frac{\text{struct} \, tag \, \& \, \overline{member_i : \tau_i}^{\, i} \, \in \, \text{Globals}}{\overline{\mathcal{C}}; \mathcal{L}; \Phi \vdash mem_val_i \Rightarrow \, \text{mem} \, \beta_{\tau_i}^{\, i}}}{\mathcal{C}; \mathcal{L}; \Phi \vdash (\, \text{struct} \, tag) \{\, \overline{. \, member_i : \tau_i = \, mem_val_i}^{\, i} \, \} \Rightarrow \, \text{obj struct} \, tag} \quad \text{Ty_PVAL_OBJ_STRUCT}}$$

 $C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$

$$\frac{x:\beta \in \mathcal{C}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{TY_PVAL_VAR_COMP}$$

$$\frac{x:\beta \in \mathcal{L}}{\mathcal{C};\mathcal{L};\Phi \vdash x \Rightarrow \beta} \quad \text{TY_PVAL_VAR_LOG}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathsf{obj}\,\beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \beta} \quad \text{TY_PVAL_OBJ}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathtt{obj}\,\beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{specified}\,object_value \Rightarrow \beta} \quad \mathsf{TY_PVAL_LOADED}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{Unit} \Rightarrow \mathtt{unit}} \quad \mathrm{TY_PVAL_UNIT}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{True} \Rightarrow \mathtt{bool}} \quad \mathtt{TY_PVAL_TRUE}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi\vdash \mathtt{False}\Rightarrow\mathtt{bool}}\quad \mathtt{TY_PVAL_FALSE}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash value_i \Rightarrow \beta}^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash \beta[\overline{value_i}^i] \Rightarrow \mathtt{list}\,\beta} \quad \mathsf{TY_PVAL_LIST}$$

$$\frac{\overline{C; \mathcal{L}; \Phi \vdash value_i \Rightarrow \beta_i}^i}{C; \mathcal{L}; \Phi \vdash (\overline{value_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \text{TY_PVAL_TUPLE}$$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{error}\,(string,pval)\Rightarrow\beta}\quad \mathsf{TY_PVAL_ERROR}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{Nil}\,\beta(\,) \Rightarrow \mathtt{list}\,\beta}$$
 TY_PVAL_CTOR_NIL

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \beta \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{list}\,\beta \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{Cons}(pval_1, pval_2) \Rightarrow \mathtt{list}\,\beta \end{array} \quad \text{TY_PVAL_CTOR_CONS}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta_i}^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Tuple}(\overline{pval_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \text{TY_PVAL_CTOR_TUPLE}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta}^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash \operatorname{Array}(\overline{pval_i}^i) \Rightarrow \operatorname{array} \beta} \quad \text{TY_PVAL_CTOR_ARRAY}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathsf{Specified}(pval) \Rightarrow \beta} \quad \mathsf{TY_PVAL_CTOR_SPECIFIED}$$

$$\frac{\texttt{struct} \, tag \, \& \, \overline{member_i : \tau_i}^{\ i} \in \texttt{Globals}}{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta_{\tau_i}^{\ i}}} \\ \frac{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta_{\tau_i}^{\ i}}}{\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash (\texttt{struct} \, tag)\{\overline{.member_i = pval_i}^{\ i}\}} \Rightarrow \texttt{struct} \, tag} \\ \text{TY_PVAL_STRUCT}$$

$$\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash ::\! \mathit{ret} \, \gg \cdot; \mathit{ret}} \quad \text{Ty_Spine_Empty}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \Pi x: \beta. arg \gg pval/x, \sigma; ret \end{array} \quad \text{TY_SPINE_COMP}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret} \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \forall x : \beta. arg \gg pval/x, \sigma; ret}$$
 TY_SPINE_LOG

$$\begin{aligned} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term \Leftarrow res \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\ & \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash x = res_term, \overline{x_i = spine_elem_i}^i :: res \multimap arg \gg res_term/x, \sigma; ret} \end{aligned}$$
 Ty_Spine_Res

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow term\right)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_{i}=spine_elem_{i}}^{i}::arg\gg\sigma;ret} \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_{i}=spine_elem_{i}}^{i}::term\supset arg\gg\sigma;ret} \\ \text{TY_Spine_Phi}$$

 $C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow y: \beta. \ y = pval} \quad \text{TY_PE_VAL}$$

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\mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathsf{loc}
                                                                      \mathcal{C}: \mathcal{L}: \Phi \vdash pval_2 \Rightarrow \text{integer}
                                                                                                                                                                                                   TY_PE_ARRAY_SHIFT
          \overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{array\_shift}\left(pval_1,\tau,pval_2\right) \Rightarrow y\mathtt{:loc.}\ y = pval_1 +_{\mathtt{ptr}}\left(pval_2 \times \mathtt{size\_of}(\tau)\right)}
                                                        C; \mathcal{L}; \Phi \vdash pval \Rightarrow loc
                                                        \operatorname{struct} tag \ \& \ \overline{member_i {:} 	au_i}^i \in \operatorname{Globals}
                                                                                                                                                                                                         TY_PE_MEMBER_SHIFT
\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{member\_shift}(pval, tag, member_i)} \Rightarrow y:\mathtt{loc}.\ y = pval +_{\mathtt{ptr}} \mathtt{offset\_of}_{tag}(member_i)
                                                                \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{not} (pval) \Rightarrow y \texttt{:bool}. \ y = \neg pval} \quad \text{TY\_PE\_NOT}
                                                                      C; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow integer
                                                                      C; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \text{integer}
                                                                                                                                                                                         TY_PE_ARITH_BINOP
                    \overline{\mathcal{C};\mathcal{L};\Phi \vdash pval_1 \ binop_{arith} \ pval_2 \Rightarrow y\text{:integer.} \ y = (pval_1 \ binop_{arith} \ pval_2)}
                                                                        \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathtt{integer}
                                                                        \mathcal{C}: \mathcal{L}: \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                              \frac{\mathcal{C}, \mathcal{L}, \text{ } r \mid pval_2 \Rightarrow \text{ integer}}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{rel} \ pval_2 \Rightarrow y \text{:bool.} \ y = (pval_1 \ binop_{rel} \ pval_2)}
                                                                                                                                                                                    TY_PE_REL_BINOP
                                                                           \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow bool
                                                                          \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow bool
                                                                                                                                                                                    TY_PE_BOOL_BINOP
                          \overline{\mathcal{C};\mathcal{L};\Phi\vdash pval_1\ binop_{bool}\ pval_2\Rightarrow y\text{:bool.}\ y=(pval_1\ binop_{bool}\ pval_2)}
                                                 name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in Globals
                                                \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma y: \beta. term \wedge I
                                                                                                                                                                              TY PE CALL
                                                                 C; \mathcal{L}; \Phi \vdash name(\overline{pval_i}^i) \Rightarrow y:\beta. \ \sigma(term)
```

$$\frac{\overline{pattern_i:\beta} \leadsto \overline{C_i \text{ with } term_i}^i}{\operatorname{Array}(\overline{pattern_i}^i) : \operatorname{array}\beta \leadsto \overline{\overline{C_i}}^i \text{ with } [|\overline{term_i}^i|]} \quad \operatorname{TY_PAT_COMP_ARRAY}$$

$$\frac{pattern: \beta \leadsto \mathcal{C} \, \mathtt{with} \, term}{\mathtt{Specified}(pattern): \beta \leadsto \mathcal{C} \, \mathtt{with} \, term} \quad \mathtt{TY_PAT_COMP_SPECIFIED}$$

 $ident_or_pattern:eta\leadsto\mathcal{C}$ with term

$$\frac{}{x:\beta\leadsto\cdot,x:\beta\text{ with }x}\quad\text{TY_PAT_SYM_OR_PATTERN_SYM}$$

$$\frac{pattern: \beta \leadsto \mathcal{C} \, \text{with} \, term}{pattern: \beta \leadsto \mathcal{C} \, \text{with} \, term} \quad \text{Ty_Pat_Sym_Or_Pattern_Pattern}$$

$$\Phi \vdash res' = \mathtt{strip_ifs}(res)$$

$$\frac{}{\Phi \vdash \texttt{emp} = \texttt{strip_ifs}(\texttt{emp})} \quad \text{TY_PAT_RES_STRIPIFS_EMPTY}$$

$$\overline{\Phi \vdash pt = \mathtt{strip_ifs}\left(pt\right)} \quad \text{Ty_Pat_Res_StripIfs_PointsTo}$$

$$\overline{\Phi \vdash qpt = \mathtt{strip_ifs}\left(qpt\right)} \quad \text{Ty_Pat_Res_StripIfs_QpointsTo}$$

$$\frac{}{\Phi \vdash res_1 * res_2 = \mathtt{strip_ifs}(res_1 * res_2)} \quad \text{Ty_Pat_Res_StripIfs_SepConj}$$

$$\Phi \vdash \exists x:\beta. res = strip_ifs(\exists x:\beta. res)$$
 TY_PAT_RES_STRIPIFS_EXISTS

 $\overline{\Phi \vdash term \land res = \mathtt{strip_ifs} \, (term \land res)} \quad \text{Ty_Pat_Res_StripIfs_TermConj}$ $\operatorname{smt}(\Phi \Rightarrow term)$ $\frac{\Phi \vdash res_1' = \text{strip_ifs}(res_1')}{\Phi \vdash res_1' = \text{strip_ifs}(\text{if} term \text{ then} res_1 \text{ else} res_2)} \quad \text{Ty_Pat_Res_StripIfs_True}$ $\operatorname{smt} (\Phi \Rightarrow \neg \operatorname{term})$ $\frac{\Phi \vdash res'_2 = \mathtt{strip_ifs}(\mathit{res}_2)}{\Phi \vdash \mathit{res}'_2 = \mathtt{strip_ifs}(\mathit{if}\,\mathit{term}\,\mathsf{then}\,\mathit{res}_1\,\mathsf{else}\,\mathit{res}_2)} \quad \mathsf{Ty_Pat_Res_StripIfs_False}$ $\overline{\Phi \vdash \texttt{if} \, term \, \texttt{then} \, res_1 \, \texttt{else} \, res_2 = \, \texttt{strip_ifs} \, \big(\texttt{if} \, term \, \texttt{then} \, res_1 \, \texttt{else} \, res_2 \big)}$ Ty_Pat_Res_StripIfs_UnderDet $\frac{1}{\Phi \vdash \alpha(\overline{pval_i}^i) = \text{strip_ifs}(\alpha(\overline{pval_i}^i))} \quad \text{Ty_Pat_Res_StripIfs_Pred}$ $\Phi \vdash \mathit{res} \; \mathsf{as} \; \mathit{res_pattern} \; \leadsto \; \mathcal{L}'; \Phi'; \mathcal{R}'$ $\Phi \vdash \text{emp as emp} \leadsto :; :; \cdot$ TY_PAT_RES_MATCH_EMPTY $\overline{\Phi \vdash \mathit{res} \; \mathsf{as} \; r \leadsto \cdot; \cdot; \cdot, r {:} \mathit{res}} \quad \mathsf{TY_PAT_RES_MATCH_VAR}$ $\Phi \vdash res_pattern_1: res_1 \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1$ $\frac{\Phi \vdash \mathit{res_pattern}_2 : \mathit{res}_2 \leadsto \mathcal{L}_2 ; \Phi_2 ; \mathcal{R}_2}{\Phi \vdash \mathit{res}_1 * \mathit{res}_2 \mathsf{as} \langle \mathit{res_pattern}_1, \mathit{res_pattern}_2 \rangle \leadsto \mathcal{L}_1, \mathcal{L}_2 ; \Phi_1, \Phi_2 ; \mathcal{R}_1, \mathcal{R}_2} \quad \mathsf{TY_PAT_RES_MATCH_SEPCONJ}$ $\frac{\Phi \vdash \mathit{res_pattern} : \mathit{res} \leadsto \mathcal{L}' ; \Phi' ; \mathcal{R}'}{\Phi \vdash \mathit{term} \land \mathit{res} \ \mathsf{as} \ \mathit{res_pattern} \leadsto \mathcal{L}' ; \Phi' , \mathit{term} ; \mathcal{R}'} \quad \mathsf{TY_PAT_RES_MATCH_CONJ}$

$$\frac{\Phi \vdash res_pattern: x/y, \cdot (res) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \exists \ y:\beta. \ res \ \text{as pack} \ (x, res_pattern) \leadsto \mathcal{L}', x:\beta; \Phi'; \mathcal{R}'} \quad \text{Ty_Pat_Res_Match_Pack}$$

$$\alpha \equiv \overline{x_i : \beta_i}^i \mapsto res \in \mathsf{Globals}$$

$$\frac{\Phi \vdash res_pattern : \overline{pval_i/x_i}, \cdot^i(res) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \alpha(\overline{pval_i}^i) \text{ as fold } (res_pattern) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Ty_Pat_Res_Match_Fold}$$

 $\Phi \vdash res_pattern:res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\frac{\Phi \vdash res' = \text{strip_ifs} (res)}{\Phi \vdash res' \text{ as } res_pattern \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}$$

$$\frac{\Phi \vdash res_pattern: res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash res_pattern: res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}$$

$$\text{TY_PAT_RES_STRIP_IFS}$$

 $\Phi \vdash \overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\frac{}{\Phi \vdash : \texttt{I} \leadsto \cdot ; \cdot ; \cdot ; \cdot} \quad \text{TY_PAT_RET_EMPTY}$$

$$\frac{ident_or_pattern:\beta \leadsto \mathcal{C}_1 \, \text{with} \, term_1}{\Phi \vdash \overline{ret_pattern_i}^{\,\,i}: term_1/y, \cdot (ret) \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{\Phi \vdash \mathsf{comp} \, ident_or_pattern, \, \overline{ret_pattern_i}^{\,\,i}: \Sigma \, y:\beta. \, ret \leadsto \mathcal{C}_1, \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \text{Ty_Pat_Ret_Comp}$$

$$\frac{\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \log y, \ \overline{\mathit{ret_pattern}_i}^i : \exists \ y : \beta. \ \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}', y : \beta; \Phi'; \mathcal{R}'} \quad \text{Ty_Pat_Ret_Log}$$

$$\frac{\Phi \vdash \mathit{res_pattern} : \mathit{res} \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1}{\Phi \vdash \mathit{ret_pattern}_i{}^i : \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{\Phi \vdash \mathit{res_res_pattern}, \mathit{ret_pattern}_i{}^i : \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2}}{\Phi \vdash \mathit{res_res_pattern}, \mathit{ret_pattern}_i{}^i : \mathit{res} \otimes \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2}} \quad \mathsf{TY_PAT_RET_RES}$$

$$\frac{\Phi \vdash \overline{\mathit{ret_pattern_i}}^i : \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \overline{\mathit{ret_pattern_i}}^i : \mathit{term} \land \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi', \mathit{term}; \mathcal{R}'} \quad \mathsf{TY_PAT_RET_PHI}$$

 $C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{undef}\ \mathit{UB_name} \Leftarrow y{:}\beta.\,\mathit{term}}\quad \mathtt{TY_TPVAL_UNDEF}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \underbrace{\text{smt} \left(\Phi \Rightarrow pval/y, \cdot (term)\right)}_{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{done } pval \Leftarrow y : \beta. \ term} \quad \text{Ty_TPVal_Done} \end{split}$$

 $C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool} \\ \mathcal{C}; \mathcal{L}; \Phi, pval = \texttt{true} \vdash tpexpr_1 \Leftarrow y : \beta. \ term \\ \mathcal{C}; \mathcal{L}; \Phi, pval = \texttt{false} \vdash tpexpr_2 \Leftarrow y : \beta. \ term \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2 \Leftarrow y : \beta. \ term \end{split} \qquad \texttt{TY_TPE_IF}$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pexpr &\Rightarrow y_1 {:} \beta_1. \ term_1 \\ ident_or_pattern {:} \beta_1 &\leadsto \mathcal{C}_1 \ \text{with} \ term \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term/y_1, \cdot (term_1) \vdash tpexpr &\Leftarrow y_2 {:} \beta_2. \ term_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \text{let} \ ident_or_pattern &= pexpr \ \text{in} \ tpexpr &\Leftarrow y_2 {:} \beta_2. \ term_2 \end{split} \quad \text{TY_TPE_LET}$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash tpexpr_1 &\Leftarrow y_1 {:} \beta_1. \ term_1 \\ ident_or_pattern {:} \beta_1 &\leadsto \mathcal{C}_1 \, \mathtt{with} \, term \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term/y_1, \cdot (term_1) \vdash tpexpr &\Leftarrow y_2 {:} \beta_2. \ term_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{let} \, ident_or_pattern {:} (y_1 {:} \beta_1. \ term_1) &= tpexpr_1 \, \mathtt{in} \, tpexpr_2 &\Leftarrow y_2 {:} \beta_2. \ term_2 \end{split} \quad \text{TY_TPE_LETT} \end{split}$$

$$\begin{split} & \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_1}{\underbrace{pattern_i : \beta_1 \leadsto \mathcal{C}_i \, \text{with} \, term_i}^i} \\ & \frac{\overline{\mathcal{C}, \mathcal{C}_i; \mathcal{L}; \Phi, term_i = pval \vdash tpexpr_i \Leftarrow y_2 : \beta_2. \, term_2}^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathsf{case} \, pval \, \mathsf{of} \, \overline{\mid pattern_i \Rightarrow tpexpr_i}^i \, \mathsf{end} \Leftarrow y_2 : \beta_2. \, term_2} \end{split} \quad \mathsf{TY_TPE_CASE} \end{split}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret$

$$\begin{array}{c} \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow \mathsf{integer} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\vdash \mathsf{create}\,(pval,\tau)\Rightarrow \Sigma\,y_p; \mathsf{loc}.\,\mathsf{representable}\,(\tau*,y_p)\,\wedge\,\mathsf{alignedI}\,(pval,y_p)\,\wedge\,\exists\,y;\beta_\tau,y_p(1)\overset{\check{\hookrightarrow}}{\to}_\tau\,y\otimes\mathsf{I} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi\vdash pval_0\Rightarrow \mathsf{loc} \\ \mathsf{smt}\,(\Phi\Rightarrow pval_0=pval_1) \\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash pval_1(1)\overset{\check{\hookrightarrow}}{\mapsto}_\tau\,pval_2\Leftarrow pval_1(1)\overset{\check{\hookrightarrow}}{\mapsto}_\tau\,pval_2 \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{load}\,(\tau,pval_0,\neg,pval_1(1)\overset{\check{\hookrightarrow}}{\mapsto}_\tau\,pval_2)\Rightarrow \Sigma\,y;\beta_\tau.\,\,y=pval_2\wedge\,pval_1(1)\overset{\check{\hookrightarrow}}{\mapsto}_\tau\,pval_2\otimes\mathsf{I} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi\vdash pval_0\Rightarrow \mathsf{loc} \\ \mathcal{C};\mathcal{L};\Phi\vdash pval_1\Rightarrow \beta_\tau \\ \mathsf{smt}\,(\Phi\Rightarrow \mathsf{representable}\,(\tau,pval_1)) \\ \mathsf{smt}\,(\Phi\Rightarrow \mathsf{representable}\,(\tau,pval_1)) \\ \mathsf{smt}\,(\Phi\Rightarrow pval_2=pval_0) \\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{pval}_2(1)\mapsto_\tau _ \Leftarrow pval_2(1)\mapsto_\tau _ \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{store}\,(_,\tau,pval_0,pval_1,_,pval_2(1)\mapsto_\tau _)\Rightarrow \Sigma.\mathsf{:unit.}\,pval_2(1)\overset{\check{\hookrightarrow}}{\mapsto}_\tau\,pval_1\otimes\mathsf{I} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash pval_1(1)\mapsto_\tau _ \Leftarrow pval_1(1)\mapsto_\tau _ \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{pval}_1(1)\mapsto_\tau _ \Leftrightarrow pval_1(1)\mapsto_\tau _ \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{kill}\,(\mathsf{static}\,\tau,pval_0,pval_1(1)\mapsto_\tau _)\Rightarrow \Sigma.\mathsf{:unit.}\,\mathsf{I} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{kill}\,(\mathsf{static}\,\tau,pval_0,pval_1(1)\mapsto_\tau _)\Rightarrow \Sigma.\mathsf{:unit.}\,\mathsf{I} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{kill}\,(\mathsf{static}\,\tau,pval_0,pval_1(1)\mapsto_\tau _)\Rightarrow \Sigma.\mathsf{:unit.}\,\mathsf{I} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \hline\\ \mathsf{TY_ACTION_KILL_STATIC} \\ \\ \mathsf{TY_ACTION_KILL_STATIC} \\ \\ \mathsf{TY_ACTION_KILL_STATIC} \\ \\ \mathsf{TY_ACTION_KILL_STATIC} \\ \\ \mathsf{TY_ACTION_KILL_ST$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret$

$$C;\mathcal{L};\Phi\vdash pval_1\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_2\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_2\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_2\Rightarrow \text{Loc} \\ C;\mathcal{L};\Phi\vdash pval_1\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_2\Rightarrow \text{Loc} \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash \text{intFromPtr}(\tau_1,\tau_2,pval)\Rightarrow \Sigma y\text{:integer},y=\text{cast.ptr.to.int}pval \wedge \mathbf{I} \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow \text{integer} \\ C;\mathcal{L};\Phi\vdash pval_0\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_0\Rightarrow \text{loc} \\ \text{smt}(\Phi\Rightarrow pval_1=pval_0) \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash pval_1(.)\stackrel{\checkmark}{\to}_{\tau^-}\Rightarrow \Sigma y\text{:bool},y=\text{cast.int.to.ptr}pval_1\land \mathbf{I} \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash pval_1(.)\stackrel{\checkmark}{\to}_{\tau^-}\Rightarrow \Sigma y\text{:bool},y=\text{aligned}(\tau,pval_1)\land pval_1(.)\stackrel{\checkmark}{\to}_{\tau^-}\otimes \mathbf{I} \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash pval_1\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi;\mathcal{L};\Phi\vdash pval_1\Rightarrow \text{loc} \\ C;\mathcal{L};\Phi\vdash pval_1\Rightarrow \text{loc}$$

$$\begin{array}{c} \mathcal{C};\mathcal{L};\Phi \vdash pval \Rightarrow \beta \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, pval, \, \overline{spine_elem_i}^{\; i} \Leftarrow \Sigma \, y : \beta. \, ret} \end{array} \quad \text{TY_TVAL_COMP} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi \vdash pval \Rightarrow \beta}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, pval, \, \overline{spine_elem_i}^{\; i} \Leftarrow \exists \, y : \beta. \, ret} \quad \text{TY_TVAL_LOG} \\ \\ \frac{\mathsf{smt} \, (\Phi \Rightarrow term)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, spine \Leftarrow ret} \\ \hline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R} \vdash \mathsf{done} \, spine \Leftarrow term \land ret} \quad \text{TY_TVAL_PHI} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1 \vdash res_term \Leftarrow res}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_2 \vdash \mathsf{done} \, \overline{spine_elem_i}^{\; i} \Leftarrow ret} \quad \text{TY_TVAL_RES} \\ \hline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1,\mathcal{R}_2 \vdash \mathsf{done} \, res_term, \, \overline{spine_elem}^{\; i} \Leftarrow res \otimes ret} \quad \text{TY_TVAL_RES} \\ \\ \frac{\mathsf{smt} \, (\Phi \Rightarrow \mathsf{false})}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1 \vdash \mathsf{undef} \, UB_name \Leftarrow ret} \quad \text{TY_TVAL_UNDEF} \\ \end{array}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret$

$$\begin{split} ident: & arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals} \\ & \underline{\mathcal{C}}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret} \\ & \overline{\mathcal{C}}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{ccall}\left(\tau, ident, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \end{split} \quad \texttt{TY_SEQ_E_CCALL} \\ & name: arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals} \\ & \underline{\mathcal{C}}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret} \\ & \overline{\mathcal{C}}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pcall}\left(name, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \end{split} \quad \texttt{TY_SEQ_E_PROC}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash memop (mem_op) \Rightarrow ret} \quad \text{TY_IS_E_MEMOP}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Action}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{neg}\,mem_action \Rightarrow ret} \quad \mathsf{TY_IS_E_NEG_ACTION}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash seg_texpr \Leftarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret} \quad \text{TY_SEQ_TE_TVAL}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pexpr \Rightarrow y : \beta. \ term \\ ident_or_pattern : \beta \leadsto \mathcal{C}_1 \ \text{with} \ term_1 \\ \hline \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term_1/y, \cdot (term); \mathcal{R} \vdash texpr \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{let} \ ident_or_pattern = pexpr \ \mathtt{in} \ texpr \Leftarrow ret \end{array}$$
 TY_SEQ_TE_LETP

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash tpexpr &\Leftarrow y : \beta. \ term \\ ident_or_pattern : \beta \leadsto \mathcal{C}_1 \ \text{with} \ term_1 \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term_1/y, \cdot (term); \mathcal{R} \vdash texpr &\Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{let} \ ident_or_pattern : (y : \beta. \ term) = tpexpr \ \text{in} \ texpr &\Leftarrow ret \end{split}$$
 TY_Seq_TE_LetPT

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}' &\vdash seq_expr \Rightarrow ret_1 \\ \Phi &\vdash \overline{ret_pattern_i}^i : ret_1 \rightsquigarrow \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 &\vdash texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} &\vdash \text{let} \overline{ret_pattern_i}^i = seq_expr \text{in} texpr \Leftarrow ret_2 \end{split}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}' \vdash texpr_1 \Leftarrow ret_1 \\ \Phi \vdash \overline{ret_pattern_i}^i : ret_1 \rightsquigarrow \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr_2 \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \underline{ret_pattern_i}^i : ret_1 = texpr_1 \text{ in } texpr_2 \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_1 \\ \hline \underline{c}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \underline{ret_pattern_i}^i : ret_1 = texpr_1 \text{ in } texpr_2 \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_1 \\ \hline \underline{c}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{case } pval \text{ of } \boxed{pattern_i} \Rightarrow texpr_i \triangleq ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{case } pval \text{ of } \boxed{pattern_i} \Rightarrow texpr_i \triangleq \text{ond} \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi, pval \Rightarrow \text{bool} \\ \mathcal{C}; \mathcal{L}; \Phi, pval = \text{true}; \mathcal{R} \vdash texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi, pval = \text{false}; \mathcal{R} \vdash texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi, pval = \text{false}; \mathcal{R} \vdash texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{if } pval \text{ then } texpr_1 \text{ else } texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{if } pval \text{ then } texpr_1 \text{ else } texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{L}; \Phi; \vdash run i dent \underbrace{pval_i}^i : arg \gg \sigma; \text{false } \land \mathbf{I} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{is } texpr \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{bound } [int] \underbrace{(is _texpr)}^i \Leftarrow \text{ret} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{bound } [int] \underbrace{(is _texpr)}^i \Leftarrow \text{ret} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}' \vdash \text{is } texpr \Rightarrow ret_1 \\ \Phi \vdash \overline{ret_pattern_i}^i : ret_1 \rightsquigarrow \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \hline \mathcal{C}; \mathcal{L}; \mathcal{L}; \mathcal{L}; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash \text{texpr} \Leftarrow \text{ret}_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in } texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in } texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in } texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in } texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in } texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \text{let } \text{strong } \overline{\text{ret } pattern_i}^i = is _expr \text{ in }$$

C; L; Φ ; $R \vdash is_texpr \Leftarrow ret$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret} \quad \text{TY_TE_IS}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret} \quad \text{TY_TE_SEQ}$$

 $pattern = pval \leadsto \sigma$

$$\overline{x:=pval \leadsto pval/x}$$
, Subs_Decons_Value_Sym_Annot

$$\begin{aligned} pattern_1 &= pval_1 \leadsto \sigma_1 \\ pattern_2 &= pval_2 \leadsto \sigma_2 \\ \overline{\mathsf{Cons}(pattern_1, pattern_2) = \mathsf{Cons}(pval_1, pval_2) \leadsto \sigma_1, \sigma_2} \end{aligned} \quad \text{Subs_Decons_Value_Cons}$$

$$\frac{\overline{pattern_i = pval_i \leadsto \sigma_i}^i}{\text{Tuple}(\overline{pattern_i}^i) = \text{Tuple}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Tuple}$$

$$\frac{\overline{pattern_i = pval_i \leadsto \sigma_i}^i}{\operatorname{Array}(\overline{pattern_i}^i) = \operatorname{Array}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Array}$$

$$\frac{pattern = pval \leadsto \sigma}{\texttt{Specified}(pattern) = pval \leadsto \sigma} \quad \texttt{Subs_Decons_Value_Specified}$$

 $ident_or_pattern = pval \leadsto \sigma$

 $\overline{x = pval \leadsto pval/x}$. Subs_Decons_Value'_Sym

 $\frac{pattern = pval \leadsto \sigma}{pattern = pval \leadsto \sigma}$ Subs_Decons_Value'_Pattern

 $res_pattern = res_term \leadsto \sigma$

 $\frac{}{\text{emp} = \text{emp} \leadsto}$ Subs_Decons_Res_Emp

 $ident = res_term \leadsto res_term/ident$, Subs_Decons_Res_Var

 $res_pattern_1 = res_term_1 \leadsto \sigma_1$ $res_pattern_2 = res_term_2 \leadsto \sigma_2$ $\overline{\langle res_pattern_1, res_pattern_2 \rangle} = \overline{\langle res_term_1, res_term_2 \rangle} \leadsto \sigma_1, \sigma_2$ SUBS_DECONS_RES_PAIR

 $\overline{\left\langle res_pattern_1, res_pattern_2 \right\rangle} = \mathtt{break} \left(res_term: qpt, i \right) \leadsto \sigma_1, \sigma_2 \quad \text{Subs_Decons_Res_Break_Doesnt_Work}$

 $\frac{res_pattern = res_term \leadsto \sigma}{\texttt{pack} \, (ident, res_pattern) = \texttt{pack} \, (pval, res_term) \leadsto pval/ident, \sigma} \quad \texttt{Subs_Decons_Res_Pack}$

 $\frac{res_pattern = res_term \leadsto \sigma}{\texttt{fold} (res_pattern) = res_term \leadsto \sigma} \quad \text{Subs_Decons_Res_Fold}$

 $\overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma$

Subs_Decons_Ret_Empty

$$\frac{ident_or_pattern = pval \leadsto \sigma}{ret_pattern_i = spine_elem_i{}^i \leadsto \psi} \\ \frac{comp\:ident_or_pattern = spine_elem_i{}^i \leadsto \psi}{ret_pattern_i = spine_elem_i{}^i \leadsto \sigma, \psi} \\ \text{Subs_Decons_Ret_Comp}$$

$$\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \psi}{\log ident = pval, \ \overline{ret_pattern_i = spine_elem_i}^i \leadsto pval/ident, \psi} \quad \text{Subs_Decons_Ret_Log}$$

$$\frac{res_pattern = res_term \leadsto \sigma}{ret_pattern_i = spine_elem_i{}^i \leadsto \psi}$$
 Subs_Decons_Ret_Res
$$res_pattern = res_term, \overline{ret_pattern_i = spine_elem_i{}^i} \leadsto \sigma, \psi$$

 $\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$

 $\frac{}{::ret \gg \cdot; ret}$ Subs_Decons_Arg_Empty

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \ \overline{x_i = spine_elem_i}^i :: \Pi x:\beta. \ arg \gg pval/x, \sigma; ret} \quad \text{Subs_Decons_Arg_Comp}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \ \overline{x_i = spine_elem_i}^i :: \forall x:\beta. \ arg \gg pval/x, \sigma; ret} \quad \text{Subs_Decons_Arg_Log}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: \forall x:\beta. \ arg \gg pval/x, \sigma; ret}{x = res_term, \ \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret} \quad \text{Subs_Decons_Arg_Res}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\overline{x_i = spine_elem_i}^i :: term \supset arg \gg \sigma; ret} \quad \text{Subs_Decons_Arg_Phi}$$

 $\langle pexpr \rangle \longrightarrow \langle pexpr' \rangle$

$$\begin{array}{c} \underline{mem_ptr'} \equiv mem_ptr +_{\text{ptr}} mem_int \times \text{size_of}(\tau)}{\langle \text{array_shift}(mem_ptr, \tau, mem_int)\rangle \longrightarrow \langle mem_ptr'\rangle} & \text{Op_PE_PE_ARRAYSHIFT} \\ \\ \underline{mem_ptr'} \equiv mem_ptr +_{\text{ptr}} \text{ offset_of}_{tag}(member)}{\langle \text{member_shift}(mem_ptr, tag, member)\rangle \longrightarrow \langle mem_ptr'\rangle} & \text{Op_PE_PE_MemberShift} \\ \\ \overline{\langle \text{not}(\text{True})\rangle \longrightarrow \langle \text{False}\rangle} & \text{Op_PE_PE_Not_True} \\ \\ \overline{\langle \text{not}(\text{False})\rangle \longrightarrow \langle \text{True}\rangle} & \text{Op_PE_PE_Not_False} \\ \\ \underline{mem_int} \equiv mem_int_1 \ binop_{arith} \ mem_int_2}{\langle mem_int_1 \ binop_{arith} \ mem_int_2} & \text{Op_PE_PE_ARITH_BINOP} \\ \\ \underline{bool_value} \equiv mem_int_1 \ binop_{rel} \ mem_int_2}{\langle mem_int_1 \ binop_{rel} \ mem_int_2\rangle \longrightarrow \langle bool_value\rangle} & \text{Op_PE_PE_Rel_BINOP} \\ \\ \underline{bool_value} \equiv bool_value_1 \ binop_{bool} \ bool_value_2}{\langle bool_value_1 \ binop_{bool} \ bool_value_2\rangle \longrightarrow \langle bool_value_2} & \text{Op_PE_PE_Bool_BINOP} \\ \\ \overline{\langle \text{assert_undef}(\text{True}, \ UB_name)}\rangle \longrightarrow \langle \text{Unit}\rangle} & \text{Op_PE_PE_Assert_UNDEF} \\ \end{array}$$

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OP_PE_PE_BOOL_TO_INTEGER_TRUE
                                                                                    \overline{\langle \mathtt{bool\_to\_integer}\left(\mathtt{True}\right) \rangle \longrightarrow \langle 1 \rangle}
                                                                                                                                                                         OP_PE_PE_Bool_To_Integer_False
                                                                                  \overline{\langle \texttt{bool\_to\_integer}(\texttt{False}) \rangle \longrightarrow \langle 0 \rangle}
                                                               abbrev_1 \equiv \max_{\cdot} \inf_{\tau} - \min_{\cdot} \inf_{\tau} + 1
                                                               abbrev_2 \equiv pval \, rem_f \, abbrev_1
                                                              mem\_int' \equiv \text{if } abbrev_2 \leqslant \max\_int_{\tau} \text{ then } abbrev_2 \text{ else } abbrev_2 - abbrev_1
                                                                                                                                                                                                                                              OP_PE_PE_WRAPI
                                                                                                      \langle \mathtt{wrapI} (\tau, mem\_int) \rangle \longrightarrow \langle mem\_int' \rangle
\langle pexpr \rangle \longrightarrow \langle tpexpr:(y:\beta. term) \rangle
                                                                                               name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in Globals
                                                                                         \frac{\overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma \ y : \beta. \ term \land \mathtt{I}}{\langle name(\overline{pval_i}^i) \rangle \longrightarrow \langle \sigma(tpexpr) : (y : \beta. \ \sigma(term)) \rangle} \quad \mathsf{OP\_PE\_TPE\_CALL}
\langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle
                                                                                                       pattern_i = pval \leadsto \sigma_i
                                                                            \frac{\overbrace{\forall \: i < j. \: \text{not} \: (pattern_i = pval \leadsto \sigma_i)}}{\left\langle \mathsf{case} \: pval \: \mathsf{of} \: \middle| \: pattern_i \Rightarrow tpexpr_i \: {}^i \: \mathsf{end} \right\rangle \longrightarrow \left\langle \sigma_j(tpexpr_j) \right\rangle} \quad \mathsf{OP\_TPE\_TPE\_CASE}
                                                                         \frac{ident\_or\_pattern = pval \leadsto \sigma}{\left\langle \texttt{let}\, ident\_or\_pattern = pval\, \texttt{in}\, tpexpr \right\rangle \longrightarrow \left\langle \sigma(tpexpr) \right\rangle} \quad \text{Op\_TPE\_TPE\_Let\_Sub}
                                                                                                                   \langle pexpr \rangle \longrightarrow \langle pexpr' \rangle
                                   \frac{\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle}{\left\langle \texttt{let} \, ident\_or\_pattern = pexpr \, \texttt{in} \, tpexpr\rangle \longrightarrow \left\langle \texttt{let} \, ident\_or\_pattern = pexpr' \, \texttt{in} \, tpexpr\rangle}\right\rangle}
                                                                                                                                                                                                                                                        OP\_TPE\_TPE\_LET\_LET
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\frac{\langle pexpr\rangle \longrightarrow \langle tpexpr_1: (y:\beta.\ term)\rangle}{\langle \text{let}\ ident\_or\_pattern = pexpr\ in} \frac{\langle pexpr\rangle \longrightarrow \langle tpexpr_1: (y:\beta.\ term)\rangle}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term)}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term)}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in} \frac{\langle pexpr\rangle \longrightarrow \langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr_1\ in}{\langle \text{let}\ ident\_or\_pattern: (y:\beta.\ term) = tpexpr
                                                                                                                             \frac{ident\_or\_pattern = pval \leadsto \sigma}{\left\langle \texttt{let} \, ident\_or\_pattern: (y:\beta. \, term) = \texttt{done} \, pval \, \texttt{in} \, tpexpr \right\rangle \longrightarrow \left\langle \sigma(tpexpr) \right\rangle} \quad \text{Op\_TPE\_TPE\_Lett\_Sub}
\frac{\langle tpexpr_1'\rangle \longrightarrow \langle tpexpr_1'\rangle}{\langle \mathsf{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr_1' \,\mathsf{in}\, tpexpr_2\rangle} \quad \mathsf{OP\_TPE\_LetT\_LetT}
                                                                                                                                                                                                     \frac{}{\langle \mathtt{if}\,\mathtt{True}\,\mathtt{then}\,tpexpr_1\,\mathtt{else}\,tpexpr_2\rangle \longrightarrow \langle tpexpr_1\rangle}\quad \mathsf{OP\_TPE\_TPE\_IF\_TRUE}
                                                                                                                                                                                                \overline{\langle \mathtt{if}\,\mathtt{False}\,\mathtt{then}\,tpexpr_1\,\mathtt{else}\,tpexpr_2\rangle \longrightarrow \langle tpexpr_2\rangle} \quad \mathsf{OP\_TPE\_TPE\_IF\_FALSE}
      \langle h; seq\_expr \rangle \longrightarrow \langle h'; texpr:ret \rangle
                                                                                                                                                                                                                                                       ident:arg \equiv \overline{x_i}^i \mapsto texpr \in Globals
                                                                                                                                                                                            \frac{\overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret}{\langle h; \mathsf{ccall} \left(\tau, ident, \overline{spine\_elem_i}^i \right) \rangle \longrightarrow \langle h; \sigma(texpr) : \sigma(ret) \rangle} \quad \mathsf{OP\_SE\_TE\_CCALL}
                                                                                                                                                                                                \frac{name:arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals}}{\overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret} \\ \frac{\langle h; \texttt{pcall} \left( name, \overline{spine\_elem_i}^i \right) \rangle \longrightarrow \langle h; \sigma(texpr) : \sigma(ret) \rangle}{\langle h; \texttt{pcall} \left( name, \overline{spine\_elem_i}^i \right) \rangle \longrightarrow \langle h; \sigma(texpr) : \sigma(ret) \rangle}
  \langle h; seq\_texpr \rangle \longrightarrow \langle h'; texpr \rangle
                                                                                                                                                                                                                                                           ident:arg \equiv \overline{x_i}^i \mapsto texpr \in Globals
                                                                                                                                                                                                                                                    \frac{\overline{x_i = pval_i}^i :: arg \gg \sigma; \texttt{false} \wedge \texttt{I}}{\langle h; \texttt{run} i dent \ \overline{pval_i}^i \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op\_STE\_TE\_RUN}
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pattern_i = pval \leadsto \sigma_i
                                                                                 \frac{\sqrt[3]{i < j. \; \text{not} \left(pattern_i = pval \leadsto \sigma_i\right)}}{\langle h; \text{case} \, pval \, \text{of} \, \overline{\mid pattern_i \Rightarrow texpr_i}^i \, \text{end} \rangle \longrightarrow \langle h; \sigma_j(texpr_j) \rangle} \quad \text{Op\_STE\_TE\_CASE}
                                                                             \frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle h; \texttt{let}\, ident\_or\_pattern = pval\, \texttt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op\_STE\_TE\_Letp\_Sub}
                                   \frac{\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle}{\langle h; \mathtt{let}\, ident\_or\_pattern = pexpr\, \mathtt{in}\, texpr\rangle \longrightarrow \langle h; \mathtt{let}\, ident\_or\_pattern = pexpr'\, \mathtt{in}\, texpr\rangle} \quad \mathsf{OP\_STE\_TE\_LETP\_LETP}
                                                                                                        \langle pexpr \rangle \longrightarrow \langle tpexpr:(y:\beta.\ term) \rangle
                  \frac{\langle pexpr. \langle y.p..verm \rangle \rangle}{\langle h; \texttt{let} \, ident\_or\_pattern = pexpr \, \texttt{in} \, texpr} \rangle \longrightarrow \langle h; \texttt{let} \, ident\_or\_pattern : (y:\beta. \, term) = tpexpr \, \texttt{in} \, texpr} \rangle \quad \text{Op\_STE\_TE\_LetP\_LetP}
                                                                                                                ident\_or\_pattern = pval \leadsto \sigma
                                                      \frac{}{\langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = \mathtt{done}\, pval\,\, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op\_STE\_TE\_LetTP\_Sub}
\frac{\langle tpexpr\rangle \longrightarrow \langle tpexpr'\rangle}{\langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr\, \mathtt{in}\, texpr\rangle \longrightarrow \langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr'\, \mathtt{in}\, texpr\rangle} \quad \text{Op\_STE\_TE\_LETTP\_LETTP}
                                                            \frac{\overline{ret\_pattern_i = spine\_elem_i}^i \leadsto \sigma}{\langle h; \mathtt{let}\, \overline{ret\_pattern_i}^i : ret = \mathtt{done}\, \overline{spine\_elem_i}^i \, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_SUB}
                                   \frac{\langle h; seq\_expr \rangle \longrightarrow \langle h; texpr_1 : ret \rangle}{\langle h; \mathsf{let} \, \overline{ret\_pattern_i}^i \, = \, seq\_expr \, \mathsf{in} \, texpr_2 \rangle \longrightarrow \langle h; \mathsf{let} \, \overline{ret\_pattern_i}^i : ret \, = \, texpr_1 \, \mathsf{in} \, texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LET\_LETT}
                               \frac{\langle h; texpr_1 \rangle \longrightarrow \langle h'; texpr_1' \rangle}{\langle h; \mathsf{let} \ \overline{ret\_pattern_i}^i : ret = texpr_1 \ \mathsf{in} \ texpr_2 \rangle \longrightarrow \langle h'; \mathsf{let} \ \overline{ret\_pattern_i}^i : ret = texpr_1' \ \mathsf{in} \ texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_LETT}
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\overline{\langle h; \text{if True then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_1 \rangle}
                                                                                                                                                                                                               OP_STE_TE_IF_TRUE
                                                                                 \overline{\langle h; \text{if False then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_2 \rangle}
                                                                                                                                                                                                               OP_STE_TE_IF_FALSE
                                                                                                                                                                                                     OP_STE_TE_BOUND
                                                                                               \langle h: \mathsf{bound} [int](is\_texpr) \rangle \longrightarrow \langle h: is\_texpr \rangle
   \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                                                   bool\_value \, \equiv \, mem\_int_1 \, binop_{rel} \, mem\_int_2
                                                                                                                                                                                                             OP_MEMOP_TVAL_REL_BINOP
                                                                 \overline{\langle h; mem\_int_1 \ binop_{rel} \ mem\_int_2 \rangle \longrightarrow \langle h; \mathtt{done} \ bool\_value \rangle}
                                                                \frac{mem\_int \equiv \texttt{cast\_ptr\_to\_int} \, mem\_ptr}{\left\langle h; \texttt{intFromPtr} \left( \tau_1, \tau_2, mem\_ptr \right) \right\rangle \longrightarrow \left\langle h; \texttt{done} \, mem\_int \right\rangle}
                                                                                                                                                                                                         Op_Memop_TVal_IntFromPtr
                                                                \frac{mem\_ptr \equiv \texttt{cast\_ptr\_to\_int} \, mem\_int}{\left\langle h; \texttt{ptrFromInt} \left( \tau_1, \tau_2, mem\_int \right) \right\rangle \longrightarrow \left\langle h; \texttt{done} \, mem\_ptr \right\rangle}
                                                                                                                                                                                                        OP_MEMOP_TVAL_PTRFROMINT
\frac{bool\_value \equiv \texttt{aligned}\left(\tau, mem\_ptr\right)}{\langle h + \{mem\_ptr(\_) \overset{\checkmark}{\mapsto}_{\tau} \_\}; \texttt{ptrValidForDeref}\left(\tau, mem\_ptr, mem\_ptr(\_) \overset{\checkmark}{\mapsto}_{\tau} \_\right) \rangle \longrightarrow \langle h + \{mem\_ptr(\_) \overset{\checkmark}{\mapsto}_{\tau} \_\}; \texttt{done} \ bool\_value, mem\_ptr(\_) \overset{\checkmark}{\mapsto}_{\tau} \_\rangle}
                                                                                                                                                                                                                                                                                                                                        OP_MEMOP_TVAL_
                                                       \frac{bool\_value \equiv \mathtt{aligned} \left(\tau, mem\_ptr\right)}{\left\langle h; \mathtt{ptrWellAligned} \left(\tau, mem\_ptr\right) \right\rangle \longrightarrow \left\langle h; \mathtt{done} \ bool\_value \right\rangle} \quad \mathsf{OP\_MEMOP\_TVAL\_PTRWELLALIGNED}
                                                \frac{mem\_ptr' \equiv mem\_ptr +_{ptr} (mem\_int \times size\_of(\tau))}{\langle h; \texttt{ptrArrayShift} (mem\_ptr, \tau, mem\_int) \rangle \longrightarrow \langle h; \texttt{done} \ mem\_ptr' \rangle} \quad \text{Op\_Memop\_TVal\_PtrArrayShift}
```

 $\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle$

 $\begin{array}{l} \texttt{fresh}\,(mem_ptr) \\ \texttt{representable}\,(\tau*,mem_ptr) \\ \texttt{alignedI}\,(mem_int,mem_ptr) \\ pval: \beta_{\tau} \end{array}$

 $\frac{pear.p_{\tau}}{\langle h; \mathtt{create}\,(mem_int,\tau)\rangle \longrightarrow \langle h + \{mem_ptr(1) \overset{\times}{\mapsto_{\tau}} pval\}; \mathtt{done}\,mem_ptr, pval, mem_ptr(1) \overset{\times}{\mapsto_{\tau}} pval\rangle} \quad \mathsf{OP_ACTION_TVAL_CREATE}$

 $\frac{}{\left\langle h + \{mem_ptr(1) \overset{\checkmark}{\mapsto}_{\tau} pval\}; \texttt{load}\left(\tau, mem_ptr, _, mem_ptr(1) \overset{\checkmark}{\mapsto}_{\tau} pval\right) \right\rangle} \longrightarrow \left\langle h + \{mem_ptr(1) \overset{\checkmark}{\mapsto}_{\tau} pval\}; \texttt{done} \ pval, mem_ptr(1) \overset{\checkmark}{\mapsto}_{\tau} pval \right\rangle}$

OP_ACTION_TVAL_STO

OP_ACTION_TVAL_LOA

 $\overline{\langle h + \{mem_ptr(_) \overset{\checkmark}{\mapsto}_{\tau} _\}; \mathtt{store}\left(_, \tau, mem_ptr, pval,_, mem_ptr(_) \overset{\checkmark}{\mapsto}_{\tau} _)\rangle} \longrightarrow \langle h + \{mem_ptr(_) \overset{\checkmark}{\mapsto}_{\tau} pval\}; \mathtt{done}\, \mathtt{Unit}, mem_ptr(_) \overset{\checkmark}{\mapsto}_{\tau} pval\rangle}$

 $\overline{\langle h + \{mem_ptr(_) \mapsto_{\tau} _\}; \texttt{kill} \left(\texttt{static} \ \tau, mem_ptr, mem_ptr(_) \mapsto_{\tau} _\right) \rangle} \longrightarrow \langle h; \texttt{done} \ \texttt{Unit} \rangle \\ \text{OP_ACTION_TVAL_KILL_STATIC}$

 $\langle h; is_expr \rangle \longrightarrow \langle h'; is_expr' \rangle$

$$\frac{\langle h; mem_op \rangle \longrightarrow \langle h; tval \rangle}{\langle h; \mathsf{memop} \, (mem_op) \rangle \longrightarrow \langle h; tval \rangle} \quad \text{Op_IsE_IsE_Memop}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \text{Op_IsE_IsE_Action}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; \mathsf{neg}\, mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \text{Op_IsE_IsE_Neg_Action}$$

 $|\langle h; is_texpr \rangle \longrightarrow \langle h'; texpr \rangle$

$$\frac{\overline{ret_pattern_i} = spine_elem_i^{\ i} \leadsto \sigma}{\langle h; \mathtt{let\,strong}\, \overline{ret_pattern_i^{\ i}} = \mathtt{done}\, \overline{spine_elem_i^{\ i}} \, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op_ISTE_ISTE_LETS_SUB}$$

$$\frac{\langle h; is_expr\rangle \longrightarrow \langle h'; is_expr'\rangle}{\langle h; \mathsf{let\,strong}\,\overline{\mathit{ret_pattern_i}}^i = is_expr\,\mathsf{in}\,\mathit{texpr}\rangle \longrightarrow \langle h'; \mathsf{let\,strong}\,\overline{\mathit{ret_pattern_i}}^i = is_expr'\,\mathsf{in}\,\mathit{texpr}\rangle} \quad \text{Op_ISTE_ISTE_LETS_LETS}$$

 $\langle h; texpr \rangle \longrightarrow \langle h'; texpr' \rangle$

$$\frac{\langle h; seq_texpr \rangle \longrightarrow \langle h; texpr \rangle}{\langle h; seq_texpr \rangle \longrightarrow \langle h; texpr \rangle} \quad \text{Op_TE_TE_SEQ}$$

$$\frac{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle}{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle} \quad \text{Op_TE_TE_IS}$$

Definition rules: 233 good 0 bad Definition rule clauses: 538 good 0 bad