ident, $x, y, y_p, y_f, -$, abbrev, r, α subscripts: p for pointers, f for functions

n, i, j index variables

 $impl_const$ implementation-defined constant member C struct/union member name

Ott-hack, ignore (annotations)

nat OCaml arbitrary-width natural number

mem_ptr abstract pointer value
mem_val abstract memory value

Ott-hack, ignore (locations)

mem_iv_c OCaml type for memory constraints on integer values

 UB_name undefined behaviour

string OCaml string

Ott-hack, ignore (OCaml type variable TY)
Ott-hack, ignore (OCaml Symbol.prefix)

mem_order, _ OCaml type for memory order

linux_mem_order OCaml type for Linux memory order

Ott-hack, ignore (OCaml type variable bt)

```
Sctypes_{-}t, \tau
                                               C type
                                                  pointer to type \tau
                                               OCaml type for struct/union tag
tag
                    ::=
                          ident
β, _
                                               base types
                    ::=
                                                  unit
                          unit
                          bool
                                                  boolean
                                                  integer
                          integer
                                                  rational numbers?
                          real
                                                  location
                          loc
                          \operatorname{array} \beta
                                                  array
                          \mathtt{list}\, eta
                                                  list
                                                  tuple
                          \mathtt{struct}\,tag
                                                  struct
                          \operatorname{\mathfrak{set}} \beta
                                                  \operatorname{set}
                          opt(\beta)
                                                  option
                          \beta \to \beta'
                                                  parameter types
                                          Μ
                                                  of a C type
binop
                                               binary operators
                                                  addition
                                                  subtraction
                                                  multiplication
                                                  division
                                                  modulus
                          rem_t
                                                  remainder
                          rem_f
                                                  exponentiation
                                                  equality, defined both for integer and C types
                                                  inequality, similarly defined
                           !=
```

```
greater than, similarly defined
                                   less than, similarly defined
                                   greater than or equal to, similarly defined
                                   less than or equal to, similarly defined
                                   conjucttion
                                   disjunction
binop_{arith}
                                 arithmentic binary operators
                    rem_t
                    rem_f
binop_{rel}
                                relational binary operators
binop_{bool}
                                boolean binary operators
mem\_int
                                memory integer value
                            Μ
                            М
```

$object_value$::= 	$\begin{split} & mem_int \\ & mem_ptr \\ & \texttt{array} \left(\overline{loaded_value_i}^i \right) \\ & (\texttt{struct} ident) \{ \overline{.member_i : \tau_i = mem_val_i}^i \} \\ & (\texttt{union} ident) \{ .member = mem_val \} \end{split}$	C object values (inhabitants of object types), which can be read/stored integer value pointer value C array value C struct value C union value
$loaded_value$::=		potentially unspecified C object values
		$\verb specified object_value $	specified loaded value
value	::=		Core values
		$object_value$	C object value
	ĺ	$loaded_value$	loaded C object value
	ĺ	Unit	unit
		True	boolean true
		False	boolean false
		$eta[\overline{value_i}^i]$	list
		$(\overline{value_i}^i)$	tuple
$bool_value$::=		Core booleans
		True	boolean true
		False	boolean false
$ctor_val$::=		data constructors
		$\mathtt{Nil}\beta$	empty list
		Cons	list cons
		Tuple	tuple
		Array	C array
		Specified	non-unspecified loaded value
$ctor_expr$::=		data constructors

		Ivmax Ivmin Ivsizeof Ivalignof IvCOMPL IvAND IvOR IvXOR Fvfromint		max integer value min integer value sizeof value alignof value bitwise complement bitwise AND bitwise OR bitwise XOR cast integer to floating value
		Ivfromfloat		cast floating to integer value
name	::= 	$ident \\ impl_const$		Core identifier implementation-defined constant
pval	::= 	$ident \\ impl_const \\ value \\ \texttt{constrained}\left(\overline{mem_iv_c_i, pval_i}^i\right) \\ \texttt{error}\left(string, pval\right) \\ ctor_val\left(\overline{pval_i}^i\right) \\ (\texttt{struct}ident)\{\overline{.member_i = pval_i}^i\} \\ (\texttt{union}ident)\{.member = pval\}$		pure values Core identifier implementation-defined constant Core values constrained value impl-defined static error data constructor application C struct expression C union expression
tpval	::=	$\begin{array}{l} \text{undef} \ \ UB_name \\ \text{done} \ pval \end{array}$		top-level pure values undefined behaviour pure done
$ident_opt_\beta$::=	<i>∴</i> ;β	$binders = \{\}$	type annotated optional identifier

```
ident:\beta
                                                              binders = ident
pattern
                               ident\_opt\_\beta
                                                              binders = binders(ident\_opt\_\beta)
                              ctor\_val(\overline{pattern_i}^i)
                                                              binders = binders (\overline{pattern_i}^i)
                                                                                                    OCaml arbitrary-width integer
z
                        ::=
                                                              Μ
                                                                                                       literal integer
                              mem\_int
                                                              Μ
                              size\_of(\tau)
                                                              Μ
                                                                                                       size of a C type
                              offset_of_tag(member)
                                                                                                       offset of a struct member
                                                              Μ
                              ptr_size
                                                              Μ
                                                                                                       size of a pointer
                              \max_{-int_{\tau}}
                                                              Μ
                                                                                                       maximum value of int of type \tau
                                                              Μ
                              \min_{-int_{\tau}}
                                                                                                       minimum value of int of type \tau
                                                                                                    OCaml type for rational numbers
\mathbb{Q}, q, 
                               \frac{int_1}{int_2}
lit
                         ::=
                               ident
                               unit
                               bool
ident\_or\_pattern
                               ident
                                                              binders = ident
                              pattern
                                                              binders = binders(pattern)
array\_prop\_form
                                                                                                   array property formulas
```

```
form
                    ::=
                           \bigwedge(\overline{form_i}^i)
                           \bigvee (\overline{form_i}^i)
                           expr\_form \leq expr\_form'
                           expr\_form = expr\_form'
expr\_form
                           ident
                           z
                           expr\_form_1 \times expr\_form_2
                           expr\_form_1 + expr\_form_2
                           expr\_form_1[expr\_form_2]
bool\_op
                    ::=
                           \neg term
                           term_1 = term_2
                           term_1 \rightarrow term_2
                           \bigwedge (\overline{term_i}^i)
                           \bigvee (\overline{term_i}^i)
                           term_1 \ binop_{bool} \ term_2
                                                                        Μ
                           if term_1 then term_2 else term_3
arith\_op
                    ::=
                           term_1 + term_2
                           term_1 - term_2
                           term_1 \times term_2
                           term_1/term_2
                           term_1 \, {\tt rem\_t} \, term_2
                           term_1 \, {\tt rem\_f} \, term_2
                           term_1 \hat{} term_2
                                                                        Μ
                           term_1 binop_{arith} term_2
```

```
cmp\_op
                        term_1 < term_2
                                                               less than
                        term_1 \leq term_2
                                                               less than or equal
                                                       Μ
                        term_1 \ binop_{rel} \ term_2
list\_op
                  ::=
                        nil
                        term_1 :: term_2
                        {\tt tl}\, term
                        term^{(int)}
tuple\_op
                         (\overline{term_i}^i)
                        term^{(int)}
pointer\_op
                  ::=
                        mem\_ptr
                        term_1 +_{ptr} term_2
                        {\tt cast\_int\_to\_ptr}\, term
                        {\tt cast\_ptr\_to\_int}\, term
array\_op
                  ::=
                        [|\overline{term_i}^i|]
                        term_1[term_2]
param\_op
                  ::=
                        ident:\beta.\ term
                        term(term_1, ..., term_n)
struct\_op
                  ::=
                        term.member \\
```

```
ct\_pred
                    representable (\tau, term)
                    \bar{\mathtt{aligned}} \, (\tau, term)
                    alignedI(term_1, term_2)
term, _
                    lit
                    arith\_op
                    bool\_op
                    cmp\_op
                    tuple\_op
                    struct\_op
                    pointer\_op
                    list\_op
                    array\_op
                    ct\_pred
                    param\_op
                                                               S
                                                                      parentheses
                    (term)
                    \sigma(term)
                                                               Μ
                                                                       simul-sub \sigma in term
                                                               Μ
                    pval
                                                                    pure expressions
pexpr
                    pval
                                                                       pure values
                    ctor\_expr(\overline{pval_i}^i)
                                                                       data constructor application
                    array\_shift(pval_1, \tau, pval_2)
                                                                       pointer array shift
                    member_shift(pval,ident,member)
                                                                       pointer struct/union member shift
                    \mathtt{not}\left(pval\right)
                                                                      boolean not
                    pval_1 \ binop \ pval_2
                                                                       binary operations
                    {\tt memberof}\ (ident, member, pval)
                                                                       C struct/union member access
                    name(\overline{pval_i}^i)
                                                                      pure function call
                    assert_undef (pval, UB_name)
```

		$\begin{aligned} &\texttt{bool_to_integer} \left(pval \right) \\ &\texttt{conv_int} \left(\tau, pval \right) \\ &\texttt{wrapI} \left(\tau, pval \right) \end{aligned}$		
tpexpr	::=	$tpval \\ \texttt{case} \ pval \ \texttt{of} \ \overline{\mid tpexpr_case_branch_i}^i \ \texttt{end} \\ \texttt{let} \ ident_or_pattern = pexpr \ \texttt{in} \ tpexpr \\ \texttt{let} \ ident_or_pattern: (y_1:\beta_1. \ term_1) = tpexpr_1 \ \texttt{in} \ tpexpr_2 \\ \texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2 \\ \sigma(tpexpr) \\ \end{cases}$	bind binders $(ident_or_pattern)$ in $tpexpr$ bind binders $(ident_or_pattern)$ in $tpexpr_2$ bind y_1 in $term_1$	top-level pure expressions top-level pure values pattern matching pure let annoted pure let pure if simul-sub σ in $tpexpr$
$tpexpr_case_branch$::=	$pattern \Rightarrow tpexpr$	bind binders $(pattern)$ in $tpexpr$	pure top-level case expression top-level case expression br
m_kill_kind	::= 	$\begin{array}{l} \operatorname{dynamic} \\ \operatorname{static} \tau \end{array}$		
bool, _	::= 	true false		OCaml booleans
$int,$ _	::=	i		OCaml fixed-width integer literal integer
res_term	::= 	${f emp} \\ points_to \\ ident$		resource terms empty heap single-cell heap variable

	$egin{array}{lll} & \langle res_term_1, res_term_2 angle \ & ext{pack} \left(pval, res_term ight) \ & ext{fold} \left(res_term ight) \ & \sigma(res_term) \end{array}$	seperating-conjunction pair packing for existentials fold into recursive res. pred. M substitution for resource terms
mem_action		memory actions
	$ \begin{array}{ c c c c c } & \texttt{kill} (m_kill_kind, pval, pt) \\ & \texttt{store} (bool, \tau, pval_1, pval_2, mem_order, pt) \\ & \texttt{load} (\tau, pval, mem_order, pt) \\ & \texttt{rmw} (\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_1) \\ & \texttt{fence} (mem_order) \\ & \texttt{cmp_exch_strong} (\tau, pval_1, pval_2, pval_3, mem_order_1) \\ & \texttt{cmp_exch_weak} (\tau, pval_1, pval_2, pval_3, mem_order_1) \\ & \texttt{linux_fence} (linux_mem_order_1) \\ & \texttt{linux_load} (\tau, pval, linux_mem_order_1) \\ \end{array} $	$der_1, mem_order_2)$
	$\begin{array}{ c c c c c }\hline & \texttt{linux_store} \ (\tau, pval_1, pval_2, linux_mem_order)\\\hline & \texttt{linux_rmw} \ (\tau, pval_1, pval_2, linux_mem_order)\\\hline \end{array}$	
polarity	::= neg	polarities for memory actions (pos) sequenced by let weak and let strong only sequenced by let strong
pol_mem_action	::= polarity mem_action	memory actions with polarity
mem_op		operations involving the memory state pointer relational binary operations pointer subtraction

```
intFromPtr(	au_1,	au_2,pval)
                                                                                                                               cast of pointer value to integer value
                       \mathtt{ptrFromInt}\left(\tau_{1},\tau_{2},pval\right)
                                                                                                                               cast of integer value to pointer value
                                                                                                                               dereferencing validity predicate
                       ptrValidForDeref(	au, pval, pt)
                       ptrWellAligned (\tau, pval)
                       ptrArrayShift(pval_1, \tau, pval_2)
                       memcpy(pval_1, pval_2, pval_3)
                       memcmp(pval_1, pval_2, pval_3)
                       realloc(pval_1, pval_2, pval_3)
                       va\_start(pval_1, pval_2)
                       va\_copy(pval)
                       va\_arg(pval, \tau)
                       va_end(pval)
spine\_elem
                                                                                                                            spine element
                                                                                                                              pure or logical value
                       pval
                                                                                                                              resource value
                        res\_term
                                                             Μ
                       \sigma(spine\_elem)
                                                                                                                              substitution for spine elements / return values
                                                                                                                            spine
spine
                  ::=
                        spine\_elem_i
                                                                                                                            (effectful) top-level values
tval
                  ::=
                                                                                                                              end of top-level expression
                        {\tt done}\ spine
                                                                                                                               undefined behaviour
                        undef UB\_name
res\_pattern
                                                                                                                            resource terms
                  ::=
                                                             binders = \{\}
                                                                                                                               empty heap
                        emp
                                                             binders = ident
                                                                                                                               variable
                        ident
                       fold (res_pattern)
                                                                                                                               unfold (recursive) predicate
                                                             binders = \{\}
                       \langle res\_pattern_1, res\_pattern_2 \rangle
                                                             binders = binders(res\_pattern_1) \cup binders(res\_pattern_2)
                                                                                                                              seperating-conjunction pair
                        pack (ident, res_pattern)
                                                             binders = ident \cup binders(res\_pattern)
                                                                                                                              packing for existentials
```

$ret_pattern$::= 	compident_or_pattern logident resres_pattern	$binders = binders(ident_or_pattern)$ $binders = ident$ $binders = binders(res_pattern)$	return pattern computational variable logical variable resource variable
init,	::= 	✓ ×		initialisation status initialised uninitalised
$points_to, pt$::=	$term_1 \overset{init}{\mapsto}_{\tau} term_2$		points-to separation logic predicate
res	::=	emp $points_to$ $res_1 * res_2$ $\exists ident: \beta. res$ $term \land res$ if $term$ then res_1 else res_2 $\alpha(\overline{pval_i}^i)$ $\sigma(res)$	M	resources empty heap points-top heap pred. seperating conjunction existential logical conjuction ordered disjuction predicate simul-sub σ in res
ret, _	::=	$\Sigma ident: \beta. \ ret$ $\exists ident: \beta. \ ret$ $res \otimes ret$ $term \wedge ret$ I $\sigma(ret)$	M	return types return a computational value return a logical value return a resource value return a predicate (post-condition) end return list simul-sub σ in ret
seq_expr	::=			sequential (effectful) expressions

		$\begin{array}{l} \mathtt{ccall}\left(\tau, ident, spine\right) \\ \mathtt{pcall}\left(name, spine\right) \end{array}$		C function call procedure call
seq_texpr	::=	$tval \\ \operatorname{run} ident \overline{pval_i}^i \\ \operatorname{let} ident_or_pattern = pexpr \operatorname{in} texpr \\ \operatorname{let} ident_or_pattern: (y_1:\beta_1.\ term_1) = tpexpr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i = seq_expr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i : ret = texpr_1 \operatorname{in} texpr_2 \\ \operatorname{case} pval \operatorname{of} \overline{\mid texpr_case_branch_i}^i \operatorname{end} \\ \operatorname{if} pval \operatorname{then} texpr_1 \operatorname{else} texpr_2 \\ \operatorname{bound} [int](is_texpr) \\ \end{aligned}$	bind binders($ident_or_pattern$) in $texpr$ bind binders($ident_or_pattern$) in $texpr$ bind y_1 in $term_1$ bind binders($\overline{ret_pattern_i}^i$) in $texpr$ bind binders($\overline{ret_pattern_i}^i$) in $texpr_2$	sequential top-level (effectful) express (effectful) top-level values run from label pure let annotated pure let bind return patterns annotated bind return patterns pattern matching conditional limit scope of indet seq behaviour
$texpr_case_branch$::=	$pattern \Rightarrow texpr$	bind $binders(pattern)$ in $texpr$	top-level case expression branch top-level case expression branch
is_expr	::= 	$tval \\ ext{memop} (mem_op) \\ pol_mem_action$		indet seq (effectful) expressions (effectful) top-level values pointer op involving memory memory action
is_texpr	::= 	$\begin{array}{l} \texttt{let weak} \overline{ret_pattern_i}^{i} = is_expr \texttt{in} texpr \\ \texttt{let strong} \overline{ret_pattern_i}^{i} = is_expr \texttt{in} texpr \end{array}$	bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$ bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$	indet seq top-level (effectful) express weak sequencing strong sequencing
texpr	::=	seq_texpr is_texpr $\sigma(texpr)$	M	top-level (effectful) expressions sequential (effectful) expressions indet seq (effectful) expressions simul-sub σ in $texpr$

```
argument/function types
arg
                         \Pi ident:\beta. arg
                         \forall ident: \beta. arg
                         res \multimap arg
                         term \supset arg
                          ret
                          \sigma(arg)
                                                       Μ
                                                                simul-sub\sigma in arg
                                                             pure argument/function types
pure\_arg
                          \Pi ident:\beta. pure\_arg
                          term \supset pure\_arg
                          pure\_ret
pure\_ret
                                                             pure return types
                          \Sigma ident:\beta. pure\_ret
                         term \land pure\_ret
\mathcal{C}
                                                             computational var env
                  ::=
                         C, ident: \beta
\mathcal{L}
                                                             logical var env
                         \overline{\mathcal{L}_{i}}^{i}
\mathcal{L}, ident: \beta
\Phi
                                                             constraints env
```

```
\overline{\Phi_i}^i
\mathcal R
                                                                                                                                                                  resources env
                                            \mathcal{R}, ident:res
                                                                                                                                                                 substitutions
\sigma, \psi
                                           spine\_elem/ident, \sigma
                                          term/ident, \sigma
                                            \overline{\sigma_i}^{\ i}
                                            \sigma(\psi)
                                                                                                                                                     Μ
                                                                                                                                                                        apply \sigma to all elements in \psi
typing
                                            \mathtt{smt}\left(\Phi\Rightarrow term\right)
                                          ident:\beta \in \mathcal{C}
                                          ident:\beta \in \mathcal{L}
                                          \operatorname{struct} tag \ \& \ \overline{member_i {:} 	au_i}^i \in \operatorname{\mathsf{Globals}}
                                         lpha \equiv \overline{x_i : eta_i}^i \mapsto res \in 	ext{Globals} \ \overline{\mathcal{C}_i ; \mathcal{L}_i ; \Phi_i \vdash mem\_val_i \Rightarrow mem eta_i}^i \ \overline{\mathcal{C}_j ; \mathcal{L}_j \mid \overline{ident_{ij}}^i \vdash 	ext{guarded } (form_j)}^j \ \overline{\mathcal{C}_j ; \mathcal{L}_j \mid \overline{ident_{ij}}^i \vdash 	ext{vconstr} (form_j)}^j \ \overline{\mathcal{C}_j ; \mathcal{L}_j \mid \overline{ident_{ij}}^i \vdash 	ext{vconstr} (form_j)}^j}
                                                                                                                                                                        recursive resource predicate
                                                                                                                                                                        dependent on memory object model
                                           ident \in C; \mathcal{L}
                                            ident \in \overline{ident_i}^i
 opsem
                                            \forall i < j. \ \mathsf{not} \ (pattern_i = pval \leadsto \sigma_i)
                                            \mathtt{fresh}\left(mem\_ptr
ight)
                                            term
                                            pval:\beta
```

```
formula
                                     judgement
                                     typing
                                     opsem
                                     term \equiv term'
                                     name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in Globals
                                     name: arg \equiv \overline{x_i}^i \mapsto texpr \in Globals
heap, h, f
                                                                                                                     heaps
                                h + \{points\_to\} \\ h + f
                                                                                                                      [O] convenient for the soundness proof
wf_jtyp
                                   C; \mathcal{L} \vdash \text{guarded_e}(expr\_form)
                              \mid \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{guarded}(expr\_form)
                              \mid \quad \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded} (form)
                              \mathcal{C}; \mathcal{L} \vdash \text{vconstr\_e}(expr\_form)
                              \mid \quad \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(expr\_form) \\ \mid \quad \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(form)
                                     C; \mathcal{L} \vdash \text{well\_formed}(array\_prop\_form)
lemma\_jtype
res\_jtype
```

object_value_jtype ::=

 $\mid \quad \mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathtt{obj} \ eta$

 $pval_jtype$::=

 $| \quad \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$

 $spine_jtype \qquad \qquad ::= \\ | \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$

 $\begin{array}{ll} pexpr_jtype & ::= \\ & \mid \quad \mathcal{C}; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. \ term \end{array}$

 $comp_pattern_jtype ::= \\ | pattern: \beta \leadsto \mathcal{C} \text{ with } term$

 $| ident_or_pattern:eta \leadsto \mathcal{C} ext{ with } term$

 $\Phi \vdash res \text{ as } res_pattern \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

 $\Phi \vdash res_pattern:res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

 $ret_pattern_jtype \qquad ::= \\ | \quad \Phi \vdash \overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

 $tpval_jtype ::=$

 $| \mathcal{C}; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term$

 $tpexpr_jtype \qquad ::= \\ | \quad \mathcal{C}; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. \ term \\$

 $action_jtype ::=$

```
| \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_action \Rightarrow ret
```

$$memop_jtype ::=$$

$$| \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret$$

$$tval_jtype$$
 ::=

$$| \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$$

$$seq_expr_jtype$$
 ::=

$$\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret$$

$$is_expr_jtype$$
 ::=

$$| \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$$

$$texpr_jtype$$
 ::=

$$| \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret$$

$$| \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret$$

$$| \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret$$

$$subs_jtype ::=$$

$$| pattern = pval \leadsto \sigma$$

$$| ident_or_pattern = pval \leadsto \sigma$$

$$| res_pattern = res_term \leadsto \sigma$$

$$| ret_pattern_i = spine_elem_i^{\ i} \leadsto \sigma$$

$$| x_i = spine_elem_i^{\ i} :: arg \gg \sigma; ret$$

$$pure_opsem_jtype$$
 ::=

$$\langle pexpr \rangle \longrightarrow \langle pexpr' \rangle$$

$$\langle pexpr \rangle \longrightarrow \langle tpexpr: (y:\beta. \ term) \rangle$$

$$\langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle$$

```
opsem\_jtype
                                                                  \langle h; seq\_expr \rangle \longrightarrow \langle h'; texpr:ret \rangle
                                                     \langle h; seq\_expr\rangle \longrightarrow \langle h; texpr:ret \\ \langle h; seq\_texpr\rangle \longrightarrow \langle h'; texpr\rangle \\ \langle h; mem\_op\rangle \longrightarrow \langle h'; tval\rangle \\ \langle h; mem\_action\rangle \longrightarrow \langle h'; tval\rangle \\ \langle h; is\_expr\rangle \longrightarrow \langle h'; is\_expr'\rangle \\ \langle h; is\_texpr\rangle \longrightarrow \langle h'; texpr\rangle \\ \langle h; texpr\rangle \longrightarrow \langle h'; texpr'\rangle 
\mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(\mathit{expr\_form}\right)
                                                                                                                                                    ident \in C; \mathcal{L}
                                                                                                                                   \frac{\text{---,--}}{\mathcal{C};\mathcal{L}\vdash \mathtt{guarded\_e}\left(ident\right)}\quad \text{Wf\_GUARDED\_EXPR\_EVAR}
                                                                                                                \frac{ident \, \in \, \mathcal{C}; \mathcal{L}}{\mathcal{C}; \mathcal{L} \vdash \, \mathtt{guarded\_e} \, (z \times ident)}
                                                                                                                                                                                                       Wf_Guarded_Eexpr_Scaled_EVar
                                                                                                                            \mathcal{C}; \mathcal{L} \vdash \mathsf{guarded\_e}\left(\mathit{expr\_form}_1\right)
                                                                                                                            \mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(\mathit{expr\_form}_2\right)
                                                                                                                                                                                                                                             Wf_Guarded_Eexpr_Plus
                                                                                                        \overline{\mathcal{C}:\mathcal{L}\vdash \mathtt{guarded\_e}\left(\mathit{expr\_form}_1+\mathit{expr\_form}_2\right)}
 |C; \mathcal{L}| |\overline{ident_i}^i| \vdash \text{guarded}(expr\_form)
                                                                                                                            \mathcal{C}; \mathcal{L} \vdash \mathtt{guarded\_e}\left(\mathit{expr\_form}\right)
                                                                                                                                                                                                                          Wf_Guarded_Expr_Eexpr
                                                                                                                 \overline{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded}(expr\_form)}
                                                                                                                         \frac{ident \in \overline{ident_i}^i}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded}\left(ident\right)} \quad \text{Wf\_GUARDED\_EXPR\_UVAR}
```

 $C; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{guarded}(form)$

 $C; \mathcal{L} \vdash \text{vconstr_e}(expr_form)$

$$\frac{ident \in \mathcal{C}; \mathcal{L}}{\mathcal{C}; \mathcal{L} \vdash \mathtt{vconstr_e}(ident)} \quad \text{Wf_Vconstr_Eexpr_EVar}$$

$$\frac{ident \in \mathcal{C}; \mathcal{L}}{\mathcal{C}; \mathcal{L} \vdash \mathsf{vconstr_E}(z \times ident)} \quad \text{Wf_Vconstr_Eexpr_Scaled_EVar}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L} \vdash \text{ vconstr_e} \left(expr_form_1 \right) \\ \mathcal{C}; \mathcal{L} \vdash \text{ vconstr_e} \left(expr_form_2 \right) \\ \hline \mathcal{C}; \mathcal{L} \vdash \text{ vconstr_e} \left(expr_form_1 + expr_form_2 \right) \end{array} \\ \text{WF_VCONSTR_EEXPR_PLUS}$$

$$\begin{split} & ident \in \mathcal{C}; \mathcal{L} \\ & \frac{\mathcal{C}; \mathcal{L} \vdash \text{vconstr_e} \left(expr_form \right)}{\mathcal{C}; \mathcal{L} \vdash \text{vconstr_e} \left(ident [expr_form] \right)} \quad \text{Wf_Vconstr_Eexpr_Index} \end{split}$$

 $C; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{vconstr}(expr_form)$

$$\frac{\mathcal{C}; \mathcal{L} \vdash \mathsf{vconstr_e}\left(\mathit{expr_form}\right)}{\mathcal{C}; \mathcal{L} \mid \overline{\mathit{ident}_i}^i \vdash \mathsf{vconstr}\left(\mathit{expr_form}\right)} \quad \mathsf{WF_VCONSTR_EXPR_EXPR}$$

$$\frac{ident \in \mathcal{C}; \mathcal{L}}{ident' \in \overline{ident_i}^i}$$

$$\frac{ident' \in \overline{ident_i}^i}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(ident[ident'])}$$
WF_VCONSTR_EXPR_INDEX_UVAR

 $\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathtt{vconstr}(form)$

$$\begin{array}{c|c} \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(expr_form) \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(expr_form') \\ \hline \mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(expr_form \leq expr_form') \end{array} \\ \end{array} \\ \text{WF_VCONSTR_LEQ}$$

$$\frac{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr}(expr_form)}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr}(expr_form')} \qquad \text{Wf_Vconstr_Eq} \\ \frac{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr}(expr_form = expr_form')}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr}(expr_form = expr_form')}$$

$$\frac{\overline{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash vconstr(form_j)^j}}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash vconstr(\bigvee(\overline{form_j}^j))} \quad WF_VCONSTR_OR$$

$$\frac{\overline{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(form_j)}^j}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \text{vconstr}(\bigwedge(\overline{form_j}^j))} \quad \text{Wf_VCONSTR_AND}$$

 $\mathcal{C}; \mathcal{L} \vdash \mathtt{well_formed}\left(\mathit{array_prop_form}\right)$

$$\frac{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{guarded}\left(form_1\right)}{\mathcal{C}; \mathcal{L} \mid \overline{ident_i}^i \vdash \mathsf{vconstr}\left(form_2\right)} \quad \text{Wf_Apf_BASE}}{\mathcal{C}; \mathcal{L} \vdash \mathsf{well_formed}\left(\forall \overline{ident_i}^i . form_1 \rightarrow form_2\right)}$$

 $\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret$

$$\frac{}{::ret \leadsto \cdot; \cdot; \cdot; \cdot \mid ret} \quad Arg_Env_Ret$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \Pi \, x : \beta. \, arg \leadsto \mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Comp}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \forall x : \beta. arg \leadsto \mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Log}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{\overline{x_i}^i :: term \supset arg \leadsto \mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \mid ret} \quad \text{Arg_Env_Phi}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: res \multimap arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x: res \mid ret} \quad \text{Arg_Env_Res}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\frac{}{\cdot;\cdot;\cdot;\cdot\sqsubseteq\cdot;\cdot;\cdot;}\quad \text{Weak_Empty}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}, x : \beta; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x : \beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Cons_Phi}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x : res \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res} \quad \text{Weak_Cons_Res}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x:\beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Skip_Phi}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma): (C'; \mathcal{L}'; \Phi'; \mathcal{R}')$$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash(\cdot):(\cdot;\cdot;\cdot;\cdot)} \quad \text{Ty_Subs_Empty}$$

$$\begin{array}{ll} \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}') \\ \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta \\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}',x:\beta;\mathcal{L}';\Phi';\mathcal{R}') \end{array} \quad \text{Ty_Subs_Cons_Comp} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')}{\mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta} \quad \text{Ty_Subs_Cons_Log} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}';\mathcal{L}',x:\beta;\Phi';\mathcal{R}')}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(pval/x,\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')} \quad \text{Ty_Subs_Cons_Log} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi',term;\mathcal{R}')} \quad \text{Ty_Subs_Cons_Phi} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi',\mathcal{R}')}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')} \quad \text{Ty_Subs_Cons_Res} \\ \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash(\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}')}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R},\mathcal{R}_1\vdash(res_term/x,\sigma):(\mathcal{C}';\mathcal{L}';\Phi';\mathcal{R}',x:res)} \quad \text{Ty_Subs_Cons_Res} \\ \end{array}$$

 $\Phi \vdash res \equiv res'$

$$\overline{\Phi \vdash \mathtt{emp} \ \equiv \ \mathtt{emp}} \quad \mathrm{TY_RES_EQ_EMP}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(term_{1}=term_{1}'\right)\wedge\left(term_{2}=term_{2}'\right)\right)}{\Phi\vdash term_{1}\overset{init}{\mapsto}_{\tau}term_{2}\equiv term_{1}'\overset{init}{\mapsto}_{\tau}term_{2}'} \quad \text{Ty_Res_Eq_PointsTo}$$

$$\begin{array}{ccc} \Phi \vdash res_1 \equiv res_1' \\ \Phi \vdash res_2 \equiv res_2' \\ \hline \Phi \vdash res_1 * res_2 \equiv res_1' * res_2' \end{array} \quad \text{TY_RES_EQ_SEPCONJ}$$

$$\frac{\Phi \vdash res \equiv res'}{\Phi \vdash \exists ident: \beta. \ res \equiv \exists ident: \beta. \ res'} \quad \text{TY_RES_EQ_EXISTS}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow(\operatorname{term}\to\operatorname{term}')\wedge(\operatorname{term}'\to\operatorname{term})\right)}{\Phi\vdash\operatorname{ters}\cong\operatorname{res}'} \qquad \operatorname{Ty.Res_Eq.Term}$$

$$\frac{\Phi\vdash\operatorname{term}\wedge\operatorname{res}\cong\operatorname{term}'\wedge\operatorname{res}'}{\Phi\vdash\operatorname{term}_{1}\to\operatorname{term}_{2})\wedge(\operatorname{term}_{2}\to\operatorname{term}_{1}))} \qquad \operatorname{term}_{1}\oplus\operatorname{term}_{2}\oplus\operatorname{term}_$$

$$\begin{split} & \text{smt} \; (\Phi \Rightarrow term) \\ & \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res \\ & \quad \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow term \land res \end{split} \quad \text{Ty_Res_Conj}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow pval/y, \cdot (res)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \operatorname{pack} (pval, res_term) \Leftarrow \exists \, y : \beta. \, res} \end{split} \quad \text{TY_RES_PACK} \end{split}$$

$$\begin{split} \alpha &\equiv \overline{x_i {:} \beta_i}^i \mapsto res \in \mathtt{Globals} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta_i}^i \\ \Phi \vdash res' &= \mathtt{strip_ifs}\left(\overline{pval_i/x_i, \cdot}^i(res)\right) \\ \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res'} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{fold}\left(res_term\right) \Leftarrow \alpha(\overline{pval_i}^i)} \end{split} \quad \mathtt{TY_RES_FOLD} \end{split}$$

 $h:\mathcal{R}$

$$\frac{h:\mathcal{R}}{\vdots:::\mathcal{R}' \vdash pt \Leftarrow pt}$$

$$\frac{h:\mathcal{R}}{h + \{pt\}:\mathcal{R},\mathcal{R}'}$$

$$\text{TY_HEAP_POINTSTO}$$

 $\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathtt{obj}\,\beta$

$$\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash mem_int} \Rightarrow \mathtt{objinteger} \quad \mathrm{TY_PVAL_OBJ_INT}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash mem_ptr \Rightarrow \mathtt{objloc}} \quad \mathsf{TY_PVAL_OBJ_PTR}$$

$$\frac{\overline{\mathcal{C};\mathcal{L};\Phi \vdash loaded_value_i \Rightarrow \beta}^i}{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{array}\left(\overline{loaded_value_i}^i\right) \Rightarrow \mathtt{obj}\,\mathtt{array}\,\beta} \quad \mathsf{TY_PVAL_OBJ_ARR}$$

$$\frac{\texttt{struct} \, tag \, \& \, \overline{member_i : \tau_i}^{\, i} \, \in \, \texttt{Globals}}{\overline{\mathcal{C}}; \mathcal{L}; \Phi \vdash mem_val_i \, \Rightarrow \, \texttt{mem} \, \beta_{\tau_i}^{\, i}}}{\mathcal{C}; \mathcal{L}; \Phi \vdash (\, \texttt{struct} \, tag) \{\, \overline{.member_i : \tau_i = mem_val_i}^{\, i} \, \} \, \Rightarrow \, \texttt{obj} \, \texttt{struct} \, tag} \quad \text{Ty_Pval_Obj_Struct}$$

 $C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$

$$\frac{x:\beta \in \mathcal{C}}{\mathcal{C}:\mathcal{L};\Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Comp}$$

$$\frac{x:\beta \in \mathcal{L}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathsf{obj} \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \beta} \quad \text{Ty_Pval_Obj}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathtt{obj}\,\beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{specified}\,object_value \Rightarrow \beta} \quad \mathsf{TY_PVAL_LOADED}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{Unit} \Rightarrow \mathtt{unit}} \quad \mathtt{TY_PVAL_UNIT}$$

$$\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{True} \Rightarrow \mathtt{bool}} \quad \mathtt{TY_PVAL_TRUE}$$

$$\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{False} \Rightarrow \mathtt{bool}} \quad \mathtt{TY_PVAL_FALSE}$$

$$\frac{\overline{C}; \mathcal{L}; \Phi \vdash value_i \Rightarrow \beta^i}{C; \mathcal{L}; \Phi \vdash \beta[\overline{value_i}^i] \Rightarrow \mathbf{list} \beta} \quad \text{Ty-Pval_List}$$

$$\frac{\overline{C}; \mathcal{L}; \Phi \vdash value_i \Rightarrow \beta_i^i}{C; \mathcal{L}; \Phi \vdash (\overline{value_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \text{Ty-Pval_Tuple}$$

$$\frac{\text{smt} (\Phi \Rightarrow \text{false})}{C; \mathcal{L}; \Phi \vdash \text{error} (string, pval) \Rightarrow \beta} \quad \text{Ty-Pval_Error}$$

$$\overline{C}; \mathcal{L}; \Phi \vdash \text{nil} \beta() \Rightarrow \mathbf{list} \beta \quad \text{Ty-Pval_Ctor_Nil}$$

$$\frac{C}{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \beta \quad \text{Ty-Pval_Ctor_Cons}$$

$$\frac{C}{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathbf{list} \beta \quad \text{Ty-Pval_Ctor_Cons}$$

$$\frac{C}{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \overline{\beta_i}^i \quad \text{Ty-Pval_Ctor_Tuple}$$

$$\frac{\overline{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \overline{\beta_i}^i}{C; \mathcal{L}; \Phi \vdash \text{Tuple} (\overline{pval_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \text{Ty-Pval_Ctor_Array}$$

$$\frac{\overline{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \overline{\beta_i}^i}{C; \mathcal{L}; \Phi \vdash \text{pval} \Rightarrow \beta} \quad \text{Ty-Pval_Ctor_Array}$$

$$\frac{C; \mathcal{L}; \Phi \vdash \text{pval} \Rightarrow \beta}{C; \mathcal{L}; \Phi \vdash \text{pval} \Rightarrow \beta} \quad \text{Ty-Pval_Ctor_Specified}$$

$$\frac{c; \mathcal{L}; \Phi \vdash \text{pval} \Rightarrow \beta}{C; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \beta_{\tau_i}^i} \in \text{Globals}$$

$$\frac{\overline{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \beta_{\tau_i}^i}{C; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \beta_{\tau_i}^i} \Rightarrow \text{struct} tag \\ \frac{\overline{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \beta_{\tau_i}^i}{\overline{C}; \mathcal{L}; \Phi \vdash \text{pval}_i \Rightarrow \beta_{\tau_i}^i} \Rightarrow \text{struct} tag}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash :: ret \gg \cdot; ret} \quad \text{Ty_Spine_Empty}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \Pi \, x : \beta. \, arg \gg pval/x, \sigma; ret \end{array} \quad \text{Ty_Spine_Comp}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash x = pval, \overline{x_i = spine_elem_i}^i :: \forall \, x : \beta. \, arg \gg pval/x, \sigma; ret \end{array} \quad \text{TY_Spine_Log}$$

$$\begin{aligned} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash \underbrace{\mathit{res_term} \Leftarrow \mathit{res}}_{i} \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \underbrace{\overline{x_i = \mathit{spine_elem}_i}^i :: \mathit{arg} \gg \sigma; \mathit{ret}}_{i} \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash x = \mathit{res_term}, \underbrace{\overline{x_i = \mathit{spine_elem}_i}^i :: \mathit{res} \multimap \mathit{arg} \gg \mathit{res_term}/x, \sigma; \mathit{ret}} \end{aligned}$$
 Ty_Spine_Res

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow term\right)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_{i}=spine_elem_{i}}^{i}::arg\gg\sigma;ret} \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_{i}=spine_elem_{i}}^{i}::arg\gg\sigma;ret}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_{i}=spine_elem_{i}}^{i}::term\supset arg\gg\sigma;ret}$$
 TY_SPINE_PHI

 $C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow y: \beta. \ y = pval} \quad \text{TY_PE_VAL}$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \text{loc} \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \text{integer} \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \text{array_shift} \left(pval_1, \tau, pval_2\right) \Rightarrow y : \text{loc.} \ y = pval_1 +_{\text{ptr}} \left(pval_2 \times \text{size_of}(\tau)\right) \end{split} \quad \text{TY_PE_ARRAY_SHIFT}$$

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{loc}$$
 $\mathtt{struct} \ tag \ \& \ \overline{member_i : au_i}^i \in \mathtt{Globals}$

Ty_PE_Member_Shift

 $\overline{\mathcal{C};\mathcal{L};\Phi} \vdash \mathtt{member_shift}(pval,tag,member_i) \Rightarrow y:\mathtt{loc}.\ y = pval +_{\mathtt{ptr}} \mathtt{offset_of}_{tag}(member_i)$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{not} \, (pval) \Rightarrow y \texttt{:bool}. \, y = \neg \, pval} \quad \texttt{TY_PE_NOT}$$

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{integer}$$

 $C; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}$

 $\overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{arith} \ pval_2} \Rightarrow y : \mathtt{integer}. \ y = (pval_1 \ binop_{arith} \ pval_2)$

TY_PE_ARITH_BINOP

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{integer}$$
 $\mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}$

TY_PE_REL_BINOP

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow bool$$

 $\mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow bool$

 $\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{bool} \ pval_2 \Rightarrow y : bool. \ y = (pval_1 \ binop_{bool} \ pval_2)}{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{bool} \ pval_2 \Rightarrow y : bool. \ y = (pval_1 \ binop_{bool} \ pval_2)}$

TY_PE_BOOL_BINOP

$$\begin{array}{l} \textit{name:pure_arg} \equiv \overline{x_i}^i \mapsto \textit{tpexpr} \in \texttt{Globals} \\ \underline{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \overline{x_i = pval_i}^i :: pure_arg \gg \sigma; \Sigma \ y : \beta. \ term \land \mathtt{I}} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash name(\overline{pval_i}^i) \Rightarrow y : \beta. \ \sigma(term)} \end{array} \quad \text{TY_PE_CALL}$$

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool} \\ \texttt{smt} \ (\Phi \Rightarrow pval)$$

 $\frac{\texttt{smt} \ (\Psi \Rightarrow pval)}{\mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{assert_undef} \ (pval, \ UB_name) \Rightarrow y \text{:unit.} \ y = \texttt{unit}} \quad \texttt{Ty_PE_Assert_UNDEF}$

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool}$$

 $\overline{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{bool_to_integer}\left(pval\right)\Rightarrow y\mathtt{:integer}.\ y=\mathtt{if}\ pval\ \mathtt{then}\ 1\ \mathtt{else}\ 0}$

Ty_PE_Bool_To_Integer

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval &\Rightarrow \mathtt{integer} \\ abbrev_1 &\equiv \mathtt{max_int}_\tau - \mathtt{min_int}_\tau + 1 \\ abbrev_2 &\equiv pval\,\mathtt{rem_f}\,\,abbrev_1 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{wrapI}\,(\tau, pval) \Rightarrow y : \beta.\,\, y = \mathtt{if}\,\,abbrev_2 \leq \mathtt{max_int}_\tau\,\mathtt{then}\,\,abbrev_2\,\mathtt{else}\,\,abbrev_2 - abbrev_1 \end{split}$$

Ty_PE_WrapI

 $pattern:eta \leadsto \mathcal{C}$ with term

$$\underline{\hspace{1cm}}$$
: β : $\beta \leadsto \cdot with_-$ TY_PAT_COMP_NO_SYM_ANNOT

$$\overline{x{:}\beta{:}\beta \leadsto \cdot, x{:}\beta \, \text{with} \, x} \quad \text{Ty_Pat_Comp_Sym_Annot}$$

$$\frac{1}{\text{Nil }\beta(\cdot): \text{list }\beta \leadsto \cdot \text{with nil}} \quad \text{TY_PAT_COMP_NIL}$$

$$\frac{pattern_1:\beta \leadsto \mathcal{C}_1 \text{ with } term_1}{pattern_2: \texttt{list} \, \beta \leadsto \mathcal{C}_2 \text{ with } term_2} \\ \frac{Cons(pattern_1, pattern_2): \texttt{list} \, \beta \leadsto \mathcal{C}_1, \mathcal{C}_2 \text{ with } term_1 :: term_2}{\texttt{TY_PAT_COMP_CONS}}$$

$$\frac{\overline{pattern_i:}\beta_i \leadsto \overline{C_i \text{ with } term_i}^i}{\text{Tuple}(\overline{pattern_i}^i): \overline{\beta_i}^i \leadsto \overline{C_i}^i \text{ with } (\overline{term_i}^i)} \quad \text{Ty_Pat_Comp_Tuple}$$

$$\frac{\overline{pattern_i:\beta \leadsto \mathcal{C}_i \, \mathtt{with} \, term_i}^i}{\operatorname{Array}(\, \overline{pattern_i}^i\,) : \operatorname{array}\beta \leadsto \overline{\mathcal{C}_i}^i \, \mathtt{with} \, [|\,\, \overline{term_i}^i\,|]} \quad \text{Ty_Pat_Comp_Array}$$

$$\frac{pattern: \beta \leadsto \mathcal{C} \, \mathtt{with} \, term}{\mathtt{Specified}(pattern): \beta \leadsto \mathcal{C} \, \mathtt{with} \, term} \quad \mathsf{TY_PAT_COMP_SPECIFIED}$$

 $ident_or_pattern{:}\beta \leadsto \mathcal{C} \, \mathtt{with} \, term$

$$\frac{pattern:\beta \leadsto \mathcal{C} \text{ with } term}{pattern:\beta \leadsto \mathcal{C} \text{ with } term} \quad \text{Ty_Pat_Sym_Or_Pattern_Pattern}$$

$$\Phi \vdash res' = \mathtt{strip_ifs}(res)$$

$$\overline{\Phi \vdash \mathtt{emp} = \mathtt{strip_ifs}\,(\mathtt{emp})} \quad \mathrm{TY_PAT_RES_STRIPIFS_EMPTY}$$

$$\overline{\Phi \vdash pt = \mathtt{strip_ifs}\left(pt\right)} \quad \text{TY_PAT_RES_STRIPIFS_POINTSTO}$$

$$\frac{}{\Phi \vdash res_1 * res_2 = \mathtt{strip_ifs}(res_1 * res_2)} \quad \text{TY_PAT_RES_STRIPIFS_SEPCONJ}$$

$$\overline{\Phi \vdash \exists x : \beta. \ res = \text{strip_ifs} (\exists x : \beta. \ res)} \quad \text{TY_PAT_RES_STRIPIFS_EXISTS}$$

$$\frac{}{\Phi \vdash term \land res = \mathtt{strip_ifs} (term \land res)} \quad \text{TY_PAT_RES_STRIPIFS_TERMCONJ}$$

$$\frac{\texttt{smt}\,(\Phi\Rightarrow term)}{\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(res_1')} \\ \frac{\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(\texttt{if}\,term\,\texttt{then}\,res_1\,\texttt{else}\,res_2)}{\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(\texttt{if}\,term\,\texttt{then}\,res_1\,\texttt{else}\,res_2)}$$

$$\frac{\texttt{smt}\,(\Phi \Rightarrow \neg \textit{term})}{\Phi \vdash \textit{res}_2' = \,\texttt{strip_ifs}\,(\textit{res}_2)} \\ \frac{\Phi \vdash \textit{res}_2' = \,\texttt{strip_ifs}\,(\textit{if}\,\textit{term}\,\texttt{then}\,\textit{res}_1\,\texttt{else}\,\textit{res}_2)}{\Phi \vdash \textit{res}_2' = \,\texttt{strip_ifs}\,(\textit{if}\,\textit{term}\,\texttt{then}\,\textit{res}_1\,\texttt{else}\,\textit{res}_2)}$$

 $\overline{\Phi \vdash \text{if } term \text{ then } res_1 \text{ else } res_2 = \text{strip_ifs} (\text{if } term \text{ then } res_1 \text{ else } res_2)}$

Ty_Pat_Res_StripIfs_UnderDet

 $\frac{}{\Phi \vdash \alpha(\overline{\textit{pval}_i}^i) = \mathsf{strip_ifs}(\alpha(\overline{\textit{pval}_i}^i))} \quad \text{Ty_Pat_Res_StripIfs_Pred}$

 $\Phi \vdash \mathit{res} \ \mathsf{as} \ \mathit{res_pattern} \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

 $\overline{\Phi \vdash \mathtt{emp \, as \, emp \, \leadsto \, \cdot; \cdot; \cdot}} \quad \text{TY_PAT_RES_MATCH_EMPTY}$

 $\overline{\Phi \vdash \mathit{res} \; \mathsf{as} \; r \leadsto \cdot; \cdot; \cdot, r {:} \mathit{res}} \quad \mathsf{TY_PAT_RES_MATCH_VAR}$

 $\Phi \vdash res_pattern_1:res_1 \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1$

 $\Phi \vdash res_pattern_2 : res_2 \leadsto \mathcal{L}_2; \Phi_2; \mathcal{R}_2$

 $\frac{\Psi \vdash \mathit{res_pattern}_2 : \mathit{res}_2 \leadsto \mathcal{L}_2; \Psi_2; \mathcal{K}_2}{\Phi \vdash \mathit{res}_1 * \mathit{res}_2 \mathsf{as} \left\langle \mathit{res_pattern}_1, \mathit{res_pattern}_2 \right\rangle \leadsto \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2} \quad \text{TY_PAT_RES_MATCH_SEPCONJ}$

 $\frac{1}{\Phi \vdash term \land res \text{ as } res_pattern \leadsto \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Ty_Pat_Res_Match_Conj}$ $\Phi \vdash res_pattern:res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

 $\frac{\Phi \vdash \mathit{res_pattern} : x/y, \cdot (\mathit{res}) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \exists \, y : \beta. \, \mathit{res} \, \mathsf{as} \, \mathsf{pack} \, (x, \mathit{res_pattern}) \leadsto \mathcal{L}', \, x : \beta; \Phi'; \mathcal{R}'}$ Ty_Pat_Res_Match_Pack

$$\begin{array}{l} \alpha \equiv \overline{x_i :} \overline{\beta_i}^i \mapsto res \in \texttt{Globals} \\ \frac{\Phi \vdash res_pattern :} \overline{pval_i/x_i, \cdot}^i (res) \leadsto \mathcal{L}' ; \Phi' ; \mathcal{R}'}{\Phi \vdash \alpha (\overline{pval_i}^i) \text{ as fold } (res_pattern) \leadsto \mathcal{L}' ; \Phi' ; \mathcal{R}'} \end{array} \quad \text{Ty_Pat_Res_Match_Fold}$$

 $\Phi \vdash \mathit{res_pattern} : \mathit{res} \leadsto \mathcal{L}' ; \Phi' ; \mathcal{R}'$

$$\frac{\Phi \vdash \mathit{res'} = \mathsf{strip_ifs}\,(\mathit{res})}{\Phi \vdash \mathit{res_pattern} : \mathit{res} \leadsto \mathcal{L'}; \Phi'; \mathcal{R'}} \quad \text{TY_PAT_RES_STRIP_IFS}$$

 $\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \! \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\frac{}{\Phi \vdash : \texttt{I} \leadsto \cdot ; \cdot ; \cdot ; \cdot } \cdot \quad \text{TY_PAT_RET_EMPTY}$$

$$\frac{ident_or_pattern:\beta \leadsto \mathcal{C}_1 \, \text{with} \, term_1}{\Phi \vdash \overline{ret_pattern_i}^i : term_1/y, \cdot (ret) \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} {\Phi \vdash \mathsf{comp} \, ident_or_pattern, \, \overline{ret_pattern_i}^i : \Sigma \, y : \beta. \, ret \leadsto \mathcal{C}_1, \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \quad \text{TY_PAT_RET_COMP}$$

$$\frac{\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \log y, \, \overline{\mathit{ret_pattern}_i}^i : \exists \, y : \beta. \, \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}', y : \beta; \Phi'; \mathcal{R}'} \quad \text{Ty_Pat_Ret_Log}$$

$$\frac{\Phi \vdash \mathit{res_pattern} : \mathit{res} \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1}{\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{\Phi \vdash \mathit{res_pattern}, \overline{\mathit{ret_pattern}_i}^i : \mathit{res} \otimes \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2}}{\Phi \vdash \mathit{res_pattern}, \overline{\mathit{ret_pattern}_i}^i : \mathit{res} \otimes \mathit{ret} \leadsto \mathcal{C}_2; \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2}}$$

$$\frac{\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\Phi \vdash \overline{\mathit{ret_pattern}_i}^i : \mathit{term} \land \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi', \mathit{term}; \mathcal{R}'} \quad \mathsf{TY_PAT_RET_PHI}$$

 $C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. \overline{term}$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{undef}\ \mathit{UB_name}\Leftarrow\mathit{y}{:}\beta.\mathit{term}}\quad \mathsf{TY_TPVAL_UNDEF}$$

$$\begin{array}{l} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \underline{\text{smt} \left(\Phi \Rightarrow pval/y, \cdot (term)\right)} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{done } pval \Leftarrow y:\beta. \ term} \end{array} \quad \text{Ty_TPVal_Done} \\ \end{array}$$

 $C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool} \\ \mathcal{C}; \mathcal{L}; \Phi, pval &= \texttt{true} \vdash tpexpr_1 \Leftarrow y : \beta. \ term \\ \mathcal{C}; \mathcal{L}; \Phi, pval &= \texttt{false} \vdash tpexpr_2 \Leftarrow y : \beta. \ term \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2 \Leftarrow y : \beta. \ term \end{split} \qquad \texttt{TY_TPE_IF}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pexpr \Rightarrow y_1 {:} \beta_1. \ term_1 \\ & ident_or_pattern {:} \beta_1 \leadsto \mathcal{C}_1 \ \text{with} \ term \\ & \frac{\mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term/y_1, \cdot (term_1) \vdash tpexpr \Leftarrow y_2 {:} \beta_2. \ term_2}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{let} \ ident_or_pattern = pexpr \ \mathtt{in} \ tpexpr \Leftarrow y_2 {:} \beta_2. \ term_2} \end{split} \quad \mathtt{TY_TPE_LET} \end{split}$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash tpexpr_1 &\Leftarrow y_1 : \beta_1. \ term_1 \\ ident_or_pattern : \beta_1 &\leadsto \mathcal{C}_1 \ \mathtt{with} \ term \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term/y_1, \cdot (term_1) \vdash tpexpr &\Leftarrow y_2 : \beta_2. \ term_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{let} \ ident_or_pattern : (y_1 : \beta_1. \ term_1) &= tpexpr_1 \ \mathtt{in} \ tpexpr_2 &\Leftarrow y_2 : \beta_2. \ term_2 \end{split} \quad \texttt{TY_TPE_LETT}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_{1}}{\underbrace{\frac{pattern_{i}: \beta_{1} \leadsto \mathcal{C}_{i} \, \text{with} \, term_{i}}{\mathcal{C}, \mathcal{C}_{i}; \mathcal{L}; \Phi, term_{i} = pval \vdash tpexpr_{i} \Leftarrow y_{2}: \beta_{2}. \, term_{2}}^{i}}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathsf{case} \, pval \, \mathsf{of} \, \overline{\mid pattern_{i} \Rightarrow tpexpr_{i}}^{i} \, \mathsf{end} \Leftarrow y_{2}: \beta_{2}. \, term_{2}}}$$
 TY_TPE_CASE

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{create}\,(pval, \tau) \Rightarrow \Sigma\,y_p \mathtt{:loc.\,representable}\,(\tau *, y_p) \land \mathtt{alignedI}\,(pval, y_p) \land \exists\,y \mathtt{:} \beta_\tau.\,y_p \overset{\times}{\mapsto}_\tau y \otimes \mathtt{I}} \quad \mathsf{TY_ACTION_CREATE}$$

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\mathcal{C}: \mathcal{L}: \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                       \operatorname{smt} (\Phi \Rightarrow pval_0 = pval_1)
                                                 \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash pval_1\overset{\checkmark}{\mapsto}_{\tau}\;pval_2 \Leftarrow pval_1\overset{\checkmark}{\mapsto}_{\tau}\;pval_2}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{load}\left(\tau,pval_0, \_,pval_1\overset{\checkmark}{\mapsto}_{\tau}\;pval_2\right)\Rightarrow \Sigma\;y:\beta_{\tau}.\;y=pval_2\wedge pval_1\overset{\checkmark}{\mapsto}_{\tau}\;pval_2\otimes \mathtt{I}}
                                                                                                                                                                                                                                                                    TY_ACTION_LOAD
                                                                                                                \mathcal{C}; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                                \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \beta_{\tau}
                                                                                                                 \operatorname{smt}(\Phi \Rightarrow \operatorname{representable}(\tau, pval_1))
                                                                                                                \operatorname{smt}(\Phi \Rightarrow pval_2 = pval_0)
                                                                                                                \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_2 \mapsto_{\tau} \bot \Leftarrow pval_2 \mapsto_{\tau} \bot
                                                      \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{store} \xrightarrow{(\neg, \tau, pval_0, pval_1, \neg, pval_2 \mapsto_{\tau} \neg)} \Rightarrow \Sigma \neg \mathtt{:unit.} \ pval_2 \xrightarrow{\checkmark} pval_1 \otimes \mathtt{I}
                                                                                                                                                                                                                                                                                 Ty_Action_Store
                                                                                                         C; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                         \operatorname{smt} (\Phi \Rightarrow pval_0 = pval_1)
                                                                          \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \mapsto_{\tau_-} \Leftarrow pval_1 \mapsto_{\tau_-}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{kill} \left( \text{static} \ \tau, pval_0, pval_1 \mapsto_{\tau_-} \right) \Rightarrow \Sigma_-: \text{unit. I}} \quad \text{TY\_ACTION\_KILL\_STATIC}
C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_op \Rightarrow ret
                                                                                                                              \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathsf{loc}
                                                                                                                              C; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                                                                 TY_MEMOP_REL_BINOP
                                                          \overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash pval_1\ binop_{rel}\ pval_2\Rightarrow\Sigma\ y\text{:bool}.\ y=(pval_1\ binop_{rel}\ pval_2)\wedge\mathtt{I}}
                                                                                                                           C; \mathcal{L}; \Phi \vdash pval \Rightarrow loc
                                                                                                                                                                                                                                                                          TY_MEMOP_INTFROMPTR
                                            \overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash \mathtt{intFromPtr}\left(\tau_{1},\tau_{2},pval\right)}\Rightarrow \Sigma \ y\mathtt{:integer}. \ y=\mathtt{cast\_ptr\_to\_int} \ pval\wedge \mathtt{I}
                                                                                                                     \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}
                                                                                                                                                                                                                                                                   TY_MEMOP_PTRFROMINT
                                                 \overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash \mathsf{ptrFromInt}\left(\tau_1,\tau_2,pval\right)}\Rightarrow \Sigma\,y\mathtt{:loc}.\,\,y=\mathtt{cast\_int\_to\_ptr}\,pval\wedge\mathtt{I}
```

$$\begin{aligned} &\mathcal{C}; \mathcal{L}; \Phi \vdash pval_0 \Rightarrow \texttt{loc} \\ &\texttt{smt} \ (\Phi \Rightarrow pval_1 = pval_0) \\ &\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \overset{\checkmark}{\mapsto}_{\tau} \ _ \Leftarrow pval_1 \overset{\checkmark}{\mapsto}_{\tau} \ _ \end{aligned}$$

Ty_Memop_PtrValidForDeref

 $\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \overset{\checkmark}{\mapsto}_{\tau -} \Leftarrow pval_1 \overset{\checkmark}{\mapsto}_{\tau -}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{ptrValidForDeref}\left(\tau, pval_0, pval_1 \overset{\checkmark}{\mapsto}_{\tau -}\right) \Rightarrow \Sigma \ y \text{:bool.} \ y = \text{aligned}\left(\tau, pval_1\right) \land pval_1 \overset{\checkmark}{\mapsto}_{\tau -} \otimes \mathbf{I} }$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{loc}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrWellAligned}\left(\tau, pval\right) \Rightarrow \Sigma \ y : \mathtt{bool}. \ y = \mathtt{aligned}\left(\tau, pval\right) \wedge \mathtt{I}} \quad \mathsf{TY_MEMOP_PTRWellAligneD}$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \texttt{loc} \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \texttt{integer} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \texttt{ptrArrayShift} \left(pval_1, \tau, pval_2\right) \Rightarrow \Sigma \ y : \texttt{loc}. \ y = pval_1 +_{\texttt{ptr}} \left(pval_2 \times \texttt{size_of}(\tau)\right) \land \texttt{I} \end{split}$$
 TY_MEMOP_PTRARRAYSHIFT

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$

$$\overline{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{done} \ \Leftarrow \mathtt{I}} \quad \mathrm{TY_TVAL_I}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ \overline{spine_elem_i}^{\ i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ pval, \ \overline{spine_elem_i}^{\ i} \Leftarrow \Sigma \ y : \beta. \ ret} \end{split} \qquad \text{TY_TVAL_COMP}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ \overline{spine_elem_i}^{\ i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ pval, \ \overline{spine_elem_i}^{\ i} \Leftarrow \exists \ y : \beta. \ ret} \end{split} \quad \mathsf{TY_TVAL_LOG}$$

$$\begin{split} & \text{smt} \ (\Phi \Rightarrow term) \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{done} \ spine \Leftarrow ret \\ & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{done} \ spine \Leftarrow term \land ret \end{split} \quad \text{TY_TVAL_PHI}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash \mathit{res_term} \Leftarrow \mathit{res} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \mathsf{done} \, \overline{\mathit{spine_elem}_i}^i \Leftarrow \mathit{ret}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash \mathsf{done} \, \mathit{res_term}, \, \overline{\mathit{spine_elem}}^i \Leftarrow \mathit{res} \otimes \mathit{ret}} \end{split} \quad \mathsf{TY_TVAL_RES} \end{split}$$

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash\mathtt{undef}\ \mathit{UB_name} \Leftarrow\mathit{ret}}\quad \mathtt{TY_TVAL_UNDEF}$$

 $|\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret$

$$\begin{split} ident: & arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \texttt{ccall}\left(\tau, ident, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \end{split} \text{TY_SEQ_E_CCALL}$$

$$\begin{array}{l} \mathit{name} : \mathit{arg} \; \equiv \; \overline{x_i}^{\; i} \; \mapsto \mathit{texpr} \; \in \; \mathsf{Globals} \\ \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \; \overline{x_i = \mathit{spine_elem}_i^{\; i}} \; :: \mathit{arg} \gg \sigma; \mathit{ret}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pcall} \left(\mathit{name}, \overline{\mathit{spine_elem}_i^{\; i}} \right) \Rightarrow \sigma(\mathit{ret})} \end{array} \quad \mathsf{TY_Seq_E_PROC}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash memop \ (mem_op) \Rightarrow ret} \quad \text{Ty_Is_E_MEMOP}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Action}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash neg \ mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Neg_Action}$$

 $|\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret$

$$\frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash tval\Leftarrow ret}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash tval\Leftarrow ret} \qquad \text{Ty_Seq_TE_TVal}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi\vdash pexpr\Rightarrow y:\beta.\ term}{ident_or_pattern:\beta\rightsquigarrow\mathcal{C}_1\text{ with } term_1} \\ \mathcal{C},\mathcal{C}_1;\mathcal{L};\Phi,term_1/y,\cdot(term);\mathcal{R}\vdash texpr\Leftarrow ret} \\ \hline \mathcal{C};\mathcal{L};\Phi\vdash pexpr\Rightarrow y:\beta.\ term} \\ \frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \text{let } ident_or_pattern=pexpr\, in\, texpr\Leftarrow ret}}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \text{let } ident_or_pattern=pexpr\, in\, texpr\Leftarrow ret}} \qquad \text{Ty_Seq_TE_LetP}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi,term_1/y,\cdot(term);\mathcal{R}\vdash texpr\Leftarrow ret}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \text{let } ident_or_pattern:(y:\beta.\ term)=tpexpr\, in\, texpr\Leftarrow ret}} \\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \text{let } ident_or_pattern:(y:\beta.\ term)=tpexpr\, in\, texpr\Leftarrow ret}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}'\vdash \text{seq_expr}\Rightarrow ret_1}{\mathcal{C};\mathcal{L};\Phi,\mathcal{R}'\vdash \text{ret_pattern}_i^i:ret_1\rightsquigarrow\mathcal{C}_1;\mathcal{L}_1;\Phi_1;\mathcal{R}_1} \\ \hline \mathcal{C},\mathcal{C}_1;\mathcal{L},\mathcal{L}_1;\Phi,\Phi_1;\mathcal{R},\mathcal{R}_1\vdash texpr\Leftarrow ret_2} \\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}',\mathcal{R}\vdash \text{let}\, \overline{ret_pattern}_i^i:ret_1\rightsquigarrow\mathcal{C}_1;\mathcal{L}_1;\Phi_1;\mathcal{R}_1 \\ \hline \mathcal{C},\mathcal{L};\Phi;\mathcal{R}',\mathcal{R}\vdash \text{let}\, \overline{ret_pattern}_i^i:ret_1\Rightarrow \mathcal{C}_1;\mathcal{L}_1;\Phi_1;\mathcal{R}_1 \\ \hline \mathcal{C},\mathcal{L};\mathcal{L},\mathcal{L}_1;\Phi,\Phi_1;\mathcal{R},\mathcal{R}_1\vdash texpr_2\Leftarrow ret_2} \\ \hline \mathcal{C};\mathcal{L};\Phi,\mathcal{R}',\mathcal{R}\vdash \text{let}\, \overline{ret_pattern}_i^i:ret_1=texpr_1\, in\, texpr_2\Leftarrow ret_2} \\ \hline \mathcal{C};\mathcal{L};\Phi,\mathcal{R}',\mathcal{R}\vdash \text{let}\, \overline{ret_pattern}_i^i:ret_1=texpr_1\, in\, texpr_2\Leftarrow ret_2} \\ \hline \mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow\beta_1 \\ \hline pattern_i;\beta_1\rightsquigarrow\mathcal{C}_i\, \text{with}\, term_i^i \\ \hline \mathcal{C},\mathcal{C}_i;\mathcal{L};\Phi,\text{term}_i=pval;\mathcal{R}\vdash texpr_i\Leftarrow ret^i \\ \hline \mathcal{C};\mathcal{L};\Phi,\mathcal{R}\vdash \text{case}\, pval\, \text{of}\, \boxed{pattern}_i\Rightarrow texpr_i^i \text{ end} \Leftarrow ret} \\ \hline \text{Ty_Seq_TE_Case} \\ \hline \text{Ty_Seq_TE_Case} \\ \hline \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \text{case}\, pval\, \text{of}\, \boxed{pattern}_i\Rightarrow texpr_i^i \text{ end} \Leftarrow ret} \\ \hline \end{array}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool} \\ \mathcal{C}; \mathcal{L}; \Phi, pval = \texttt{true}; \mathcal{R} \vdash texpr_1 \Leftarrow ret \\ \mathcal{C}; \mathcal{L}; \Phi, pval = \texttt{false}; \mathcal{R} \vdash texpr_2 \Leftarrow ret \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \texttt{if} pval \texttt{then} texpr_1 \texttt{else} texpr_2 \Leftarrow ret \end{array} \quad \text{TY_SEQ_TE_IF}$$

$$\begin{array}{c} \mathit{ident} : \mathit{arg} \; \equiv \; \overline{x_i}^i \; \mapsto \mathit{texpr} \; \in \; \mathsf{Globals} \\ \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \; \overline{x_i = \mathit{pval}_i}^i \; :: \; \mathit{arg} \gg \sigma; \mathsf{false} \wedge \mathsf{I} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathsf{run} \; \mathit{ident} \; \overline{\mathit{pval}_i}^i \; \Leftarrow \; \mathsf{false} \wedge \mathsf{I} \end{array} \quad \mathsf{TY_SeQ_TE_RUN}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{bound} [int] (is_texpr) \Leftarrow ret} \quad \mathsf{TY_SEQ_TE_BOUND}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}' \vdash is_expr \Rightarrow ret_1 \\ \Phi \vdash \overline{ret_pattern_i}^i : ret_1 \leadsto \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \mathtt{let\,strong} \, \overline{ret_pattern_i}^i = is_expr \, \mathtt{in} \, texpr \Leftarrow ret_2 \end{split} \qquad \text{TY_IS_TE_LETS}$$

 $\boxed{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret}$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret} \quad \text{TY_TE_IS}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret} \quad \text{TY_TE_SEQ}$$

 $pattern = pval \leadsto \sigma$

$$\underline{} := pval \leadsto \underline{}$$
 Subs_Decons_Value_No_Sym_Annot

$$\overline{x:_=pval \leadsto pval/x,\cdot} \quad \text{Subs_Decons_Value_Sym_Annot}$$

$$\begin{aligned} & pattern_1 = pval_1 \leadsto \sigma_1 \\ & pattern_2 = pval_2 \leadsto \sigma_2 \\ & \overline{\text{Cons}(pattern_1, pattern_2) = \text{Cons}(pval_1, pval_2) \leadsto \sigma_1, \sigma_2} \end{aligned} \quad \text{SUBS_DECONS_VALUE_CONS}$$

$$\frac{\overline{pattern_i} = pval_i \leadsto \overline{\sigma_i}^i}{\text{Tuple}(\overline{pattern_i}^i) = \text{Tuple}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Tuple}$$

$$\frac{\overline{pattern_i = pval_i \leadsto \sigma_i}^i}{\operatorname{Array}(\overline{pattern_i}^i) = \operatorname{Array}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \quad \text{Subs_Decons_Value_Array}$$

$$\frac{pattern = pval \leadsto \sigma}{\texttt{Specified}(pattern) = pval \leadsto \sigma} \quad \texttt{Subs_Decons_Value_Specified}$$

 $ident_or_pattern = pval \leadsto \sigma$

$$x = pval \leadsto pval/x$$
, Subs_Decons_Value'_Sym

$$\frac{pattern = pval \leadsto \sigma}{pattern = pval \leadsto \sigma} \quad \text{Subs_Decons_Value'_Pattern}$$

 $res_pattern = res_term \leadsto \sigma$

$$\frac{}{\texttt{emp} = \texttt{emp} \leadsto \cdot} \quad \text{Subs_Decons_Res_Emp}$$

 $\overline{ident = res_term \leadsto res_term/ident}, \quad \text{Subs_Decons_Res_Var}$ $res_pattern_1 = res_term_1 \leadsto \sigma_1$ $\mathit{res_pattern}_2 = \mathit{res_term}_2 \leadsto \sigma_2$ $\frac{\textit{res_pattern}_1 \sim \sigma_2}{\langle \textit{res_pattern}_1, \textit{res_pattern}_2 \rangle = \langle \textit{res_term}_1, \textit{res_term}_2 \rangle \leadsto \sigma_1, \sigma_2} \quad \text{Subs_Decons_Res_Pair}$ $\frac{res_pattern = res_term \leadsto \sigma}{\texttt{pack} \, (ident, res_pattern) = \texttt{pack} \, (pval, res_term) \leadsto pval/ident, \sigma} \quad \texttt{Subs_Decons_Res_Pack}$ $\frac{\mathit{res_pattern} = \mathit{res_term} \leadsto \sigma}{\mathtt{fold}\,(\mathit{res_pattern}) = \mathit{res_term} \leadsto \sigma} \quad \mathsf{Subs_Decons_Res_Fold}$ $\overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma$ \longrightarrow Subs_Decons_Ret_Empty $ident_or_pattern = pval \leadsto \sigma$ $\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \psi}{\operatorname{comp} ident_or_pattern = pval, \ \overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma, \psi}$ Subs_Decons_Ret_Comp $\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \psi}{\log ident = pval, \ \overline{ret_pattern_i = spine_elem_i}^i \leadsto pval/ident, \psi} \quad \text{Subs_Decons_Ret_Log}$

 $\frac{res_pattern = res_term \leadsto \sigma}{ret_pattern_i = spine_elem_i{}^i \leadsto \psi}$ $\frac{res_pattern = res_term, \overline{ret_pattern_i = spine_elem_i{}^i} \leadsto \phi, \psi}{res_pattern = res_term, \overline{ret_pattern_i = spine_elem_i{}^i} \leadsto \sigma, \psi}$ Subs_Decons_Ret_Res

$$\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$$

$$\frac{}{::ret \gg \cdot; ret} \quad \text{Subs_Decons_Arg_Empty}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \ \overline{x_i = spine_elem_i}^i :: \Pi \, x:\beta. \ arg \gg pval/x, \sigma; ret} \quad \text{Subs_Decons_Arg_Comp}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \overline{x_i = spine_elem_i}^i :: \forall x : \beta. arg \gg pval/x, \sigma; ret}$$
 Subs_Decons_Arg_Log

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = res_term, \ \overline{x_i = spine_elem_i}^i :: res \multimap arg \gg res_term/x, \sigma; ret}$$
 Subs_Decons_Arg_Res

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\overline{x_i = spine_elem_i}^i :: term \supset arg \gg \sigma; ret} \quad \text{Subs_Decons_Arg_Phi}$$

$$\langle pexpr \rangle \longrightarrow \langle pexpr' \rangle$$

$$\frac{mem_ptr' \equiv mem_ptr +_{\text{ptr}} mem_int \times \text{size_of}(\tau)}{\langle \texttt{array_shift} (mem_ptr, \tau, mem_int) \rangle \longrightarrow \langle mem_ptr' \rangle} \quad \text{Op_PE_PE_ArrayShift}$$

$$\frac{mem_ptr' \equiv mem_ptr +_{\text{ptr}} \text{ offset_of}_{tag}(member)}{\langle \texttt{member_shift} \, (mem_ptr, tag, member) \rangle \longrightarrow \langle mem_ptr' \rangle} \quad \text{Op_PE_PE_MEMBERSHIFT}$$

$$\frac{}{\langle \mathtt{not}\,(\mathtt{True})\rangle \longrightarrow \langle \mathtt{False}\rangle} \quad \mathrm{OP_PE_PE_NOT_TRUE}$$

 $\frac{\langle tpexpr\rangle \longrightarrow \langle tpexpr'\rangle}{}$

$$\frac{pattern_{j} = pval \leadsto \sigma_{j}}{\forall i < j. \ not [pattern_{i} = pval \leadsto \sigma_{i})} \qquad \text{Op.TPE.TPE.Case}$$

$$\frac{ident_or_pattern}{\langle \text{let } ident_or_pattern = pval \leadsto \sigma} \qquad \text{Op.TPE.TPE.Let.Sub}$$

$$\frac{ident_or_pattern = pval \leadsto \sigma}{\langle \text{let } ident_or_pattern = pval \ int } pvarpr) \qquad \text{Op.TPE.TPE.Let.Sub}$$

$$\frac{\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle}{\langle \text{let } ident_or_pattern = pexpr \ int } pexpr\rangle \longrightarrow \langle \text{let } ident_or_pattern = pexpr' \ int } pexpr\rangle} \qquad \text{Op.TPE.TPE.Let.Let}$$

$$\frac{\langle pexpr\rangle \longrightarrow \langle pexpr\rangle}{\langle \text{let } ident_or_pattern = pexpr' \ int } pexpr\rangle} \qquad \text{Op.TPE.TPE.Let.Let.T}$$

$$\frac{\langle pexpr\rangle \longrightarrow \langle tpexpr_{1} : \langle y.\beta. \ term\rangle}{\langle \text{let } ident_or_pattern = pval \leadsto \sigma}} \qquad \text{Op.TPE.TPE.Let.Let.T}$$

$$\frac{ident_or_pattern = pval \leadsto \sigma}{\langle \text{let } ident_or_pattern : \langle y.\beta. \ term\rangle} = done pval \ int pexpr\rangle} \qquad \text{Op.TPE.TPE.Let.T.Sub}$$

$$\frac{\langle tpexpr_{1}\rangle \longrightarrow \langle tpexpr_{1}'\rangle}{\langle \text{let } ident_or_pattern : \langle y.\beta. \ term\rangle} = tpexpr_{1}' \ int pexpr_{2}\rangle} \qquad \text{Op.TPE.TPE.Let.T.Let.T}$$

$$\frac{\langle tpexpr_{1}\rangle \longrightarrow \langle tpexpr_{1}'\rangle}{\langle \text{let } ident_or_pattern : \langle y.\beta. \ term\rangle} = tpexpr_{1}' \ int pexpr_{2}\rangle} \qquad \text{Op.TPE.TPE.Let.T.Let.T}$$

$$\frac{\langle tpexpr_{1}\rangle \longrightarrow \langle tpexpr_{1}\rangle}{\langle \text{if } \text{True } \text{then } tpexpr_{1} \text{ int } tpexpr_{2}\rangle} \longrightarrow \langle tpexpr_{1}\rangle} \qquad \text{Op.TPE.TPE.Let.T.Let.T}$$

$$\frac{\langle tpexpr_{1}\rangle \longrightarrow \langle tpexpr_{2}\rangle \longrightarrow \langle tpexpr_{2}\rangle}{\langle tpexpr_{1}\rangle \longrightarrow \langle tpexpr_{2}\rangle} \longrightarrow \langle tpexpr_{2}\rangle} \qquad \text{Op.TPE.TPE.Let.T.Let.T.Let.T}$$

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.clem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_CCALL}$$

$$\frac{name:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.elem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_CCALL}$$

$$\frac{name:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.elem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_PCALL}$$

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = pval_i}^i : arg \gg \sigma_i false \land 1} \qquad \texttt{OP_STE_TE_RUN}$$

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = pval_i}^i : arg \gg \sigma_i false \land 1} \qquad \texttt{OP_STE_TE_RUN}$$

$$\frac{pattern_j = pval \leadsto \sigma_j}{\langle h; run \, ident \, \overline{pval_i}^i \rangle \rightarrow \langle h; \sigma(texpr) \rangle} \qquad \texttt{OP_STE_TE_CASE}$$

$$\frac{pattern_j = pval \leadsto \sigma_j}{\langle h; case \, pval \, of \, [\, pattern_i \Rightarrow texpr_i^i \, end \rangle \rightarrow \langle h; \sigma_j(texpr_j) \rangle} \qquad \texttt{OP_STE_TE_CASE}$$

$$\frac{ident.or_pattern = pval \leadsto \sigma}{\langle h; let \, ident.or_pattern = pval \, in \, texpr_i^i \, end \rangle \rightarrow \langle h; \sigma(texpr_j) \rangle} \qquad \texttt{OP_STE_TE_LETP_SUB}$$

$$\frac{(pexpr) \longrightarrow \langle pexpr' \rangle}{\langle h; let \, ident.or_pattern = pexpr \, in \, texpr' \rangle \rightarrow \langle h; let \, ident.or_pattern = pexpr' \, in \, texpr}} \qquad \texttt{OP_STE_TE_LETP_LETP}$$

$$\frac{\langle pexpr \rangle \longrightarrow \langle pexpr' \rangle}{\langle h; let \, ident.or_pattern = pexpr \, in \, texpr \rangle \rightarrow \langle h; let \, ident.or_pattern = texpr \rangle} \qquad \texttt{OP_STE_TE_LETP_LETP}$$

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\frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle h; \texttt{let} ident\_or\_pattern: (y:\beta. \ term) = \texttt{done} \ pval \ \texttt{in} \ texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op\_STE\_TE\_LetTP\_Sub}
\frac{\langle tpexpr\rangle \longrightarrow \langle tpexpr'\rangle}{\langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr\, \mathtt{in}\, texpr\rangle \longrightarrow \langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr'\, \mathtt{in}\, texpr\rangle} \quad \text{Op\_STE\_TE\_LetTP\_LetTP}
                                                          \frac{\overline{ret\_pattern_i = spine\_elem_i}^i \leadsto \sigma}{\langle h; \mathtt{let}\, \overline{ret\_pattern_i}^i : ret = \mathtt{done}\, \overline{spine\_elem_i}^i \, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_SUB}
                                   \frac{\langle h; seq\_expr \rangle \longrightarrow \langle h; texpr_1 : ret \rangle}{\langle h; \mathsf{let} \ \overline{ret\_pattern_i}^i = seq\_expr \ \mathsf{in} \ texpr_2 \rangle \longrightarrow \langle h; \mathsf{let} \ \overline{ret\_pattern_i}^i : ret = texpr_1 \ \mathsf{in} \ texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LET\_LETT}
                               \frac{\langle h; texpr_1 \rangle \longrightarrow \langle h'; texpr_1' \rangle}{\langle h; \mathsf{let} \, \overline{ret\_pattern_i}^{\,\, i} : ret = texpr_1 \, \mathsf{in} \, texpr_2 \rangle \longrightarrow \langle h'; \mathsf{let} \, \overline{ret\_pattern_i}^{\,\, i} : ret = texpr_1' \, \mathsf{in} \, texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_LETT}
                                                                                       \overline{\langle h; \text{if True then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_1 \rangle} OP_STE_TE_IF_TRUE
                                                                                    \overline{\langle h; \text{if False then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_2 \rangle}
                                                                                                                                                                                                                         OP_STE_TE_IF_FALSE
                                                                                                   \frac{}{\langle h; \mathtt{bound} \, [int] (is\_texpr) \rangle \longrightarrow \langle h; is\_texpr \rangle} \quad \mathsf{OP\_STE\_TE\_BOUND}
 \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                                    \frac{bool\_value \equiv mem\_int_1 \ binop_{rel} \ mem\_int_2}{\langle h; mem\_int_1 \ binop_{rel} \ mem\_int_2 \rangle \longrightarrow \langle h; \texttt{done} \ bool\_value \rangle}
                                                                                                                                                                                                                     OP_MEMOP_TVAL_REL_BINOP
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mem\_int \equiv \texttt{cast\_ptr\_to\_int} \ mem\_ptr
                                                                                                                                                                                           Op_Memop_TVal_IntFromPtr
                                                             \overline{\langle h; \mathtt{intFromPtr} \left(\tau_1, \tau_2, mem\_ptr\right)\rangle \longrightarrow \langle h; \mathtt{done} \ mem\_int\rangle}
                                                            \frac{mem\_ptr \equiv \texttt{cast\_ptr\_to\_int} \ mem\_int}{\langle h; \texttt{ptrFromInt} \ (\tau_1, \tau_2, mem\_int) \rangle \longrightarrow \langle h; \texttt{done} \ mem\_ptr \rangle}
                                                                                                                                                                                           OP_MEMOP_TVAL_PTRFROMINT
                                                                                                     bool\_value \equiv aligned(\tau, mem\_ptr)
\frac{\textit{bool\_value} = \texttt{aligned}\left(\tau, \textit{mem\_ptr}\right)}{\langle h + \{\textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \}; \texttt{ptrValidForDeref}\left(\tau, \textit{mem\_ptr}, \textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \right) \rangle \longrightarrow \langle h + \{\textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \}; \texttt{done}\, \textit{bool\_value}, \textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \rangle}
                                                                                                                                                                                                                                                                                             OP_MEMOP_TVAL_PTRVALID
                                                                            bool\_value \equiv \mathtt{aligned}\left(\tau, mem\_ptr\right)
                                                    \overline{\langle h; \mathtt{ptrWellAligned} \left(\tau, mem\_ptr\right) \rangle \longrightarrow \langle h; \mathtt{done} \, bool\_value \rangle}
                                                                                                                                                                                     Op_Memop_TVal_PtrWellAligned
                                             \frac{mem\_ptr' \equiv mem\_ptr +_{ptr} (mem\_int \times size\_of(\tau))}{\langle h; ptrArrayShift (mem\_ptr, \tau, mem\_int) \rangle \longrightarrow \langle h; done mem\_ptr' \rangle}
                                                                                                                                                                                                 Op_Memop_TVal_PtrArrayShift
   \langle h; mem\_action \rangle \longrightarrow \langle h'; tval \rangle
                                                                                               fresh(mem_ptr)
                                                                                               representable (\tau *, mem\_ptr)
                                                                                               alignedI (mem_int, mem_ptr)
                                                                                                                                                                                                                                         OP_ACTION_TVAL_CREATE
                           \overline{\langle h; \mathtt{create}\,(mem\_int,\tau)\rangle \longrightarrow \langle h + \{mem\_ptr \overset{\times}{\mapsto}_{\tau}\,pval\}; \mathtt{done}\,mem\_ptr,pval,mem\_ptr \overset{\times}{\mapsto}_{\tau}\,pval\rangle}
                                                                                                                                                                                                                                                                                  OP_ACTION_TVAL_LOAD
\frac{}{\langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \texttt{load} \ (\tau, mem\_ptr, \_, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval) \rangle} \longrightarrow \langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \texttt{done} \ pval, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval \rangle}
                                                                                                                                                                                                                                                                                       OP_ACTION_TVAL_STORE
\frac{}{\langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_\}; \mathtt{store} \left( \_, \tau, mem\_ptr, pval, \_, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_ \right) \rangle} \longrightarrow \langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \mathtt{done} \ \mathtt{Unit}, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval \rangle}
```

 $\frac{}{\langle h + \{mem_ptr \mapsto_{\tau_-}\}; \texttt{kill} \, (\texttt{static} \, \tau, mem_ptr, mem_ptr \mapsto_{\tau_-}) \rangle} \longrightarrow \langle h; \texttt{done} \, \texttt{Unit} \rangle \\ \text{OP_ACTION_TVAL_KILL_STATIC}$

 $\langle h; is_expr \rangle \longrightarrow \langle h'; is_expr' \rangle$

$$\frac{\langle h; mem_op\rangle \longrightarrow \langle h; tval\rangle}{\langle h; \mathtt{memop}\,(mem_op)\rangle \longrightarrow \langle h; tval\rangle} \quad \text{Op_IsE_IsE_Memop}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \text{Op_IsE_IsE_Action}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; \mathsf{neg}\, mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \text{Op_IsE_IsE_Neg_Action}$$

 $\langle h; is_texpr \rangle \longrightarrow \langle h'; texpr \rangle$

$$\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma}{\langle h; \mathtt{let strong} \, \overline{ret_pattern_i}^i = \mathtt{done} \, \overline{spine_elem_i}^i \, \mathtt{in} \, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \mathsf{OP_ISTE_ISTE_LETS_SUB}$$

$$\frac{\langle h; is_expr\rangle \longrightarrow \langle h'; is_expr'\rangle}{\langle h; \mathsf{let} \, \mathsf{strong} \, \overline{ret_pattern_i}^{\,\, i} \, = is_expr \, \mathsf{in} \, texpr\rangle \longrightarrow \langle h'; \mathsf{let} \, \mathsf{strong} \, \overline{ret_pattern_i}^{\,\, i} \, = is_expr' \, \mathsf{in} \, texpr\rangle} \quad \mathsf{OP_ISTE_ISTE_LETS_LETS}$$

 $\overline{\langle h; texpr \rangle} \longrightarrow \langle h'; texpr' \rangle$

$$\frac{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle}{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle} \quad \text{Op_TE_TE_SEQ}$$

$$\frac{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle}{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle} \quad \text{Op_TE_TE_IS}$$

Definition rules: 233 good 0 bad Definition rule clauses: 525 good 0 bad