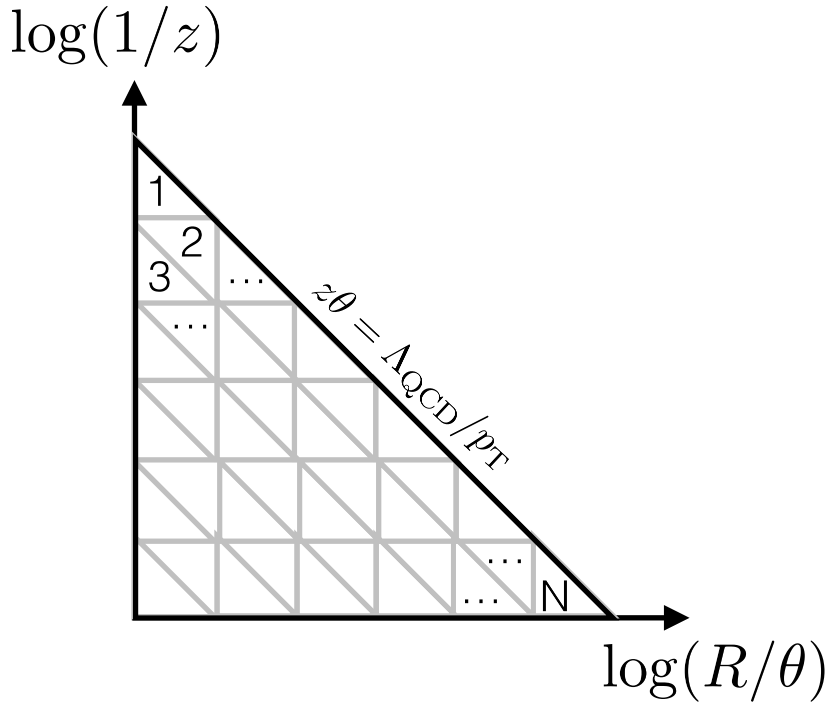

n_i = number of emissions in triangle i.

$$\Pr(n_i = n) = \frac{\lambda^n e^{-\lambda}}{n!} \qquad i \in \{q, g\}$$

$$\lambda = \frac{2\alpha_s C_i \triangle}{\pi} \qquad C_q = C_F \qquad C_g = C_A$$

~full probability distribution~

$$\Pr_i(n_1, n_2, \ldots, n_N) = \prod_{j=1}^{N} \frac{\lambda_i^{n_j} e^{-\lambda_i}}{n_j!}$$

(eikonal soft gluon approximation)

The optimal quark/gluon classier based only on Casimirs is a threshold cut on the full likelihood ratio:

$$f(n_1, \ldots, n_N) = \frac{\Pr_q(n_1, \ldots, n_N)}{\Pr_g(n_1, \ldots, n_N)} \propto \left(\frac{\lambda_q}{\lambda_g}\right)^{\sum_{j=1}^{N} n_j}$$

$\sum_{j=1}^{N} n_j$ is just (perturbative) multiplicity and is finite in the $\triangle \to 0$ limit.

Since (pert.) multiplicity is monotonically related with the likelihood ratio classifier, **it is also optimal**!

Including the running coupling: $\alpha_s(zp_{\mathrm{T}}R) \sim \frac{1}{b\log(zp_{\mathrm{T}}R/\Lambda)}$

$$\mathrm{Pr}_i(N_1, ..., n_N) = \prod_{i=1}^{N} \frac{\lambda_{ij}^{n_j} e^{-\lambda_{ij}}}{n_j!} \quad \text{where } \lambda_{ij} = \frac{2C_i}{\pi b \log(z_j p_{\mathrm{T}} R/\Lambda)}$$

$$f(n_1, ..., n_n) = \prod_{j=1}^{N} \left(\frac{\lambda_{qj}}{\lambda_{gj}}\right)^{n_j} e^{-\lambda_{qj}+\lambda_{gj}}$$

$$\propto \left(\frac{C_F}{C_A}\right)^{\sum_{j=1}^{N} n_j} e^{\sum_{j=1}^{N}(C_A - C_F)\log(z_j p_{\mathrm{T}} R/\Lambda)}$$

$$\propto \left(\frac{C_F}{C_A}\right)^{\sum_{j=1}^{N} n_j} \prod_{j=1}^{N} z_j^{C_A - C_F}$$

This is still monotonically related with multiplicity - the second term is just an overall constant!