1 Kernel definitions

$$
\begin{aligned}
P_{q\to q_{hard}g_{soft}} &= z\frac{\mathcal{C}}{\mathcal{S}}\frac{2(1-z)}{(1-z)^2+\kappa^2}\left(1+K\alpha_s(\kappa^2 m_d^2)\right)\left(\frac{1}{z}s_{-1}+s_0+zs_1+z^2s_2\right) && (1)\\
&\quad \exp\left(\frac{1}{z}e_{s,-1}+e_{s,0}+ze_{s,1}+z^2e_{s,2}\right) && (2)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}\frac{2\kappa^2}{(1-z)^2+\kappa^2}\left(\frac{1}{z}k_{-1}+k_0+zk_1+z^2k_2\right)\exp\left(\frac{1}{z}e_{k,-1}+e_{k,0}+ze_{k,1}+z^2e_{k,2}\right) && (3)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}(1-z)\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) && (4)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}f && (5)\\
P_{q\to g_{hard}q_{soft}} &= (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{2(1-z)}{(1-z)^2+\kappa^2}\left(1+K'\alpha_s(\kappa^2 m_d^2)\right)\left(\frac{1}{z}s_{-1}+s_0+zs_1+z^2s_2\right) && (6)\\
&\quad \exp\left(\frac{1}{z}e_{s,-1}+e_{s,0}+ze_{s,1}+z^2e_{s,2}\right) && (7)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{2\kappa^2}{(1-z)^2+\kappa^2}\left(\frac{1}{z}k_{-1}+k_0+zk_1+z^2k_2\right)\exp\left(\frac{1}{z}e_{k,-1}+e_{k,0}+ze_{k,1}+z^2e_{k,2}\right) && (8)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}(1-z)\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) && (9)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}f && (10)\\
P_{g\to g_{1,hard}g_{2,soft}} &= z\frac{\mathcal{C}}{\mathcal{S}}\frac{(1-z)}{(1-z)^2+\kappa^2}\left(1+K\alpha_s(\kappa^2 m_d^2)\right)\left(\frac{1}{z}s_{-1}+s_0+zs_1+z^2s_2\right) && (11)\\
&\quad \exp\left(\frac{1}{z}e_{s,-1}+e_{s,0}+ze_{s,1}+z^2e_{s,2}\right) && (12)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}\frac{\kappa^2}{(1-z)^2+\kappa^2}\left(\frac{1}{z}k_{-1}+k_0+zk_1+z^2k_2\right)\exp\left(\frac{1}{z}e_{k,-1}+e_{k,0}+ze_{k,1}+z^2e_{k,2}\right) && (13)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}\frac{1}{2}z(1-z)\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) && (14)\\
&+ z\frac{\mathcal{C}}{\mathcal{S}}\frac{1}{2}f && (15)\\
P_{g\to g_{2,hard}g_{1,soft}} &= (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{(1-z)}{(1-z)^2+\kappa^2}\left(1+K'\alpha_s(\kappa^2 m_d^2)\right)\left(\frac{1}{z}s_{-1}+s_0+zs_1+z^2s_2\right) && (16)\\
&\quad \exp\left(\frac{1}{z}e_{s,-1}+e_{s,0}+ze_{s,1}+z^2e_{s,2}\right) && (17)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{\kappa^2}{(1-z)^2+\kappa^2}\left(\frac{1}{z}k_{-1}+k_0+zk_1+z^2k_2\right)\exp\left(\frac{1}{z}e_{k,-1}+e_{k,0}+ze_{k,1}+z^2e_{k,2}\right) && (18)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{1}{2}z(1-z)\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) && (19)\\
&+ (1-z)\frac{\mathcal{C}}{\mathcal{S}}\frac{1}{2}f && (20)\\
P_{g\to q_{1,hard}q_{2,soft}} &= z\frac{\mathcal{C}}{\mathcal{S}}\left[(1-z)^2+z^2\right]\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) && (21)
\end{aligned}
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$$+ \quad z\frac{\mathcal{C}}{\mathcal{S}}f \tag{22}$$

$$P_{g\to q_{2,hard}q_{1,soft}} = (1-z)\frac{\mathcal{C}}{\mathcal{S}}\left[(1-z)^2+z^2\right]\left(\frac{1}{z}c_{-1}+c_0+zc_1+z^2c_2\right)\exp\left(\frac{1}{z}e_{c,-1}+e_{c,0}+ze_{c,1}+z^2e_{c,2}\right) \tag{23}$$

$$+ \quad (1-z)\frac{\mathcal{C}}{\mathcal{S}}f \tag{24}$$

Some further definitions are

$$2\pi K = (67/18-\pi^2/6)C_A - 10/9N_FT_R \tag{25}$$

$$2\pi K' = (67/18-\pi^2/6)C_A - 10/9N_FT_R \quad \text{heuristically at LO, not because of soft gluon arguments} \tag{26}$$

$$\mathcal{C}_{q\to q} = C_F \tag{27}$$

$$\mathcal{S}_{q\to q} = 1 \tag{28}$$

$$\mathcal{C}_{q\to g} = C_F \tag{29}$$

$$\mathcal{S}_{q\to g} = 1 \tag{30}$$

$$\mathcal{C}_{g\to g} = 2C_A \tag{31}$$

$$\mathcal{S}_{g\to g} = 1/2 \tag{32}$$

$$\mathcal{C}_{g\to q} = T_R \tag{33}$$

$$\mathcal{S}_{g\to q} = 1/2 \tag{34}$$