Some of my solutions to understanding analysis chapter 2 first edition

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Problem 2.3.3

We want to prove that there's an $N \in \mathbb{N}$ such that.

$$|y_n - l| < \epsilon$$

for any choice of $\epsilon > 0$.

As $(x_n) \to l$ there's an $N_1 \in \mathbb{N}$ such that $|x_n - l| < \epsilon_1$ for any choice of $\epsilon_1 > 0$.
Likewise for z_n — there's an $N_2 \in \mathbb{N}$ such that $|z_n - l| < \epsilon_2$ for any choice of $\epsilon_2 > 0$.

Now, we can let those epsilons ϵ_1, ϵ_2 be equal to epsilon. $\epsilon_1 = \epsilon_2 = \epsilon$. If we could find N_1 and N_2 for ϵ_1 and ϵ_2, we can also find them for the epsilon we are interested in for y_n because the definition guarantees it.

By definition we know that z_n is inside $(-\epsilon + l, \epsilon + l)$ for some N_2. Or $-\epsilon + l < z_n < \epsilon + l$. Because $y_n \leq z_n \Rightarrow y_n < \epsilon + l$.

Likewise for x_n in which we obtain $y_n > -\epsilon + l$. Therefore there exists an N, namely $N = max(N1, N2)$ (to make sure both inequalities hold) such that y_n is inside the epsilon neighbourhood. This means that there's a limit for y_n and it is l.