# Some of my solutions to understanding analysis chapter 2 first edition

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## Problem 2.5.6

Let $s$ be the supremum of the set $S$. This means that for any $\epsilon > 0$ there exists a $s' \in S$ such that $s' > s - \epsilon$. Due to the definition of $S$, for $s'$ to exist there must be a subsequence with infinitely many terms greater than $s'$. Let this subsequence be $a_{n_k}$. If $a_{n_k}$ exists there is a $k_0$ such that $a_{n_k} > s'$ whenever $k \geq k_0$.

With this inequality we get to:

$$a_{n_k} > s' > s - \epsilon \Rightarrow a_{n_k} > s - \epsilon$$

.

With this we get a lower bound for $a_{n_k}$.

Now, as $s$ is upper bound this means that $s \geq x \ \forall x \in S$. For the purpose of contradiction let's assume that for every $\epsilon > 0$ there's a subsequence with infinitely many terms bigger than $s + \epsilon$. This means that there exists a $k_1$ such that $a_{n_k} > s + \epsilon$ whenever $k \geq k_1$.

This in turn implies that $s + \epsilon$ is an element of the set $S$. However as we assumed that $s$ is an uppebound, $s \geq x \ \forall x \in S$ but $s < s + \epsilon$. This is a contradiction and thus there is no subsequence with infinitely many terms bigger than the supremum.

That is, there exists a $k_1$ such that $a_{n_k} \leq s + \epsilon$ whenever $k \geq k_1$.

Therefore, for every $\epsilon > 0$ we can select a number $K$, namely $K = \max(k_0, k_1)$ such that $s - \epsilon \leq a_{n_k} \leq s + \epsilon$, or $|a_{n_k} - s| < \epsilon$. So there's a convergent subsequence with limit $s$.