From NP-Complete to NP-Hard:

Exploiting Mathematical Synergies between Electric Power Grid Operation and ML Verification

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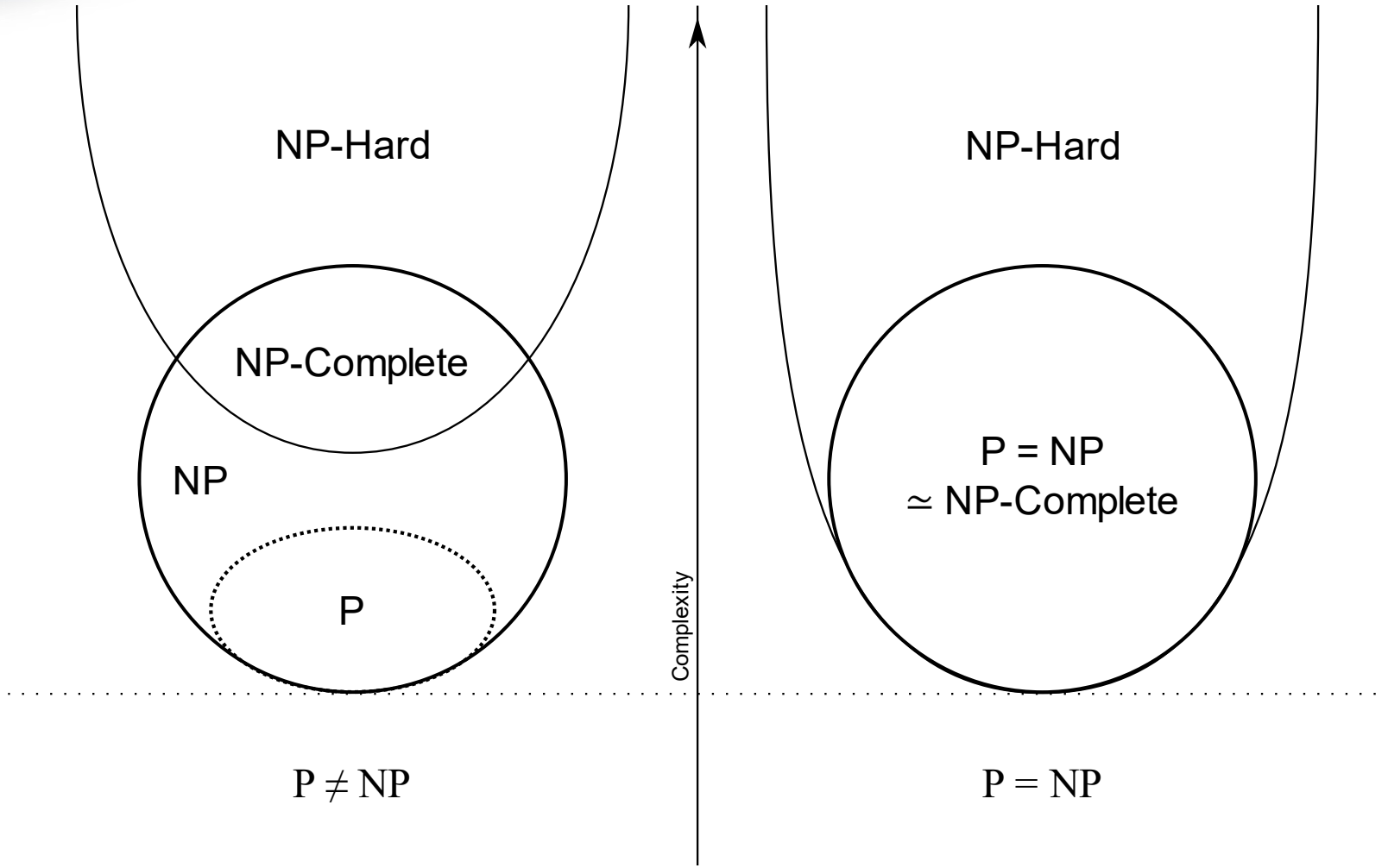
> Verifying DNNs is a difficult problem. DNNs are large, non-linear, and non-convex, and verifying even simple properties about them is an NP-complete problem (see Section I of the appendix). DNN verification is experimentally

Wikipedia:



[1] Katz, Guy, et al. "Reluplex: An efficient SMT solver for verifying deep neural networks." *CAV 2017.*

From NP-Complete to (NP-Hard $\wedge \notin$ NP)

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(Part of) What my lab at UVM focuses on

prove: (a) Operational guarantee

st: (b) neural network

(c) power grid physics

A Quick Historical Note on α,β-CROWN

Provably Bounding Neural Network Preimages

General Cutting Planes for Bound-Propagation-Based Neural Network Verification

Formal Verification for Neural Networks with General Nonlinearities via Branch-and-Bound

Beta-CROWN: Efficient Bound Propagation with Per-neuron Split Constraints for Neural Network

Automatic Perturbation Analysis for Scalable Certified Robustness and Beyond

A Branch and Bound Framework for Stronger Adversarial Attacks of ReLU Networks

⋮

αβ CROWN

Winner of International Verification of Neural Networks Competitions
(VNN-COMP 2021, 2022, 2023)

Efficient Neural Network Robustness Certification with General Activation Functions

Huan Zhang[1,3,†,*] Tsui-Wei Weng[2,†] Pin-Yu Chen[3] Cho-Jui Hsieh[1] Luca Daniel[2]

My PhD Advisor!

Lecture (Talk?) Outline

①

Challenges

...in actual, and future, power systems.

②

Tools

...for overcoming these challenges (ML + verification)

③

Synergies

...between ML verification and grid operation.

Lecture (Talk?) Outline

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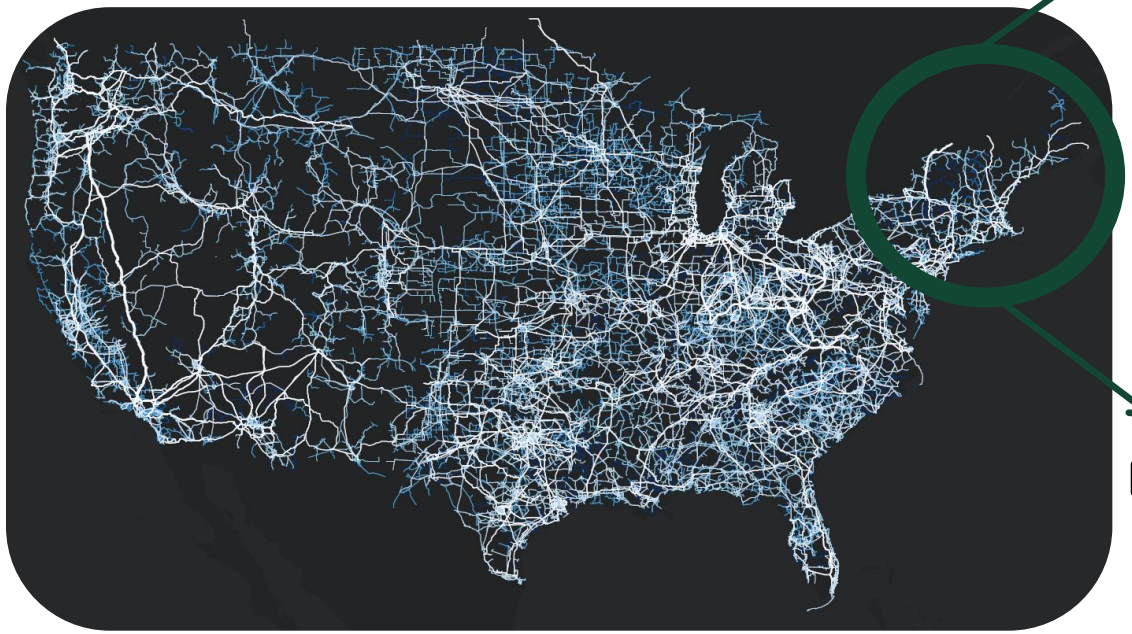
Synergies

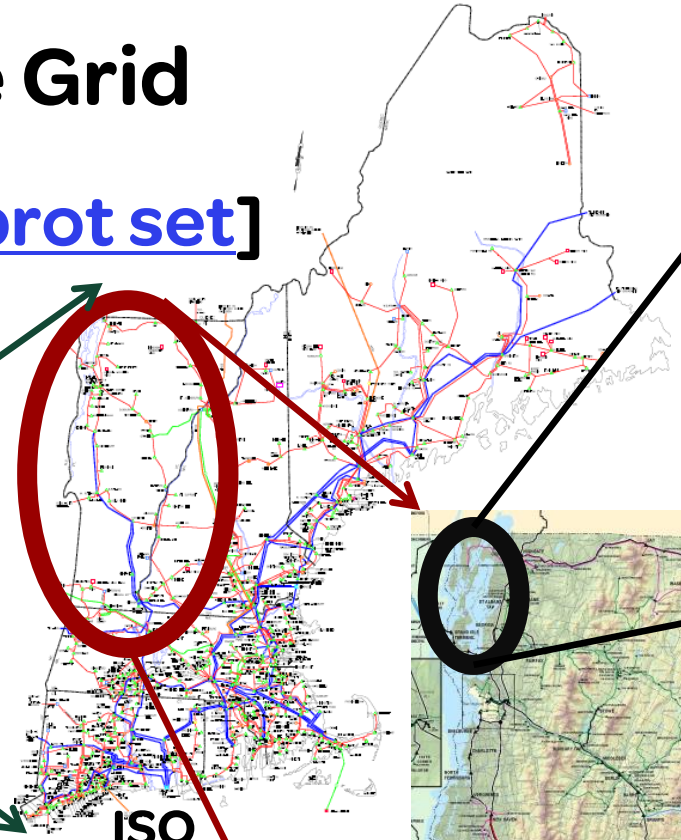
...between ML verification and grid operation.

Challenges in Operating the Grid

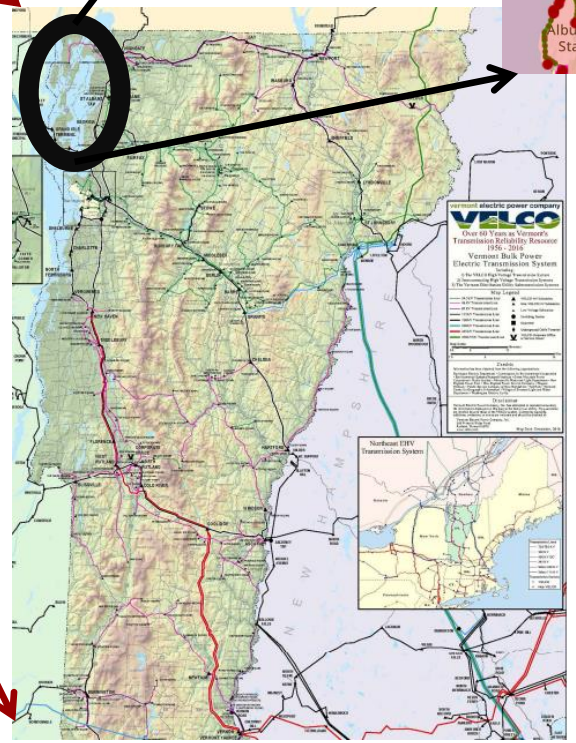
The “grid” is **[a]** huge **[Mandelbrot set]**



Western Interconnect, Eastern Interconnect, and Texas



ISO New England



VELCO (Vermont)

VEC (South Alburgh) Almost 10k nodes!

Challenges in Operating the Grid

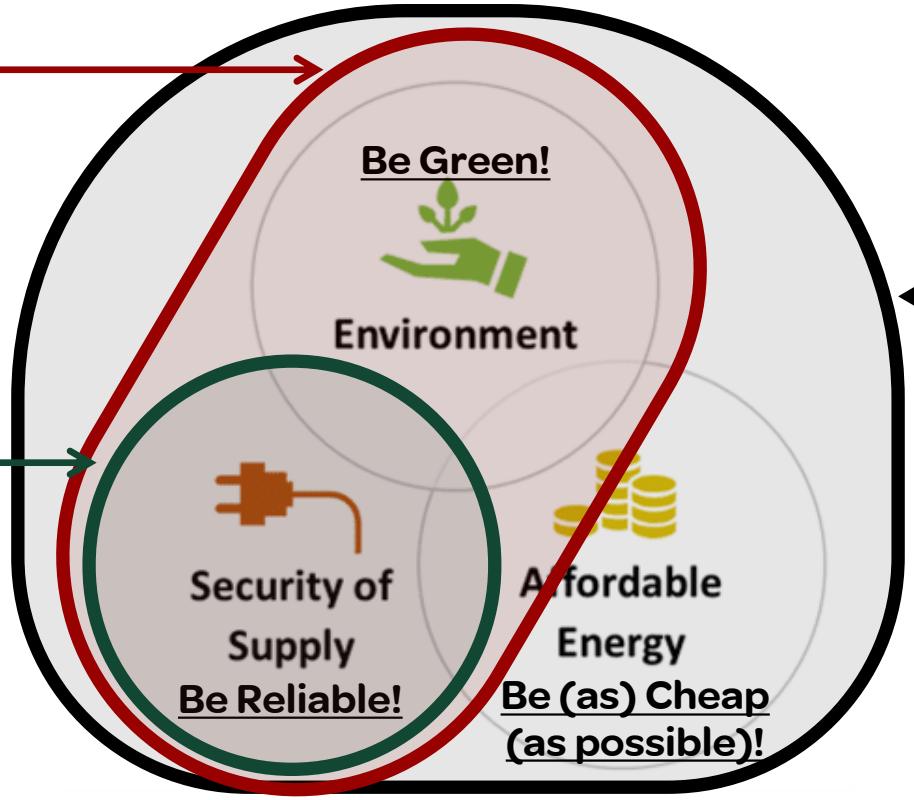
Grids operation has three **competing** goals: cheap, reliable, green.

- Which one do you think **dominates**?

More Constraints! Solution can be verified quickly, but still NP-Hard(er)

A given solution can be verified in Polynomial time, but still NP-Hard [1]

[one-in-three 3-SAT problem]



NP-Hard and NOT verifiable in polynomial time

$\ldots \wedge \notin \mathbf{NP}$

[1] Bienstock, Daniel, and Abhinav Verma. "Strong NP-hardness of AC power flows feasibility." *Operations Research Letters* 47.6 (2019).

Quick Evidence of NP-Hardness…

GRID OPTIMIZATION COMPETITION

ARPA-E GO Competition: the hardest, meanest, ugliest gird optimization challenge in the world.

Given 6+ hours on a super-computer, no team could find **feasible** solutions on the largest, 23k-bus power system in multiple scenarios

- Feasible solutions do, theoretically, exist

model	scenario	SW	Best Team	Best Score	ARPA-e Benchmark	Artelys_Columbia	GOT-BSI-OPF	quasiGrad	TIM-GO	YongOptimization
C3E4N23643D1	**3**	1	TIM-GO	104,686,125	61,152,061	63,340,112	99,123,868	48,878,908	104,686,125	104,073,565
C3E4N23643D1	4	1	GOT-BSI-OPF	91,584,129	0	70,444,810	91,584,129	0	39,494,965	69,793,156
C3E4N23643D2	**3**	1	YongOptimization	600,577,211	0	353,804,820	589,696,576	598,132,224	588,991,290	600,577,211
C3E4N23643D2	4	1	**none**	0	0	0	0	0	0	0
C3E4N23643D3	**3**	1	YongOptimization	2,158,212,496	0	1,162,674,657	2,093,288,847	2,143,434,855	2,112,949,852	2,158,212,496
C3E4N23643D3	4	1	**none**	0	0	0	0	0	0	0
C3E4N23643	**6**	#	**(all) ensemble**	**2,955,059,960**	**61,152,061**	**1,650,264,399**	**2,873,693,419**	**2,790,445,988**	**2,846,122,232**	**2,932,656,427**
C3E4N23643D1	**2**	#	**D1 ensemble**	**196,270,254**	61,152,061	133,784,922	190,707,997	48,878,908	144,181,090	173,866,721
C3E4N23643D2	**2**	#	**D2 ensemble**	**600,577,211**	0	353,804,820	589,696,576	598,132,224	588,991,290	600,577,211
C3E4N23643D3	**2**	#	**D3 ensemble**	**2,158,212,496**	0	1,162,674,657	2,093,288,847	2,143,434,855	2,112,949,852	2,158,212,496

...and the savings could be huge!

GRID OPTIMIZATION COMPETITION

- In the early 2000s, power system operators began switching from **Lagrangian Relaxation** to **Mixed Integer Program** solvers

  – e.g., Gurobi, CPLEX, etc.

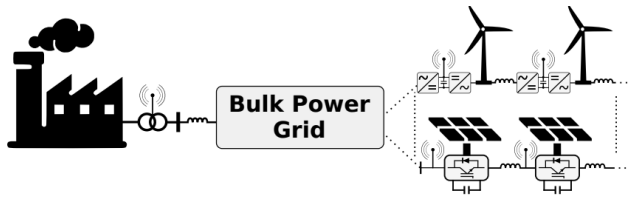
- By 2011, this was saving US grid operators **$500 million** per year [1]

- Robert Bixby (CLPEX and Gurobi founder) met Dick O'Neil at the 1999 Discrete Mathematics and Theoretical Computer Science meeting, and MIP-based **Unit Commitment** was born

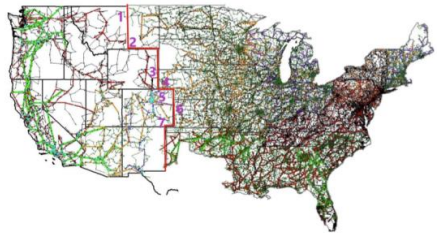
[1] Cain, M. B., R. P. O'Neill, and A. Castillo. "Optimal power flow papers: Paper 1. History of optimal power flow and formulations." *Federal Energy Regulatory Commission, Tech. Rep* (2013).

Challenges in Operating the Grid

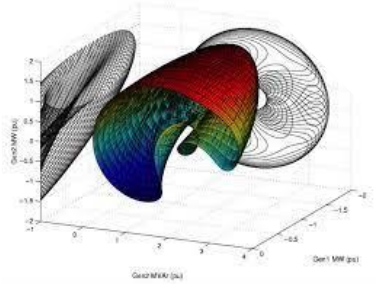
The grid has well-modeled, but complicated, physics



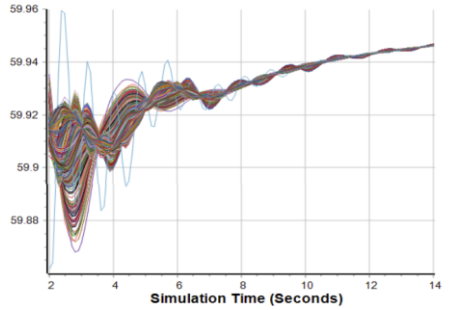
Renewable dynamics



Network flows



Voltage stability



Transient response

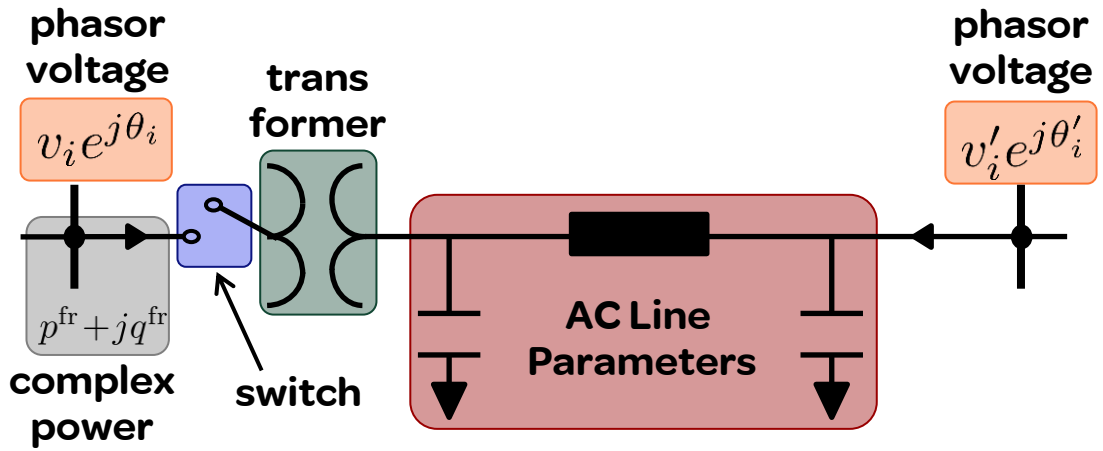
In this talk, we will focus on "quasi-steady state" **power flow** physics:

$$
\begin{aligned}
p^{\mathrm{fr}}_{jt} &= u^{\mathrm{on}}_{jt}\left(\left(g^{\mathrm{sr}}_j + g^{\mathrm{fr}}_j\right) v^2_{it}/\tau^2_{jt} + \left(-g^{\mathrm{sr}}_j \cos\left(\theta_{it} - \theta_{i't} - \phi_{jt}\right)\right.\right. \\
&\quad \left.\left. - b^{\mathrm{sr}}_j \sin\left(\theta_{it} - \theta_{i't} - \phi_{jt}\right)\right) v_{it} v_{i't}/\tau_{jt}\right) \\
q^{\mathrm{fr}}_{jt} &= u^{\mathrm{on}}_{jt}\left(\left(-b^{\mathrm{sr}}_j - b^{\mathrm{fr}}_j - b^{\mathrm{ch}}_j/2\right) v^2_{it}/\tau^2_{jt} + \left(b^{\mathrm{sr}}_j \cos\left(\theta_{it} - \theta_{i't} - \phi_{jt}\right)\right.\right. \\
&\quad \left.\left. - g^{\mathrm{sr}}_j \sin\left(\theta_{it} - \theta_{i't} - \phi_{jt}\right)\right) v_{it} v_{i't}/\tau_{jt}\right)
\end{aligned}
$$

ACOPF Polar form: $\sim v_i v_j \cos(\theta)$

Rectangular form: $\sim V_r V_i$

} **Pick your poison!**



Challenges in Operating the Grid

Thankfully, simplifications are used! Out of computational *necessity:*

$$p_{ii'} = v_i v_{i'} (g_{ii'} \cos(\theta_i - \theta_{i'}) + b_{ii'} \sin(\theta_i - \theta_{i'}))$$

$$\approx b_{ii'} (\theta_i - \theta_{i'})$$ **: Line flow is proportional to angle differences**

- This is the linear "DC" power flow approximation, and it is basis of network optimization. Example: the DC "optimal power flow" (OPF) dispatches generators to meet demand, while respecting flow limits:

$$\min p_g^T c_g$$

$$\text{s.t.} \sum p_g = \sum p_d : \lambda$$

$$\underline{p}_g \leq p_g \leq \overline{p_g}$$

$$\underline{p}_f \leq B p_{g-d} \leq \overline{p}_f : \mu$$

Power Transfer Distribution Factor (PTDF) matrix

Using the duals at optimality, Locational Marginal Prices (LMPs = cost of power at any point in the grid) "pop" out!

$$\text{LMP} = \lambda + B^T \mu$$

"There is a need for fundamental changes in the ways society views electric energy. Electric energy must be treated as a **commodity** which can be bought, sold, and traded, taking into account its time- and space-varying values and costs... The framework is based on the use of spot prices." **Schweppe**, Fred C., et al. *Spot pricing of electricity*. 1988.

Challenges in Operating the Grid

This is not just academic fluff: this is how grids are actually run (e.g., Texas):

The SCED Optimization Objective Function and Constraints

The SCED optimization objective function is as given by the following:

Minimize { Cost of dispatching generation
+ Penalty for violating Power Balance constraint
+ Penalty for violating transmission constraints}

which is:

Minimize {sum of (offer price * MW dispatched)
+ sum (Penalty * Power Balance violation MW amount)
+ sum (Penalty *Transmission constraint violation MW amount) }

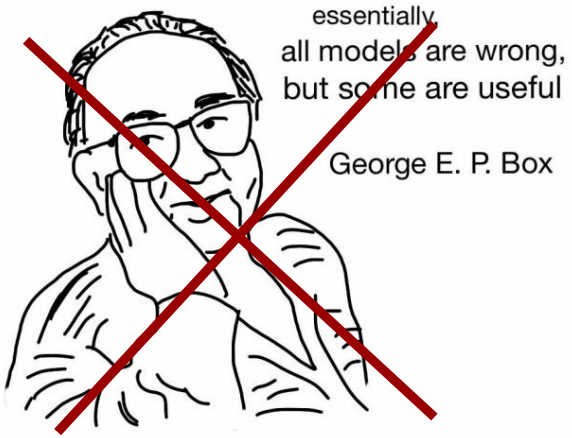
The objective is subject to the following constraints:

- Power Balance Constraint
  sum (Base Point) + under gen slack – over gen slack = Generation To Be Dispatched
- Transmission Constraints
  sum(Shift Factor * Base Point) – violation slack ≤ limit
- Dispatch Limits
  LDL ≤ Base Point ≤ HDL

ERCOT Public

Based on the SCED dispatch the LMP at each Electrical Bus is calculated as

$$LMP_{bus,t} = SP_{demand,t} - \sum_{c} SF_{bus,c,t} \cdot SP_{c;t}$$



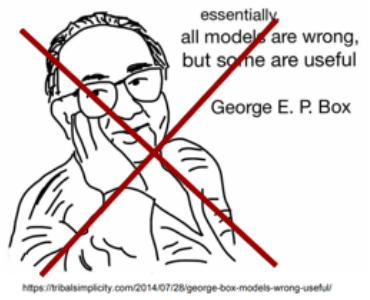
https://tribalsimplicity.com/2014/07/28/george-box-models-wrong-useful/

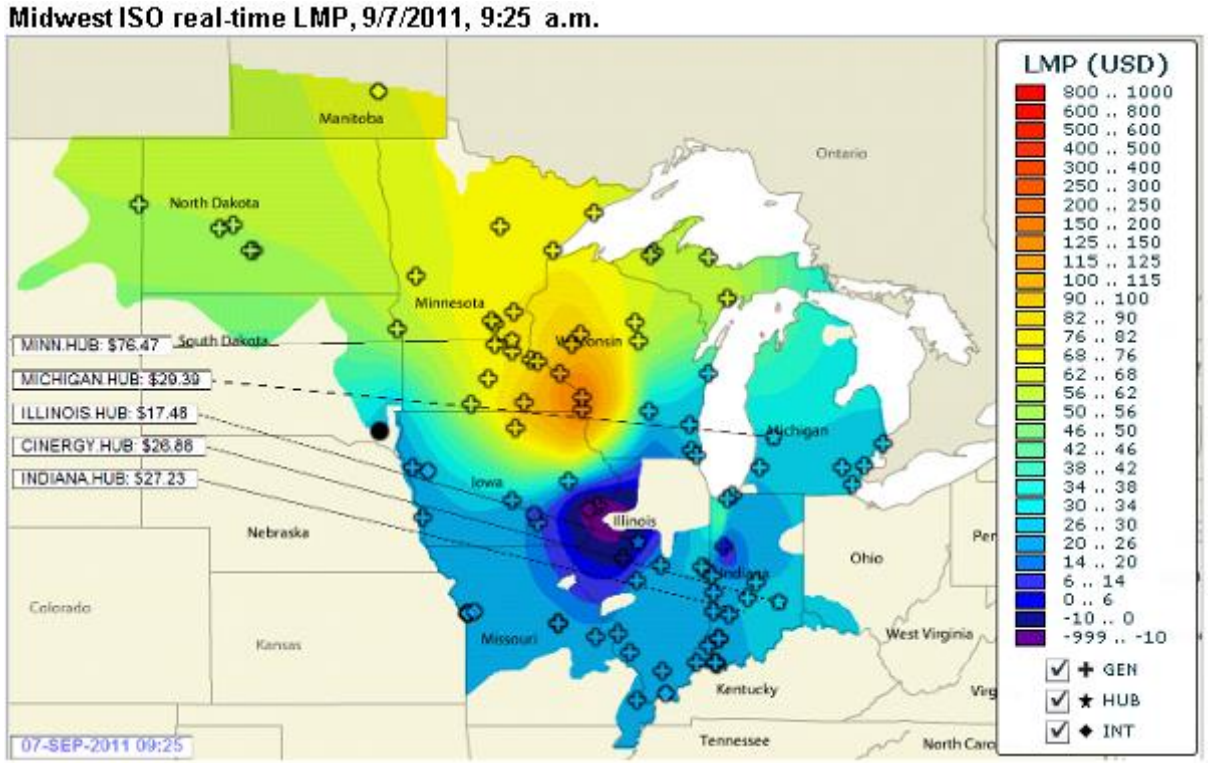
Challenge I: Keep Prices Low

The dual variables of the DCOPF problem provide network prices, paid for by wholesale market participants (generators, utilities; not you)

- Remember: global price minimization is not in NP!

- ERCOT prices
- PJM Prices
- MISO (you!) prices





Challenge I: Keep Prices Low

NONCONVEX MARKETS DRIVE POWER SYSTEMS

MISO

Solution Accuracy Requirement:

"DA MIP relative gap ≤ 0.1%"

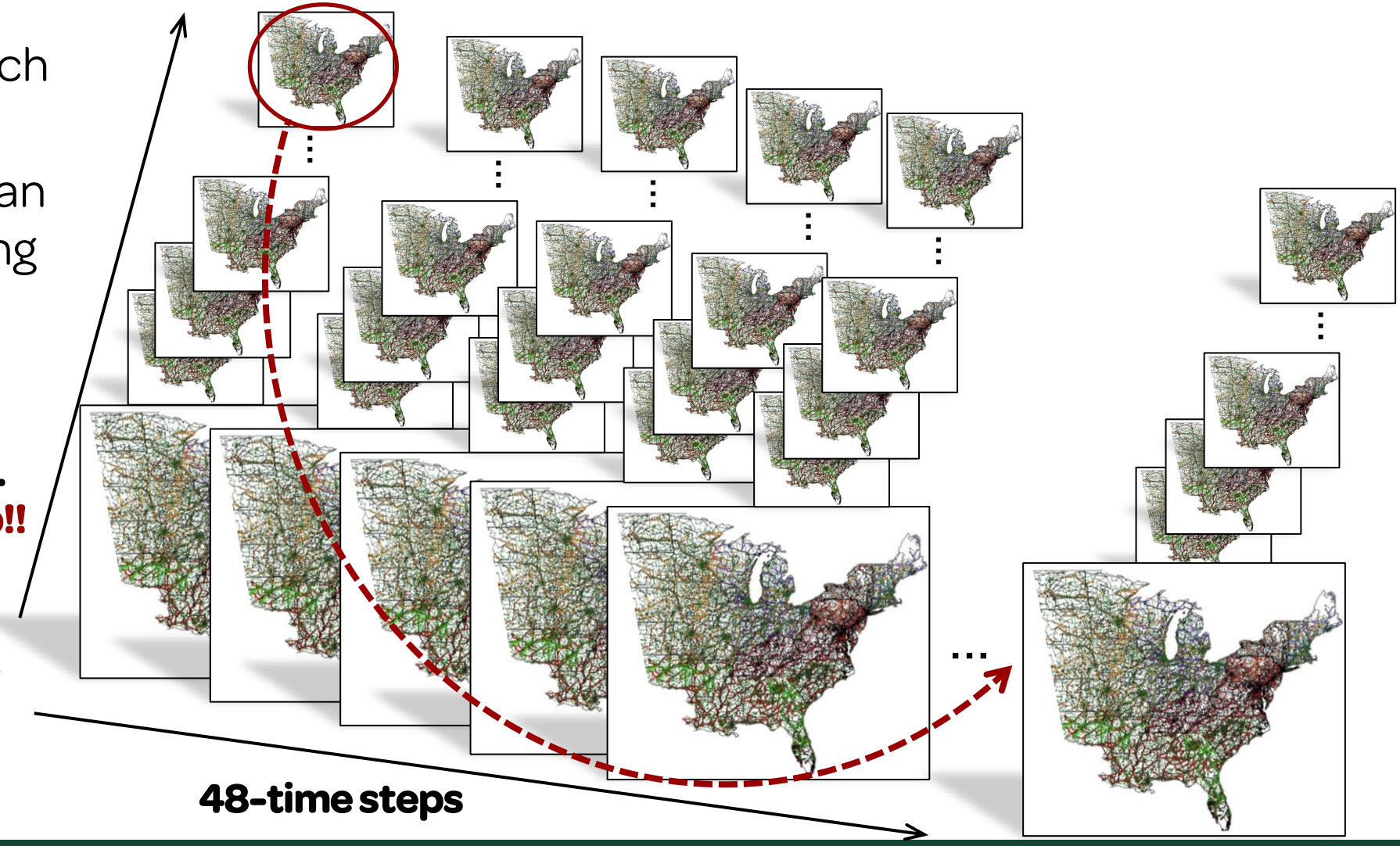
"What-if" analysis on commitment justifications

- Avoid participants' dispute on "out-of-the-money" resources.
- May lead to adjustment on disputable commitment results due to MIP gap
  - e.g. not committing wind units with close to 0 cost since the impact to the objective is less than MIP gap

Challenge II: "Security Constraints" for Network Reliability

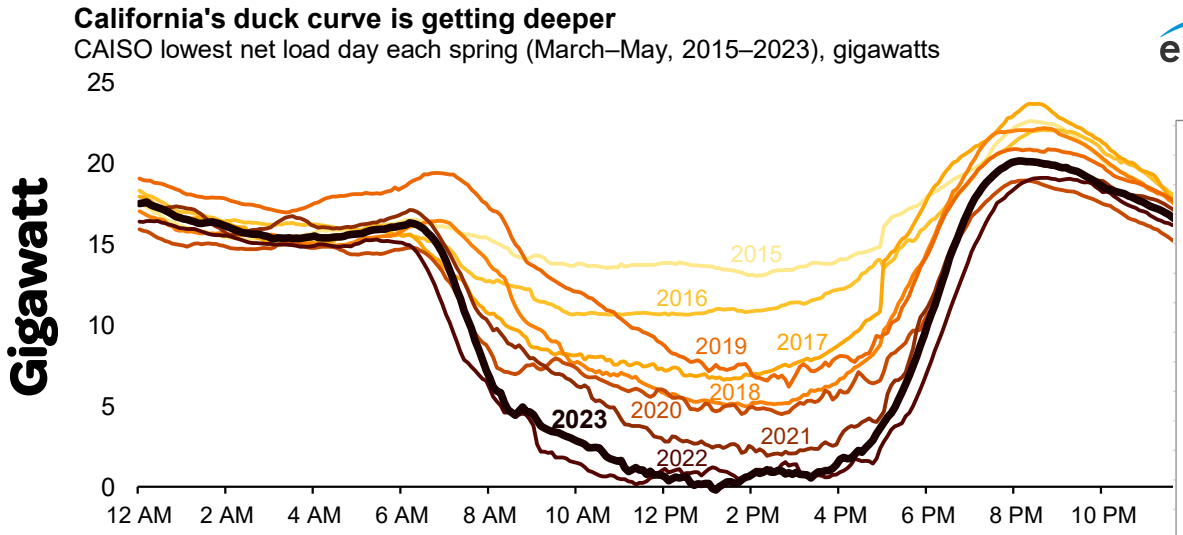
- Challenge: find an operating condition which is "N-1" secure
- Meaning: any **one line** can be lost without sacrificing network integrity
- Largest GO testcase:
  23,000+ contingencies…
  at each time step!!
- Hard problem, but its solution is easy to verify

  "$\in$ NP"



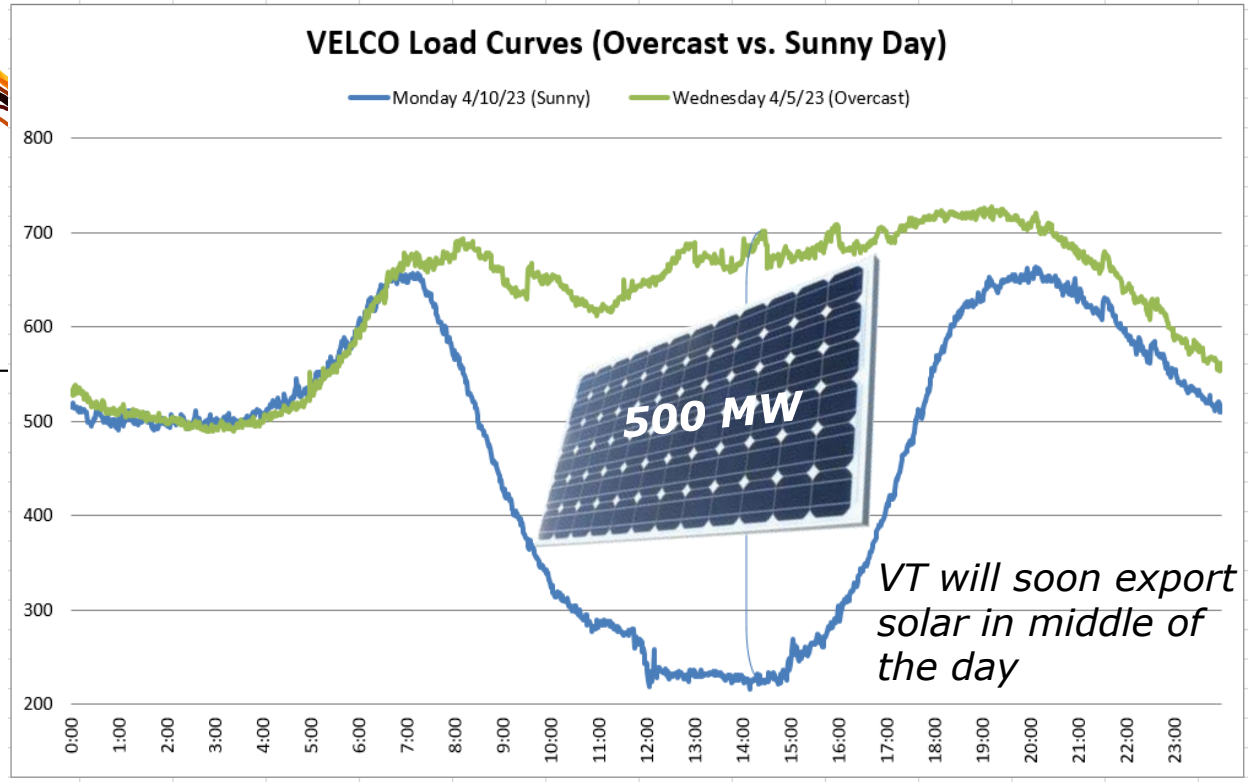
Challenge III: Renewables are coming…

California's "duck" curve is coming for you:



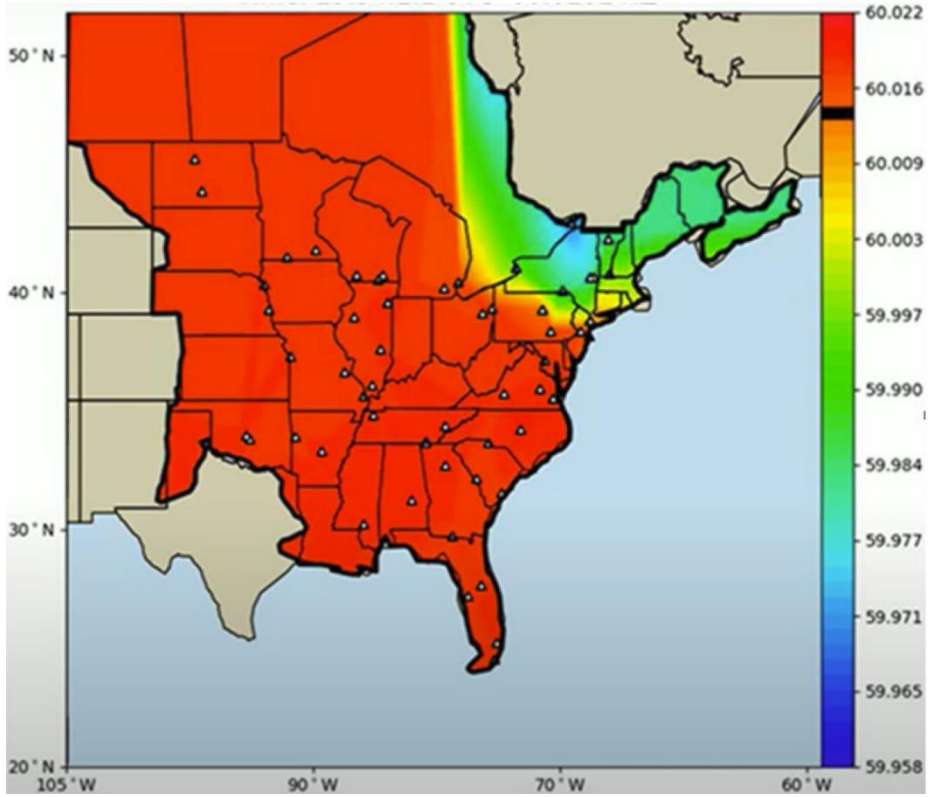
eia

And so is Vermont's Champ curve!!



Challenge III: Renewables are coming…

Potential consequences: the transmission grid is at the mercy of **distributed energy resource (DER)** behavior



- Summer of 2022: HVDC line connecting NY and Canada tripped offline, leading to DER trip
- Vermont net load **increased by 10%,** because it lost so much small-scale solar
- DERs will soon dominate the grid

Open Challenges in Operating the Grid [Sam's top 4]

1. Convex relaxation-based global optimization solutions (i.e., Semidefinite Programming) are generally not "tight"/feasible
   - Also, Mixed Integer SDP solvers don't really exist

     MOSEK can perform mixed-integer linear (MILO) and conic (MICO) problems, except for mixed-integer semidefinite problems.

2. AC transmission line switching is extremely hard.
   - Relaxations are useless. Most GO teams didn't even bother.

3. If a problem is true bilevel optimization problem, forget it
   - In a day of bitcoin and data centers, how do we solve this? Loads are flexible functions of LMP! Nasty.

4. Faster bound tightening [more later]

$$\min c_g^T p_g$$
$$\text{s.t.} \sum p_g = \sum p_d$$
$$\underline{p} \leq B p_{g-d} \leq \overline{p}$$
$$p_{d,i} = f(\text{LMP}_i)$$

Lecture (Talk?) Outline

①

Challenges

...in actual, and future, power systems.

②

Tools

...for overcoming these challenges (ML + verification)

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Synergies

...between ML verification and grid operation.

[Quick aside 1: Verification is like Optimization]

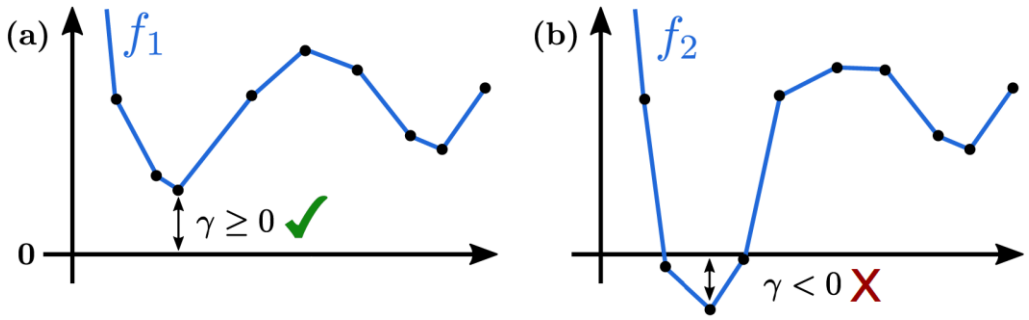
- Every verification problem can be posed in this form:

$$\gamma = \min_{x \in \mathcal{C}} f(x)$$

- If the metric is everywhere satisfied, the NN is "verified":

$\gamma \geq 0 \Rightarrow$ metric verified **(pass)** ✓

$\gamma < 0 \Rightarrow$ counter example found **(fail)** X



✓The **sign** of γ represents a verification result (yes/no)

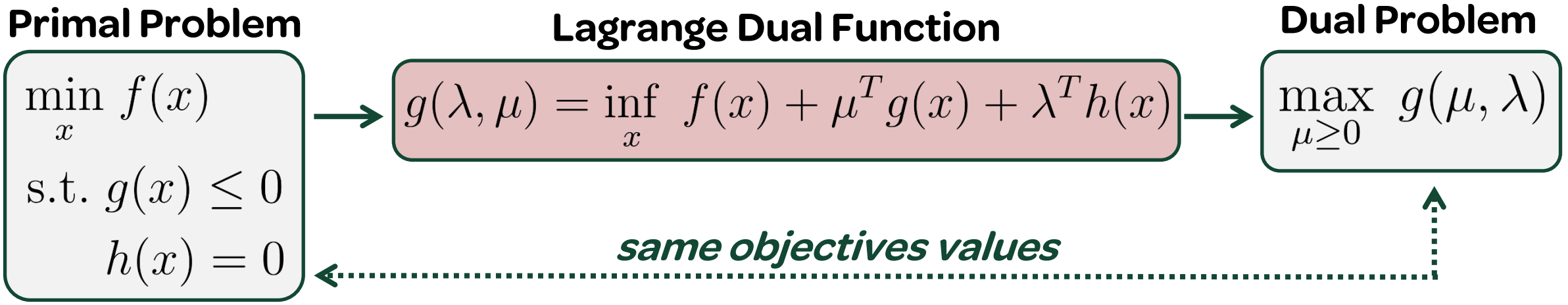
✓The **value** of γ represents a "worst case performance guarantee"

- **Challenge:** NNs are "spikey" and can yield highly nonconvex objectives

[Quick aside 2: Verification Exploits Dual Formulations]

Given a convex* problem, the **primal** and **dual** objectives are equal in value

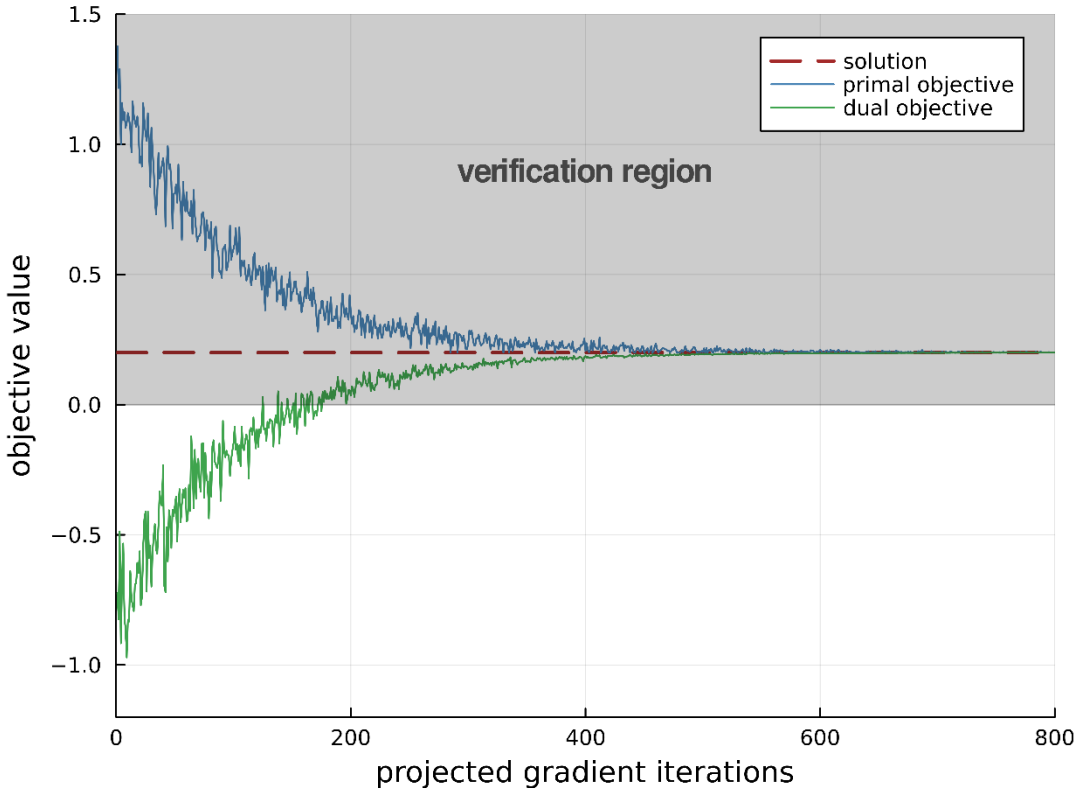
- From Boyd & Vandenberghe:



- In the NN verification problem, we care about **objective bounds**, so the primal and the dual problems can both be useful

[Quick aside 3: Verification Exploits Dual Formulations]

Assume you can solve the **primal** or **dual** with a projected gradient method:



Primal: $\min\ f(x),\ \text{s.t...}$

✓ obvious to formulate

✓ feasible iterates give a "worst incumbent"

X "bound" only guaranteed at convergence

Dual: $\max\ g(\mu, \lambda),\ \text{s.t...}$

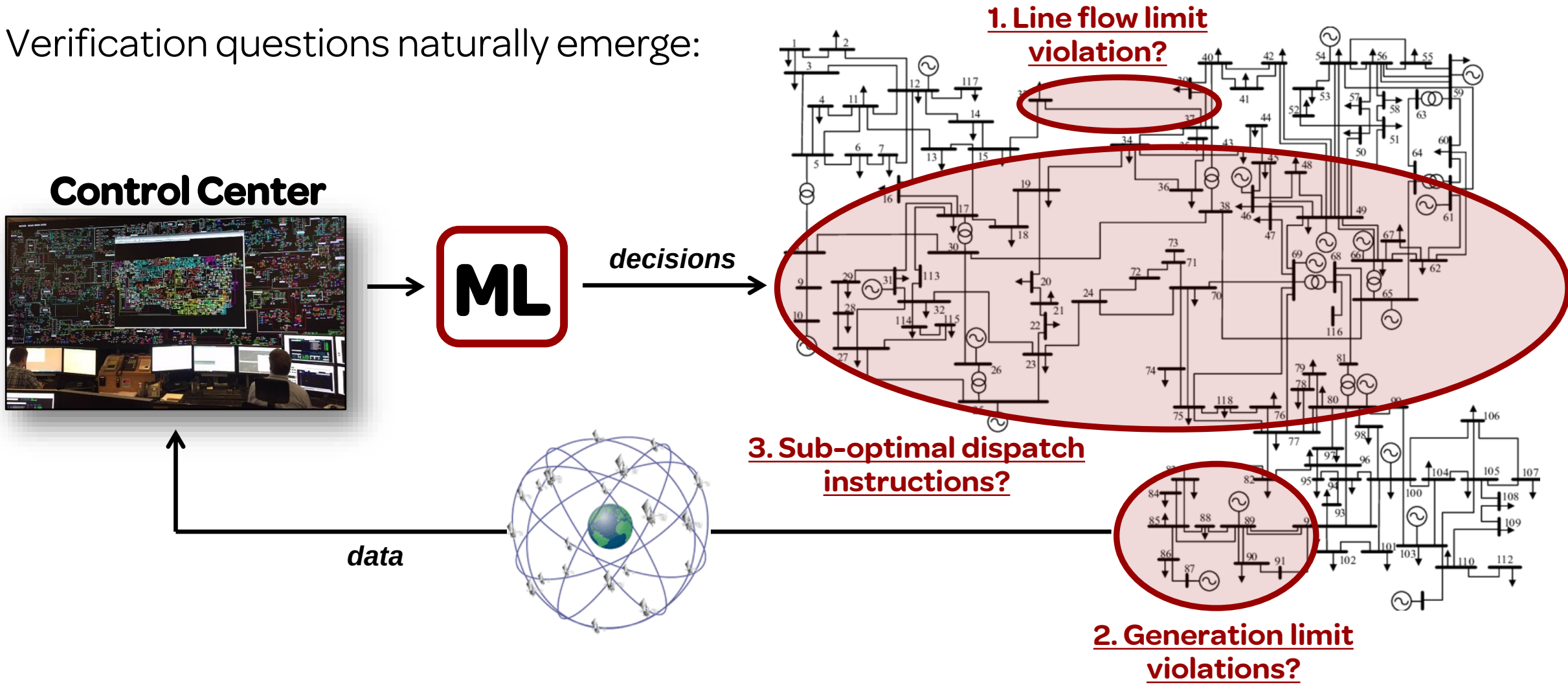
X "what the heck is the dual...?" - AP

✓ feasible iterates lower bound performance

✓ terminate early if crosses 0 (pre-converge)!

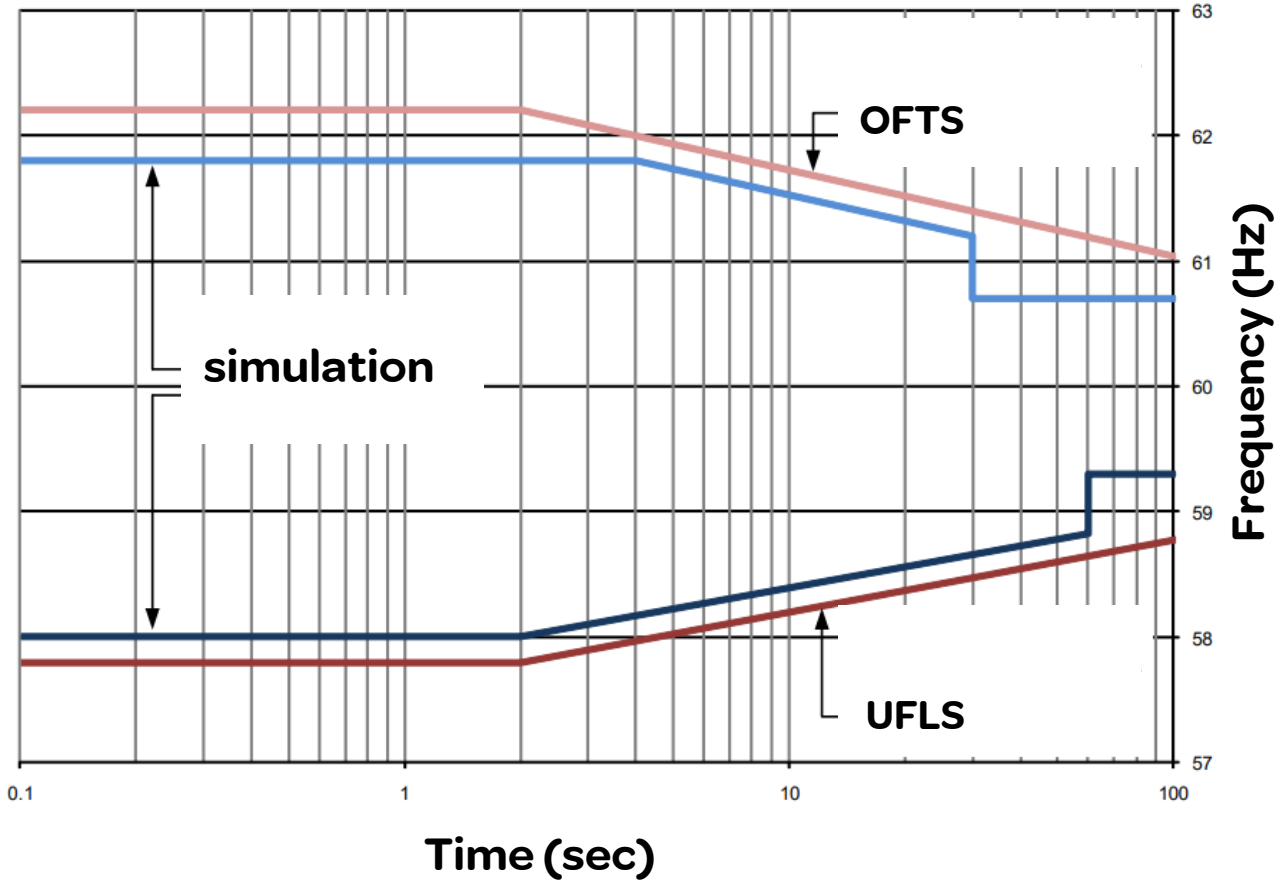
Tools: ML to the Rescue…?

- Future grids envision **ML** models "in the operational loop"
- Verification questions naturally emerge:



Tools: ML to the Rescue...? Compliance is king!!!

PRC6: Automatic Underfrequency Load Shedding

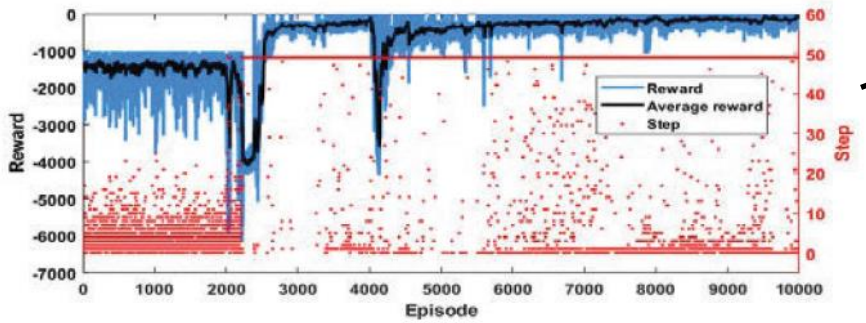


- Vermont Transmission Operators want adaptive UFLS parameter tuning to limit DER shedding
- Adaptive UFLS may be better than traditional tuning, but it must be **guaranteed to meet compliance targets**, enforced by NERC
- **Verification** of data-driven schemes can be used to ensure compliance

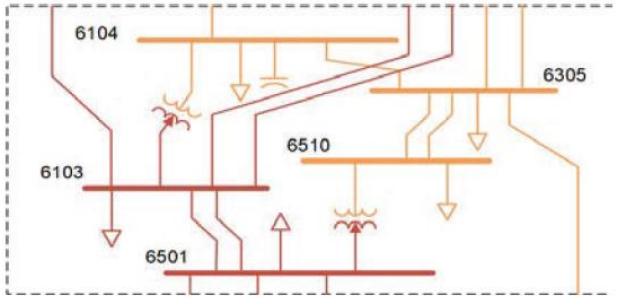
Tools: ML to the Rescue...?

Reinforcement Learning Based Voltage Control Using Multiple Control Devices

Yuling Wang[1], Vijay Vittal[1], Xiaochuan Luo[2], Slava Maslennikov[2], Qiang Zhang[2], Mingguo Hong[2], Song Zhang[3]
[1]School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ, USA
[2]Advanced Technology Solutions, ISO New England, Holyoke, MA, USA
[3]Energy & Utilities, Amazon Web Services, Seattle, WA, USA



Train an RL agent to make complex voltage control decisions



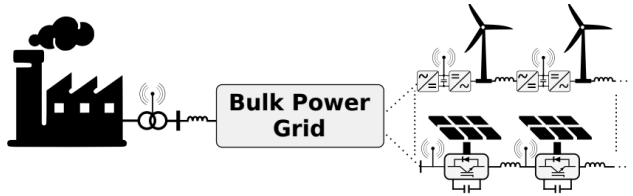
But, how can they trust it...? How about we solve this:

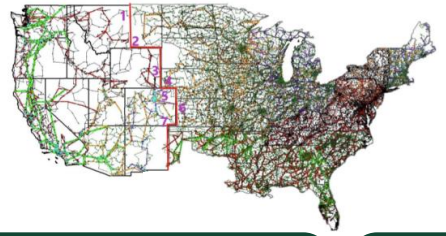
$$
\begin{aligned}
\max \quad & \textbf{network voltage violation} \\
\text{s.t.} \quad & G, B = \mathrm{NN}(v, p, q) \\
& p_i = v_i \sum v_k (G_{ik} \cos(\theta_{ik}) + B_{ik} \sin(\theta_{ik})) \\
& q_i = v_i \sum v_k (G_{ik} \sin(\theta_{ik}) - B_{ik} \cos(\theta_{ik}))
\end{aligned}
$$

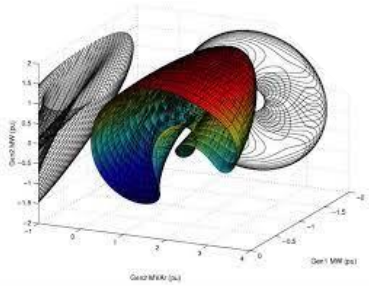
→ Negative system impacts
→ RL decisions
→ Network physics

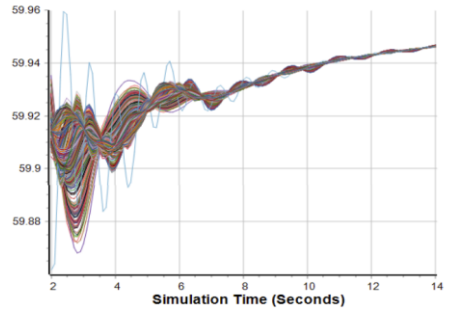
Verification in Power Systems — Opportunities

Key observation: In power systems, we know the physics!









Worst-Case Performance:

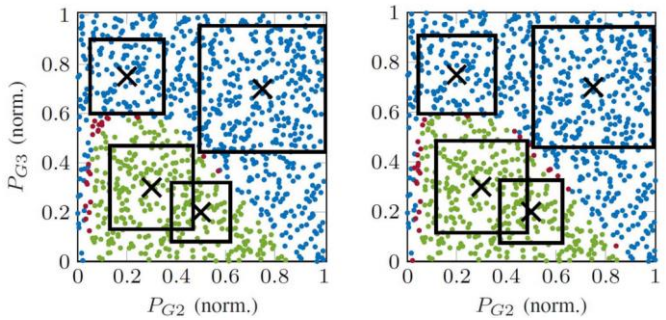
$$\max_{x \in \mathcal{X}} \|\mathbf{physics}(x) - \mathbf{ML}(x)\|$$

We thus have a **unique** opportunity to compare ML model predictions to the ground-truth physics (directly, or indirectly)

- *Solution gives a performance guarantee*
- There is no analog in many other ML applications, e.g., image classification models have no first-principles comparison

Verification in Power Systems — A 6-Paper Journey

1. Verification of NN behavior: Formal guarantees for power system applications
   - (Venzke, Chatzi., TSG'20): Trained NNs to predict regions of small signal/N-1 stability



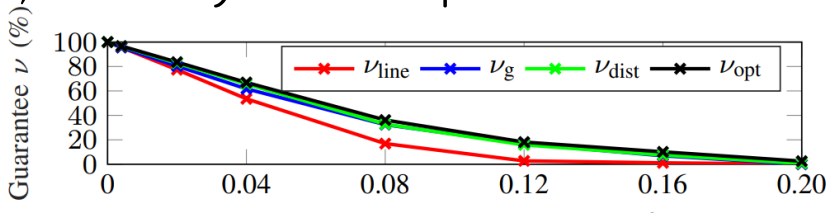
$$\min_{\mathbf{x},\hat{\mathbf{z}},\mathbf{z},\mathbf{y},\epsilon} \quad \epsilon \qquad (10a)$$

$$\text{s.t.} \quad (2),\ (3),\ (8a)-(8e) \qquad (10b)$$

correct safe · correct unsafe · misclassified · verification samples

Regression Error

2. Learning Optimal Power Flow: Worst-Case Guarantees for Neural Networks
   - (Venzke, Qu, Low, Chatzi., SGC'20): Verified performance of NN-based DCOPF solvers



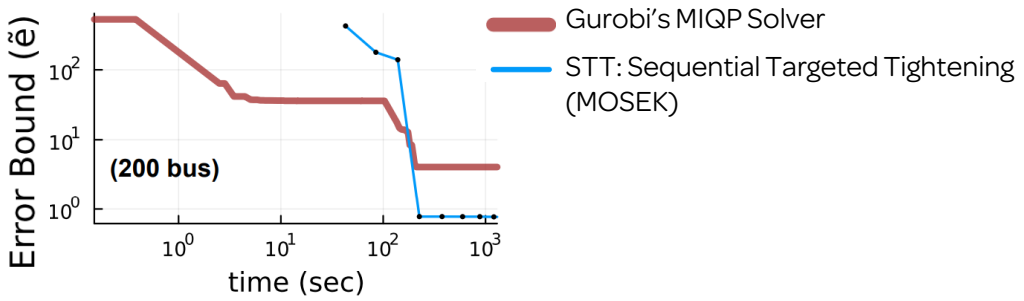
Nonlinearity

3. Physics-Informed Neural Networks for AC Optimal Power Flow
   - (Nellikkath, Chatzi., PSCC'22): Worst-case performance of NN-based ACOPF solvers
     - » *"We were unable to compute the worst-case line flow constraint violation since the MIQCQP problem could not be solved to zero optimality gap within 5 hr."*

Scale (Bounds)

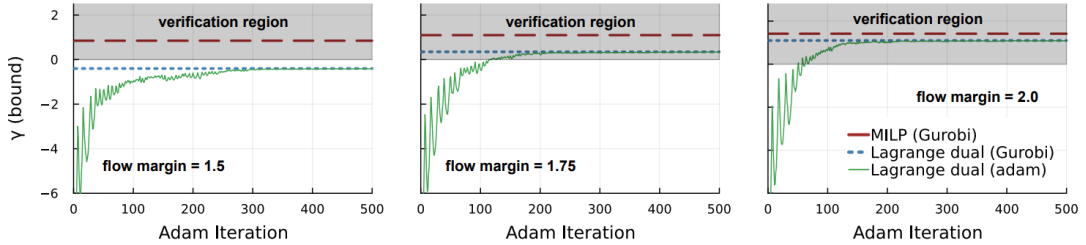
Verification in Power Systems – Key Papers

4. Global Performance Guarantees for Neural Network Models of AC Power Flow
   - (Chevalier, Chatzi., submitted '24): Targeted SDP cuts to enhance solver scalability



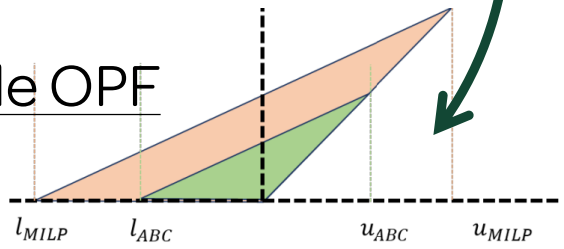
GPUs

5. GPU-Accelerated Verification of Machine Learning Models for Power Systems
   - (Chevalier, Murzakhanov, Chatzi., HICSS'24): Exploited GPU-based tools from ML community



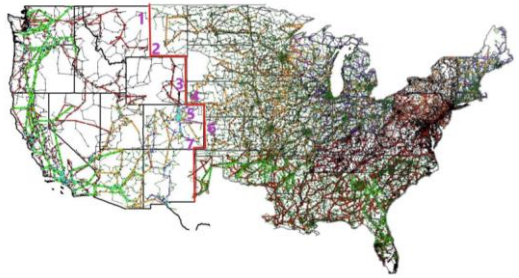
GPU + CPU

6. Scalable Exact Verification of Optimization Proxies for Large-Scale OPF
   - (Nellikkath, et al, Van Hentenryck, submitted '24): GPU-based tools to accelerate MILP convergence on CPU-based Gurobi

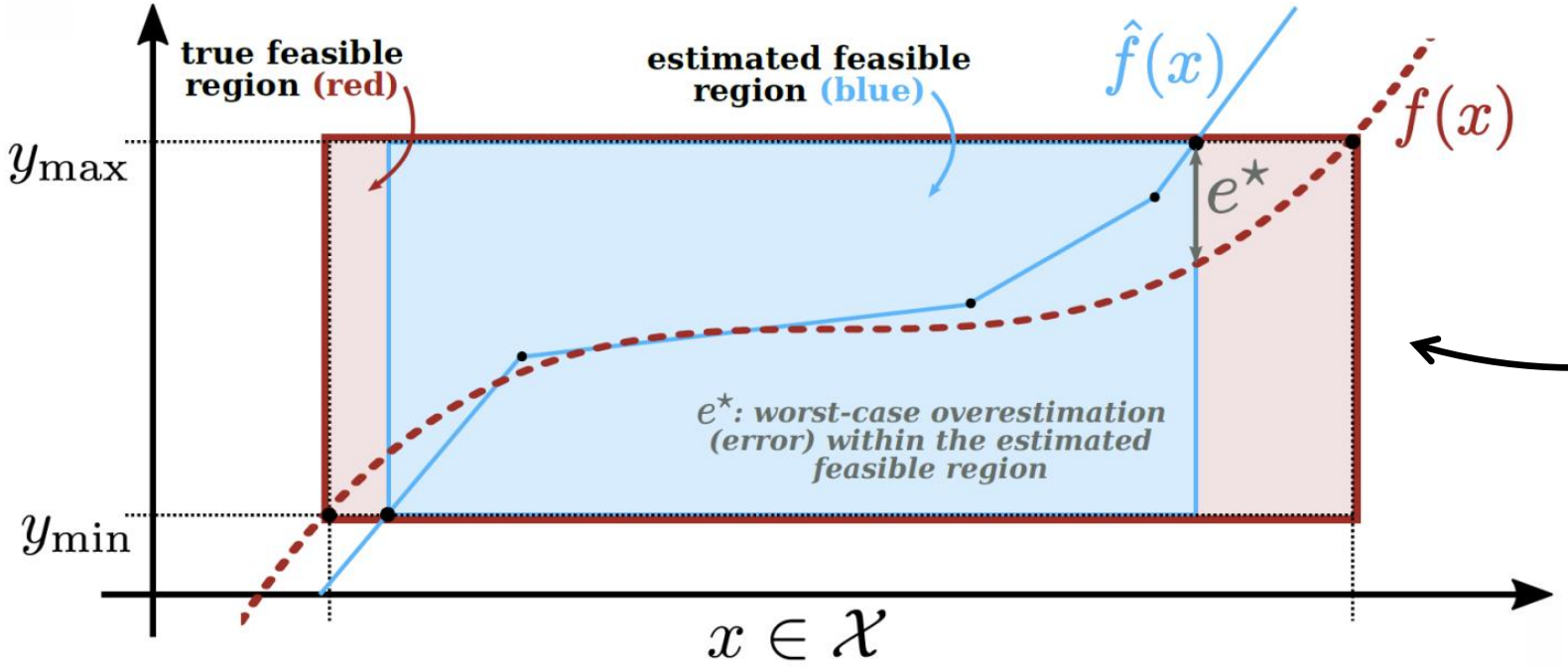


Paper: Global Performance Guarantees for NN Models of AC Power Flow

- Consider a NN power flow mapping: $\hat{f}(\underbrace{v_r, v_i}_{x}) \rightarrow (\underbrace{\mathrm{V}^2, p^{\mathrm{inj}}, q^{\mathrm{inj}}, l^{\mathrm{ft}}, l^{\mathrm{tf}}}_{y})$



- In our paper, we ask, "What is **the worst case performance** of a NN surrogate model relative to the ground truth?"



Quadratic (power)

$$e_i^\star = \max \; f_i(x) - \hat{f}_i(x)$$
$$\text{s.t.} \; \hat{f}(x) \geq 0$$

MILP (NN)

Paper: Global Performance Guarantees for NN Models of AC Power Flow

$$
\begin{aligned}
e_i^\star = \max_x \quad & x^T \boldsymbol{M}_i x - c_i^T x \\
\text{s.t.} \quad & \boldsymbol{A}x - b \geq 0 \\
& x_b \in \{0,1\}^d
\end{aligned}
$$

Nonconvex MIQP (Gurobi)

Double **Semidefinite Programming (SDP) Relaxation (binaries and quadratic terms!)**

$$
\begin{aligned}
e_i^\star \leq \max_x \quad & \mathrm{tr}(\boldsymbol{M}_i \boldsymbol{X}) - c_i^T x \\
\text{s.t.} \quad & \boldsymbol{A}x - b \geq 0 \\
& (\boldsymbol{A}\boldsymbol{X}\boldsymbol{A}^T - \boldsymbol{A}xb^T - (\boldsymbol{A}x)^T b + bb^T \geq 0)_i,\ i \in \mathcal{S}_c \\
& \begin{bmatrix} 1 & x^T \\ x & \boldsymbol{X} \end{bmatrix} \succeq 0
\end{aligned}
$$

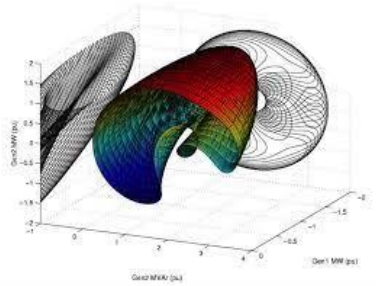
Convex SDP (MOSEK)

- **Tighten relaxation with "RLT" cuts! Only a few are needed, it turns out (which?)**

$$(\boldsymbol{A}x - b)(\boldsymbol{A}x - b)^T = \boldsymbol{A}\underbrace{xx^T}_{\boldsymbol{X}}\boldsymbol{A}^T - \boldsymbol{A}xb^T - (\boldsymbol{A}x)^T b + bb^T \geq 0$$

Paper: GPU-Accelerated Verification of ML Models for Power Systems

- In power systems, many ML models will need to be constrained to obey (not satisfy, per se) grid physics
  - Example: DERs in a distribution grid: they make a control decision, and voltages react, but reaction lives on a manifold



$$\gamma = \min_{x,y,z} \quad c^T z$$
$$\text{s.t.} \quad y = \text{NN}(x)$$
$$Ay = z \ : \ \lambda$$
$$Bz \leq h \ : \ \mu,$$

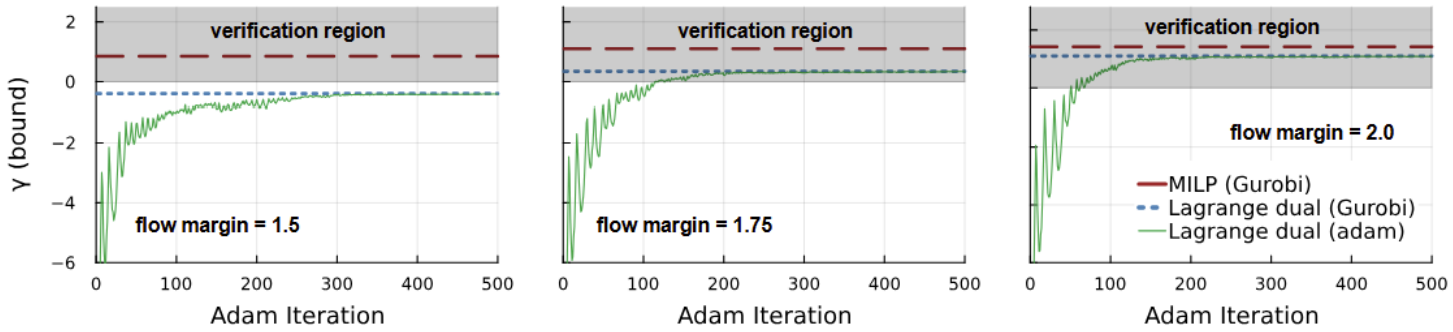
Dualize network physics

$$\gamma = \max_{\mu \geq 0, \lambda} \min_{x,y,z} \quad \lambda^T A y + (c + B^T \mu - \lambda)^T z - \mu^T h$$
$$\text{s.t.} \quad y = \text{NN}(x).$$

Now, apply class tricks to solve this ☺

ECE598HZ: you all have pretty good tools for dealing with this part!!!

AB-CROWN, AutoLirpa, etc…



Lecture (Talk?) Outline

①

Challenges

...in actual, and future, power systems.

②

Tools

...for overcoming these challenges (ML + verification)

③

Synergies

...between ML verification and grid operation.

Synergies: Grid Operation and ML Verification

& Problems...

Power System Operation and ML Verification are both "MINLPs"

Power Grid Operation

- Network is **sparse**
- Highly structured (**physics**)
- Dominated by binaries and bilinearities
- Local optimality is ok, but need primal feasibility
- Similar problems solved over and over and over on same network

ML Verification

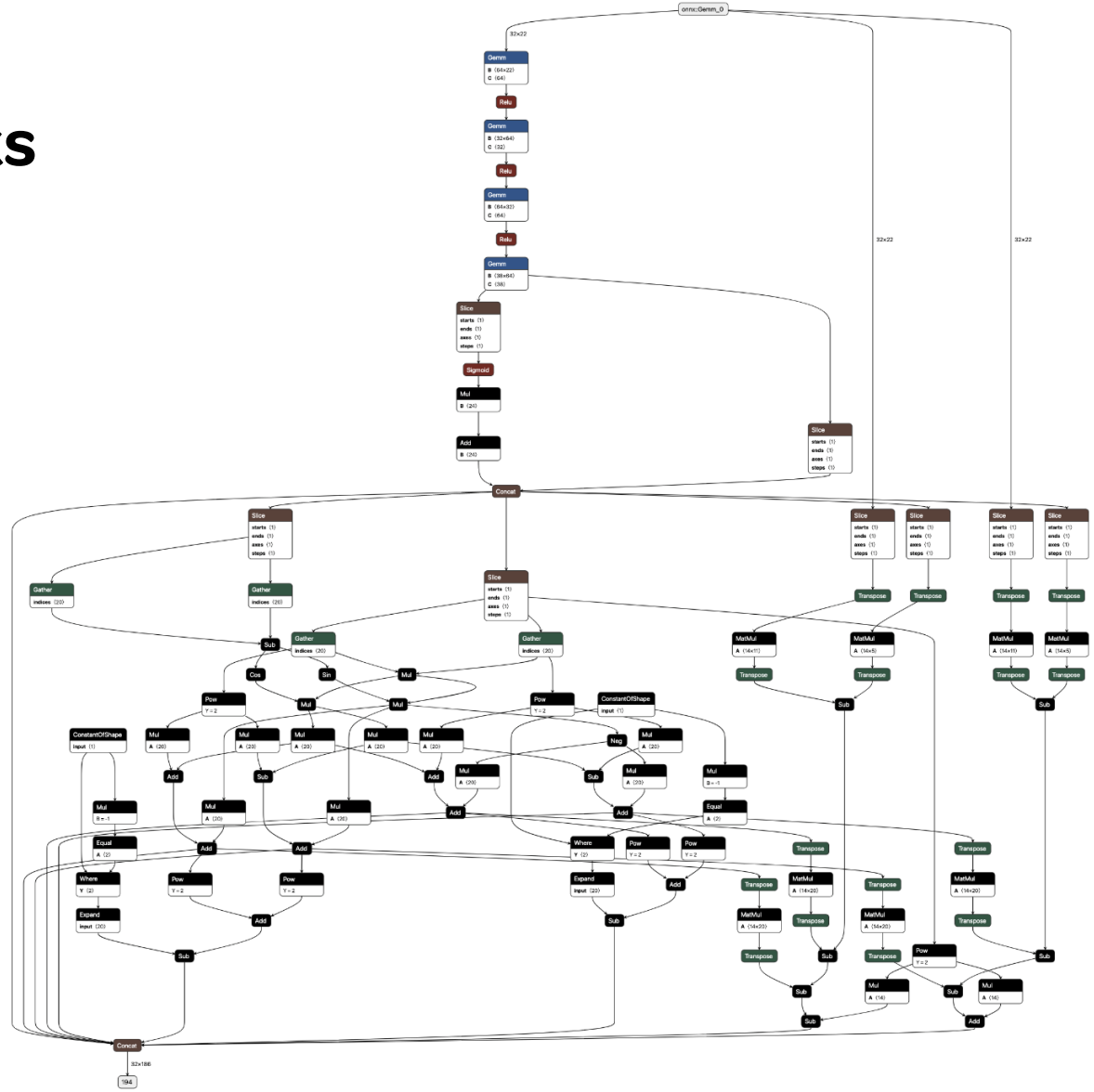
- Network is generally **dense**
- Highly structured (eg, **forward pass**)
- Dominated by binaries and many types of nonlinearities
- Need a global bound, or a local adversarial perturbation
- Similar problems solved on perturbed network (i.e., after retraining)

Synergy I: Computational Graphs

Both power grids and Neural Networks are computational graphs!

This is the “ML4ACOPF” NN, which is a VNNComp benchmark network:

- It takes a power system load, predicts an Optimal Power Flow solution, and then uses direct calculation to infer if the solution violates any constraints
- A verifier can then “search” for a loading condition which leads to violation.
- First half of the network is ML (dense); second half is sparse (physics)



Synergy I: Computational Graphs

Problem: Forward Pass in a Grid…?

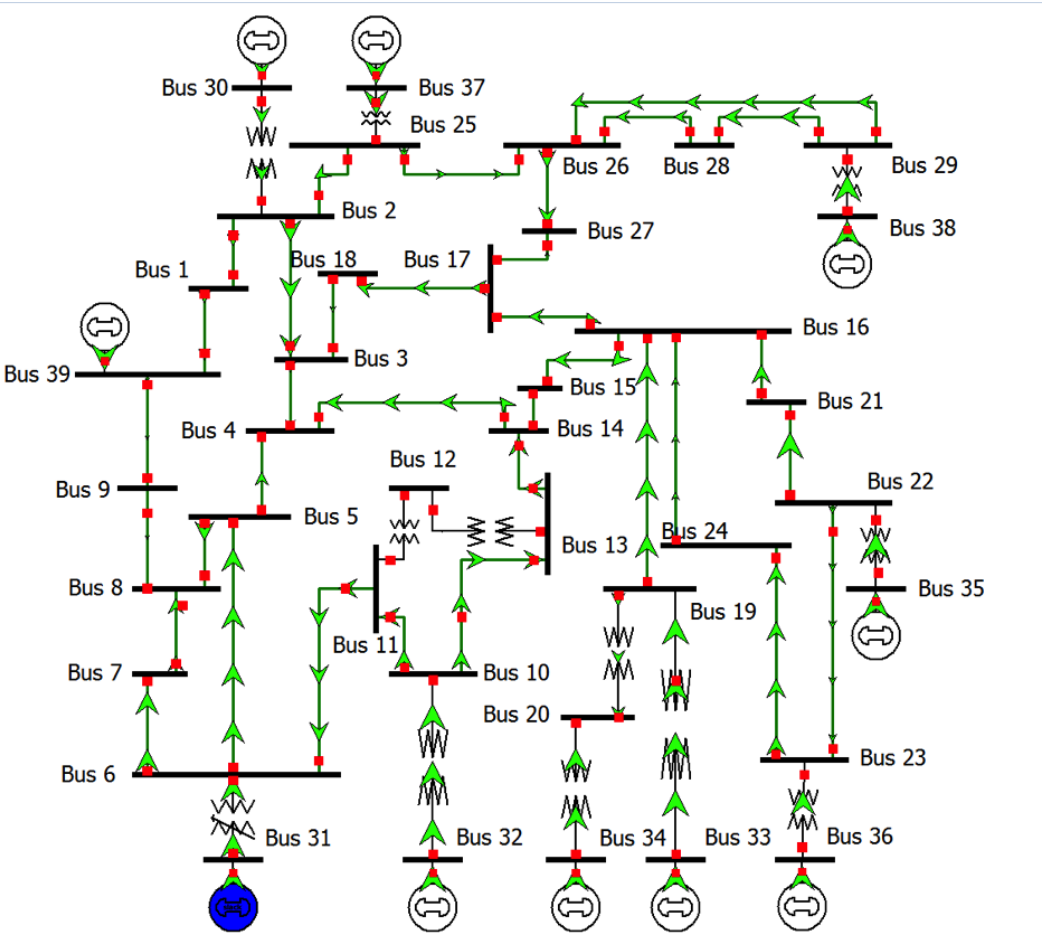
Unlike a NN, the definition of a "forward pass" is not obviously defined in a power grid

Nodal voltages cause currents to flow, engendering power flow injections.

One option:

- given V, compute the rest (**forward pass**)
- given the rest, compute V (**backward pass**)

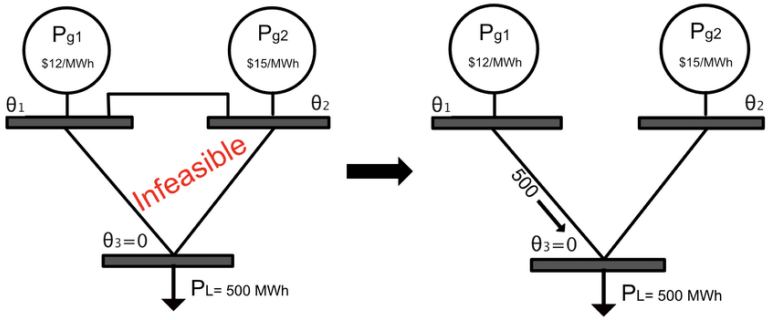
Do you have a better idea? Awesome implications, if so (next slides…)



Synergy II: Both "Need" Branch-and-Bound Solvers

Electric Power Grids

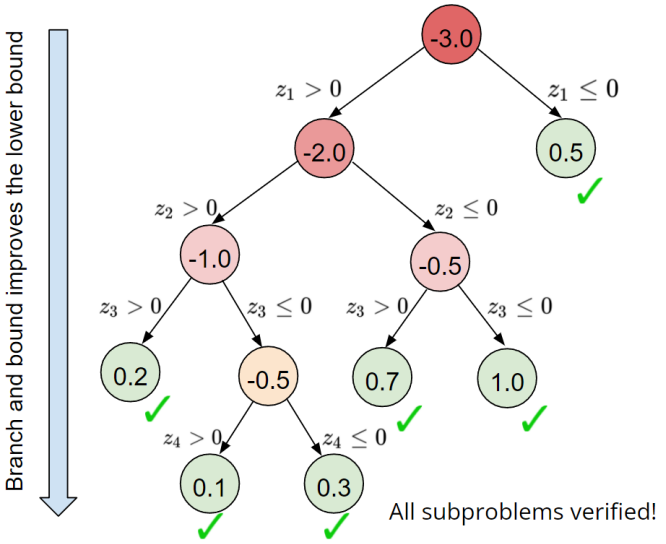
- Used to switch transmission lines, turn generators on/off, switch voltage controllers, branch over bilinear nonconvexities, etc.
- DOMINATES industry (MISO, ERCOT, ISO-NE etc)



Crozier

ML Verification

- Used to toggle activation functions on and off, in search of a lower bound (or, to find better adversarial inputs to the model)



Synergy II: Both "Need" Branch-and-Bound Solvers

- α,β-CROWN's amazing gradient-based, dual-formulated B&B performance directly inspired "**QuasiGrad**", the solver I built to compete in the GO competition

A Parallelized, Adam-Based Solver for Reserve and Security Constrained AC Unit Commitment

While Adam has been a successful tool for solving large-scale nonlinear programming (NLP) problems, it has also recently been used to solve sub-problems in massive Mixed-Integer Linear Programming (MILP) Branch-and-Bound (BaB) problems. The α, β-CROWN solver [7], which has won the most recent International Verification of Neural Networks Competitions (VNN-COMPs) [8], uses a GPU-accelerated Adam solver to verify the performance of ex-

Rank	Team	Division 2 Score	$k
	Ensemble	**163,579,841,300**	
1	GOT-BSI-OPF	162,941,475,726	100
2	TIM-GO	162,270,256,651	90
3	YongOptimization	160,165,088,341	80
4	Artelys_Columbia	157,359,267,058	70
5	GravityX	156,131,225,903	60
	ARPA-e Benchmark	156,014,230,887	
	quasiGrad	155,168,735,676	
	Occams razor	145,494,618,835	
	Electric-Stampede	139,357,283,507	
	LLGoMax	116,812,192,654	
	The Blackouts	114,098,832,983	
	Gatorgar	10,263,109,863	

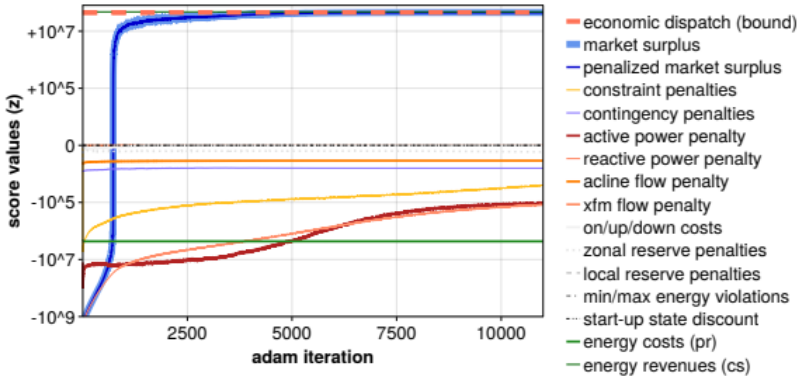


Fig. 1: Illustrated is an Adam solve on a 617-bus, 18 time period, real-time market clearing test case (integers relaxed); this system is initialized with a copper pate economic dispatch solution (LP), whose upper bound is given as the orange dashed line. Within several thousand iterations, Adam finds an AC network solution to within 1% of this global bound. A single back-propagation (i.e., gradient calculation) through this entire system, include all 18×562 contingencies, takes ~24ms when parallelized on 6 CPU threads.

- Results were very mixed, but I did solve power systems with **100 million+ variables**

Synergy II: Both "Need" Branch-and-Bound Solvers

The QuasiGrad approach has 2 general problems:

- **Problem I: Termination.**
  - ML verification B&B terminates when (i) a lower bound is proved >0, or (ii) an adversarial input is identified
  - Power system B&B terminates when the **global optimality** is reached: ***way harder***

- **Problem II: Nonlinearity (switches!!)**
  - Integers in power systems generally represent discontinuous switches, while ReLUs are continuous

Synergy III: Both Seek to Protect Against Adversarial Attacks

Electric Power Grids

- Specifically **enumerates** "contingencies" and finds a single operating point which guards against all of them
- Grid operators may utilize:
  - "**security constraints**" to guard against network component (line) loss
  - "**reserve constraints**" to ensure enough spinning reserve, etc, in grid zones
  - "**chance constraints**" to guard against probabilistic overloading or renewables

ML Verification

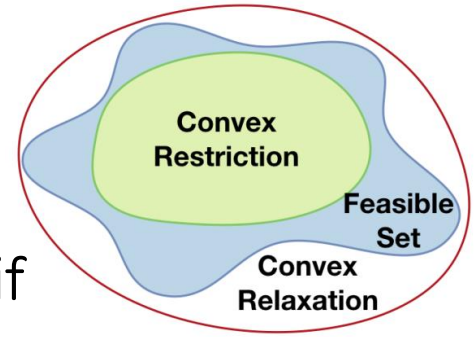
- Given a model that "does something", verification typically searches for a conceivable violation of some metric
- These violations may include:
  - Adversarial perturbations
  - Correctness violation
  - Lack of monotonicity or
  - Violation of terms of constraints

Searches for working operating point in an apriori broken model ⟷ **Searches for broken operating point in an apriori working model**

Synergy IV: Convex Restrictions Can be Useful!

Electric Power Grids

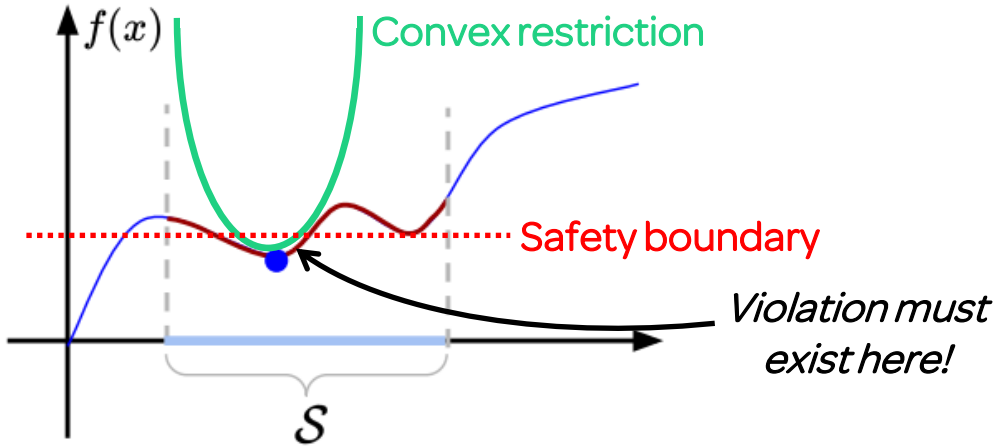
- Convex restrictions can guarantee the existence of a "robust" solution within some operational space [1]
- Fixed point theorems (e.g., Browers) can guarantee that, under some bounded perturbation, a **feasible solution will still exist**, even if we don't explicitly find it



Proves the existence of something good.

ML Verification

- Convex restrictions can guarantee the existence of an adversarial input, even if we never actually find it – useful?



Proves the existence of something bad.

[1] D. Lee, H. D. Nguyen, K. Dvijotham and K. Turitsyn, "Convex Restriction of Power Flow Feasibility Sets," in *IEEE TCNS*, Sept. 2019.

Synergy V: Bounding of Nonlinearities

Electric Power Grids

- Can we tighten over power system nonlinearities? How about **bilinearity**?

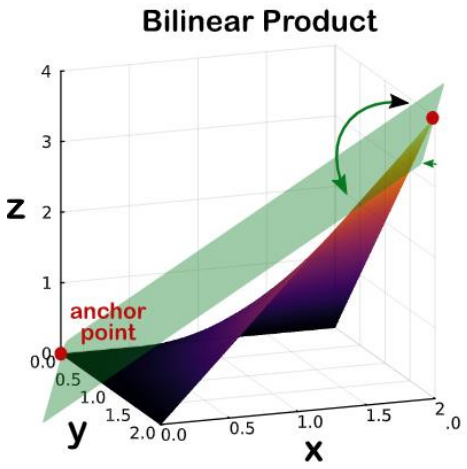
$$z \triangleq xy = \alpha_1 x + \alpha_2 y + \beta$$

$$\alpha_1 = y$$

$$\alpha_2 = x$$

$$\underbrace{\beta = -\alpha_1 \alpha_2}$$

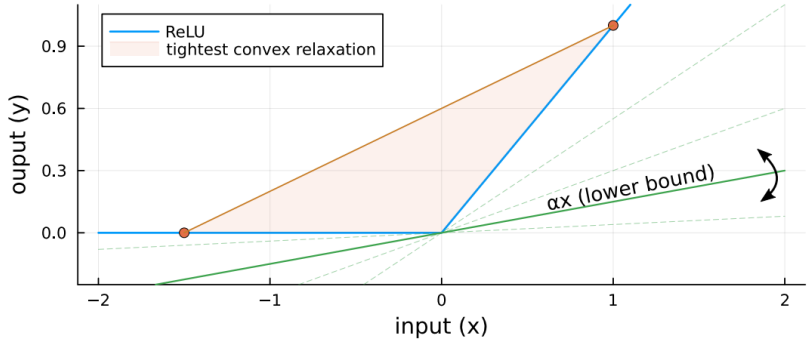
$$z = \alpha_1 x + \alpha_2 y - \alpha_1 \alpha_2$$



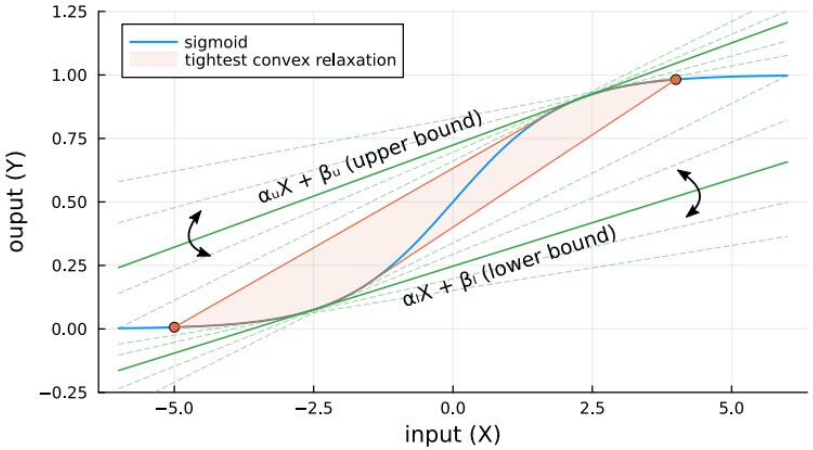
Problem: convexifying ugly non-convexitites in power systems pushes you far from feasibility

ML Verification

- Tighten over ReLUS via α-CROWN:



- Tighten over sigmoid via α-sig:



Synergy VI: Find Intermediate Bounds! ❤

Electric Power Grids

- Bounding intermediate variables in a big problem has huge benefits
  - **Big-M** values in MILP optimization,
  - Upper/lower bounds in bilinear McCormick **envelopes**,
  - Sequential/optimization-based bound tightening (OBBT) for state estimation and global opt.
  - Faster B&B tree trimming, even
- Power system researchers are historically clever, but technologically constrained on this task (**no GPUs**!)

ML Verification

- Auto-LiRPA is **very good** at this sort of thing. However, it isn't 1:1 applicable to bounding variable in the problem

$$\begin{aligned} &\min\ c(x) \\ &\text{s.t. } f(x) = 0 \\ &\quad\ g(x) \leq 0 \end{aligned}$$

- General Cutting Plane (GCP-CROWN) is a step in the right direction...
- ...but **INVPROP** is a game changer

Synergy VI: Find Intermediate Bounds! ❤

Let's assume you **don't know** good bounds on the input (e.g., voltage)

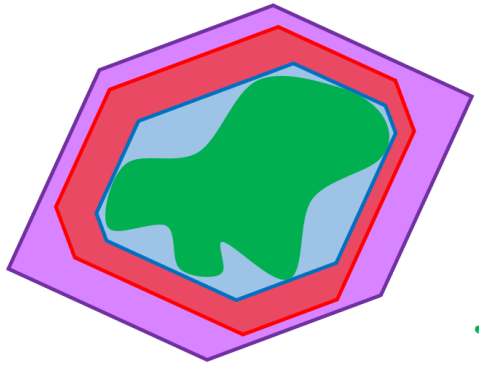
- Can you take an output bound and propagate backwards to the input?
  - **Sure!** Just use INVPROP [1].

Bounded Input (Conventional)

$$\min_{x \in \mathcal{S}_{\text{in}}} \; c^T y$$
$$\text{s.t. } y = \text{NN}(x)$$

Bounded Output (INVPROP)

$$\min_{x} \; c^T x$$
$$\text{s.t. } \text{NN}(x) \in \mathcal{S}_{\text{out}}$$



$$\mathcal{S}_{\text{over}} \supseteq$$
$$\mathcal{S}_{\text{LP}} \supseteq$$
$$\mathcal{S}_{\text{MILP}} \supseteq$$
$$f^{-1}(\mathcal{S}_{\text{out}})$$

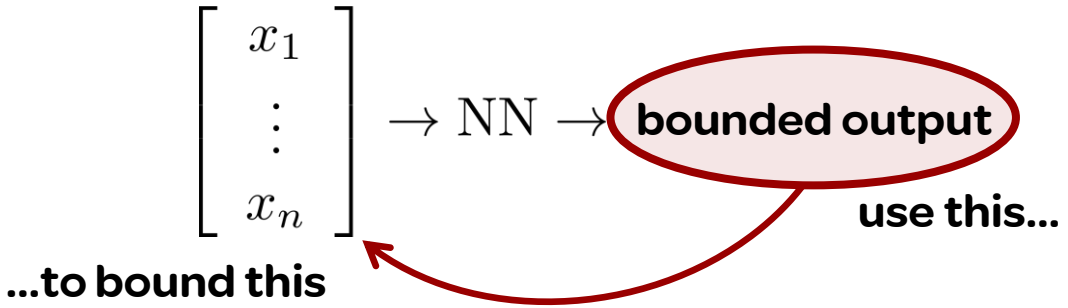
  - **Most exciting:** you can bound any input, or any arbitrary cut!

[1] Kotha, Suhas, et al. "Provably bounding neural network preimages." *NeurIPS* (2024).

Synergy VI: Find Intermediate Bounds! ❤

Most exciting: you can bound any input, or any arbitrary cut!

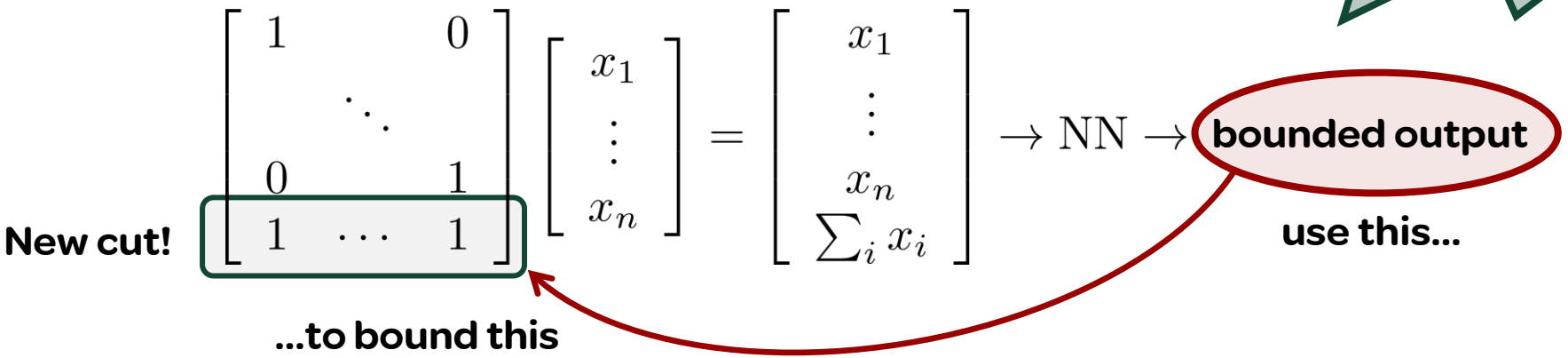
1. Direct bounding of inputs:



Potential (Non-ML) Uses

McCormick Bounds
Big-M Tightening
Reachability Analysis
Feasibility Tests...

2. Bounding of new, useful **cuts**!



1. Trained NN to emulate DC-OPF

$$\mathrm{NN}(p_d) \rightarrow p_g$$

2. Given output generation limits:

$$\underline{p}_g \leq p_g \leq \overline{p}_g$$

↓

INVPROP

↓

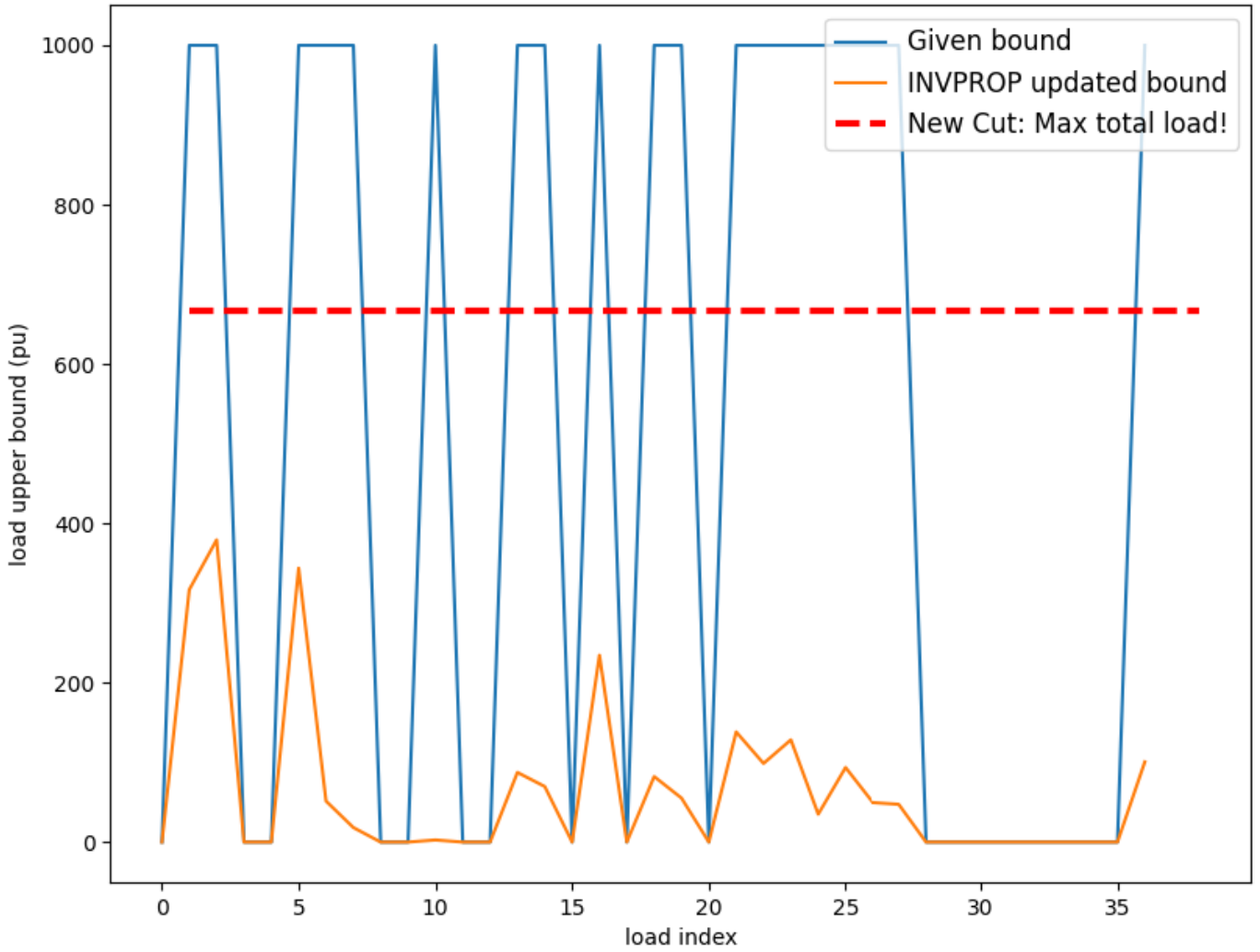
Improved blue bound to orange bound (load inputs)!

3. I then asked INVPROP to compute a new cut:

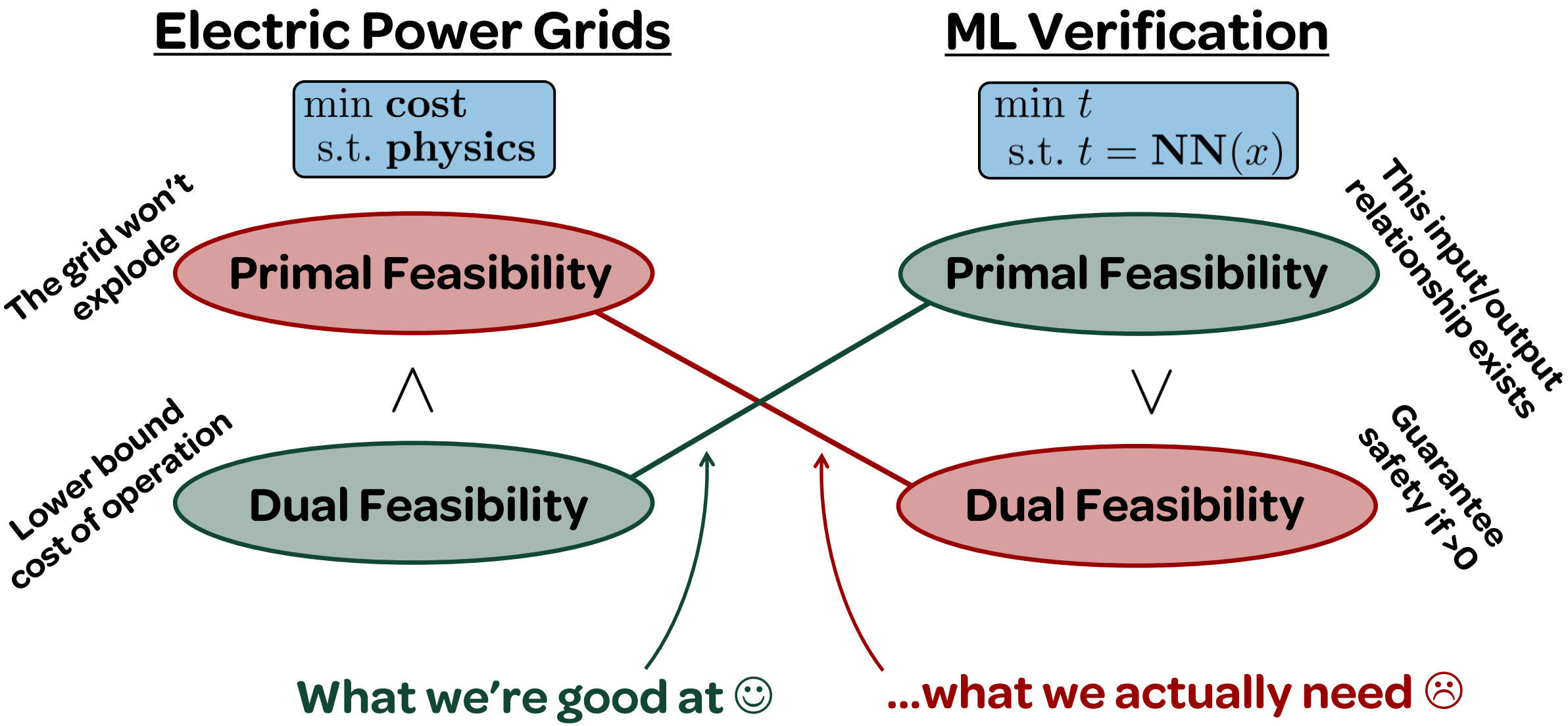
$$\sum p_d \leq ?$$

↓

Red bound! (3x+ tighter than just $\sum$[orange bound])



(Anti?)Synergies: Grid Operation and ML Verification



Summary

- Neural Network verification is a hard, **NP-complete** problem
- Power grid reliability/security is also **NP-compete**, in the sense that a potential solution is checkable in polynomial time
- Grid **optimization**, however, is generally **not in NP**, because we want a solution which is "globally optimal" and secure
  – If a solution is not optimal, market participants may **not be happy**…

- If ML is embraced by system operators, we will want guarantees that it is **reliable** (NP-complete) and, in some many, **optimal** (not in NP)
- Verification and grid optimization share many exciting congruent + dual characteristics, and the **future is bright**

- Finally, if you want to **work on these problems**…

Come to Vermont!

(For a PhD)

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