NRICH

Polite Numbers

Age 16 to 18
Challenge Level

A polite number is a number which can be written as the sum of two or more consecutive positive integers.
For example, $21 = 10 + 11$ is polite as it is the sum of 2 consecutive positive integers, and $10 = 1 + 2 + 3 + 4$ is polite as it is the sum of four consecutive positive integers.
Here are some questions to think about:
Is 63 a polite number?

If you add up three consecutive integers, what sort of answers do you get?

Are all multiples of 5 polite?
An impolite number is one that cannot be written as a sum of two or more consecutive positive integers.
Can you find an impolite number?

Can an impolite number be odd?
Can you find a rule for identifying impolite numbers?
Show hint

Can you explain why your rule works?
Show hint

When you have explored this problem, you might like to take a look at the different proofs offered in the problem Impossible Sums https://nrich.maths.org/14954 .
Did you know ... ?

Although number theory - the study of the natural numbers - does not typically feature in school curricula it plays a leading role in university at first year and beyond. Having a good grasp of the fundamentals of number theory is useful across all disciplines of mathematics. Moreover, problems in number theory are a great leisure past time as many require only minimal knowledge of mathematical 'content'.

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