Lab Assignment 1

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Task 1

Use `SymPy` to solve the differential equation $y' = -y^2 \sin(x)$, with $y(0) = 1$, and plot the solution.

In [4]:

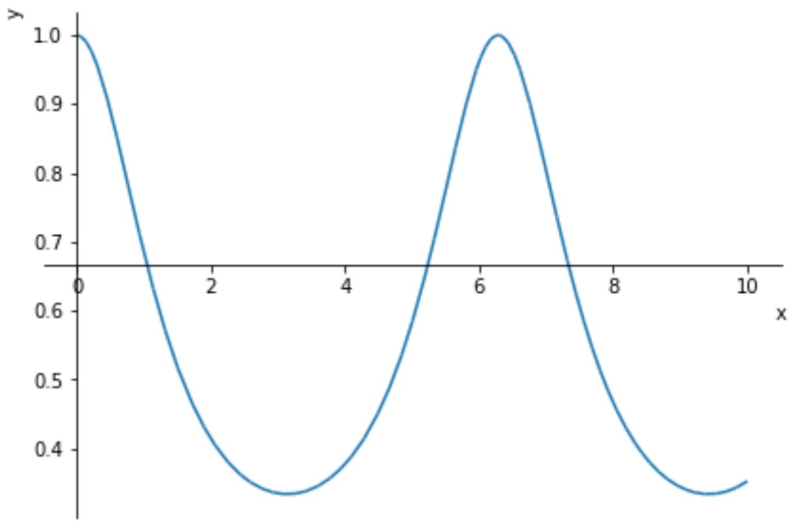
```
from sympy import *
init_printing()
from IPython.display import display_latex

# Define symbols for sympy to work with.
x = symbols("x")
y = Function("y")
y_prime = y(x).diff(x)

# Define the differential equation and print it into the console.
diff_eq = Eq(y_prime,
             -(y(x)**2)*sin(x))

print("Equation:")
display_latex(diff_eq)

# Solve it and print the solution into the console.
sol = dsolve(diff_eq, ics={y(0):1})

print("Has solution (for y(0) = 1):")
display_latex(sol)

# Plot the solution
plotting.plot(sol.rhs, (x,0,10), xlabel = 'x', ylabel = 'y')
```

```
Equation:
```

$$\frac{d}{dx} y(x) = -y^2(x) \sin(x)$$

```
Has solution (for y(0) = 1):
```

$$y(x) = -\frac{1}{\cos(x) - 2}$$



Out[4]: `<sympy.plotting.plot.Plot at 0x19bc7c9d4c0>`

Task 2

Use `SciPy` 's `odeint` function to solve the system of equations

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = x - x^3$$

Produce a plot of the solutions for $0 \leq t \leq 10$ with initial conditions $x(0) = 0$ and $y(0) \in \{0, 0.5, 1, \ldots, 3\}$.

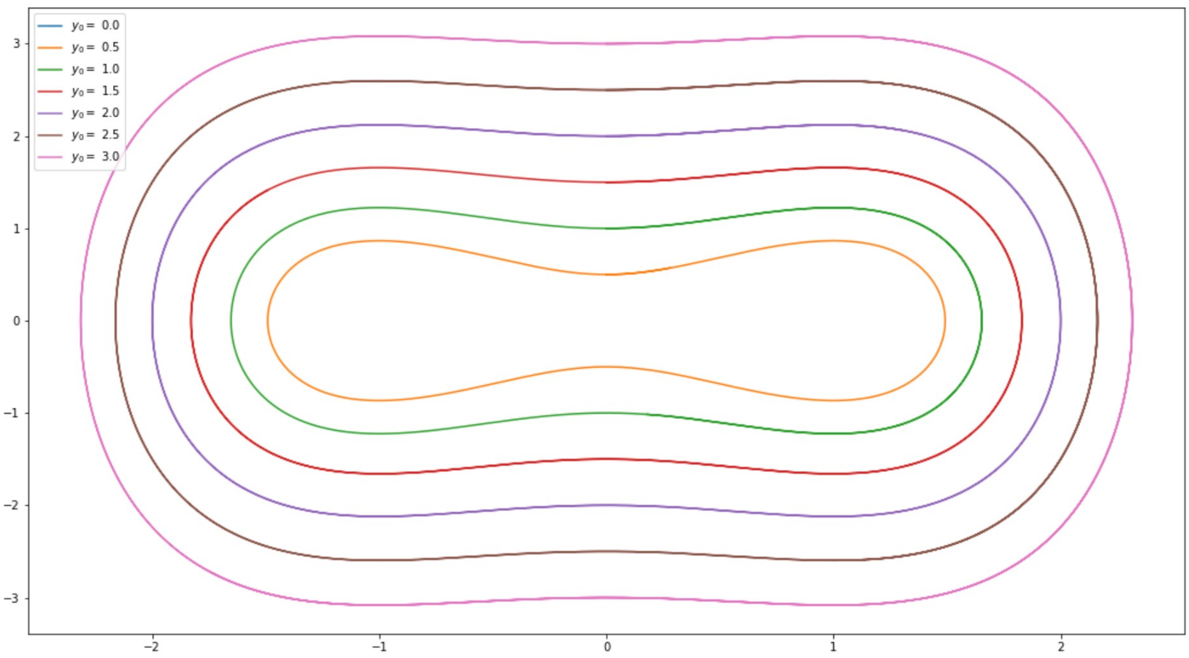
How many curves do you expect to see plotted? How many do you actually see, and why is this?

In [3]:

```python
# Imports for numerical integration and plotting
from scipy.integrate import odeint
import numpy as np
import matplotlib.pyplot as plt

# Setup a figure and axes
fig, ax = plt.subplots(figsize=(18, 10))



#
def dX_dt(X, t):
    x, y = X
    return (y, x - x**3)


t_range = np.linspace(0, 10, 1000)

for i in range(7):
    y_0 = i/2
    X = odeint(dX_dt, (0, y_0), t_range)
    x_list, y_list = X.T
    ax.plot(x_list, y_list, label=f"$y_0 =$ {y_0}")

ax.legend()
```

Out[3]:

```
<matplotlib.legend.Legend at 0x19bc786a6a0>
```



We don't see the solution plotted for $(x_0, y_0) = (0, 0)$. This is because at this point both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are 0; so the x and y values of our solution won't change as time progresses.

Note that our differential equation gives the level curves to:

$$F(x, y) = \frac{1}{4}(2y^2 + x^4 - 2x^2)$$

Hence why our plot looks like a contour plot (because it is).

In [ ]: