

# Relaxation for Efficient Asynchronous Queues

Samuel Baldwin 

Bucknell University, USA

Cole Hausman 

Bucknell University, USA

Mohamed Bakr 

Bucknell University, USA

Edward Talmage  

Bucknell University, USA

## 1 Introduction

This paper proposes a new algorithm for implementing a standard FIFO queue in a fully asynchronous message passing mode. To our knowledge we have not encountered any previous work that suggested a queue algorithm for this specific model. Our algorithm utilizes vector clock timestamps that allow individual processes to hold some view of the time steps of other processes have taken at points of communication. Using this technique to timestamp invocations of the two operations of the queue, then linearizing all of the queue invocations based on the ascending lexicographic order of these timestamps creates a valid permutation that meets the specifications of a FIFO Queue. The goal of establishing this queue method is to design a system with full replication. There are existing systems that handle the asynchronous queue (server client model), but are incapable of replication. With replication, and (INSERT THE CITATION TO THE PAPER HERE) we hope that this queue algorithm can be used in fault tolerant systems in the future.

### 1.1 Related Work

## 2 Model and Definitions

### 2.1 Asynchronous System Model

We assume a fully asynchronous message passing model, with a set of  $n$  processes  $\Pi = [p_0, \dots, p_{n-1}]$  modeled as state machines. These state machines are time-free, meaning that their output is only described through their input and state transitions without any specific time bound. All inter-process communication is assumed to be reliable i.e any message that is sent will always be received at its destination process. Additionally, communication channels between processes are considered to be treated as FIFO, in that any message sent from process  $A$  to process  $B$  will be processed prior to any other message from the same pair sent later in an execution. This can be accomplished by assuming that each process has an incoming buffer for each other process, and that inter-process messages are marked numerically, and out of order messages are stored in the buffer until they can be retrieved in order. We also assume no processes crash, and none of the processes have access to a hardware clock.

### 2.2 Queue Definitions

We introduce the definition for a standard FIFO Queue abstract data type. We will use the special character  $\perp$  to represent an empty queue.

► **Definition 1.** A Queue over a set of values  $V$  is a data type with two operations:



© Samuel Baldwin, Cole Hausman, Mohamed Bakr, Edward Talmage;  
licensed under Creative Commons License CC-BY 4.0

42nd Conference on Very Important Topics (CVIT 2016).

Editors: John Q. Open and Joan R. Access; Article No. 23; pp. 23:1–23:7

Leibniz International Proceedings in Informatics



LIPIC Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

## 23:2 Asynchronous Relaxed Queues

- 42 ■ *Enqueue(val, -)* adds the value  $X$  to the queue
- 43 ■ *Dequeue(-, val)* returns the oldest value in the queue.

44 A sequence of queue operations is legal iff it satisfies the following conditions. The empty  
45 sequence is a legal sequence. Each Enqueue value is unique. Once a value has been enqueued  
46 to the queue once, it can not be enqueued again. If  $\rho$  is a legal sequence of operation instances,  
47 then  $\rho \cdot \text{Dequeue}(-, val), val \neq \perp$  is legal iff *Enqueue(val, -)* is the first Enqueue instance in  $\rho$   
48 that does not already have a matching *Dequeue(-, val)* in  $\rho$ . Furthermore,  $\rho \cdot \text{Dequeue}(-, \perp)$   
49 is legal iff every *Enqueue(val, -)* in  $\rho$  has a matching *Dequeue(-, val)* in  $\rho$ .

### 3 Asynchronous FIFO Queues

#### 3.1 Description

52 Each process stores a vector clock timestamp that holds its local view of the clocks at all  
53 other processes. Thus the vector clock timestamp at  $v_i$ , dictating the vector clock view of  
54 process  $i$  is an array of size  $n$  (number of processes in this system) that is initially 0 at all  
55 indices. At the point when a process starts an *Enqueue()* or *Dequeue()* invocation, that  
56 process will increment their own index in the local clock timestamp, which marks a step  
57 or clock tick. Processes also update the local view of the vector clocks when they receive  
58 a message containing a timestamp from another process. This occurs each Enqueue and  
59 Dequeue invocation and response, as as a part of the payload of each of these messages,  
60 the timestamp at the point in which it was invoked is included. This results in processes  
61 regularly updating the local views of their clocks, resulting in processes being as updated as  
62 possible. To update the local vector clock, values at corresponding indexes in the local clock  
63 and message clock are compared, and the larger of the two values is set to the local view. By  
64 adjusting each of these indices, this guarantees that a local clock will have the largest of the  
65 two indices at all indices in the local clock, and therefore has the most advanced possible clock  
66 at that point in the execution. For any two vector timestamps  $v_i$  and  $v_j$ , such that  $v_i \prec v_j$ ,  
67 states that  $v_i$  is a strictly smaller timestamp than  $v_j$ , if  $v_i[x] < v_j[x], \forall x \in [0, \dots, n-1]$ . If  
68 this is not true, there is some index  $k$ , the first element that is different between the two  
69 vector timestamps  $v_i$  and  $v_j$  i.e. for  $x = 0$  to  $k-1, v_i[x] < v_j[x]$  but at index  $k, v_i[k] \geq v_j[k]$ .  
70 Then we say that  $v_i$  is lexicographically smaller than  $v_j, v_i \ll v_j$  if  $v_i[i] < v_j[i]$ . Notice that  
71  $v_i \prec v_j$  implies that  $v_i \ll v_j$  but not the opposite.

72 Within each process there is a local version of an augmented minimum priority queue  
73 keyed on lexicographic timestamp order. This priority queue can perform three operations:  
74 *insert(value, vclock)*, *get(position)*, *remove(value)*. The insert operation adds the value to  
75 the queue based on the vclock as a priority. The *get(position)* function allows the user to  
76 peek into the element at the passed position without removing a value from the queue. The  
77 *remove(value)* function removes the specific value passed to it from the queue which can  
78 be at any location in the queue. Ordering elements in FIFO order is not a straight-faced  
79 task in a distributed setting since defining which invocation happened first is not as clear as  
80 in a linear setup. Consequently using a priority queue keyed on lexicographic timestamp  
81 order allows us to ensure some order locally to guarantee FIFO consistency to the user once  
82 linearized.

##### 3.1.1 Confirmation Lists

84 The main structure of this algorithm it to utilize a structure we will define called a *Con-*  
85 *firmation List* to track the responses for a given dequeue locally for each process. Upon

receiving a dequeue request message, a given process will either declare that dequeue "safe" or "unsafe" if that process is or isn't in the active process of dequeuing an element. If a process will declare that message safe, it sends a message confirming that to the process that invoked the dequeue, and if it will declare the message unsafe, it sends a message stating that to all processes in the system. Both safe and unsafe messages will be marked with the invoking process, the responding process, and the invoking dequeue vector clock.

Upon hearing about a dequeue request, which are sent globally, a process will instantiate a Confirmation List in its local memory, indexed by the timestamp of the invoking processes dequeue. For the dequeue invoking process, the confirmation list will fill by receiving messages from the other processes in the system with a corresponding timestamp. A safe message will be marked as a 1 in the corresponding index of the process in the Confirmation List and an unsafe will be marked as a 2. Once the Confirmation List has no undefined indices, the number of 2's within the list will be counted, and the queue will be accessed at that index, and the element removed.

For processes that are not the invoking process, only the unsafe messages are received, and marked as 2 in the local Confirmation Lists. To fill the rest of the list, messages with timestamps strictly greater than the invoking timestamp will be treated as implicit "safe" messages, as there cannot be an unsafe message that has not been received from that process. Thus, with the same number of unsafe messages, the non-invoking processes will naturally come to the same conclusion and remove the same element as the invoking processes. Confirmation lists, like enqueue requests are sorted by timestamp order, as when they are created, the invoking dequeue timestamp is included. The general structure of a of conf list is as follows [[Flags for responses from a given process][Invoking Process ID][Timestamp of invocation]]

## 3.2 Algorithm

## 3.3 Correctness

► **Lemma 2.** *Every invocation has a matching response.*

**Proof.** Based on the specifications of the algorithm, if there is an *Enqueue*, the request for that is sent globally and responded to. In the case of a *Dequeue*, the request is sent globally, and the response for that *Dequeue* is also sent globally. In all cases of requests being sent, there is a delay of  $2d$ , where  $d$  is the maximum message delay. ◀

► **Lemma 3.** *The local views of the queues are equivalent at the time of Dequeue.*

**Proof.** For this algorithm, we will consider the entire history of the execution, such that all elements that were at one point in the queue are considered a part of the prefix of the queue. Once an element is dequeued, that element remains a part of the prefix, but no longer can be accessed, and is marked unavailable for dequeues.

While processes may have different complete histories for the histories of their enqueues at some point in time, given they may be in different local states with respect to active enqueues that they may have not received, the prefix of all local views up to the timestamp of the local execution of a dequeue will be the same. This is due to the fact that there is an agreed upon order for sorting the enqueues, and as such all processes will agree on the order. Furthermore, at the time of a dequeue, we can confirm that a given process will have heard of all prior enqueues. This is due to the fact that in order to locally execute a dequeue, a given process will have to have heard all messages prior to that dequeue, via the Confirmation List structure. The nature of the communication between processes being FIFO results in

## 23:4 Asynchronous Relaxed Queues

131 the knowledge that there cannot have been earlier messages that have gone unrelieved if a  
132 later message has been processed and therefore, if a process is actively completing a dequeue  
133 invocation, at the point of that invocation every message prior has been received. Thus, we  
134 can formally state that at such a point, the prefix of all messages and queue states are equal  
135 across all processes. ◀

136 ▶ **Lemma 4.** *In order to dequeue an element, there must be a corresponding Enqueue.*

137 **Proof.** Proof by contradiction. Assume that some process invokes a dequeue, receives  
138 responses and locally executes, removing some element that was not enqueued. That element,  
139 by the algorithm definition, must be held within the local view of the queue. This is because  
140 dequeues are accessed via the index provided by the unsafes. Therefore, the element that  
141 was dequeued by the process must exist in the local queue. The only way that can occur if  
142 the element was enqueued. Additionally, when removing an element from the queue, it is  
143 important to not remove elements with a larger timestamp than the dequeue. In the case  
144 in which there is a dequeue request and a timestamp where  $dqts < eqts$ , and the dequeue  
145 executes locally, given we cannot linearize the enq before the deq, we must return  $\perp$  and not  
146 the element. This behavior is defined in the algorithm (IM LYING ITS NOT WE NEED  
147 TO ADD IT :D), and allows us to establish a specified order. ◀

148 ▶ **Lemma 5.** *Elements that are dequeued are unique. (No double dequeue.)*

149 **Proof.** Given that at the time of execution we have proved that the head of the list of  
150 enqueues, combined with the algorithm definition guaranteeing that the local conf list of  
151 all processes contains the same information, we can state that all processes at the time of  
152 local execution will act on the same information to select an index, and therefore will select  
153 the same index. Given that, we can state that they will remove the same element from the  
154 queue, regardless of the time of execution locally. ◀

155 ▶ **Theorem 6.** *Algorithm ?? correctly implements a FIFO queue.*

156 **Proof.** Based on the information of the lemmas, we can conclude that the algorithm definition  
157 for the queue follows the specifications for the ADT. Enqueues have corresponding dequeues,  
158 no double dequeues, we can linearize by timestamp such that the first element in the queue  
159 is always removed first. ◀

## 160 3.4 Complexity

## 161 4 Asynchronous Out-of-Order Queues

### 162 4.1 Description

### 163 4.2 Algorithm

### 164 4.3 Correctness

165 In the case where  $k > n$ , since the number of messages required to have a successful dequeue  
166 in bounds is less than the number of permitted dequeues ooo, dequeues can be instant. This  
167 information is based off of PUT THE PAPER NAME HERE which utilizes prior labeling of  
168 elements within the queue structure with a process that is allowed to instantly dequeue it.  
169 Given each element is distinct and can only be dequeued once, the labeling must be distinct,  
170 in that each process agrees on the labelling that a given value in the queue receives. Whereas  
171 in PAPER NAME AGAIN this is done via a process of synchronization of clocks, we will be

■ **Algorithm 1** Code for each process  $p_i$  to implement a Queue with out-of-order  $k$ -relaxed *Dequeue*, where  $k \geq n$  and  $l = \lceil k/n \rceil$

---

```

1: function ENQUEUE( $val$ )
2:    $EnqCount = 0$ 
3:    $incrementTS(v_i)$ 
4:    $Enq_{ts} = v_i$ 
5:   send ( $EnqReq, val, i, Enq_{ts}$ ) to all processes
6: end function
7: function RECEIVE( $EnqReq, val, j, Enq_{ts}$ ) from  $p_j$ 
8:    $updateTS(v_i, Enq_{ts})$ 
9:    $lQueue.insertByTS(val, j, Enq_{ts})$ 
10:  for  $confirmationList$  in  $PendingDequeues$  do
11:    if  $confirmationList.ts \prec Enq_{ts}$  then
12:       $confirmationList.responses[j] = Safe$ 
13:    end if
14:  end for
15:  send ( $EnqAck, i$ ) to  $p_j$ 
16: end function
17: function RECEIVE( $EnqAck, j$  from  $p_j$ )
18:    $EnqCount++ = 1$ 
19:   if  $EnqCount == n$  then return  $EnqResponse$ 
20:   end if
21: end function

```

---

■ **Algorithm 2** Continued, part 2

---

```

1: function DEQUEUE
2:    $incrementTS(v_i)$ 
3:   let  $Deq_{ts} = v_i$ 
4:   if  $localQueue.peekByLabel(p_i) \neq \perp$  then
5:     let  $ret = localQueue.deqByLabel(p_i)$ 
6:     send ( $Deq_f, ret, i, Deq_{ts}$ ) to all processes
7:   else
8:     send ( $Deq_s, null, i, Deq_{ts}$ ) to all processes
9:   end if
10: end function
11: function RECEIVE( $(op, val, j, Deq_{ts})$  from  $p_j$ )
12:    $updateTs(v_i, Deq_{ts})$ 
13:   let  $p_{invoker} = p_j$ 
14:   if  $Deq_{ts}$  is not in  $PendingDequeues$  then
15:      $PendingDequeues.insertByTs(createList(Deq_{ts}, p_{invoker}))$ 
16:   end if
17:   if  $Deq_{ts} \neq 0$  and  $Deq_{ts} \prec v_i$  then
18:     let  $safetyFlag = Unsafe$ 
19:   else
20:     let  $safetyFlag = Safe$ 
21:   end if
22:   send ( $op, val, safetyFlag, Deq_{ts}, i, p_{invoker}$ ) to all processes
23: end function

```

---

---

**Algorithm 3** Continued, part 3
 

---

```

1: function RECEIVE( $(op, val, safetyFlag, Deq_{ts}, i, p_{invoker})$  from  $p_j$ )
2:   if  $Deq_{ts}$  not in PendingDequeues then
3:     PendingDequeues.insertByTs(createList( $Deq_{ts}, p_{invoker}$ ))
4:   end if
5:   propagateEarlierResponses(PendingDequeues)
6:   for ( $index, confirmationList$ ) in PendingDequeues do
7:     if  $confirmationList.ts == Deq_{ts}$  then
8:        $confirmationList.responses[j] = safetyFlag$ 
9:     end if
10:    if  $confirmationList.full()$  and not  $confirmationList.handled$  then
11:       $confirmationList.handled = True$ 
12:      if  $op == Deq_f$  and  $p_i \neq p_{invoker}$  then
13:         $lQueue.remove(val)$ 
14:      end if
15:      updateUnsafes(PendingDequeues,  $index$ )
16:      if  $op == Deq_f$  then
17:        return
18:      end if
19:      if  $localQueue.peekByLabel(p_{invoker}) \neq \perp$  then
20:        let  $ret = lQueue.deqByLabel(p_{invoker})$ 
21:      else
22:        let  $pos = \text{Number of Unsafe flags in } confirmationList.responses$ 
23:        let  $ret = lQueue.deqByIndex(pos)$ 
24:      end if
25:      labelElements( $p_{invoker}$ )
26:      updateUnsafes(PendingDequeues,  $index$ )
27:      if  $p_i == p_{invoker}$  then
28:        return  $ret$ 
29:      end if
30:    end if
31:  end for
32: end function

```

---

adjusting the labelling to occur at points when we can guarantee that the view of distinct processes agree, at the point of dequeuing. As we have discussed, at the point in which a process dequeues, the information prior to that dequeue, the prefix to the execution  $\rho$  to that point will be the same for all processes. Given that all processes will agree on a select set of information at the same relative point in their executions, we can treat this as a sort of synchronization, and make decisions as to the labelling of elements at this point.

For this relaxation to still follow the established queue definition, it must only return elements once. Therefore, we must prove that a) Each process dequeues each element only once and b) that elements removed are within the first  $n$  elements within the queue. Given that the labeling is agreed upon by every process, and done by calculations with an invariant, we can state that each process will agree on the elements that are "fast dequeued" with respect to which process is allowed to "fast dequeue" them. Given that processes agree which elements are "reserved" for a fast dequeue by another process, we can furthermore state that a process will not slow dequeue an element that has been marked.

Thus, elements can not be dequeued by processes that they have not been marked for, so we can guarantee that they will ONLY be dequeued by the assigned process. By the definition of the algorithm on line GIVE ME THE LINE NUMBER HERE COLE we can also state that the second criterion is satisfied, as at a point where a given process will attempt to fast dequeue, the position of the element that is attempting to be fast dequeued is checked, and the process is only allowed to execute the operation in the case where the element is within the legal bounds for dequeuing.

#### 4.3.1 Less Relaxation than Number of Processes

In the case in which the  $X$  of out of order is less than the number of processes, there isn't an order of magnitude benefit to relaxation. The benefit of relaxation is that rather than forcing a wait of 2 times the maximum message delay, the wait can be cut to 2 times the  $(n-x+1)$ th slowest message delay. In a close connection system, the benefit is minimal as the difference between the max message delay and the  $(n-x+1)$ th is minimal, but in an actualized system with nonstandard message delays, this could have some substantial benefits.

## 4.4 Complexity

## 5 Conclusion