Computational Modelling of Physical Systems
Worksheet 2

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Contents

1 Introduction

2 Numerical Integration

2.1 The Trapezium Rule

2.1.1 Theory

The trapezium rule can be used as a method of numerically evaluating integrals. It involves splitting a function $f(x)$ into n trapeziums of width h, and using their area to approximate the integral of $f(x)$. As $h \to 0$, the estimated value converges to the exact integral value. The trapezium rule is given as follows:

$$\int_{x_0}^{x_n} f(x)dx \simeq \frac{1}{2}h\left[f(x_0) + f(x_n) + 2(f(x_1) + f(x_2) + ... + f(x_(n-1)))\right] \tag{1}$$

Where h, the width of the interval is defined by:

$$h = \frac{x_n - x_0}{n} \tag{2}$$

Due to the nature of the trapezium rule, there is naturally an error associated with values calculated using it. There exists an analytic expression for this error and is given by the following equation [1]:

$$E_n^T = -\frac{h^2\,(b-a)}{12}f''(c_n) \tag{3}$$

Which can be adapted to give the maximum error on the estimated value for the integral of $f(x)$:

$$\left|E_n^T\right| \leq \frac{h^2\,(b-a)}{12}\max_{a\leq x\leq b}\left|f''(x)\right| \tag{4}$$

2.2 Simpson's Rule

2.2.1 Theory

3 Numerical Integration with GSL

4 Appendices

References

[1] Kendall E. Atkinson
http://homepage.math.uiowa.edu/~atkinson/ftp/ENA_Materials/Overheads/sec_5-2.pdf
Accessed at 11-2013