IVP Analytic vs. Numerical Solution

June 24, 2020

1 Problem

Compare the analytic and numerical solution of η of the following shallow water problem:

$$\begin{aligned} \eta &= e^{-(x-3.5)^2} \\ u &= 0 \\ h &= x \\ m &= \infty \end{aligned}$$

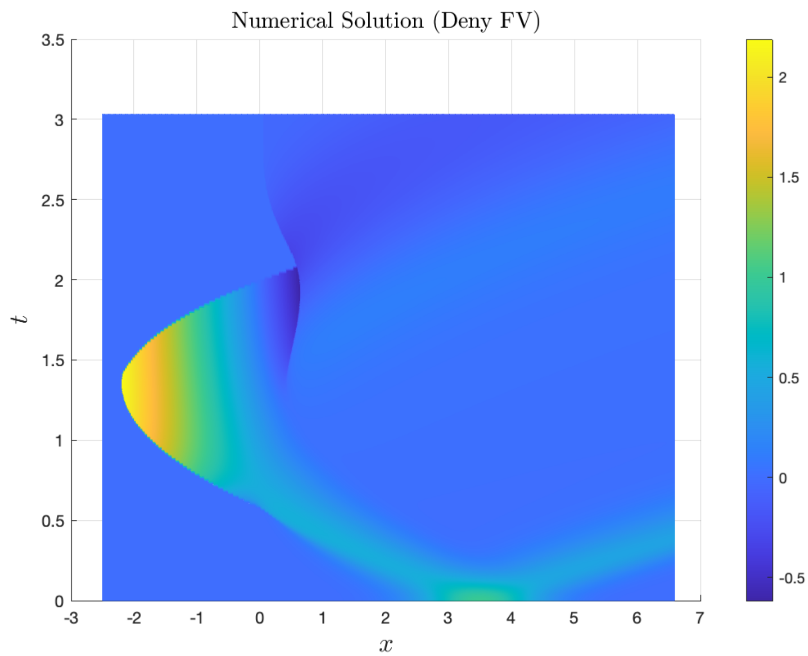
In other words, a Gaussian initial wave with no initial velocity, and a plane-inclined shape (y^{∞}). This reduces to a 1-1 SWE. We can reproduce this with a different slope and initial conditions easily.

2 Setup

Statistical comparison was done on an equally spaced grid of 1000 points in time on [0,3] and at 1000 points in x on [-2.5, 6.5]

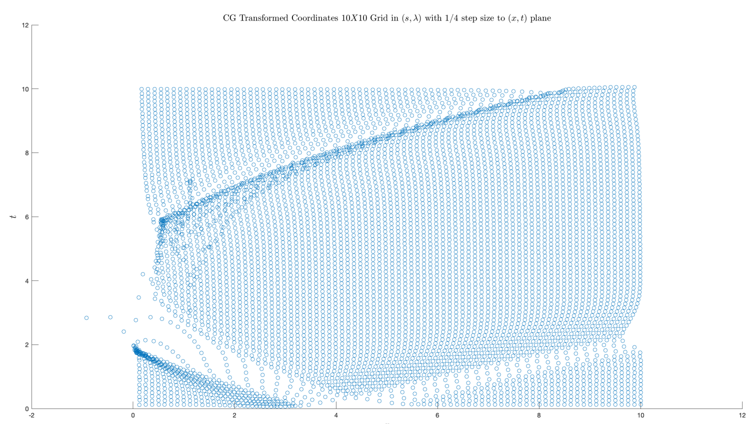
2.1 Numerical

I set Deny's Catalina 1 "runwave.m" with the initial conditions. The following displays eta in the (x, t) plane



2.2 Anaylytic

Chebfun was used to calculate the Hankel transform solution to the CG transform on a grid in (s, λ) then CG transform to (x, t)
The following figure shows the a grid in (s, λ) transformed to (x, t)

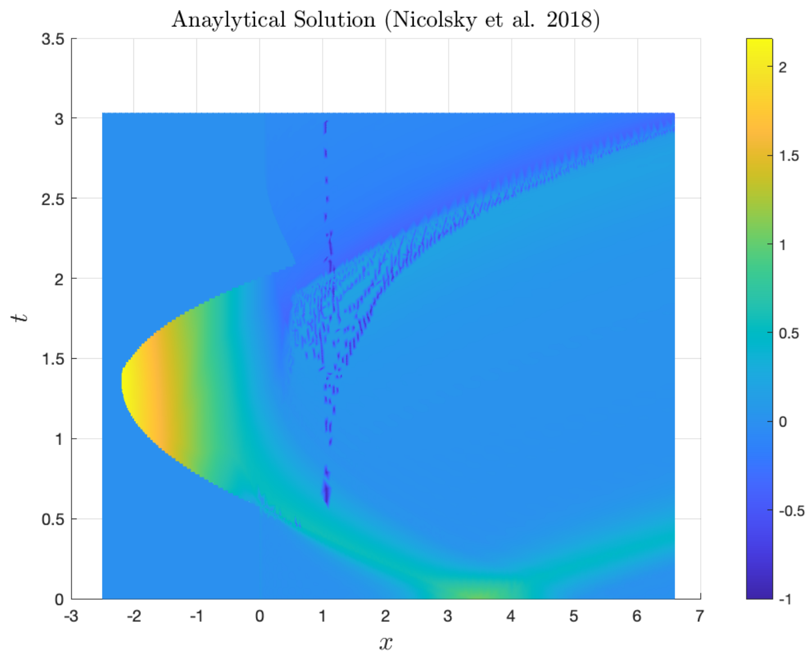


Note the distinct non-linear nature caused by the $-u^2$ of η
The analytical solution of η was computed using formulas in Nicolsky (2018)

$$\psi(s,\lambda) = \int_0^\infty (a(k)cos(\beta * k * \lambda) + b(k) * sin(\beta * k * \lambda))dk$$
$$\phi(s,\lambda) =$$

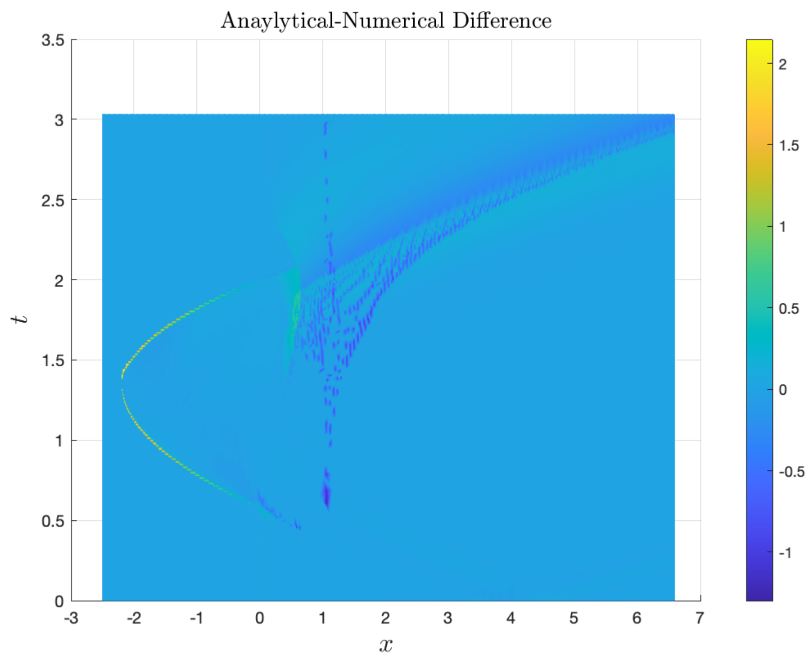
where

$$a(k) = e^{-(x-3.5)^2}$$
$$b(k) =$$
$$a(k) = e^{-(x-3.5)^2}$$
$$b(k) =$$

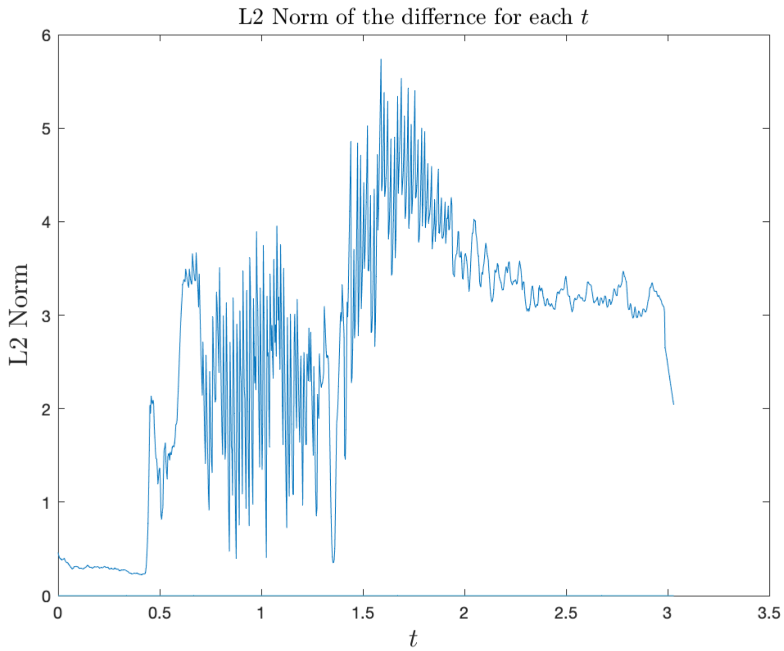


3 Statistical Analysis

This is the difference between the two ie. numerical - anaytical



The following is the L2 norm at each value of t. The difference increases in a sporadic fashion at the beginning and end of run-up. The primary explaination for this is problems with the computation of the analytic solution.



4 Further Problems

1. Analytic solution stability.
2. Comparison of the speed wasn't completed.
3. Different initial conditions.
4. NOAA anaytic solution.