IVP Analytic vs. Numerical Solution

June 18, 2020

1   Problem

Compare the analytic and numerical solution of η of the following shallow water problem:

$$\eta = e^{-(x-3.5)^2}$$
$$u = 0$$
$$h = x$$
$$m = \infty$$

In other words, a Gaussian initial wave with no initial velocity, and a plane-inclined shape (y^∞). This reduces to a 1-1 SWE. We can reproduce this with a different slope and initial conditions easily.

2   Setup

Statistical comparison was done on an equally spaced grid of 1000 points in time on [0,3] and at 1000 points in x on [-2.5, 6.5]
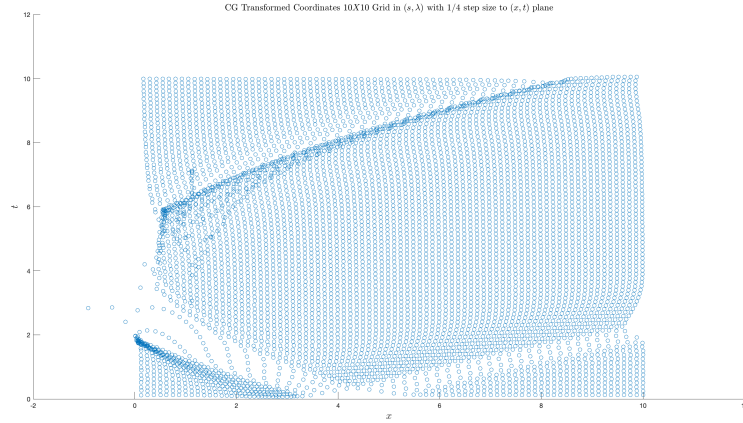
2.1   Numerical

I set Deny's Catalina 1 "runwave.m" with the initial conditions. The following displays eta in the (x, t) plane

Numerical Solution (Deny FV)
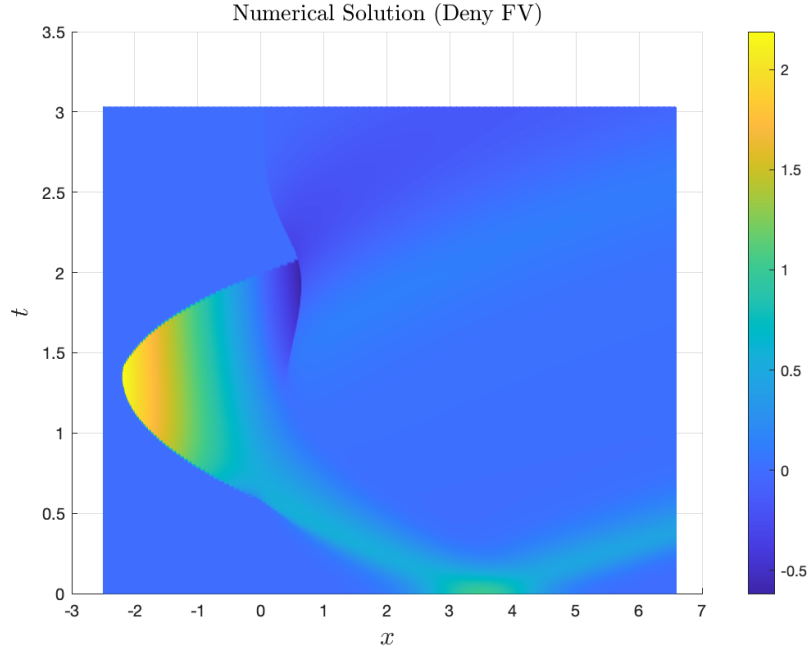
2.2   Anaylytic

Chebfun was used to calculate the Hankel transform solution to the CG transform on a grid in (s, λ) then CG transform to (x, t)

The following figure shows the a grid in (s, λ) transformed to (x, t)

CG Transformed Coordinates 10X10 Grid in (s, λ) with 1/4 step size to (x, t) plane

Note the distinct non-linear nature caused by the $-u^2$ of η

The following is the analytical solution of η

Anaylytical Solution (Nicolsky et al. 2018)
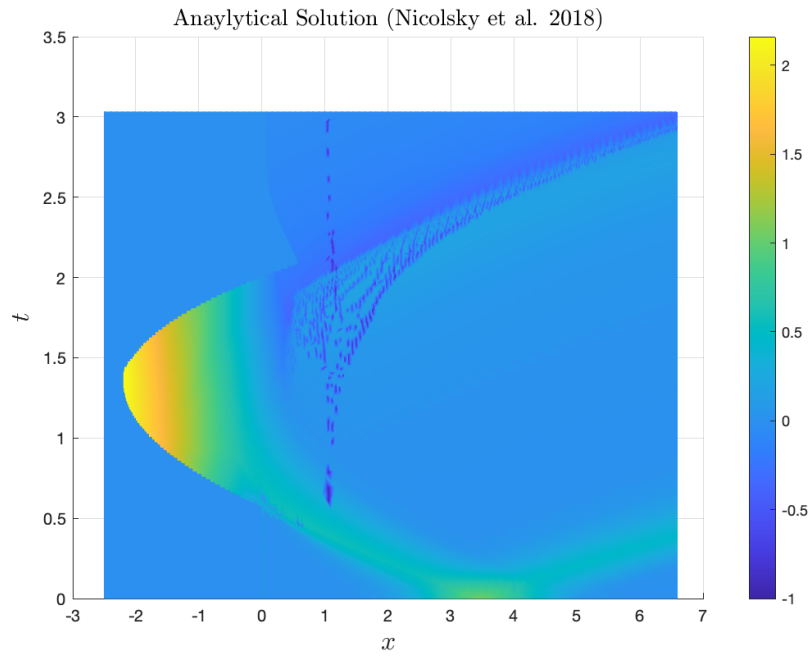
3   Statistical Analysis

This is the difference between the two ie. numerical - anaytical

Anaylytical-Numerical Difference
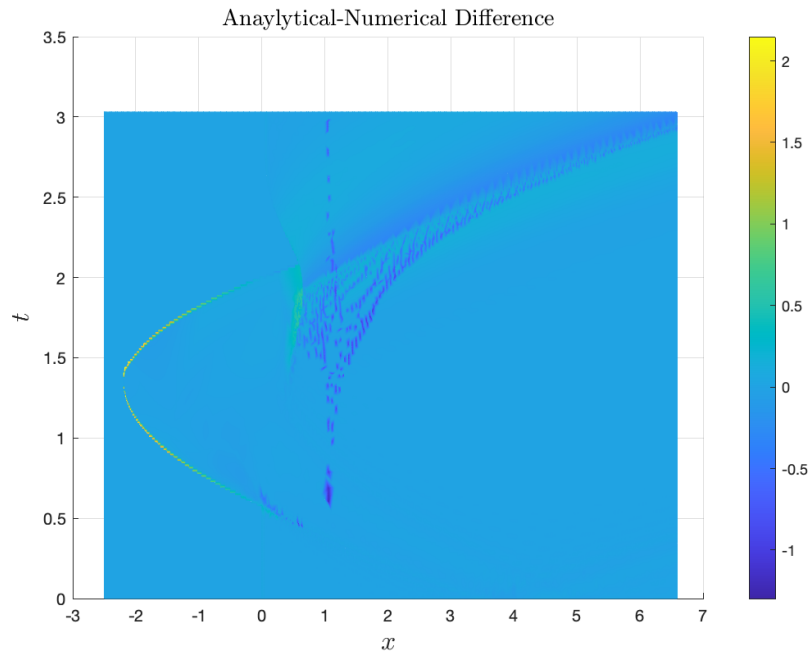
The following is the L2 norm at each value of t. The difference increases in a sporadic fashion at the beginning and end of run-up. The primary explaination for this is problems with the computation of the analytic solution.

L2 Norm of the differnce for each t
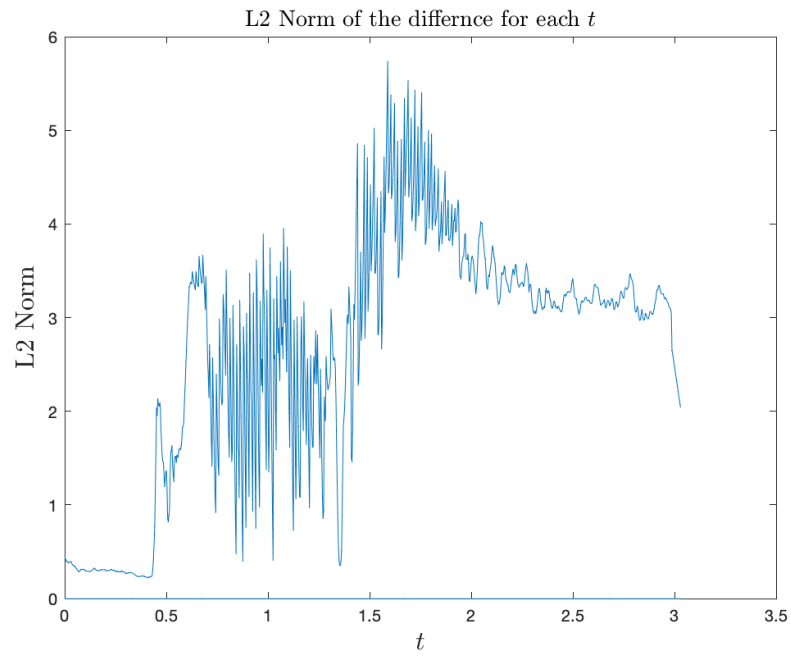
4 Further Problems

1. Analytic solution stability.
2. Comparison of the speed wasn't completed.
3. Different initial conditions.
4. NOAA anaytic solution.