CSci 435: Formal Languages and Automata

Instructor: Dr. M. E. Kim Date: December 8th, 2020

**Due: by the end of the day, 12/15 (Tue.)**

**Final Exam: 120 points + 30 points (optional)**

Name: \_Sam Dressler \_

1. Write your answer below the corresponding question.
2. Do not include the photo image of your handwriting for a text.

Abbreviation:

REG: Regular Language, REC: Recursive Language, RE: Recursive Enumerable Language,

(D)CFL: (Deterministic) Context-Free Language, FA: Finite Automata, TM: Turing Machine,

UG: Unrestricted Grammar, CSG: Context-Sensitive Grammar, etc.

Mark the followings.

Difficulty:

Very Easy: \_\_\_\_\_\_ Easy: \_\_\_\_\_\_ Moderate: \_\_\_\_\_\_ Difficult: \_\_\_\_\_\_ Very Difficulty: \_\_\_\_\_\_

Time:

\_\_\_\_\_\_\_\_\_ Hours and \_\_\_\_\_\_\_\_\_ minutes

Q1. [10] For a language *L*(*aa*\*(*ab*+*a*)\*),

1. [5] Find a regular grammar that generates L.
2. [5] Construct the minimal DFA that accepts L.

Hint: the number of states is four.

Q2. [10] Prove that L= { *anbn ambm* | *n, m* ≥ 1 } is Context-Free Language but not Linear.

Hint: Construct its CFG and prove its non-linearity using P.L. for linear language.

Q3. [10] Using adders, subtracters, comparers, copiers, or multipliers, construct a Turing Machine that compute the functions: *f*(*x*) = 2*n .* Draw its block diagram.

Q4. [10] Design a TM that computes the function: *f*(*x*) = 2*x* + 1, where *x* is given in unary notation with 1’s only to TM.

Use the tape symbol Γ = {1, x, €}. Draw its transition function.

Q5. [10] Prove that the complement of a Context-Free Language must be Recursive.

Q6. [10] Give the language generated by the following unrestricted grammar in a formal expression.

e.g.) { *anbn* | *n* ≥ 1 }

S → AB, A → *a*Ab, bB → bbbB, *a*Ab → *aa*, B→λ

Q7. [10] Find a context-sensitive grammar for L = {*anbna2*n | *n* ≥1} and give a derivation of any string *w*∈ L by your grammar: S ⇒\* *w*.

Q8. [10] Let M1 and M2 be arbitrary Turing machines. Show that the problem “L(M1) ⊆ L(M2)" is undecidable.

Q9. [10] Show that for A = {*wi*| 10, 00, 11, 01} and B = {*vi* | 0, 001, 1, 101}, there exists a Pot Correspondence solution.

Q10. [10] Determine whether the given Boolean expression is satisfiable or not.

Q11. [10] Show that L = {*www* | w ∈{*a, b*}+} is in DTIME(*n*). Explain how a Deterministic-TM runs on *w* ∈ L in O(*n*).

Q12. [10] Explain a Halting Problem in detail.

Q13. [10, optional]

(A) Explain P-Problem, NP-Problem and NP-Complete Problem, respectively (P/NP/NP-Complete-language, equivalently).

(B) Give one example of NP-Complete problem and define/explain it.

Q14. [20, optional] In the given table of the closure property, mark whether each language family is closed (O) under the given operation or not closed (X) at (1) – (35). E.g.) ∀L1, L2 ∈ REG → L1 ∪ L2 ∈REG?

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| --- | --- | --- | --- | --- | --- |
|  | **REG** | **DCFL** | **CFL** | **REC** | **RE** |
| **Union** | (1) | (8) | (15) | (22) | (29) |
| **Complement** | (2) | (9) | (16) | (23) | (30) |
| **Intersection** | (3) | (10) | (17) | (24) | (31) |
| **Concatenation** | (4) | (11) | (18) | (25) | (32) |
| **Kleene Star (L\*)** | (5) | (12) | (19) | (26) | (33) |
| **Reversal (LR)** | (6) | (13) | (20) | (27) | (34) |
| **Intersection**  **with REG** |  | (14) | (21) | (28) | (35) |