Astron 1221 Written Report 1

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1 Introduction

This is a written report for Astronomy1221. In this project, we started with the solar system and studied the orbital speed of planets around sun. We applied it to the Milky Way, hoping to explore the rotational orbital speed for the Milky Way central black hole and compare the calculated data with the observed data.

2 Theory

In physics, Newton's first law states that an object will change its trajectory only when it is acted upon by a force, and an object with a zero net force will maintain its original state of motion. In astronomy, planets orbit the sun and their directions of motion and velocity are constantly changing, which means that the planets are constantly subject an external forces from the sun. This force should be the sun's gravitational pull on the planets. Newton's law of universal gravitation states that the formula for gravity is:

$$F = \frac{GMm}{r^2}$$

Where F is force, G refers to the gravitational constant $6.67 * 10^{-11} m^3 kg^{-1} s^{-2}$, M and m refer to the masses of the two objects affected by gravity, and r is the distance between the two objects. Newton also proposed the formula for uniform circular motion (which is also the motion of planetary revolution):

$$F = m\frac{V^2}{r}$$

Where F refers to the centripetal force, V refers to the speed of circular motion, m refers to the mass of the moving object, and r refers to the distance between the object and the center of the circle. Combining these two formulas, we can get:

$$G\frac{Mm}{r^2} = m\frac{V^2}{r}$$

After simplification, we can get:

$$V = \sqrt{\frac{GM}{r}}$$

From this, we get a formula that can be applied to planet velocity curves and Milky Way orbital velocity curves. As long as we know the center mass M, orbital radius r and gravitational constant G, we can calculate the orbital velocity V.

3 Methods

In this project, we utilized Python programming to perform calculations and generate plots related to orbital velocities in the Solar System and the Milky Way galaxy. The primary goal was to model the rotational velocities of celestial bodies and compare our calculated data with observed measurements.

3.1 Orbital Velocity Calculation

The foundational equation used for calculating the orbital velocity v of an object orbiting a central mass M at a distance R is derived from Newton's law of universal gravitation and the formula for centripetal force:

$$v = \sqrt{\frac{GM}{R}}$$

where G is the gravitational constant. This equation was implemented in Python through the function `orbital_velocity(M, R)`, allowing us to compute orbital velocities for various masses and radii.

3.2 Validation with the Solar System

We first applied the orbital velocity function to the eight planets in the Solar System. The orbital radii were given in astronomical units (AU), and the mass of the Sun was used as the central mass. Using Python, we calculated the orbital velocities and plotted them against their distances from the Sun to validate our model. (figure 1)

3.3 Modeling the Milky Way Galaxy

3.3.1 Galactic Components

We considered three primary components of the Milky Way:

1. **Galactic Bulge**: Modeled as a point mass with $M_{\text{bulge}} = 1 \times 10^{10}, M_{\odot}$.
2. **Galactic Disk**: Treated as a flat disk with total mass $M_{\text{disk}} = 5.2 \times 10^{10}, M_{\odot}$ and radius $R_{\text{disk}} = 10, \text{kpc}$.
3. **Dark Matter Halo**: Approximated as a sphere with mass $M_{\text{halo}} = 1.2 \times 10^{11}, M_{\odot}$ and radius $R_{\text{halo}} = 30, \text{kpc}$.

3.3.2 Calculating Enclosed Masses

For each component, we calculated the enclosed mass $M(R)$ within a radius R:

Bulge Since it's a point mass, the enclosed mass is constant: $M_{\text{bulge}}(R) = M_{\text{bulge}}$.

Disk The surface mass density σ_{disk} is: $\sigma_{\text{disk}} = \frac{M_{\text{disk}}}{\pi R_{\text{disk}}^2}$. The enclosed mass for $R \leq R_{\text{disk}}$ is:

$$M_{\text{disk}}(R) = \pi R^2 \sigma_{\text{disk}} = M_{\text{disk}} \left(\frac{R}{R_{\text{disk}}} \right)^2.$$

Halo The volume mass density ρ_{halo} is: $\rho_{\text{halo}} = \frac{M_{\text{halo}}}{\frac{4}{3}\pi R_{\text{halo}}^3}$. The enclosed mass for $R \leq R_{\text{halo}}$ is:

$$M_{\text{halo}}(R) = \frac{4}{3}\pi R^3 \rho_{\text{halo}} = M_{\text{halo}} \left(\frac{R}{R_{\text{halo}}} \right)^3.$$

3.3.3 Orbital Velocity Calculations

We calculated orbital velocities at radii ranging from 1 to 30 kiloparsecs (kpc) in 1 kpc increments:

1. **Bulge** $v_{\textbf{bulge}}(R)$:$v_{\text{bulge}}(R) = \sqrt{\frac{GM_{\text{bulge}}}{R}}$.
2. **Disk** $v_{\textbf{disk}}(R)$:$v_{\text{disk}}(R) = \sqrt{\frac{GM_{\text{disk}}(R)}{R}}$.
3. **Halo** $v_{\textbf{halo}}(R)$:$v_{\text{halo}}(R) = \sqrt{\frac{GM_{\text{halo}}(R)}{R}}$.
4. **Total** $v_{\textbf{total}}(R)$:$v_{\text{total}}(R) = \sqrt{\frac{G[M_{\text{bulge}}+M_{\text{disk}}(R)+M_{\text{halo}}(R)]}{R}}$.

3.4 Data Acquisition

We imported observed rotational velocity data from `galaxy_rotation_2006.txt`, which includes measurements of orbital velocities at various radii and their associated uncertainties. This data allowed us to compare our theoretical models directly with empirical observations.

4 Results

4.1 Solar System Validation

The calculated orbital velocities of the eight planets showed an inverse relationship with their distances from the Sun, as expected from Kepler's laws. The plot demonstrates that planets closer to the Sun have higher orbital velocities, validating our approach.

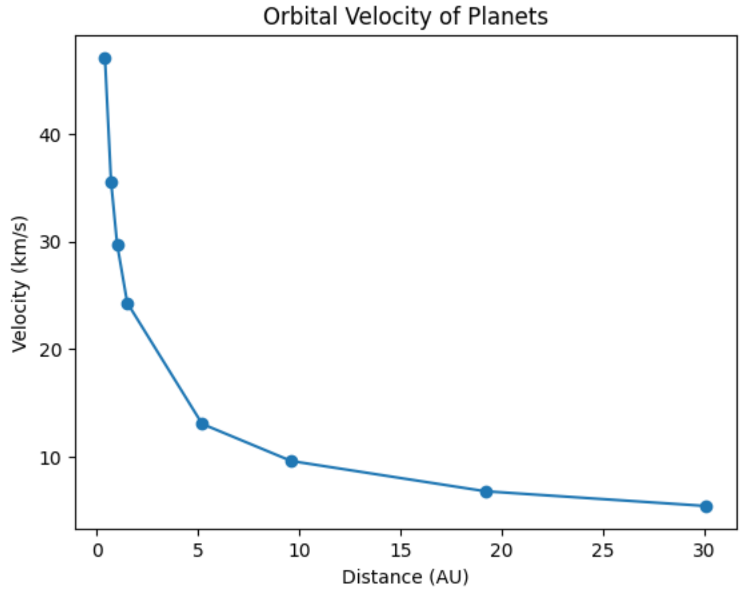

Figure 1: Orbital velocities of the eight planets plotted against their distances from the Sun.

4.2 Galactic Rotation Curve

4.2.1 Individual Component Contributions

Bulge The orbital velocity due to the bulge decreases sharply with increasing radius and is significant only within the inner 2 kpc. This is evident in the steep decline of the bulge's rotation curve.

Disk The disk's contribution increases with radius up to $R_{\text{disk}} = 10, \text{kpc}$, after which it remains constant. The rotation curve due to the disk shows a rise in velocities in the inner regions, plateauing beyond 10 kpc.

Halo The halo's contribution becomes dominant at larger radii. The calculated velocities due to the halo increase with radius, which initially seems inconsistent with the observed flat rotation curve.

4.2.2 Combined Rotation Curve

The combined rotation curve (bulge + disk + halo) underestimated the observed velocities beyond 8 kpc. (figure 2)

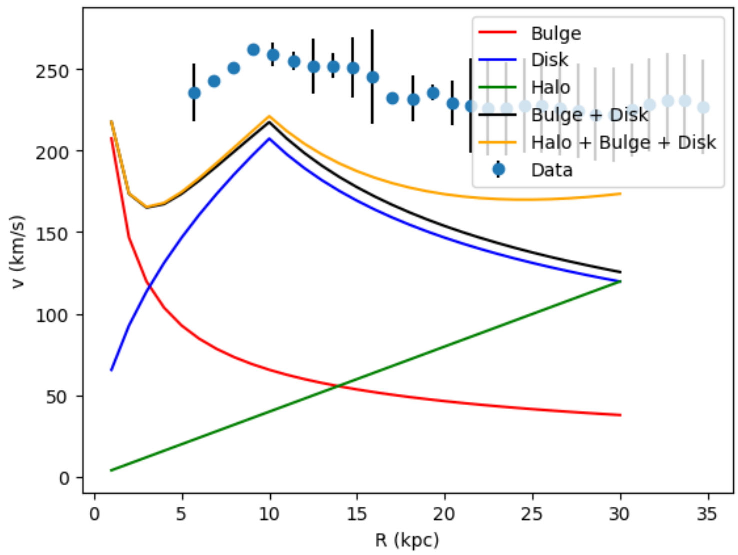

Figure 2: Orbital velocities due to the bulge, disk, halo, bulge+halo, and bulge+halo+disk components plotted against radius.

4.2.3 Assumptions and Limitations

- **Spherical Symmetry**: We assumed spherical symmetry for the halo, which may oversimplify the actual distribution of dark matter.
- **Constant Densities**: Uniform densities were assumed for the disk and halo, whereas in reality, these densities may vary with radius.

5 Conclusion

In summary, after calculating the bulge and disk, we found that when the distance is very large, our calculated data does not match the observed data. This means that there must be something we cannot see, that is the dark matter halo. So we calculated our results with the halo this time. Then we multiplied the mass of the disk, bulge, and halo by a factor or 1.6 so that we finally got a curve that fit the observed data. (figure 3)

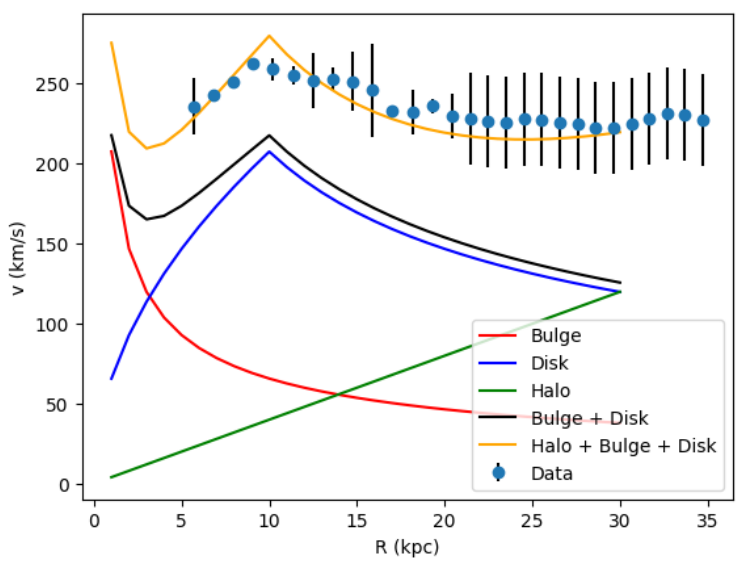

Figure 3: Orbital velocities due to the bulge, disk, halo, bulge+halo, and bulge+halo+disk components plotted against radius shifted up by a factor

6 AI and Contribution

6.1 AI

We did not use AI.

6.2 Contribution

Theory and Conslusion: Wentao Zhong
Methods and Results: Sam Grobelny
Slide: Sacha Wible