Astron 1221 Project 3 Written Report

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November 5, 2024

1 Introduction

This is a written report for Astronomy1221. Gravitational wave provide a new way to observe the universe. Unlike electromagnetic radiation(light), gravitational waves are ripples in the fabric of space-time caused by massive, accelerating bodies, such as black holes in the process of merging(just like the wave in the water caused by a stone). In this report, we studied the calculations and methods used to estimate black hole mass, distance, and energy released during a merger event, specifically focusing on data derived from the LIGO (Laser Interferometer Gravitational-Wave Observatory) and similar gravitational wave detectors. Our aim is to illustrate how gravitational waves allow us to uncover critical information about cosmic phenomena that are otherwise invisible to traditional telescopic observations, such as how far a black hole leave us.

2 Theory

The theoretical basis of this study lies in Einstein's general theory of relativity, which predicts that accelerating masses produce gravitational waves that propagate through space-time. For a binary black hole merger, the emitted gravitational waves carry information about the black holes' masses, their orbital dynamics, and the total energy radiated during the merger.

Schwarzschild Radius and Black Hole Properties: $R = \frac{2GM}{C^2}$

The Schwarzschild radius R represents the boundary beyond which nothing, not even light, can escape the black hole's gravitational field. This radius scales with mass, helping astronomers understand the extent and scale of black holes in merger events.

Energy Release Due to Mass Loss: $E = \Delta mc^2$

According to Einstein's mass-energy equivalence principle $E = mc^2$, where Δm represents the mass lost in the merger. This mass is converted into gravitational wave energy and carries away significant information about the system, particularly about the dynamics of the black hole merger. The amount of energy released can be in the range of several solar masses.

3 Methods

3.1 Masses and Distance

To get the black hole masses and distances, we first had to find the orbital period P of the two merging black holes by looking at the period right before merging. We can then see that, when the black holes merge, their orbital distance is twice the Schwarzschild radius. This can be represented as the equation $2Rsch$ with $Rsch = \frac{2GM}{c^2}$. We also know that the orbital velocity is represented as the equation $v = \sqrt{\frac{GM}{\alpha}}$. After combining these equations, we can see the following:

$$\delta t = 2\pi\alpha/v = 2\pi\frac{4GM}{c^2}/(c/2) = 16\pi GM/c^3$$

We can then rearrange this to find that $M = \frac{\delta tc^3}{16G\pi}$. This gets us the mass of the black holes.

To find the distance, we can use the equation $h = R_{bh}/distance$ and then rearrange it to get that $distance = R_{bh}/h$. R_{bh} can be represented as the equation $\frac{GM}{c^2}$ that was used in the masses equations. We will also define h as a constant $1e-21$

3.2 Energy Release due to Mass Loss

To calculate the energy release after the merger, we need to have the mass of each black hole before the merger and the final mass of the black hole after the merger. The equation used for calculating this mass loss is as follows:

$$loss = M1 + M2 - M_{final}$$

We can then obtain the value for energy released by using the loss value in the equation:

$$released = loss * c^2$$

4 Results

Using our event of LIGO 173609, and the aforementioned equations, we obtained a final mass of about 54.2 solar masses, a distance of $7.3e9$ parsecs, and an estimated energy released of about $7.15e54$ erg

5 Conclusion

After comparing the values obtained from our calculations to the ones in the database, we can see that all our values are within the ranges given in the database which means that our equations are fairly accurate.

6 AI Statement

We did not use AI to assist in any of our calculations or any other part of the project.

7 Contributions

Introduction, Theory, and Conclusion: Wentao Zhong
Methods and Results: Sam Grobelny
Presentation Slides: Sach Wible

Gravitational Waves

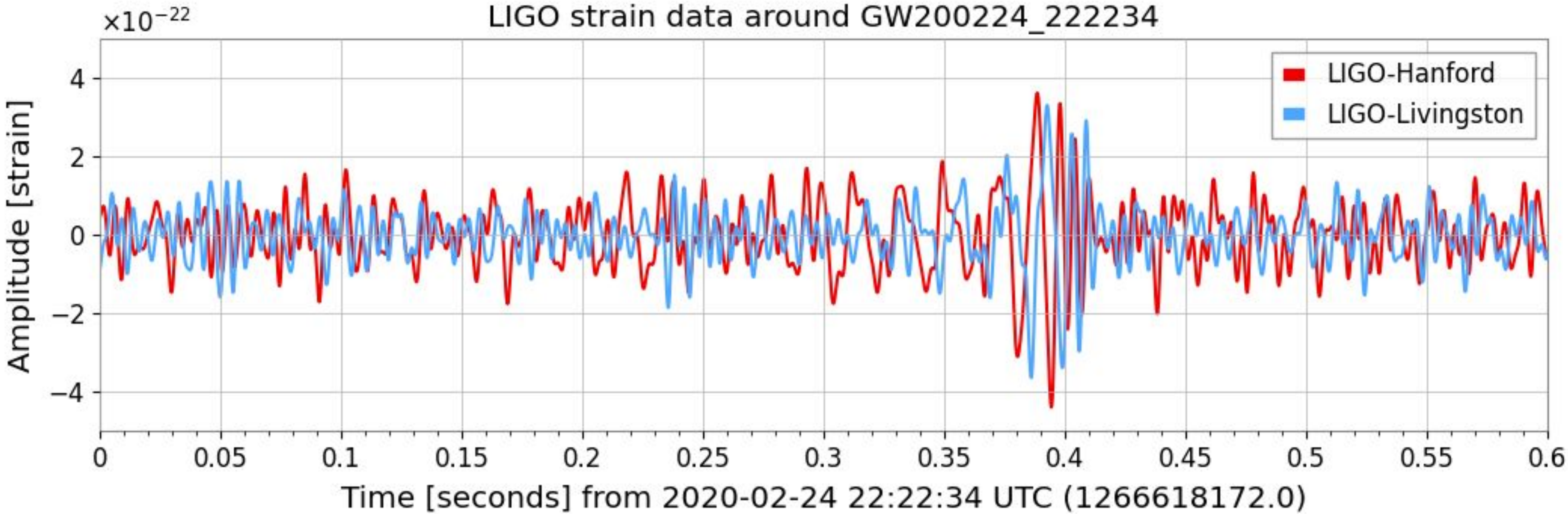
Sam Grobelny, Sacha Wible, Wentao Zhong

Motivations

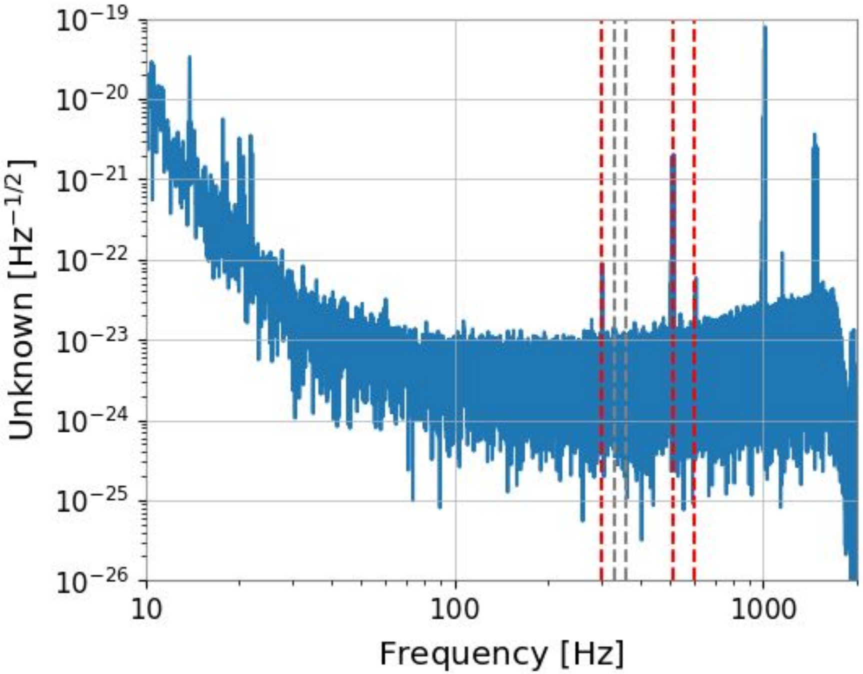
By studying black hole mergers via gravitational waves, we can figure out the distance from Earth. Comparing this to redshift of the light from these events allows us to better understand the universe's expansion.

Methods + Assumptions

By using gravitational wave data collected from LIGO we should be able to calculate the masses of the two black holes that merged in order to cause the wave.


LIGO strain data around GW200224_222234
×10^−22
Amplitude [strain]
LIGO-Hanford
LIGO-Livingston
4
2
0
−2
−4
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
Time [seconds] from 2020-02-24 22:22:34 UTC (1266618172.0)

Fast Fourier transform - GW200224_222234




```python
# Find the index of the maximum power
max_index = np.argmax(hq.value)

# Get the frequency and time corresponding to the maximum power
peak_frequency = hq.frequencies.value[max_index // hq.times.size]
peak_time = hq.times.value[max_index % hq.times.size]

print("Peak Frequency:", peak_frequency, "Hz")
print("Peak Time:", peak_time, "s")
```

```
Peak Frequency: 603.0042 Hz
Peak Time: 1266618172.3880148 s
```

```python
orbital_period = 1 / (peak_frequency) # converting to seconds

print("Orbital Period:", orbital_period, "seconds")
```

Black Hole Mass

We can also use the peak frequency to derive the orbital period, which can be used with the orbital velocity and Schwarzschild radius formula to get

$$M = \frac{T * c^3}{16\pi G}$$

```
orbital_period = 1 / (peak_frequency) # converting to seconds

print("Orbital Period:", orbital_period, "seconds")

c = 299792458
G = 6.67408e-11

M = orbital_period * c**3 / (16 * np.pi * G)
print("Black Hole Mass =", (M*u.kg).to(u.solMass))
```

```
Orbital Period: 0.001803412813454974 seconds
Black Hole Mass = 7.284337946081171 solMass
```

Black Hole Distance

We can also use the mass of the black holes and the Schwarzschild radius formula again to get

$$R_{sch} = \frac{2GM}{c^2} \qquad D = \frac{R_{sch}}{strain}$$

```
max_amplitude_hanford = np.max(np.abs(hfilt.value))
print("Maximum Amplitude (Hanford):", max_amplitude_hanford)

schwarz_rad = 2 * G * M / c**2
print("Schwarzschild Radius:", (schwarz_rad*u.m).to(u.km))

D = schwarz_rad / max_amplitude_hanford
print("Distance:", (D*u.m).to(u.Mpc))
#
```

```
Maximum Amplitude (Hanford): 4.388426591569063e-22
Schwarzschild Radius: 21.51176249396066 km
Distance: 1588.6075092108929 Mpc
```

Energy Lost

We can use Einstein's formula to estimate the amount of energy released in the black hole merger.

$E = mc^2$

```
m = 0.1 * M
E = m * c**2
print("Energy:", (E*u.J))
```

```
Energy: 1.3017794224413516e+47 J
```

Conclusion

Black holes are big. Gravitational waves tell us a lot about the universe, such as how far things can be. Measuring the distance of objects in the universe, combined with redshift data, could also help us get a handle on the age of the universe.

AI Acknowledgement

I used AI to generate the code for the peak frequency, and max strain.