Function Sum

$\{y \geq 0\}$

```
z := x;
n := y;
WHILE n != 0 DO
    z := z + 1;
    n := n - 1;
 Loop
```

$\{z = x + y\}$

{Sequencing Rule}

1) $\{I\}$

```
WHILE n != 0 DO
    z := z + 1;
    n : n - 1;
LOOP
```

$\{z = x + y\}$

{while rule}

1.1) $\{I \wedge n != 0\}$

```
z := z + 1;
n := n - 1;
```

$\{I\}$

{Apply loop Invariant}

$I = [z + n = x + y]$

$\{z + n = x + y \wedge n != 0\}$

```
z := z + 1;
n := n - 1;
```

$\{z + n = x + y\}$

{Sequencing Rule}

1.1.1) $\{R\}$

```
n := n + 1
```

$\{z + n = x + y\}$

{Assignment axiom}

$R = (z+n = x+y)[n-1/n]$
$= (z + n-1 = x+y)$

1.1.2) $\{z+n = x+y \wedge n! = 0\}$
$z := z+1$
$\{z+n-1 = x+y\}$

{Assignment Axiom}

$(z+n-1 = x+y)[z+1/z]$
$\Rightarrow z+1+n-1 = x+y$

{Arithmetic}

$\Rightarrow z+n = x+y$

{precondition Strengthing}

1.1.2.1)
~~$[z+n = x+y] \longrightarrow$~~
$[z+n = x+y \wedge n!=0] \longrightarrow [z+n = x+y]$

{pure logic}

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1.2) $[z+n = x+y \wedge \neg(n != 0)] \longrightarrow [z = x+y]$

{Pure logic}

$[z+0 = x+y] \longrightarrow [z = x+y]$

{Reflexivity}

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2) $\{Q\}$
$n := y$
$\{z+n = x+y\}$

{Assignment Axiom}

$Q = (z+n = x+y)[y/n]$
$= z+y = x+y$

3) {y >= 0}

z := x

{z + y = x + y}

{Assignment Axiom}

(z + y = x + y)[x/z]

=> x + y = x + y

{Simplify}

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[y >= 0] -> [True]

{Precondition strengthening}

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{Pure Logic}

{Q.E.D}