

Function min

{true}

if $x < y$ then

$z := x;$

else

$z := y$

$\{(x < y \wedge z = x) \vee (x > y \wedge z = y)\}$

{2-Armed conditional Rule}

1) $\{x < y \wedge P\}$

$z := x;$

$\{(x < y \wedge z = x) \vee (x > y \wedge z = y)\}$

{Assignment Axiom}

$$P = ((x < y \wedge z = x) \vee (x > y \wedge z = y)) [x/z]$$

$$= (x < y \wedge x = x) \vee (x > y \wedge x = y)$$

$$= \text{True} \vee \text{False}$$

$$= \text{True}$$

2) $\{x > y \wedge P\}$

$z := y;$

$\{(x < y \wedge z = x) \vee (x > y \wedge z = y)\}$

{Assignment Axiom}

$$P = ((x < y \wedge z = x) \vee (x > y \wedge z = y)) [y/z]$$

$$= (x < y \wedge y = x) \vee (x > y \wedge y = y)$$

$$= \text{False} \vee \text{True}$$

$$= \text{True}$$

{Precondition strengthening}

$\{\text{True}\} \rightarrow \{\text{True}\}$

$\therefore T$

{Q.E.D}