Less Than

$\{a \geqslant 0 \wedge b \geqslant 0\}$

```
x := a;
y := b;

WHILE x != 0 ∧ y != 0 DO
    x := x - 1;
    y := y - 1;
LOOP
IF x = 0 ∧ y != 0 THEN
    p := 1;
ELSE
    p := 0;
END
```

$\{(x=0 \wedge y != 0 \wedge p=1) \vee ((x != 0 \vee y=0) \wedge p=0)\}$

{Sequencing Rule}

1) $\{Q\}$

```
IF x = 0 ∧ y != 0 THEN
    p := 1;
ELSE
    p := 0;
END
```

$\{(x=0 \wedge y!=0 \wedge p=1) \vee ((x != 0 \vee y=0) \wedge p=0)\}$

{Two-Armed Conditional Rule}

1.1) $\{Q \wedge (x=0 \wedge y != 0)\}$

p := 1;

$\{(x=0 \wedge y != 0 \wedge p=1) \vee ((x != 0 \vee y=0) \wedge p=0)\}$

{Assignment Axiom}

$((x=0 \wedge y != 0 \wedge p=1) \vee ((x != 0 \vee y=0) \wedge p=0))[1/p] =$

$= (x=0 \wedge y != 0 \wedge 1=1) \vee (x != 0 \wedge y=0 \wedge 1=0)$

$= (x=0 \wedge y != 0) \vee \text{false} = \underline{(x=0 \wedge y != 0)}$

Less Than

1.1.1) $(x=0 \wedge y!=0)$

$\{Precondition\ Strengthening\}$

$\{Q \wedge (x=0 \wedge y!=0)\} \longrightarrow (x=0 \wedge y!=0)$

$\{Pure\ Logic\}$

$\top$

1.2) $\{Q \wedge \neg(x=0 \wedge y!=0)\}$

$p := 0$

$\{(x=0 \wedge y!=0 \wedge p=1) \vee ((x!=0 \vee y=0) \wedge p=0)\}$

$\{Assignment\ Axiom\}$

$((x=0 \wedge y!=0 \wedge p=1) \vee ((x!=0 \vee y=0) \wedge p=0))[0/p] =$

$= (x=0 \wedge y!=0 \wedge 0=1) \vee ((x!=0 \vee y=0) \wedge 0=0)$

$= false \vee (x!=0 \vee y=0)$

$= (x!=0 \vee y=0)$

1.2.1) $\{Q \wedge \neg(x=0 \wedge y!=0)\} \longrightarrow \{x!=0 \vee y=0\}$

$\neg(x=0 \wedge y!=0) = (x!=0 \vee y=0)$

$\{Precondition\ Strengthening\}$

$\{Q \wedge (x!=0 \vee y=0)\} \longrightarrow \{x!=0 \vee y=0\}$

$\{Pure\ Logic\}$

$\top$

1.3) $Q = ((x=0 \wedge y!=0 \wedge p=1) \vee ((x!=0 \vee y=0) \wedge p=0))[0/p] \vee$
$(x=0 \wedge y!=0 \wedge p=1) \vee ((x!=0 \vee y=0) \wedge p=0))[1/p]$

$= (x!=0 \vee y=0) \vee (x=0 \wedge y!=0)$

Less Than

2) $\{Q_1\}$

WHILE $x != 0 \wedge y != 0$ DO

$x := x - 1;$

$y := y - 1;$

LOOP

$\{(x != 0 \vee y = 0) \vee (x = 0 \wedge y != 0)\}$

{While Rule}

2.1) Q_1 = loop Invariant = $a - x = b - y$

$\{a - x = b - y \wedge (x != 0 \wedge y != 0)\}$

$x := x - 1;$

$y := y - 1;$

$\{a - x = b - y\}$

{Sequencing Rule}

2.1.1) $\{Q_2\}$

$y := y - 1;$

$\{a - x = b - y\}$

{Assignment Axiom}

$Q_2 = (a - x = b - y)[y-1/y] = a - x = b - y + 1$

2.1.2) $\{a - x = b - y \wedge (x != 0 \wedge y != 0)\}$

$x := x - 1;$

$\{a - x = b - y + 1\}$

{Assignment Axiom}

$(a - x = b - y + 1)[x-1/x] = a - x + 1 = b - y + 1$

$= a - x = b - y$

{Precondition Strengthening}

2.1.2.1) $\{a - x = b - y \wedge (x != 0 \wedge y != 0)\} \rightarrow (a - x = b - y)$

{Pure Logic}

$= T$

Less Than

2.2) $\{a - x = b - y \wedge \neg(x != 0 \wedge y != 0)\} \rightarrow \{a - x = b - y\}$

T $\{$Pure Logic$\}$

3) $\{Q_3\}$

$y := b;$

$\{a - x = b - y\}$

$\{$Assignment Axiom$\}$

$(a - x = b - y)[b/y] = a - x = 0 = a = x$

4) $\{a \geq 0 \wedge b \geq 0\}$

$x := a;$

$\{a = x\}$

$\{$Assignment Axiom$\}$

$(a = x)[a/x] = a = a = T$

$\{$Precondition Strengthening$\}$

$\{a \geq 0 \wedge b \geq 0\} \rightarrow T$

T $\{$Q.E.D$\}$