Factorial

$$\{n \geq 0\}$$

    a := 1

    b := 1

    WHILE   a < n   DO

        a := a + 1

        b := b × a

   LOOP

$$\{b = n!\}$$

{Sequencing Rule}

1) $\{Q\}$

     WHILE   a < n   DO

        a := a + 1

        b := b × a

    LOOP

$$\{b = n!\}$$

{While Rule}

1.1) Q = loop invariant = $b = a!$

$$\{b = a! \wedge a < n\}$$

     a := a + 1

      b := b × a

$$\{b = a!\}$$

{Sequencing Rule}

1.1.1) $\{Q_1\}$

      b := b × a

$$\{b = a!\}$$

{Assignment Axiom}

$$Q_1 = (b = a!)[b \times a / b] = \{b \times a = a!\}$$

$$= b = \frac{a!}{a}$$

Factorial

1.1.2) $\{b = a! \land a < n\}$

$$a := a + 1$$

$$\{b = \frac{a!}{a}\}$$

$$(b = \frac{a!}{a})[a+1 / a] = b = \frac{(a+1)!}{(a+1)}$$

$$= b = \frac{(a+1)(a+1-1)!}{(a+1)} = (a+1-1)! = a!$$

1.1.2.1) $P = \{b = a! \land a < n\}$ $\{$Precondition Strengthening$\}$
$P' = \{b = a!\}$

$$P \rightarrow P'$$

$\{$Pure Logic$\}$

$$T$$

1.2) $\{b = a! \land \neg a < n\} \rightarrow \{b = n!\}$

$$a = n \quad \therefore a! = n!$$

$$T \qquad \{Pure\ Logic\}$$

2) $\{Q_z\}$

$$b := 1$$

$$\{b = a!\}$$

$\{$Assignment Axiom$\}$

$$Q_z = (b = a!)[1 / b] = 1 = a!$$

Factorial

3) $\{n \geq 0\}$

$\quad\quad a := 1$

$\quad \{a! = 1\}$

$\quad\quad 1! = 1 \quad\quad 1 \times (0!) = 1$

$\quad\quad\quad\quad\quad\quad\quad 1 \times 1 = 1$

$\quad\quad\quad T$

$\{n \geq 0\} \rightarrow \quad T$

$\quad\quad\quad T$

$\{$ Assignment Axiom $\}$

$\{$ Reflexivity $\}$

$\{$ Precondition Strengthening $\}$

$\{$ Pure logic $\}$

$\{ Q \cdot E \cdot D \}$