Factorial

$\{n \geq 0\}$

```
a := 1
b := 1
c := 1
WHILE a < n DO
    c := c x a
    a := a + 1
    b := b x a
LOOP
```

$\{b = c \times a\}$

{Sequencing Rule}

1) $\{Q\}$

```
WHILE a < n DO
    c := c x a
    a := a + 1
    b := b x a
LOOP
```

$\{b = c \times a\}$

{While Rule}

1.1) Q = loop invariant = $b = c \times a$

$\{b = c \times a \wedge a < n\}$

```
c := c x a
a := a + 1
b := b x a
```

$\{b = c \times a\}$

Factorial

1.1.1) $\{Q_1\}$

$b := b \times a$

$\{b = c \times a\}$

{Assignment Axiom}

$Q_1 = (b = c \times a)[b \times a / b] = b \times a = c \times a$

1.1.2) $\{Q_2\}$

$a := a + 1$

$\{b \times a = c \times a\}$

{Assignment Axiom}

$Q_2 = (b \times a = c \times a)[a + 1 / a] = b \times (a+1) = c \times (a+1)$

1.1.3) $\{b = c \times a \wedge a > n\}$

$c := c \times a$

$\{b \times (a+1) = c \times (a+1)\}$

$(b \times (a+1) = c \times (a+1))[c \times a / c]$ {Assignment Axiom} $= b(a+1) = (c \times a)(a+1)$

$= b = c \times a$

{Precondition Strengthening}

1.1.3.1) $P = \{b = c \times a \wedge a > n\}$

$P' = \{b = c \times a\}$

$P \longrightarrow P'$

{Pure Logic}

$\top$

Factorial

1.2) $\{b = c \times a \ \wedge \ \neg a > n\} \longrightarrow \{b = c \times a\}$

{Pure Logic}

$\top$

2) $\{Q_3\}$

$c := 1$

$\{b = c \times a\}$

{Assignment Axiom}

$Q_3 = (b = c \times a)[1/c] = \{b = a\}$

2) $\{Q_4\}$

$b := 1$

$\{b = a\}$

{Assignment Axiom}

$Q_4 = (b = a)[1/b] = \{a = 1\}$

3) $\{n \geq 0\}$

$a := 1$

$\{a = 1\}$

{Assignment Axiom}

$(a = 1)[1/a] = 1 = 1 = \top$

{Precondition Strengthening}

$\{n \geq 0\} \longrightarrow \top$

{Pure Logic}

$\top$

{Q.E.D}