Func 2

$\{x \geq 0 \wedge y > 0\}$

```
r := x;
q := 0;
WHILE r ≥ y do
    q := q+1;
    r := r - y;
Loop
```

$\{x = yq + r\}$

{Sequencing Rule}

1. $\{Q\}$

```
WHILE r ≥ y do
    q := q+1;
    r := r-y;
Loop
```

$\{x = yq + r\}$

{While Rule}

1.1 $\{Q \wedge r \geq y\}$

```
q := q+1;
r := r-y;
```

$\{Q\}$

Q = loop invariant

$\therefore Q = (x = yq + r)$

{apply loop invariant}

$\{Q \wedge \neg\, r \geq y\}$

```
q := q+1;
r := r- y;
```

$\{x = yq + r\}$

{Sequencing Rule}

1.1.1 $\{Q\}$

$r := r - y;$

$\{x = yq + r\}$

{Assignment Axiom}

$Q = (x = yq + r)[r-y/r]$

$= x = yq + r - y$

1.1.2 $\{Q\}$

$q := q + 1;$

$\{x = yq + r - y\}$

{Assignment Axiom}

$Q = (x = yq + r - y)[q+1/q]$

$= x = y(q+1) + r - y$

{arithmetic}

$= x = yq + r$

1.2 $\{x = yq + r \wedge r < y\} \rightarrow \{x = qy + r\}$

{relationship between r and y always satisfies this}

$(x = yq + r) = (x = yq + r)$

{reflexivity}

T

2 $\{Q\}$

$q := 0;$

$\{x = yq + r\}$

{Assignment Axiom}

$Q = (x = yq + r)[0/q] = (x = y(0) + r)$

3. $\{P\}$

$r := x$

$\{x = y(0) + r\}$

{assignment Axiom}

$P = (x = y(0) + r)[x/r]$

$= x = y(0) + x$

{arithmetic}

$x = x$

{Reflexivity}

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{Q.E.D}.