

Factorial

$\{n \geq 0\}$

$a := 1$

$b := 1$

$c := 1$

WHILE $a < n$ DO

$c := c \times a$

$a := a + 1$

$b := b \times a$

LOOP

$\{b = c \times a\}$

{Sequencing Rule}

1) $\{Q\}$

WHILE $a < n$ DO

$c := c \times a$

$a := a + 1$

$b := b \times a$

LOOP

$\{b = c \times a\}$

{While Rule}

1.1) $Q = \text{loop invariant} = b = c \times a$

$\{b = c \times a \wedge a < n\}$

$c := c \times a$

$a := a + 1$

$b := b \times a$

$\{b = c \times a\}$

Factorial

{Sequencing Rule}

1.1.1) $\{Q_1\}$

$$\begin{array}{l} b := b \times a \\ \{b = c \times a\} \end{array}$$

{Assignment Axiom}

$$Q_1 = (b = c \times a) [b \times a / b] = b \times a = c \times a$$

1.1.2) $\{Q_2\}$

$$\begin{array}{l} a := a + 1 \\ \{b \times a = c \times a\} \end{array}$$

{Assignment Axiom}

$$Q_2 = (b \times a = c \times a) [a + 1 / a] = b \times (a + 1) = c \times (a + 1)$$

1.1.3) $\{b = c \times a \wedge a > n\}$

$$\begin{array}{l} c := c \times a \\ \{b \times (a + 1) = c \times (a + 1)\} \end{array}$$

{Assignment Axiom}

$$(b \times (a + 1) = c \times (a + 1)) [c \times a / c] = b(a + 1) = (c \times a)(a + 1)$$

$$= b = c \times a$$

{Precondition Strengthening}

$$\begin{array}{l} 1.1.3.1) \quad P = \{b = c \times a \wedge a > n\} \\ \quad \quad \quad P' = \{b = c \times a\} \end{array}$$

$$P \rightarrow P'$$

{Pure Logic}

\top

Factorial

$$1.2) \{b = c \times a \wedge \vdash a > n\} \longrightarrow \{b = c \times a\}$$

{Pure Logic}

\overline{T}

$$2) \{Q_3\}$$

$$\begin{array}{c} c := 1 \\ \{b = c \times a\} \end{array}$$

{Assignment Axiom}

$$Q_3 = (b = c \times a)[1/c] = \{b = a\}$$

$$2) \{Q_4\}$$

$$\begin{array}{c} b := 1 \\ \{b = a\} \end{array}$$

{Assignment Axiom}

$$Q_4 = (b = a)[1/b] = \{a = 1\}$$

$$3) \{n \geq 0\}$$

$$\begin{array}{c} a := 1 \\ \{a = 1\} \end{array}$$

{Assignment Axiom}

$$(a = 1)[1/a] = 1 = 1 = \overline{T}$$

{Precondition Strengthening}

$$\{n \geq 0\} \longrightarrow \overline{T}$$

{Pure Logic}

\overline{T}

{Q.E.D}