Function Subtract

$\{y \geq 0\}$

$z := x;$

$n := y;$

WHILE $n \mathrel{!}= 0$ DO

$z := z - 1;$

$n := n - 1;$

Loop

$\{z = x - y\}$

{Sequencing Rule}

1) $\{I\}$

WHILE $n \mathrel{!}= 0$ DO

$z := z - 1;$

$n := n - 1;$

Loop

$\{z = x - y\}$

{While Rule}

1.1) $\{I \wedge n \mathrel{!}= 0\}$

$z := z - 1;$

$n := n - 1;$

$\{I\}$

{Apply loop Invariant}

$I = [z - n = x - y]$

$\{z - n = x - y \wedge n \mathrel{!}= 0\}$

$z := z - 1;$

$n := n - 1;$

$\{z - n = x - y\}$

{Sequencing Rule}

1.1.1) $\{R\}$

$n := n - 1;$

$\{z - n = x - y\}$

{Assignment Axiom}

$R = (z - n = x - y)[n-1/n]$
$\Rightarrow (z - n + 1 = x - y)$

1.1.2) $\{z - n = x - y \wedge n != 0\}$
$z := z - 1;$
$\{z - n + 1 = x - y\}$

{Assignment Axiom}

$(z - n + 1 = x - y)[z-1/z]$
$\Rightarrow z - 1 - n + 1 = x - y$

{Arithmetic}

$\Rightarrow z - n = x - y$

{Precondition Strengthening}

1.1.2.1) $[z - n = x - y \wedge n != 0] \rightarrow [z - n = x - y]$

{Pure logic}

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1.2) $[z - n = x - y \wedge n = 0] \rightarrow [z = x - y]$

{Pure logic}

$[z = x - y] \rightarrow [z = x - y]$

{Reflexivity}

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2) $\{Q\}$
$n := y$
$\{z - n = x - y\}$

{Assignment Axiom}

$Q = (z - n = x - y)[y/n]$
$= z - y = x - y$

3) $\{y \geq 0\}$

$z := x$

$\{z - y = x - y\}$

{Assignment Axiom}

$(z - y = x - y)[x/z]$

$\Rightarrow x - y = x - y$

{Simplify}

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{precondion strengthening}

$[y \geq 0] \rightarrow [True]$

{Pure logic}

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{Q.E.D}