More Than

$\{a \geqslant 0 \wedge b \geqslant 0\}$

```
x := a;
y := b;

WHILE x != 0 ∧ y != 0 DO
    x := x - 1;
    y := y - 1;
LOOP
IF x != 0 ∧ y = 0 THEN
    p := 1;
ELSE
    p := 0;
END
```

$\{(x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0)\}$

{Sequencing Rule}

1) $\{Q\}$

```
IF x != 0 ∧ y = 0 THEN
    p := 1;
ELSE
    p := 0;
END
```

$\{(x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0)\}$

{Two-Armed Conditional Rule}

1.1) $\{Q \wedge (x != 0 \wedge y = 0)\}$

p := 1;

$\{(x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0)\}$

{Assignment Axiom}

$((x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0))[1/p] =$

$= (x != 0 \wedge y = 0 \wedge 1 = 1) \vee (x = 0 \vee y != 0 \wedge 1 = 0)$

$= (x != 0 \wedge y = 0) \vee \text{false} = \underline{(x != 0 \wedge y = 0)}$

More Than

1.1.1) $(x != 0 \wedge y = 0)$

{Precondition Strengthening}

$\{Q \wedge (x != 0 \wedge y = 0)\} \longrightarrow (x != 0 \wedge y = 0)$

{Pure Logic}

T

1.2) $\{Q \wedge \neg(x != 0 \wedge y = 0)\}$

$p := 0$

$\{(x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0)\}$

{Assignment Axiom}

$((x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0))[0/p] =$

$= (x != 0 \wedge y = 0 \wedge 0 = 1) \vee ((x = 0 \vee y != 0) \wedge 0 = 0)$

$= false \vee (x = 0 \vee y != 0)$

$= (x = 0 \vee y != 0)$

1.2.1) $\{Q \wedge \neg(x != 0 \wedge y = 0)\} \longrightarrow \{x = 0 \vee y != 0\}$

$\neg(x != 0 \wedge y = 0) = (x = 0 \vee y != 0)$

{Precondition Strengthening}

$\{Q \wedge (x = 0 \vee y != 0)\} \longrightarrow \{x = 0 \vee y != 0\}$

{Pure Logic}

T

1.3) $Q = ((x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0))[0/p]$ ✓

$((x != 0 \wedge y = 0 \wedge p = 1) \vee ((x = 0 \vee y != 0) \wedge p = 0))[1/p]$

$= ((x != 0 \wedge y = 0 \wedge 0 = 1) \vee ((x = 0 \vee y != 0) \wedge 0 = 0))$ ✓

$((x != 0 \wedge y = 0 \wedge 1 = 1) \vee ((x = 0 \vee y != 0) \wedge 1 = 0))$

$= (x = 0 \vee y != 0) \vee (x != 0 \wedge y = 0)$

More Than

2) $\{Q_1\}$

WHILE $x != 0 \wedge y != 0$ DO

$x := x - 1;$

$y := y - 1;$

LOOP

$\{(x = 0 \vee y != 0) \vee (x != 0 \wedge y = 0)\}$

{While Rule}

2.1) Q_1 = loop Invariant = $a - x = b - y$

$\{a - x = b - y \wedge (x != 0 \wedge y != 0)\}$

$x := x - 1;$

$y := y - 1;$

$\{a - x = b - y\}$

{Sequencing Rule}

2.1.1) $\{Q_2\}$

$y := y - 1;$

$\{a - x = b - y\}$

{Assignment Axiom}

$Q_2 = (a - x = b - y)[y-1/y] = a - x = b - y + 1$

2.1.2) $\{a - x = b - y \wedge (x != 0 \wedge y != 0)\}$

$x := x - 1;$

$\{a - x = b - y + 1\}$

{Assignment Axiom}

$(a - x = b - y + 1)[x - 1/x] = a - x + 1 = b - y + 1$

$= a - x = b - y$

{Precondition Strengthening}

2.1.2.1) $\{a - x = b - y \wedge (x != 0 \wedge y != 0)\} \rightarrow (a - x = b - y)$

{Pure Logic}

$= \quad T$

More Than

2.2) $\{a - x = b - y \wedge \neg(x != 0 \wedge y != 0)\} \rightarrow \{a - x = b - y\}$

T $\{$Pure Logic$\}$

3) $\{Q_3\}$

$y := b;$

$\{a - x = b - y\}$

$\{$Assignment Axiom$\}$

$(a - x = b - y)[b/y] = a - x = 0 = a = x$

4) $\{a \geq 0 \wedge b \geq 0\}$

$x := a;$

$\{a = x\}$

$\{$Assignment Axiom$\}$

$(a = x)[a/x] = a = a = T$

$\{$Precondition Strengthening$\}$

$\{a \geq 0 \wedge b \geq 0\} \longrightarrow T$

T $\{$Q.E.D$\}$