Exe 8.1B

The diets sheet contains information about weight loss under different diets. It provides statistical summaries which are relevant for hypothesis testing.

Sample Size (n): 50

Mean weight loss: 5.3412

Standard Deviation (SD): 2.5356

Step 1: State the Hypotheses

Null Hypothesis (H_0): The mean weight loss for Diet A is equal to the population mean of 5.0

Alternative Hypothesis (H_1): The mean weight loss for Diet A is not equal to the population mean

Significance Level (α): 0.05 (5%). The mean weight loss for Diet A is not equal to the population mean

Step 2: Set the Criteria for the Decision:

Significance Level (α): 0.05 (5%).

This is a two tailed test because of the alternative hypothesis

Step 3: Compute the Test Statistic

Sample Mean (M): 5.3412

Population Mean (μ_0): 5.0

Sample Standard Deviation (SD): 2.5356

Sample Size (n): 50

Test Statistic (z) = $\frac{M - \mu_0}{\frac{SD}{\sqrt{n}}} = \frac{5.3412 - 5.0}{\frac{2.5356}{\sqrt{50}}} = \frac{0.3412}{0.3585} \approx 0.952$

Step 4: Make a Decision

The critical z-values for a two-tailed test at α = 0.05 are ± 1.96.

Since z does not exceed the critical z values there is not enough evidence to conclude that the mean weight loss for Diet A differs significantly from 5.0 hence a failure to reject the null hypothesis.

Exe 8.2B

The diets sheet contains data about weight loss and some additional summary statistics.

Step 1: State the Hypotheses

Null Hypothesis (H_0): The mean weight loss for Diet A is equal to the population mean of 5.0

Alternative Hypothesis (H_1): The mean weight loss for Diet A is not equal to the population mean

Step 2: Set the Criteria for the Decision

Significance Level (α): 0.05 (5%).

Step 3: Compute the Test Statistic

Sample Mean (M): 5.34

Hypothesized Mean (μ): 5.0

Sample Standard Deviation (SD): 2.54

Sample Size (n): 50

Test Statistic (z) = $\frac{M-\mu}{\frac{SD}{\sqrt{n}}} = \frac{5.34-5.0}{\frac{2.54}{\sqrt{50}}} \approx 0.95$

Step 4: Make a decision

Degrees of freedom (df): n – 1 = 49

Using a t-distribution table the approximate critical t-value is 2.01.

Using a t-distribution calculator the approximate p value is 0.35

Since the p-value is greater than the significance level and the critical t-value is greater than the test statistic value we fail to reject the null hypothesis meaning the weight loss observed is not significantly different from the hypothesized mean.

Exe 8.3D

The spreadsheet contains data comparing the popularity of brands of cereals across different areas and includes some summary data

Step 1: State the Hypotheses

Null Hypothesis (H_0): The mean frequency of purchases is the same across brands

Alternative Hypothesis (H_1): At least one brand has a mean frequency that is different

Step 2: Set the Criteria for the Decision

To conduct an ANOVA test you need the frequency of purchases for each brand and the total number of observations for each brand

Significance Level (α): 0.05 (5%).

Brand	Area 1	Area 2	Total
Brand A	11	19	30
Brand B	17	30	47
Other	42	41	83

Sample Size (n): 160

Overall Mean (M): 1.0

Brand A Mean (AM): 15

Brand B Mean (BM): 23.5

Other Brand Mean (OM): 41.5

Step 3: Compute the Test Statistic

ANOVA involves partitioning the variability into the differences between the brand means and the variations with each brand group

$$F = \frac{differences\ between\ the\ brand\ means}{variations\ with\ each\ brand\ group}$$

If the F-statistic is large and the corresponding p-value is less than the significance level we reject the null hypothesis

$$SSB = 2 \times (15-1)^2 + 2 \times (23.5-1)^2 + 2 \times (41.5-1)^2 = 4685$$

$$SSW = \frac{(19-11)^2}{12} + \frac{(30-17)^2}{12} + \frac{(42-41)^2}{12} \times 2 = 39$$

Degrees of freedom between group = k – 1 = 3 – 1 = 2 (Number of groups – 1)

Degrees of freedom within group = N – k = 6 – 3 = 3 (Total observations – number of groups)

$$MSB = \frac{SSB}{Degrees\ of\ freedom\ between\ group} = \frac{4685}{2} = 2342.5$$

$$MSW = \frac{SSW}{Degrees\ of\ freedom\ with\ group} = \frac{39}{3} = 13$$

$F - statistic = \frac{MSB}{MSW} = \frac{2342.5}{13}$ = 180.192307692

Using a f-distribution table the approximate p value is 2.264

Step 4: Make a decision

Given p is greater than 0.05 we fail to reject the null hypothesis

Exe 8.4G

The excel file contains data about 2 possible filter agents used in the production of a chemical product

Step 1: State the Hypotheses

Null Hypothesis (H_0): The mean of Agent 2 is equal 8.0

Alternative Hypothesis (H_1): The mean Agent 2 is not equal to 8.0

Step 2: Set the Criteria for the Decision

Significance Level (α): 0.05 (5%).

Step 3: Compute the Test Statistic

Sample Mean (M): 8.68

Hypothesized Mean (μ): 8.0

Sample Standard Deviation (SD): 1.04

Sample Size (n): 12

Test Statistic (z) = $\frac{M-\mu}{\frac{SD}{\sqrt{n}}} = \frac{8.68-8.0}{\frac{1.04}{\sqrt{12}}} \approx 2.28$

Using a t-distribution calculator the approximate p value is 0.044

Step 4: Make a decision

Given p is less than 0.05 we fail to reject the null hypothesis meaning there is enough evidence to suggest the mean of agent 2 is significantly different from 8.0 at the 5% significance level.

Exe 8.6C

The data in the spreadsheet compares income across 60 men and 60 women with the incomes ranging from 30 to 100.9, a mean of 48.57 and a standard deviation of 15.13.

Step 1: State the Hypotheses

Null Hypothesis (H_0): The mean income is equal to 50.0

Alternative Hypothesis (H_1): The mean income is not equal to 50.0

Step 2: Set the Criteria for the Decision

Significance Level (α): 0.05 (5%).

Step 3: Compute the Test Statistic

Sample Mean (M): 48.57

Hypothesized Mean (μ): 50.0

Sample Standard Deviation (SD): 15.13

Sample Size (n): 120

Test Statistic (z) = $\frac{M-\mu}{\frac{SD}{\sqrt{n}}} = \frac{48.57-50}{\frac{15.13}{\sqrt{120}}} \approx -1.03$

Using a t-distribution calculator the approximate p value is 0.304

Step 4: Make a decision

Since the p-value is greater than the significance level we fail to reject the null hypothesis meaning there is no statistically significant evidence to suggest the mean income differs from the hypothesised mean of 50.