Course Notes Summary

Focus Area: Brief summary of main formulas

Source:	BayesTheorem.pdf
Generated:	2025-09-05 14:45:25
Summary Type:	Comprehensive

Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

Brief summary of main formulas — Bayes' theorem (study notes)

1. Core definitions - Event intersection: A B = both A and B occur. - Conditional probability: P(B | A) = probability B occurs given A has occurred. - Prior probability: the initial probability of an event before new evidence (notation: P(A)). - Posterior probability: the revised probability after observing evidence (notation: P(A | B)). - Likelihood: P(B | A) — probability of observed evidence B under hypothesis A.

2. Fundamental formulas - Conditional probability (definition) - P(B | A) = P(A B) / P(A), provided P(A) > 0.

- Symmetry for intersection - P(A B) = P(A) P(B | A) = P(B) P(A | B).

- Bayes' theorem (basic two-event form) - P(A | B) = [P(A) P(B | A)] / P(B).

- Law of total probability (useful for the denominator) - If A1, A2, ..., Ak is a partition (mutually exclusive and exhaustive), then - P(B) = $_{i=1}^{k} P(Ai)P(B|Ai)$.

- Bayes' theorem (generalized / k events / partition form) - For Aj in a partition A1, ..., Ak, - P(Aj | B) = [P(Aj) P(B | Aj)] / $_{i=1}^{k} P(Ai)P(B|Ai)$.

3. Derivation idea (short) - From P(A B) = P(A) P(B | A) = P(B) P(A | B) P(A | B) = P(A) P(B | A) / P(B). - Denominator P(B) expanded by law of total probability when multiple causes exist.

4. Intuitive / counting method (recommended for calculation) - Steps: 1. Choose a convenient total population N (e.g., 100, 1,000 or 10,000). 2. Convert priors P(Ai) into counts: count(Ai) = N × P(Ai). 3. For each Ai, compute counts of evidence B: count(Ai and B) = count(Ai) × P(B | Ai). 4. Sum counts for B: count(B) = count(Ai and B). 5. Posterior P(Aj | B) = count(Aj and B) / count(B). - This produces the same numeric result as the formula and is less error-prone.

5. Worked examples (compact)

Example A — Industrial quality (ELT manufacturers) - Given: - P(A)=0.80, P(B)=0.15, P(C)=0.05 - P(D | A)=0.04, P(D | B)=0.06, P(D | C)=0.09 (D = defective) - Using Bayes' theorem for P(A | D): - Numerator = 0.80 × 0.04 = 0.032 - Denominator = 0.032 + (0.15 × 0.06) + (0.05 × 0.09) = 0.032 + 0.009 + 0.0045 = 0.0455 - P(A | D) = 0.032 / 0.0455 0.703 - Counting method (N = 10,000) gives the same: 320 defective from A out of 455 total defective 320/455 0.703.

Example B — Smoking and gender (Orange County) - Given: - P(Male)=0.51, P(Female)=0.49 - P(Cigar | Male)=0.095, P(Cigar | Female)=0.017 - Compute P(Male | Cigar): - Numerator = 0.51 × 0.095 = 0.04845 - Denominator = 0.04845 + (0.49 × 0.017 = 0.00833) = 0.05678 - P(Male | Cigar) 0.04845 / 0.05678 0.854 - Counting method (N = 100,000) — 4,845 cigar-smoking males / 5,678 total cigar smokers 0.854.

6. Important relationships insights - Posterior Prior × Likelihood: - P(Aj | B) P(Aj) × P(B | Aj). Normalize by dividing by $_{i} P(Ai)P(B|Ai). - Bayes' theorem is a formal statement of updating belief about hyp$ $Law of total probability is essential to compute the marginal probability of evidence P(B). - When the partition is onl$ $event form.$

7. Common conditions and pitfalls - Denominator must be > 0 (P(B) 0). - The set A1,...,Ak used in the denominator must be mutually exclusive and collectively exhaustive (a partition). - Confuse P(B | A) with P(A | B) — they are generally not equal. - Use counts/intuitive table to reduce algebraic substitution errors.

8. Quick checklist for solving Bayes problems 1. Identify hypotheses Aj and evidence/event B. 2. Write priors P(Aj) and likelihoods P(B | Aj). 3. Compute marginal P(B) = P(Ai) P(B | Ai). 4. Compute posterior P(Aj | B) = P(Aj) P(B | Aj) / P(B). 5. Optionally, verify with a counting/table approach.

9. Useful mnemonic - "Posterior Prior × Likelihood" — multiply prior probability by how likely the evidence is under that hypothesis, then normalize.

Use this sheet to revise formulas quickly and to guide computations; when in doubt, convert probabilities to counts and construct a small contingency table — it's often the fastest, least error-prone route.