

Course Notes Summary

Focus Area: Summary of main formulas please

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Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

Summary of Main Formulas — Bayes' Theorem (study / revision sheet)

1. Key definitions - Conditional probability: $P(B | A)$ = probability that B occurs given A has occurred. - Prior probability: initial probability of an event before new information is used (denote $P(A)$). - Likelihood: $P(\text{new information} | \text{hypothesis})$ — e.g., $P(B | A)$. - Posterior probability: revised probability after using new information — e.g., $P(A | B)$. - Evidence (or marginal likelihood): overall probability of the new information, $P(B)$.
 2. Fundamental formulas - Conditional probability (basic definition) $P(B | A) = P(A \cap B) / P(A)$, provided $P(A) > 0$.
 - Multiplication rule (from conditional probability) $P(A \cap B) = P(A) P(B | A) = P(B) P(A | B)$.
 - Independence (special case) A and B independent $P(B | A) = P(B)$ $P(A \cap B) = P(A) P(B)$.
 3. Bayes' theorem — two-event form - Bayes' formula (general two-event) $P(A | B) = [P(B | A) P(A)] / P(B)$.
 - Using the complement A (two-category partition) $P(A | B) = [P(B | A) P(A)] / [P(B | A) P(A) + P(B | A) P(A)]$.
 - Mnemonic form posterior prior \times likelihood (then normalize by dividing by evidence $P(B)$).
 4. Total probability theorem (needed for denominator / evidence) - For a partition A_1, A_2, \dots, A_n of the sample space, $P(B) = \sum_j P(B | A_j) P(A_j)$. - For two events A and A : $P(B) = P(B | A) P(A) + P(B | A) P(A)$.
 5. Bayes' theorem — generalized (n events / partition) - For events A_1, A_2, \dots, A_n forming a partition, $P(A_i | B) = [P(A_i) P(B | A_i)] / \sum_{j=1..n} [P(A_j) P(B | A_j)]$. - Each posterior $P(A_i | B)$ is nonnegative and $\sum_i P(A_i | B) = 1$.
 6. Intuitive (frequency/table) method — practical solving tip - Pick a convenient total N (e.g., 1000, 10,000, 100,000). - Convert each prior $P(A_i)$ to counts : $\text{count}(A_i) = N \cdot P(A_i)$. - Convert likelihoods to counts : $\text{count}(A_i | B) = \text{count}(A_i) \cdot P(B | A_i)$. - Sum counts of B across all A to get total count $N \cdot P(B)$. - Posterior $P(A_i | B) = \text{count}(A_i | B) / \text{count}(B)$. - This avoids algebra errors and gives intuitive contingency counts.
 7. Step-by-step procedure for a Bayes problem 1. Identify hypotheses/partitions A_1, A_2, \dots (priors $P(A_i)$). 2. Identify observed evidence B and likelihoods $P(B | A_i)$. 3. Compute evidence $P(B)$ using total probability $P(B) = \sum_i P(A_i) P(B | A_i)$. 4. Compute posterior(s) : $P(A_i | B) = [P(A_i) P(B | A_i)] / P(B)$. 5. Optionally, construct counts via a frequency table.
 8. Short worked examples (compact)
 - Example A: Cigar-smoking / gender (from text) Given: $P(\text{Male})=0.51$, $P(\text{Female})=0.49$, $P(\text{Cigar} | \text{Male})=0.095$, $P(\text{Cigar} | \text{Female})=0.017$. Compute $P(\text{Male} | \text{Cigar})$: $P(\text{Cigar}) = 0.51 \times 0.095 + 0.49 \times 0.017 = 0.04845 + 0.00833 = 0.05678$. $P(\text{Male} | \text{Cigar}) = (0.51 \times 0.095) / 0.05678 = 0.04845 / 0.05678 = 0.853$ (85.3)
 - Example B: ELT manufacturers and defective units (three-category) Given: $P(A)=0.80$, $P(B)=0.15$, $P(C)=0.05$; defect rates $P(D | A)=0.04$, $P(D | B)=0.06$, $P(D | C)=0.09$. $P(D) = 0.80 \times 0.04 + 0.15 \times 0.06 + 0.05 \times 0.09 = 0.032 + 0.009 + 0.0045 = 0.0455$. $P(A | D) = (0.80 \times 0.04) / 0.0455 = 0.032 / 0.0455 = 0.703$ (70.3) (Alternative counts: assume $N=10,000 \rightarrow$ defective counts: A:320, B:90, C:45 \rightarrow posterior = $320/455 = 0.703$.)
 9. Important relationships checks - Consistency: $P(A \cap B)$ computed either way must match: $P(A)P(B | A) = P(B)P(A | B)$. - Posterior normalization: $\sum_i P(A_i | B) = 1$ when A_i is a partition. - Edge case (zero denominators) : Bayes' formula requires $P(B) > 0$. - Independence test : if $P(A | B) = P(A)$, then B gives no information about A. - Beware base-rate fallacy : high likelihood $P(B | A)$ alone does not imply high posterior $P(A | B)$.
 10. Common pitfalls and tips - Always identify priors and likelihoods explicitly before algebra.
 - Use the total-probability denominator — forgetting terms is the common error. - Use frequency tables for intuition and arithmetic checks. - Round only at the final step to avoid cumulative rounding errors. - For small probabilities or many events, use precise arithmetic or software.
 11. Quick formula cheat-sheet (memorize these) - $P(B | A) = P(A \cap B) / P(A)$ - $P(A \cap B) = P(A) P(B | A)$ - $P(B) = \sum_j P(B | A_j) P(A_j)$ - $P(A_i | B) = [P(A_i) P(B | A_i)] / \sum_j [P(A_j) P(B | A_j)]$
- Use this sheet to:
- Set up Bayes problems quickly (identify priors, likelihoods). - Convert to counts for sanity checks.
 - Remember posterior prior \times likelihood and then normalize.

If you want, I can convert this into a one-page printable study card or produce a few more worked examples with contingency tables.