# Course Notes Summary

Focus Area: Summary of main formulas please

| | |
|---|---|
| **Source:** | BayesTheorem.pdf |
| **Generated:** | 2025-09-05 16:09:40 |
| **Summary Type:** | Comprehensive |

Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

## 1 Key definitions

- Prior probability: initial probability before new information. Example: $P(A)$.
- Posterior probability: revised probability after new information. Example: $P(A \mid B)$.
- Conditional probability: probability of $B$ given $A$. Formula: $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$.
- Likelihood: $P(B \mid A)$, the probability of observed data $B$ under hypothesis $A$.
- Evidence (marginal probability): $P(B)$, the total probability of $B$ across all hypotheses.
- Joint probability: $P(A \cap B)$, probability both $A$ and $B$ occur.
- Independence: $A$ and $B$ independent if $P(A \mid B) = P(A)$ (equivalently $P(A \cap B) = P(A)P(B)$).

## 2 Core formulas

- Conditional probability (definition):

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}.$$

- Joint probability (equivalent forms):

$$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B).$$

- Bayes' theorem (two events):

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}.$$

- Law of total probability (for a partition $\{A_i\}$):

$$P(B) = \sum_i P(B \mid A_i)P(A_i).$$

- Generalized Bayes' theorem (for partition $\{A_i\}$):

$$P(A_k \mid B) = \frac{P(B \mid A_k)P(A_k)}{\sum_i P(B \mid A_i)P(A_i)}.$$

## 3 Short application procedure (useful checklist)

1. Identify mutually exclusive, exhaustive hypotheses $A_i$ and their priors $P(A_i)$.
2. Compute likelihoods $P(B \mid A_i)$ for the observed evidence $B$.
3. Compute evidence: $P(B) = \sum_i P(B \mid A_i)P(A_i)$.
4. Compute posterior for hypothesis $A_k$: $P(A_k \mid B) = \frac{P(B \mid A_k)P(A_k)}{P(B)}$.

## 4 Intuitive (frequency/table) method

- Choose a convenient total $N$ (e.g., 100, 1,000, 100,000).
- Convert priors to counts: $\text{count}(A_i) = N \cdot P(A_i)$.
- Convert likelihoods to counts: $\text{count}(A_i \cap B) = \text{count}(A_i) \cdot P(B \mid A_i)$.
- Evidence count: $\text{count}(B) = \sum_i \text{count}(A_i \cap B)$.
- Posterior as frequency: $P(A_k \mid B) = \dfrac{\text{count}(A_k \cap B)}{\text{count}(B)}$.

## 5 Important relationships and notes

- Posterior = (Likelihood × Prior) / Evidence.
- Bayes updates beliefs: prior → incorporate data $B$ → posterior.
- The Law of Total Probability provides the denominator for Bayes' rule.
- For independent events, Bayes' update is unnecessary: $P(A \mid B) = P(A)$.
- Use the frequency method to reduce algebra mistakes.

## 6 Examples (formula + calculation)

- Two-group Bayes (cigar example):
  Given $P(\text{Male}) = 0.51$, $P(\text{Cigar} \mid \text{Male}) = 0.095$, $P(\text{Cigar} \mid \text{Female}) = 0.017$. Then

  $$P(\text{Male} \mid \text{Cigar}) = \frac{P(\text{Cigar} \mid \text{Male})P(\text{Male})}{P(\text{Cigar} \mid \text{Male})P(\text{Male}) + P(\text{Cigar} \mid \text{Female})P(\text{Female})}$$

  Numerically,

  $$= \frac{0.095 \cdot 0.51}{0.095 \cdot 0.51 + 0.017 \cdot 0.49} = \frac{0.04845}{0.04845 + 0.00833} \approx 0.853 \ (\approx 85.3\%).$$

- Three-manufacturer defect example:
  $P(A) = 0.80$, $P(B) = 0.15$, $P(C) = 0.05$.
  $P(D \mid A) = 0.04$, $P(D \mid B) = 0.06$, $P(D \mid C) = 0.09$. Then

  $$P(A \mid D) = \frac{P(D \mid A)P(A)}{P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)}$$

  Numerically,

  $$= \frac{0.04 \cdot 0.80}{0.04 \cdot 0.80 + 0.06 \cdot 0.15 + 0.09 \cdot 0.05} = \frac{0.032}{0.032 + 0.009 + 0.0045} = \frac{0.032}{0.0455} \approx 0.703 \ (\approx 70.3\%).$$

## 7 Quick reference summary (formulas only)

- $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$
- $P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$

- $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$
- $P(B) = \sum_i P(B \mid A_i)P(A_i)$
- $P(A_k \mid B) = \dfrac{P(B \mid A_k)P(A_k)}{\sum_i P(B \mid A_i)P(A_i)}$

## 8 When to use which method

- Use algebraic Bayes when you have probabilities and want symbolic clarity.
- Use the frequency/table method for intuitive understanding and to avoid substitution errors.
- Use generalized Bayes when multiple hypotheses partition the sample space.