

# Course Notes Summary

Focus Area: Summary of main formulas please

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Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

## Summary of Main Formulas — Bayes' Theorem (study / revision sheet)

1. Key definitions - Conditional probability:  $P(B | A)$  = probability that B occurs given A has occurred. - Prior probability: initial probability of an event before new information is used (denote  $P(A)$ ). - Likelihood:  $P(\text{new information} | \text{hypothesis})$  — e.g.,  $P(B | A)$ . - Posterior probability: revised probability after using new information — e.g.,  $P(A | B)$ . - Evidence (or marginal likelihood): overall probability of the new information,  $P(B)$ .

2. Fundamental formulas - Conditional probability (basic definition)  $P(B | A) = P(A \cap B) / P(A)$ , provided  $P(A) > 0$ .

- Multiplication rule (from conditional probability)  $P(A \cap B) = P(A) P(B | A) = P(B) P(A | B)$ .

- Independence (special case) A and B independent  $P(B | A) = P(B)$   $P(A \cap B) = P(A) P(B)$ .

3. Bayes' theorem — two-event form - Bayes' formula (general two-event)  $P(A | B) = [P(B | A) P(A)] / P(B)$ .

- Using the complement A (two-category partition)  $P(A | B) = [P(B | A) P(A)] / [P(B | A) P(A) + P(B | A^c) P(A^c)]$ .

- Mnemonic form posterior = prior  $\times$  likelihood (then normalize by dividing by evidence  $P(B)$ ).

4. Total probability theorem (needed for denominator / evidence) - For a partition  $A_1, A_2, \dots, A_n$  of the sample space,  $P(B) = \sum_j P(B|A_j)P(A_j)$ . - For two events A and A' :  $P(B) = P(B|A)P(A) + P(B|A')P(A')$ .

5. Bayes' theorem — generalized (n events / partition) - For events  $A_1, A_2, \dots, A_n$  forming a partition,  $P(A_i|B) = [P(A_i)P(B|A_i)] / \sum_{j=1..n} [P(A_j)P(B|A_j)]$ . - Each posterior  $P(A_i|B)$  is nonnegative and  $\sum_i P(A_i|B) = 1$ .

6. Intuitive (frequency/table) method — practical solving tip - Pick a convenient total N (e.g., 1000, 10,000, 100,000). - Convert each prior  $P(A_i)$  to counts :  $\text{count}(A_i) = N \cdot P(A_i)$ . - Convert likelihoods to counts :  $\text{count}(A_i B) = \text{count}(A_i) \cdot P(B|A_i)$ . - Sum counts of B across all A to get total count  $N \cdot P(B)$ . - Posterior  $P(A_i|B) = \text{count}(A_i B) / \text{count}(B)$ . - This avoids algebra errors and gives intuitive contingency.

7. Step-by-step procedure for a Bayes problem 1. Identify hypotheses/partitions  $A_1, A_2, \dots$  (priors  $P(A_i)$ ). 2. Identify observed evidence B and likelihoods  $P(B|A_i)$ . 3. Compute evidence  $P(B)$  using total probability theorem. 4. Compute posterior(s) :  $P(A_i|B) = [P(A_i)P(B|A_i)] / P(B)$ . 5. Optionally, construct counts via a contingency table.

8. Short worked examples (compact)

- Example A: Cigar-smoking / gender (from text) Given:  $P(\text{Male})=0.51$ ,  $P(\text{Female})=0.49$ ,  $P(\text{Cigar} | \text{Male})=0.095$ ,  $P(\text{Cigar} | \text{Female})=0.017$ . Compute  $P(\text{Male} | \text{Cigar})$ :  $P(\text{Cigar}) = 0.51 \times 0.095 + 0.49 \times 0.017 = 0.04845 + 0.00833 = 0.05678$ .  $P(\text{Male} | \text{Cigar}) = (0.51 \times 0.095) / 0.05678 = 0.04845 / 0.05678 = 0.853$  (85.3%)

- Example B: ELT manufacturers and defective units (three-category) Given:  $P(A)=0.80$ ,  $P(B)=0.15$ ,  $P(C)=0.05$ ; defect rates  $P(D | A)=0.04$ ,  $P(D | B)=0.06$ ,  $P(D | C)=0.09$ .  $P(D) = 0.80 \times 0.04 + 0.15 \times 0.06 + 0.05 \times 0.09 = 0.032 + 0.009 + 0.0045 = 0.0455$ .  $P(A | D) = (0.80 \times 0.04) / 0.0455 = 0.032 / 0.0455 = 0.703$  (70.3%) (Alternative counts: assume  $N=10,000 \rightarrow$  defective counts: A:320, B:90, C:45  $\rightarrow$  posterior =  $320/455 = 0.703$ .)

9. Important relationships checks - Consistency:  $P(A \cap B)$  computed either way must match:  $P(A)P(B | A) = P(B)P(A | B)$ . - Posterior normalization:  $\sum_i P(A_i|B) = 1$  when  $A_i$  is a partition. - Edge case (zero denominators) : Bayes' formula requires  $P(B) > 0$ . - Independence test : if  $P(A|B) = P(A)$ , then B gives no information about A. - Beware base-rate fallacy : high likelihood  $P(B|A)$  alone does not imply high probability of A.

10. Common pitfalls and tips - Always identify priors and likelihoods explicitly before algebra. - Use the total-probability denominator — forgetting terms is the common error. - Use frequency tables for intuition and arithmetic checks. - Round only at the final step to avoid cumulative rounding errors. - For small probabilities or many events, use precise arithmetic or software.

11. Quick formula cheat-sheet (memorize these) -  $P(B | A) = P(A \cap B) / P(A)$  -  $P(A \cap B) = P(A) P(B | A) = P(B) P(A | B)$  -  $P(B) = \sum_j P(B|A_j)P(A_j)$  -  $P(A_i|B) = [P(A_i)P(B|A_i)] / \sum_j [P(A_j)P(B|A_j)]$

Use this sheet to: - Set up Bayes problems quickly (identify priors, likelihoods). - Convert to counts for sanity checks. - Remember posterior = prior  $\times$  likelihood and then normalize.

If you want, I can convert this into a one-page printable study card or produce a few more worked examples with contingency tables.