Course Notes Summary

Focus Area: Summary of main formulas please

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Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

1 Key definitions and concepts

1.1 Definitions

- Conditional probability: $P(B \mid A)$ = probability of B given A has occurred.
- Prior probability: the initial probability of an event (before new information).
- Posterior probability: the revised probability after new information.
- Likelihood: $P(B \mid A)$, the probability of the observed evidence B under hypothesis A.
- Marginal (or total) probability: $P(B)$ = overall probability of B.
- Joint probability: $P(A \text{ and } B)$ = probability both A and B occur.
- Independence: A and B independent if $P(A \mid B) = P(A)$ (equivalently $P(A \text{ and } B) = P(A)P(B)$).

1.2 Basic relationships

- $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$
- $P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$
- If A and B independent: $P(A \text{ and } B) = P(A)P(B)$

2 Main formulas

2.1 Bayes' theorem (two events)

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

2.2 Law of total probability (denominator for Bayes)

For a partition $\{A_1, A_2, \ldots, A_n\}$:

$$P(B) = \sum_i P(B \mid A_i)P(A_i)$$

Two-event special case:

$$P(B) = P(B \mid A)P(A) + P(B \mid A^c)P(A^c)$$

2.3 Bayes' theorem (generalized for multiple events)

For partition $\{A_1, \ldots, A_n\}$:

$$P(A_k \mid B) = \frac{P(B \mid A_k)P(A_k)}{\sum_j P(B \mid A_j)P(A_j)}$$

2.4 Odds form (useful for likelihood ratios)

Posterior odds = Likelihood ratio × Prior odds:

$$\frac{P(A \mid B)}{P(A^c \mid B)} = \frac{P(B \mid A)}{P(B \mid A^c)} \times \frac{P(A)}{P(A^c)}$$

3 Computation methods and examples

3.1 Intuitive (frequency/table) method — steps

1. Assume a convenient total N (e.g., $N = 100{,}000$).
2. Convert probabilities to counts by multiplying N.
3. Fill a contingency table with counts for each combination.
4. Compute posterior as $\dfrac{\text{count}(A \text{ and } B)}{\text{count}(B)}$.

3.2 Example: cigar smoking (two-category Bayes via table)

- Given: $P(\text{male}) = 0.51$, $P(\text{female}) = 0.49$, $P(\text{cigar} \mid \text{male}) = 0.095$, $P(\text{cigar} \mid \text{female}) = 0.017$.
- Assume $N = 100{,}000$.
- Males $= 51{,}000$. Cigar-smoking males $= 0.095 \times 51{,}000 = 4{,}845$.
- Females $= 49{,}000$. Cigar-smoking females $= 0.017 \times 49{,}000 = 833$.
- Total cigar smokers $= 4{,}845 + 833 = 5{,}678$.
- $P(\text{male} \mid \text{cigar}) = \dfrac{4{,}845}{5{,}678} \approx 0.853$.

3.3 Example: defective ELT (generalized Bayes)

- Given: $P(A) = 0.80$, $P(B) = 0.15$, $P(C) = 0.05$.
- Defect rates: $P(D \mid A) = 0.04$, $P(D \mid B) = 0.06$, $P(D \mid C) = 0.09$.
- Numerator for $P(A \mid D)$:
$$P(D \mid A)P(A) = 0.04 \times 0.80 = 0.032$$
- Denominator:
$$0.032 + 0.06 \times 0.15 + 0.09 \times 0.05 = 0.032 + 0.009 + 0.0045 = 0.0455$$
- $P(A \mid D) = \dfrac{0.032}{0.0455} \approx 0.703$.

4 Important relationships and connections

- Bayes requires the law of total probability to compute $P(B)$ (denominator).
- Posterior becomes new prior for sequential updating with additional evidence.
- Independence simplifies conditional expressions and Bayes calculations.
- Likelihood ratios $(P(B \mid A)/P(B \mid A^c))$ quantify how evidence shifts beliefs.
- Frequency/table method avoids algebraic substitution errors and aids intuition.

5 Quick reference (compact formulas)

- Conditional: $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$
- Joint: $P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$
- Law of total probability: $P(B) = \sum_i P(B \mid A_i)P(A_i)$
- Bayes (two events): $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$
- Bayes (multiple): $P(A_k \mid B) = \dfrac{P(B \mid A_k)P(A_k)}{\sum_j P(B \mid A_j)P(A_j)}$
- Odds form: $\dfrac{P(A \mid B)}{P(A^c \mid B)} = \dfrac{P(B \mid A)}{P(B \mid A^c)} \times \dfrac{P(A)}{P(A^c)}$