EDVANCE BY JEC

Course Notes Summary

Focus Area: Summary of main formulas please

Source:	BayesTheorem.pdf
Generated:	2025-09-05 15:40:56
Summary Type:	Comprehensive

Prepared for quick revision and reference

Use this sheet as a step-by-step guide when solving problems.

1 SUMMARY: MAIN FORMULAS FOR BAYES' THEOREM (concise study sheet)

2 KEY DEFINITIONS

- Conditional probability: $P(B \mid A)$ = probability that B occurs given A has occurred.
- Prior probability: $P(A)$ — the initial probability of hypothesis/event A before new evidence.
- Likelihood: $P(E \mid A)$ — probability of observing evidence E if A is true.
- Posterior probability: $P(A \mid E)$ — updated probability of A after observing evidence E.
- Evidence (marginal probability): $P(E)$ — total probability of observing E under all hypotheses.

3 CORE FORMULAS (quick reference)

- Definition of conditional probability:
  $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, provided $P(A) > 0$.
- Law of total probability (for a partition $A_1, A_2, \ldots, A_n$):
$$P(E) = \sum_{i=1}^{n} P(A_i)\, P(E \mid A_i).$$
- Bayes' theorem (general form for hypothesis A_i given evidence E):
$$P(A_i \mid E) = \frac{P(A_i)\, P(E \mid A_i)}{\sum_{j=1}^{n} P(A_j)\, P(E \mid A_j)}.$$
- Two-hypothesis (A and $\neg A$) Bayes formula:
$$P(A \mid E) = \frac{P(A)\, P(E \mid A)}{P(A)\, P(E \mid A) + P(\neg A)\, P(E \mid \neg A)}.$$
- Odds form (useful for comparing two hypotheses A vs. $\neg A$):
  Posterior odds = Prior odds × Likelihood ratio, i.e.
$$\frac{P(A \mid E)}{P(\neg A \mid E)} = \frac{P(A)}{P(\neg A)} \times \frac{P(E \mid A)}{P(E \mid \neg A)}.$$

4 STEP-BY-STEP PROCEDURE FOR APPLYING BAYES

1. Identify hypotheses $(A_1, A_2, \ldots, A_n)$ and which one you want the posterior for.
2. Obtain priors $P(A_i)$ for each hypothesis.
3. Obtain likelihoods $P(E \mid A_i)$ for the observed evidence E.
4. Compute evidence: $P(E) = \sum_i P(A_i) P(E \mid A_i)$.
5. Compute posterior: $P(A_k \mid E) = \dfrac{P(A_k)\, P(E \mid A_k)}{P(E)}$.
6. (Optional) Convert to odds if comparing two hypotheses.

5 INTUITIVE / FREQUENTIST APPROACH (useful for checking)

- Choose a convenient total N (e.g., 1000 or 100000).
- Compute expected counts: $\text{count}(A_i) = N \times P(A_i)$;
  $\text{count}(E \cap A_i) = \text{count}(A_i) \times P(E \mid A_i)$.
- Evidence count $= \sum_i \text{count}(E \cap A_i)$.
- Posterior $= \dfrac{\text{count}(E \cap A_k)}{\text{evidence count}}$.
- This method avoids algebra errors and is easy for practice.

6 EXAMPLES (worked, stepwise)

6.1 Example A — ELT defect (from text)

- Hypotheses: $A =$ Altigauge $(P(A) = 0.80)$, $B =$ Bryant $(P(B) = 0.15)$, $C =$ Chartair $(P(C) = 0.05)$.
- Likelihoods (defective D): $P(D \mid A) = 0.04,\ P(D \mid B) = 0.06,\ P(D \mid C) = 0.09$.
- Evidence:
$$P(D) = 0.80 \times 0.04 + 0.15 \times 0.06 + 0.05 \times 0.09 = 0.032 + 0.009 + 0.0045 = 0.0455.$$
- Posterior:
$$P(A \mid D) = \frac{0.032}{0.0455} \approx 0.703 \ (\approx 70.3\%).$$

6.2 Example B — Cigar smokers (Orange County)

- Priors: $P(\text{male}) = 0.51,\ P(\text{female}) = 0.49$.
- Likelihoods: $P(\text{cigar} \mid \text{male}) = 0.095,\ P(\text{cigar} \mid \text{female}) = 0.017$.
- Evidence:
$$P(\text{cigar}) = 0.51 \times 0.095 + 0.49 \times 0.017 = 0.04845 + 0.00833 = 0.05678.$$
- Posterior:
$$P(\text{male} \mid \text{cigar}) = \frac{0.04845}{0.05678} \approx 0.8529 \ (\approx 85.3\%).$$
- Frequentist check with $N = 100000$: cigar-smoking males $= 4845$; cigar-smoking females $= 833$;
  posterior $= 4845/(4845 + 833) \approx 0.8529$.

7 IMPORTANT RELATIONSHIPS & REMINDERS

- Posterior depends on both prior and likelihood. A large likelihood ratio can overcome a small prior.
- Law of total probability is required to compute evidence in the denominator of Bayes' formula.
- For mutually exclusive and exhaustive hypotheses A_i, $\sum_i P(A_i) = 1$.
- Use consistent units (decimals vs. percentages) throughout computations.
- The intuitive counts/table method is often less error-prone than plugging into formulas.

8 COMMON PITFALLS

- Forgetting to compute the evidence (denominator) as the total probability of the evidence.
- Mixing up $P(E \mid A)$ and $P(A \mid E)$. These are not the same.
- Using non-exhaustive or overlapping hypotheses without adjusting structure (hypotheses should partition the sample space).
- Rounding too early in intermediate steps; carry precision until the final answer.

9 QUICK CHEAT-SHEET (copyable)

- $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$
- $P(E) = \sum_i P(A_i)\, P(E \mid A_i)$
- $P(A_i \mid E) = \dfrac{P(A_i)\, P(E \mid A_i)}{P(E)}$
- Odds form: posterior odds = prior odds $\times$ likelihood ratio

10 STUDY & REVISION TIPS

- Practice both algebraic and frequency-table solutions for the same problem.
- Always label events clearly (what is hypothesis, what is evidence).
- Solve small numerical examples (two-hypothesis and multiple-hypothesis) until steps are routine.
- Memorize the structure: prior $\times$ likelihood $\rightarrow$ normalize by evidence $\rightarrow$ posterior.

If you want, I can convert this into a one-page printable cheat-sheet (compact layout) or make 5–10 flashcards with example problems and solutions.