Inference for Data Science

Learning goals:

- **Understand what a random sample is.**
- **Appreciate the importance of random sampling.**
- **Understand what a confidence interval states.**
- **Learn common use cases for confidence intervals.**
- **Compare big data and a random sample.**

COGS 108 Fall 2019

Lecture 10

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OH: Wed 10-11a in SSRB 100

Who is Sam?

2nd year Ph.D. student in Cog Sci advised by Philip Guo

Research: computational tools to teach data science

Previously taught data science @ Berkeley
(TA 5 times, Instructor 2 times)

Wrote a textbook for data science: textbook.ds100.org

- [ ] Why random samples are important
- [ ] What a random sample is
- [ ] What a confidence interval is
- [ ] Why confidence intervals are important

The Literary Digest

NEW YORK — OCTOBER 31, 1936

Topics of the day

LANDON, 1,293,669; ROOSEVELT, 972,897

Final Returns in The Digest's Poll of Ten Million Voters

Well, the great battle of the ballots in the Poll of ten million voters, scattered throughout the forty-eight States of the

lican National Committee purchased THE LITERARY DIGEST?" And all types and varieties, including: "Have the Jews purchased

returned and let the people of the Nation draw their conclusions as to our accuracy. So far, we have been right in every Poll. Will we be right in the current Poll? That, as Mrs. Roosevelt said concerning the President's reelection, is in the 'lap of the gods.'

"We never make any claims before election but we respectfully refer you to the opinion of one of the most quoted citizens


Chicago Daily Tribune
DEWEY DEFEATS TRUMAN
G. O. P. Sweep Indicated in State; Boyle Leads in City

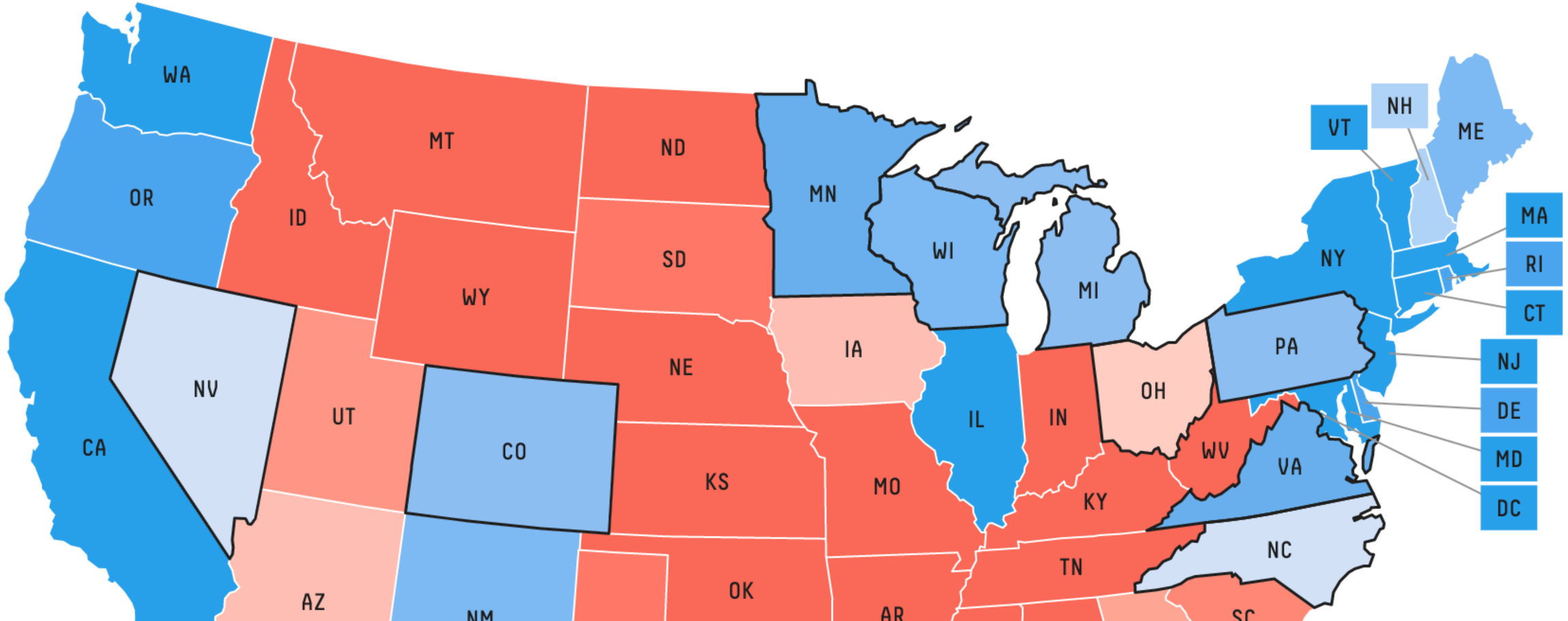
Chance of winning

Hillary Clinton

71.4%

Donald Trump

28.6%



What went wrong?

1936 Election

Literary Digest predicted results of last 5 elections.

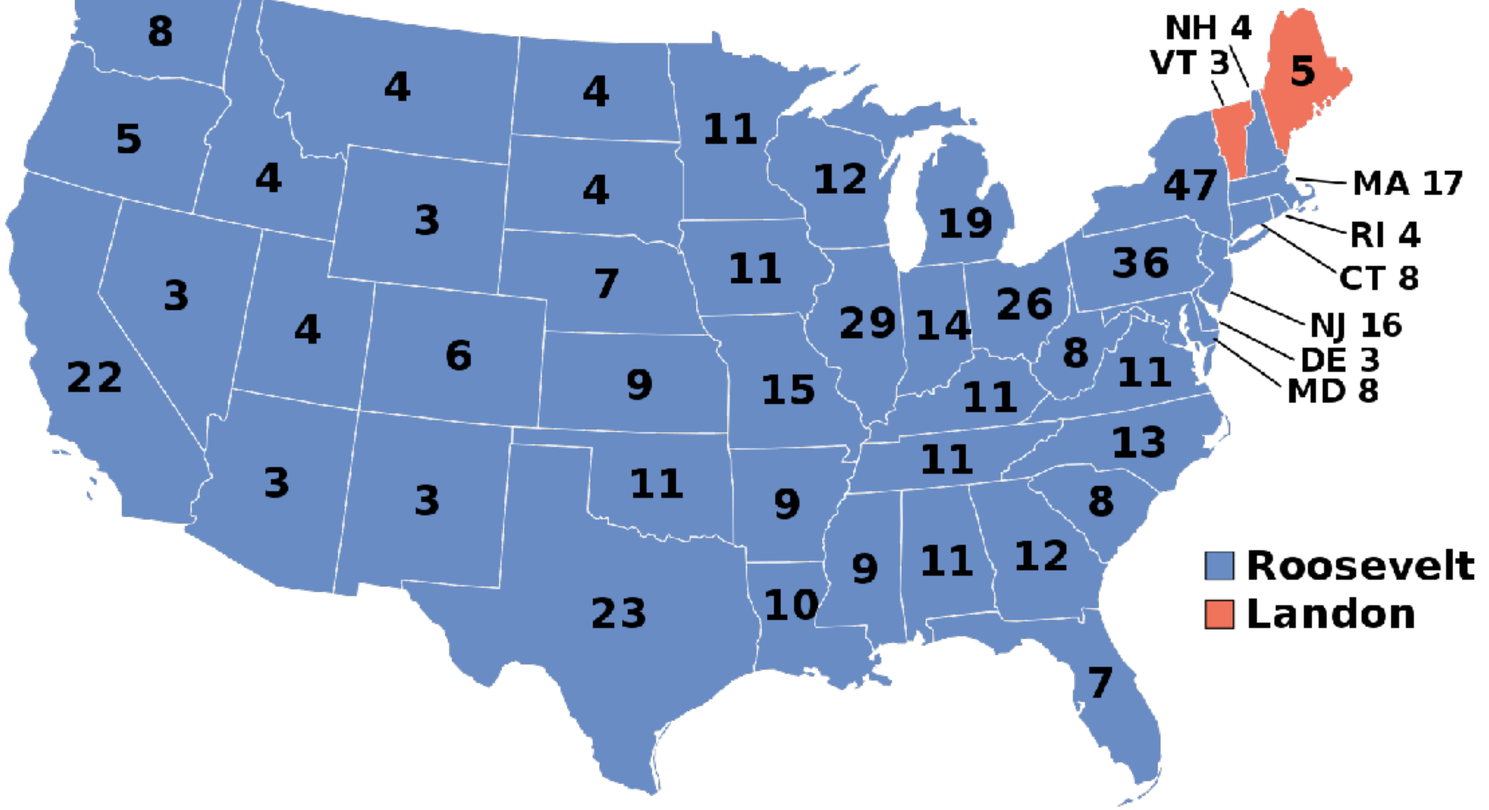
Mailed 10 million questionnaires to readers.

2.3 million responded.

Results off by 20%!

Gallup Poll correct with just 50,000 responses.

What happened?



1948 Election

Gallup Poll used quota sampling: interviewers had to match right distribution of sex, ethnicity, income level, age, etc.

Sampled 3,250 people.

Results off by 10%

What happened?

2016 Election

538 used a combination of many local polls with an emphasis on likely voters.

>1000 polls with over 1 million likely voters. Special adjustments for bias in individual polls.

Predicted 30% chance of Trump win.

What happened?

https://fivethirtyeight.com/features/a-users-guide-to-fivethirtyeights-2016-general-election-forecast/

Making good predictions requires a **random sample** from the **population of interest**.

(Or, collect your entire population in a **census**.)

Non-random samples will be **biased** in unpredictable ways.

Data science and statistics **cannot fix** this intrinsic bias.

- [x] Why random samples are important
- [ ] What a random sample is
- [ ] What a confidence interval is
- [ ] Why confidence intervals are important

Types of Samples

Suppose I want to know the average height of COGS108 students.

Population of interest:	Heights of COGS108 students.
Census:	Measuring height of every student in 108.
Non-Random Sample:	Picking my favorite 20 students and measuring their heights.
Random Sample:	Mixing all student names in a hat, drawing 20 students, then measuring those 20.

Random sample:

Every individual in the population has an **exact probability** of appearing in the sample.

What about my final project dataset?

Dataset	Pop'n of Interest	Verdict
US Census income report	US citizens	Census
US Census income report	California residents	Census
All traffic accidents in San Diego from 2015-2019	Traffic accidents in SD in 2018	Census
All traffic accidents in San Diego from 2015-2019	Future traffic accidents in SD	Non-random sample

What about my final project dataset?

Dataset	Pop'n of Interest	Verdict
Top 100 songs on Spotify in 2018	Popular Spotify songs	Non-random sample
Top 100 songs on Spotify in 2018	Popular Spotify songs in 2018	Non-random sample
All daily trending YouTube videos in 2019	Trending YouTube videos in Sept 2019	Census
All daily trending YouTube videos in 2019	Future trending YouTube videos	Non-random sample

How to be less wrong

Be **precise** about your population and your sample.

If you have a census, **don't use p-values** or confidence intervals.

If you have a non-random sample:
Explicitly say so, outline potential sources of bias, and anticipate how bias might affect your results.

"In this analysis, we assume that our dataset of YouTube videos this year is representative of YouTube videos next year as well. This can cause the following problems with our analysis…"

- [x] Why random samples are important
- [x] What a random sample is
- [ ] What a confidence interval is
- [ ] Why confidence intervals are important

5 min break

Fill out Stats Rorschach Test:
bit.ly/rorstats

What is inference?

Probability tells you about the sample

Population

48% voted for Clinton

46% voted for Trump

6% voted for other

Sample of 100 voters

What's the probability that sample will have **≥51** people who voted for Clinton?

Inference tells you about the population

Population

??% voted for Clinton

Sample of 100 voters

51 people in sample voted for Clinton. What percent of population voted for Clinton?

We can estimate single values...

Population

51% voted for Clinton

Sample of 100 voters

51 people in sample voted for Clinton. What percent of population voted for Clinton?

...but estimating ranges is more useful.

Population

51% ± 2% voted for Clinton.

Or: **between 49% and 53%** voted for Clinton.

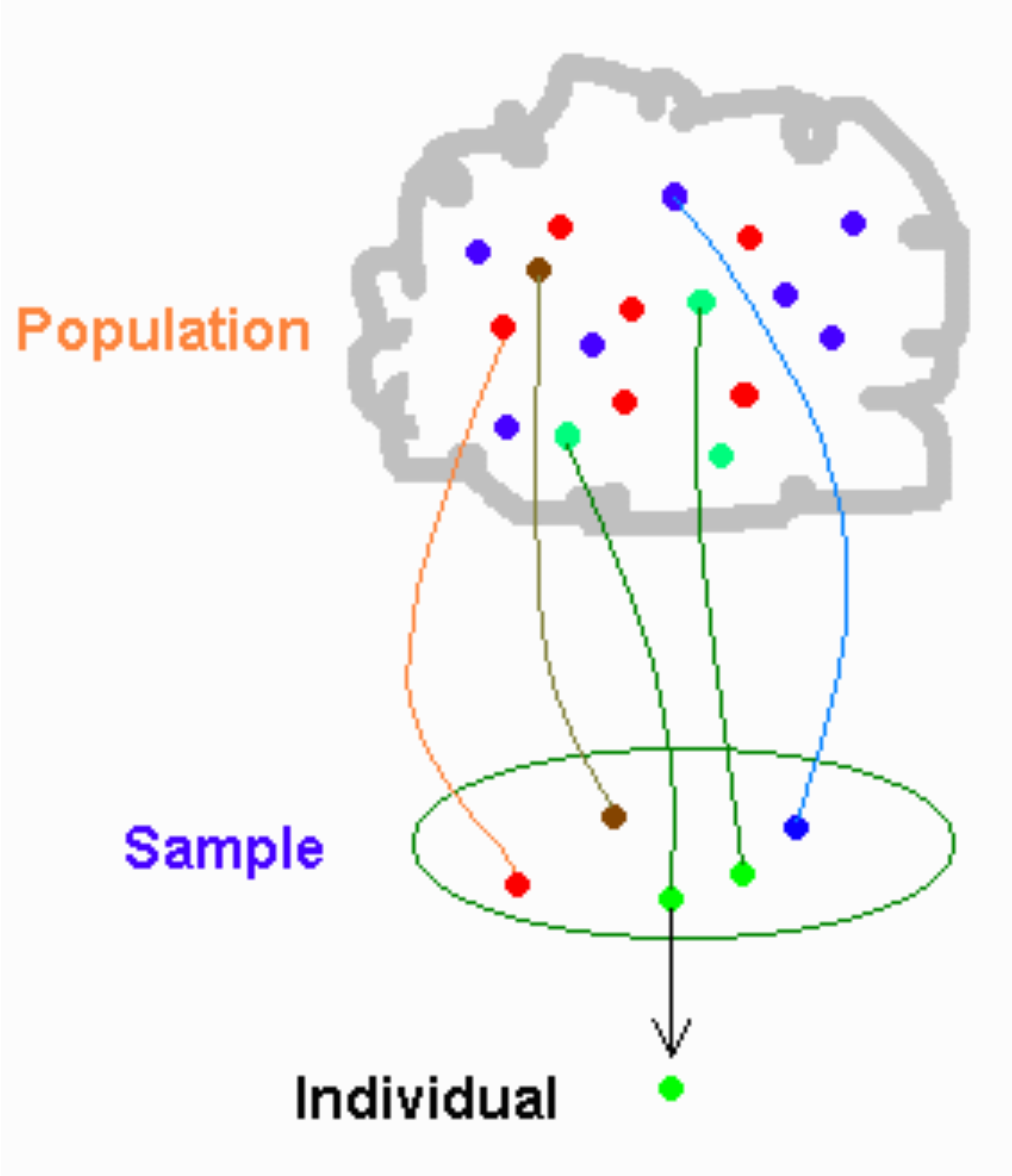
Sample of 100 voters

51 people in sample voted for Clinton. What percent of population voted for Clinton?

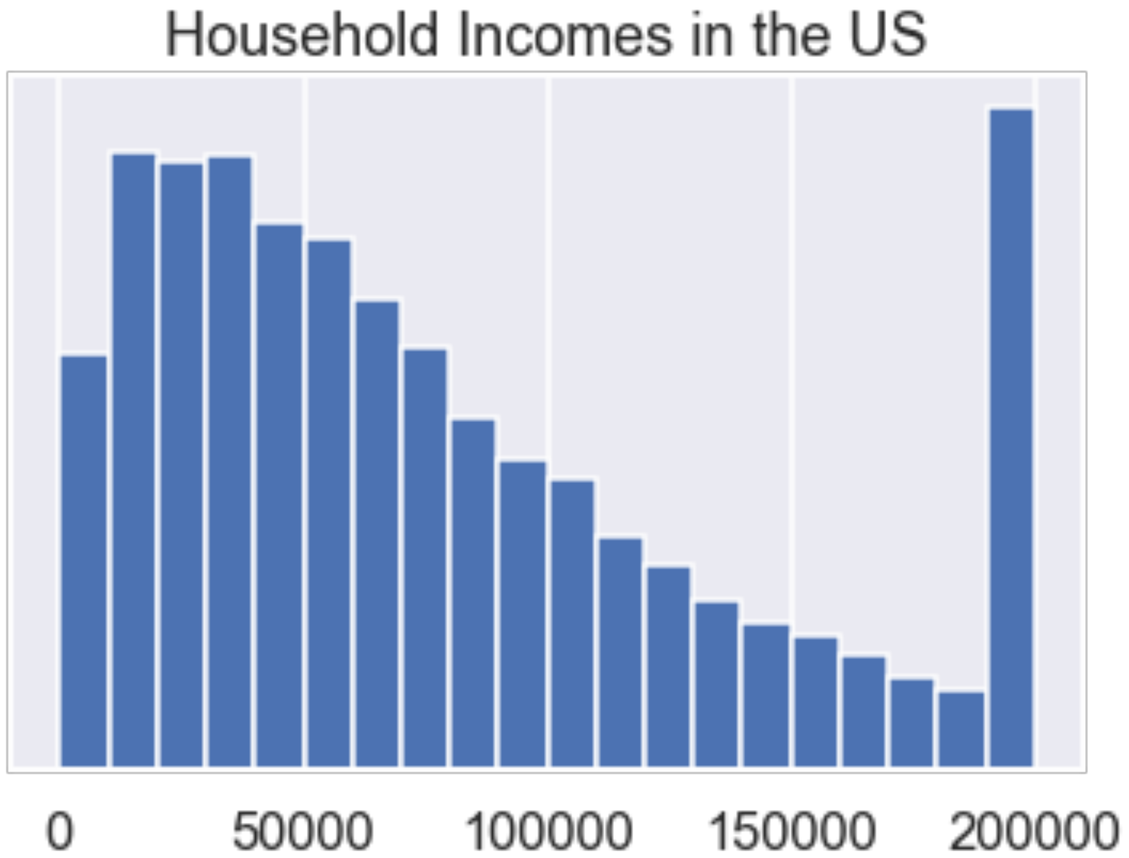
Prediction error comes from:

Sampling bias

Sampling variability

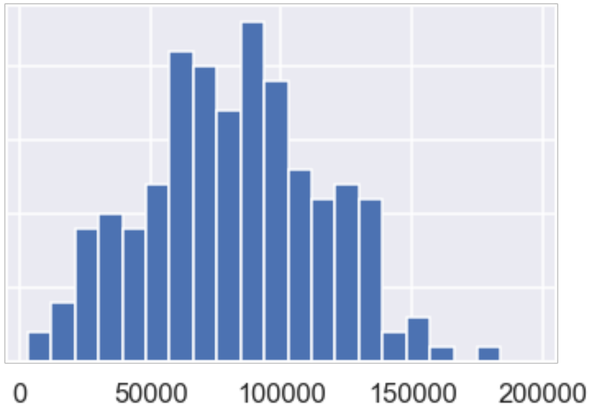


Prediction error comes from:

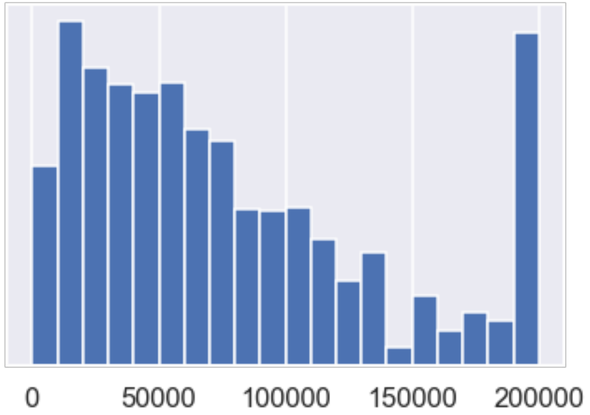


https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-hinc/hinc-01.html

Sampling bias

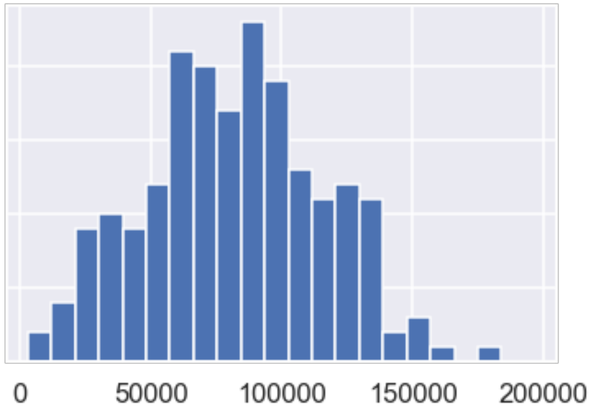


Sampling variability



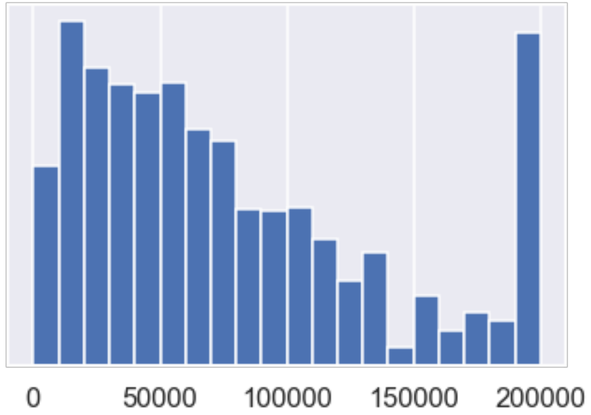
Inference only accounts for sampling variability

Sampling bias



You need to re-examine your data collection strategy.
Statistics will not help you (much).

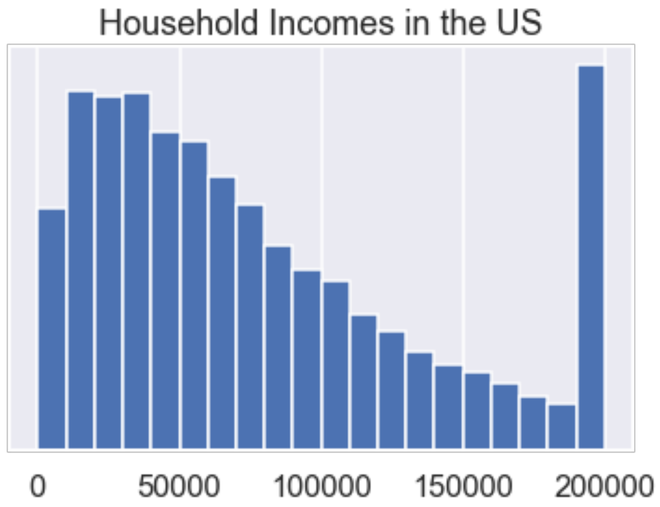
Sampling variability



You can account for this uncertainty using inference techniques like **confidence intervals.**

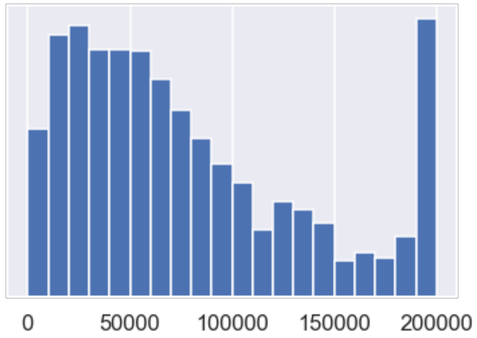
Making Confidence Intervals

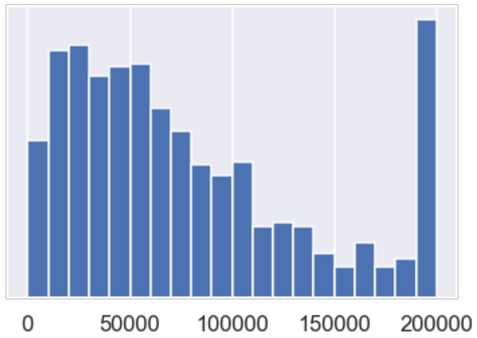
Population

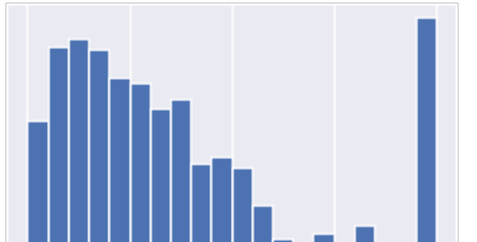


Mean: $63,000

Samples







Sample means

$62,000

$61,000

$65,000

Making Confidence Intervals

Sample means

$62,000

$61,000

$65,000

$59,000

$61,000

$60,000

$63,000

$61,000

$65,000

$65,000

Take the middle 95%

The 95% confidence interval for mean population income is ($60000, $65000).

Collecting many samples is expensive…

…so we use statistical techniques to make CIs using a **single sample**.

In Practice

$$\overline{X} \pm t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

My CI is centered at my sample mean.

In Practice

$$\overline{X} \pm t_{\alpha/2,n-1} \frac{S}{\sqrt{n}}$$

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If I want more confidence, I need to make my interval wider.

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If my sample is large, my CI will be more narrow.

In Practice

$$\overline{X} \pm t_{\alpha/2,n-1} \frac{S}{\sqrt{n}}$$

My CI is centered at my sample mean.

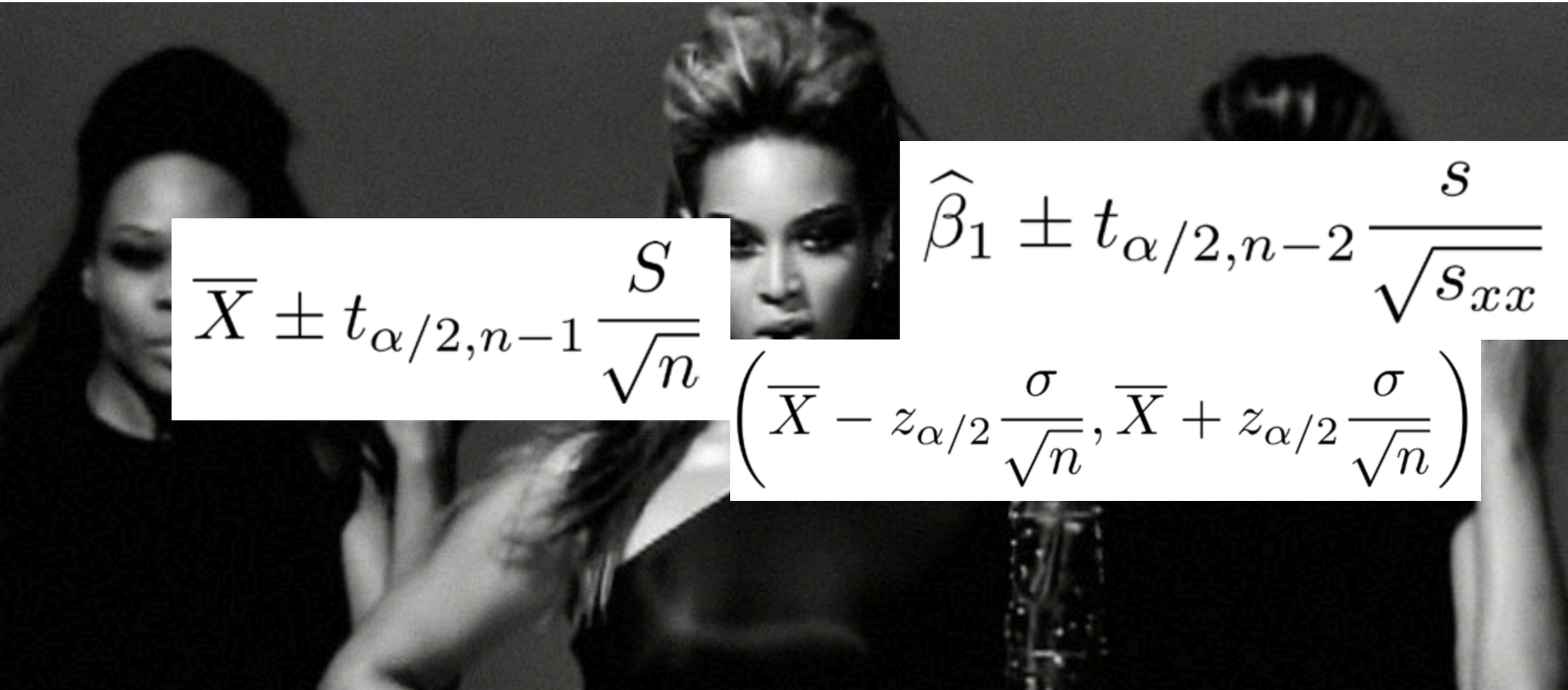
If I want more confidence, I need to make my interval wider.

If my sample is large, my CI will be more narrow.

Many other CI formulas follow this basic pattern.

If you like it...

...put an interval on it.



$$\overline{X} \pm t_{\alpha/2,n-1} \frac{S}{\sqrt{n}}$$

$$\widehat{\beta}_1 \pm t_{\alpha/2,n-2} \frac{s}{\sqrt{s_{xx}}}$$

$$\left(\overline{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

Takeaways

Confidence intervals quantify uncertainty from sampling.

(They do not fix broken samples.)

There's a confidence interval formula for almost any statistic you can imagine: mean, median, slope of regression line, etc.

One useful technique: the bootstrap lets you make confidence intervals for many types of statistics (learn more in DSC 10).

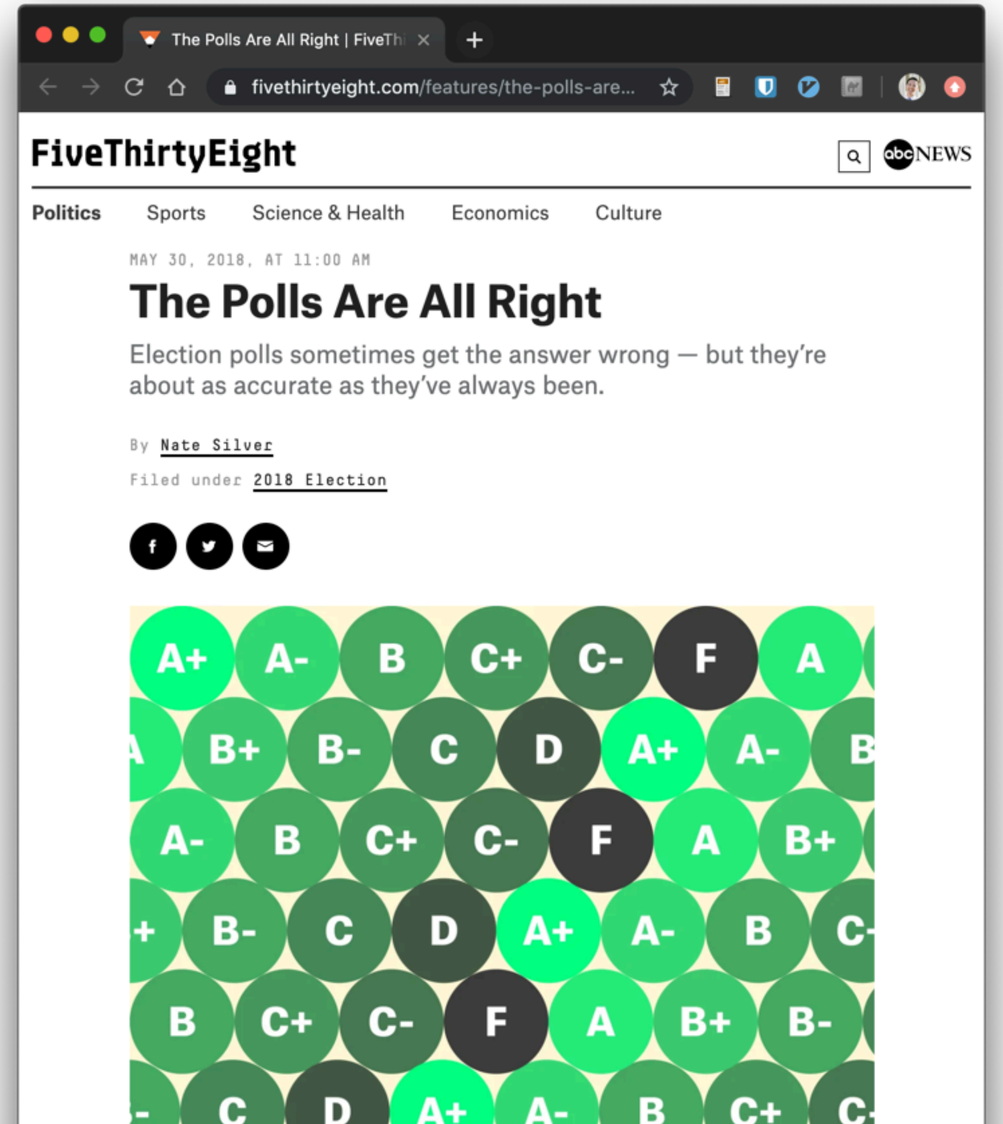
- [x] Why random samples are important
- [x] What a random sample is
- [x] What a confidence interval is
- [ ] Why confidence intervals are important

CIs let you account for sampling uncertainty

538 was wrong for 2016 election.

But error fell within historical polling errors.

CIs were actually accurate for previous elections.



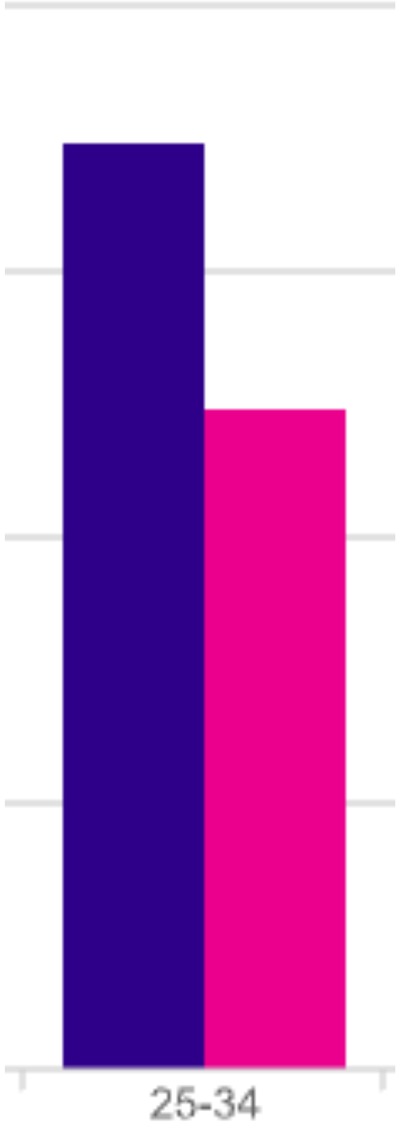
https://fivethirtyeight.com/features/the-polls-are-all-right/

CIs let you compare two quantities

Do more males smoke than females?

Make a CI for the **difference in means**.

If that CI **does not contain 0**, then you have evidence for a difference.

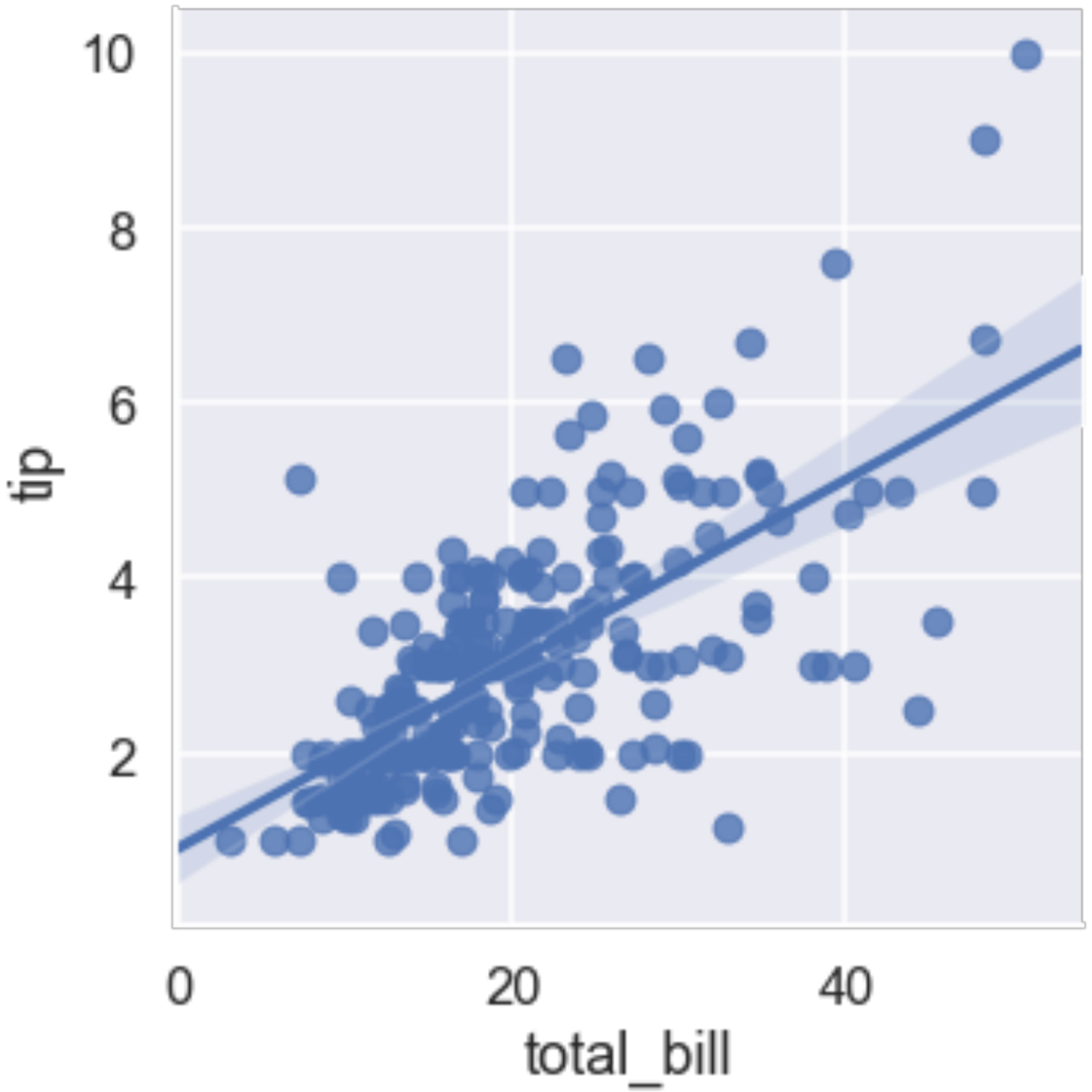


CIs let you examine associations

Do people tip more if they buy more food?

Make a CI for the **slope of the regression line**.

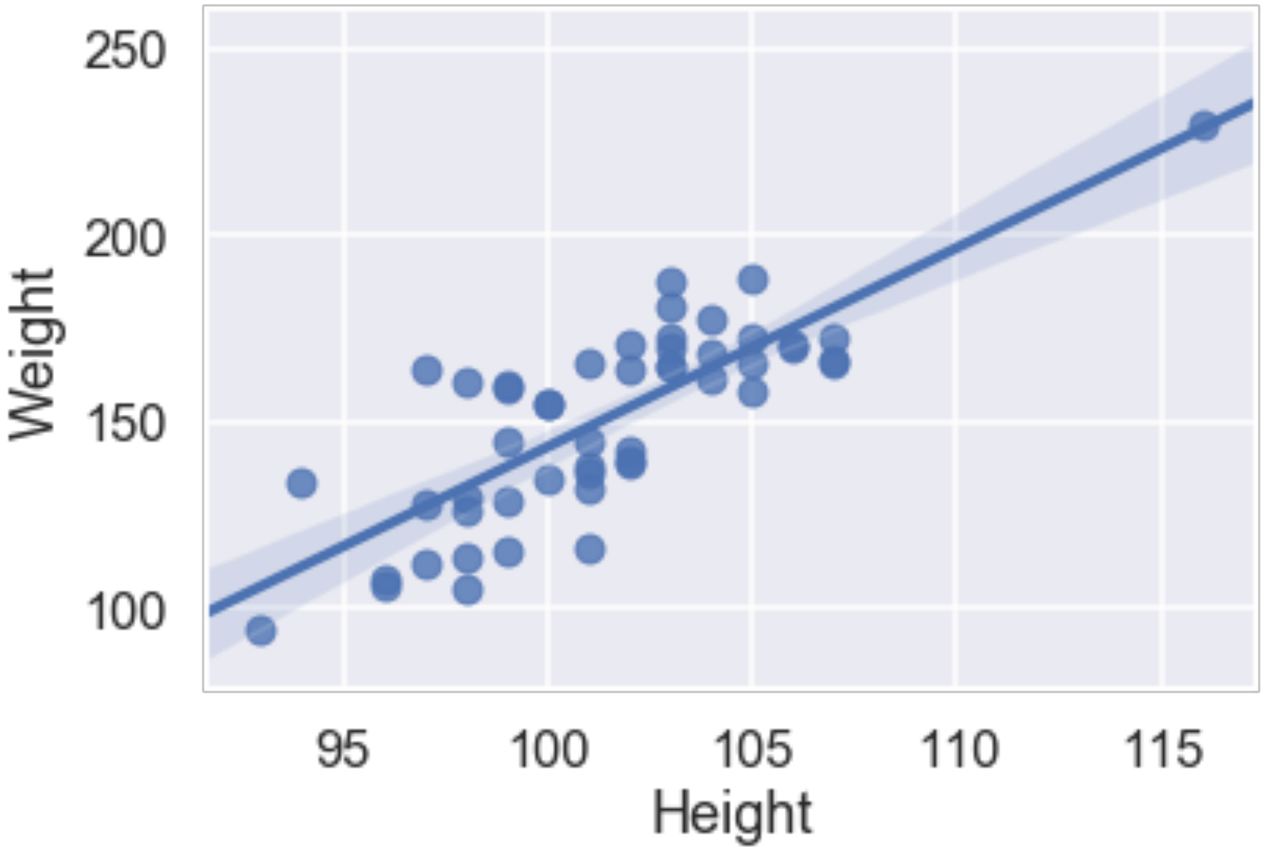
If the slope does not contain 0, you have evidence for an association.



CIs let you examine predictions

How heavy will a donkey be if she has a height of 110 inches?

Make a CI for the **prediction of a regression model**.

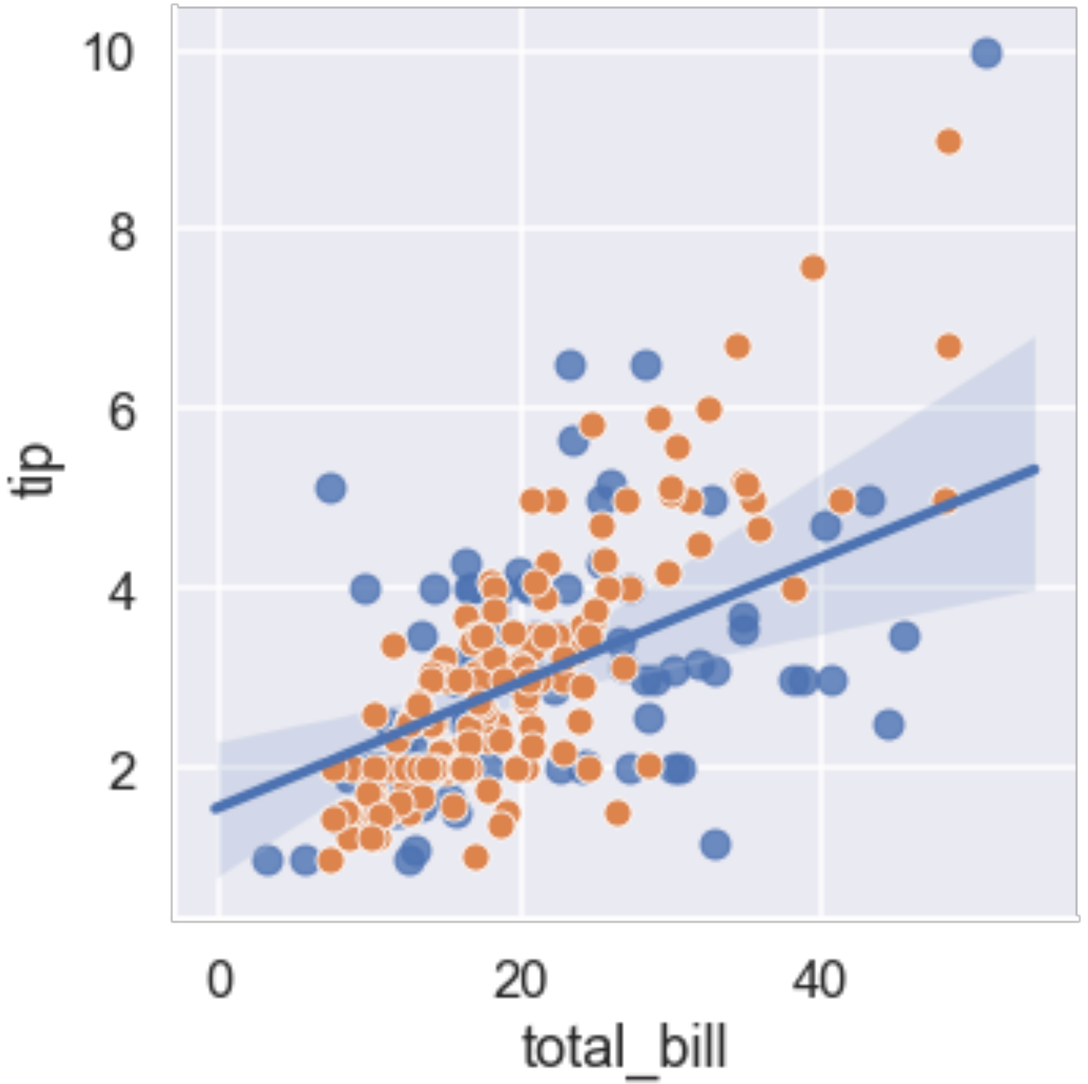


CIs let you examine error

How well will your model perform on unseen data?

Make a CI for the error of your model **on your test set**.

This lets you estimate the **population error** of your model.



- [x] Why random samples are important
- [x] What a random sample is
- [x] What a confidence interval is
- [x] Why confidence intervals are important

Can big data overcome the need for a random sample?

What would you trust more?

Random sample of size 400 (0.3% of U.S. voters)

Non-random sample of size 60,000,000 (50% of U.S. voters)

Keep your hands up for SRS

Random sample of size 400

Non-random sample of 80% of U.S. voters

Non-random sample of 90% of U.S. voters

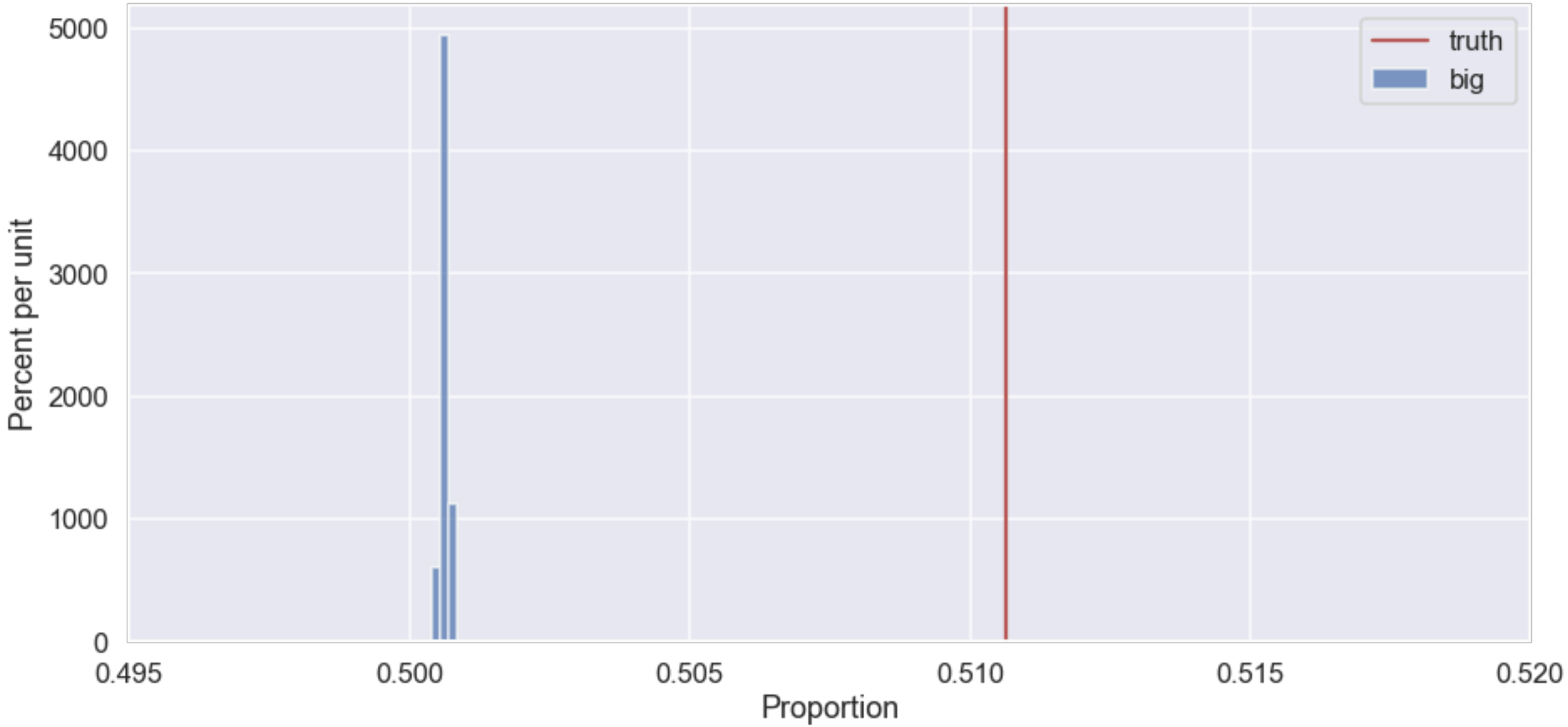
Non-random sample of 99% of U.S. voters

Create 10000 confidence intervals from a SRS of size 400.

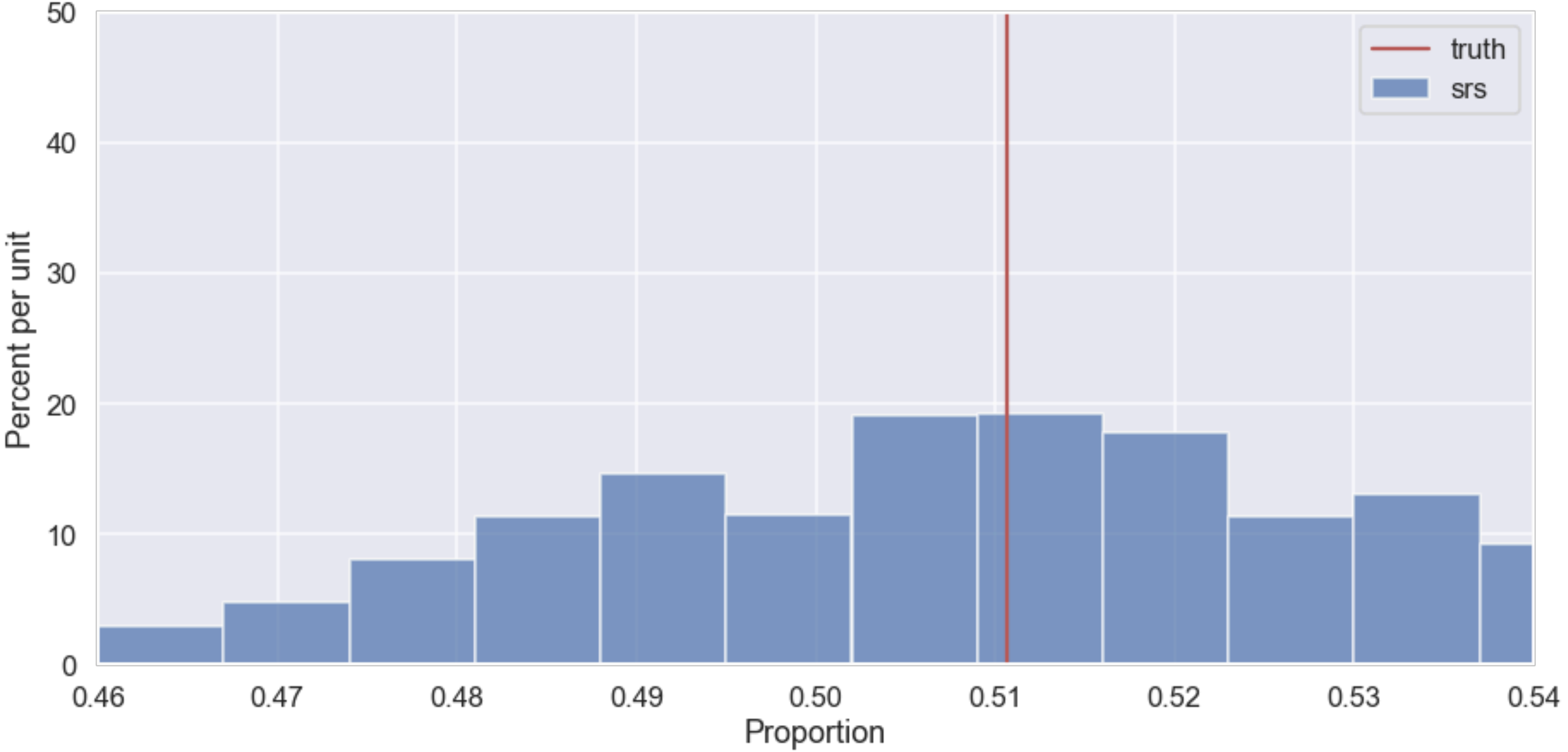
Create 10000 confidence intervals from a non-random sample of size 60mil with a 1% bias.

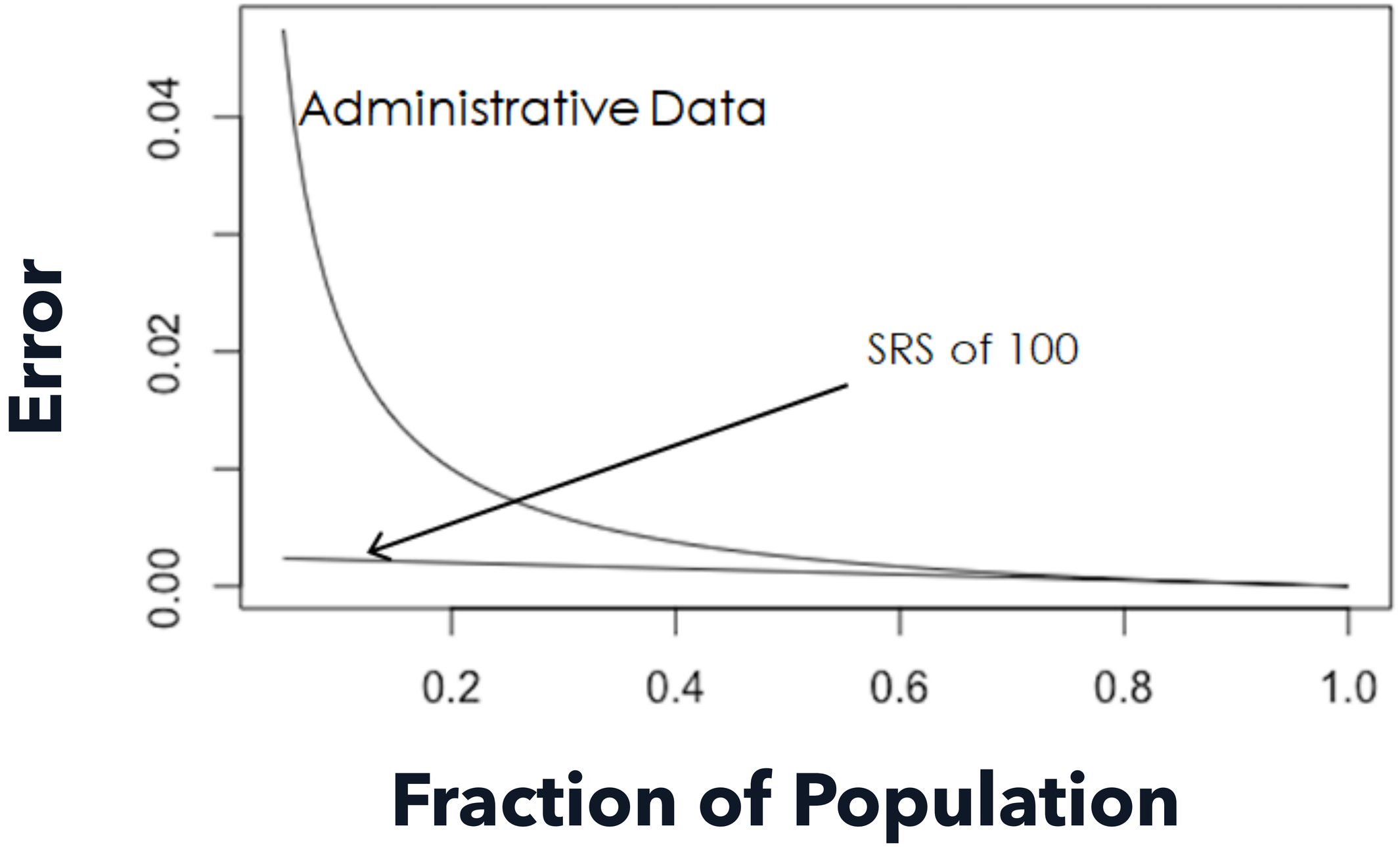
See which sample gives more accurate confidence intervals.

"Big data"


truth
big
5000
4000
3000
2000
1000
0
Percent per unit
0.495
0.500
0.505
0.510
0.515
0.520
Proportion

Simple Random Sample




Error
0.04
0.02
0.00
Administrative Data
SRS of 100
0.2
0.4
0.6
0.8
1.0
Fraction of Population

Summary

The **quality of a sample** matters a lot more than the quantity.

If a sample is non-random and biased, more data makes your predictions **worse**!

Use random samples or acknowledge potential sources of bias.

Don’t estimate single numbers: **make confidence intervals**.