$\boxed{\text{Ex 7.1}}$

First we pick $\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$ so

$$\begin{bmatrix} L_{00}^T & \lambda_{10} e_L & 0 \\ 0 & 1 & 0 \\ 0 & \upsilon_{12} e_F & U_{22}^T \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$L_{00}^T x_0 + \lambda_{10} e_L x_1 = 0$$
$$x_1 = 1$$
$$\upsilon_{12} e_F x_1 + U_{22}^T x_2 = 0$$

This give us

$$L_{00}^T x_0 = -\lambda_{10} e_L \quad (1)$$
$$U_{22}^T x_2 = -\upsilon_{12} F \quad (2)$$

Then the original equation become:

$$\begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & \upsilon_{12} e_F^T \\ 0 & 0 & U_{22} \end{bmatrix} \begin{bmatrix} D_{00} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hitroy

Since $L_{00}$, $U_{22}$ are bidiagonal, $x_0$, $x_2$ can be computed by solving ① and ②

$L_{00}$ has size $k \times k$ therefore $x_0$ can be solved in $O(k)$ flops.

$U_{22}$ is $(n-k-1) \times (n-k-1)$ so solving $x_1$ require $O(n-k-1)$

~~En~~ Overall compute $x$ need $O(n-1)$ flop.

Finally, for each eigenvalues
$B - \lambda I$ takes $O(n)$, $LDL^T$ needs $O(n)$ flops
$UEU^T$ needs $O(n)$ flops. Find smallest $\phi_i$ needs $O(n)$ flops, compute eigenvalue from twisted factorization takes $O(n)$ flops.

The total cost is $n \times O(n) = \boxed{O(n^2) \text{ flops}}$