$\boxed{\text{Ex } 6.1}$

Multiply out $A = LDL^T$

$$\Rightarrow \begin{bmatrix} A_{00} & \alpha_{10} e_L & 0 \\ \alpha_{10} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ 0 & \alpha_{21} e_F & A_{22} \end{bmatrix} = \begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{bmatrix} \begin{bmatrix} D_{00} & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & D_{22} \end{bmatrix} \begin{bmatrix} L_{00}^T & \lambda_{10} e_L & 0 \\ 0 & 1 & \lambda_{21} e_F \\ 0 & 0 & L_{22} \end{bmatrix}$$

$$= \begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{bmatrix} \begin{bmatrix} D_{00} L_{00}^T & \lambda_{10} D_{00} e_L & 0 \\ 0 & \delta_1 & \lambda_{21} \delta_1 e_F^T \\ 0 & 0 & D_{22} L_{22}^T \end{bmatrix}$$

After multification, we will result α_{11} as

$$\boxed{\alpha_{11} = \lambda_{10}^2 D_{00} e_L e_L^T + \delta_1} \Rightarrow ①$$

Also, we can multiply $A = UEU^T$

$$A = \begin{bmatrix} U_{00} & \upsilon_{01} e_L & 0 \\ 0 & 1 & \upsilon_{12} e_F^T \\ 0 & 0 & U_{22} \end{bmatrix} \begin{bmatrix} E_{00} & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & E_{22} \end{bmatrix} \begin{bmatrix} U_{00}^T & 0 & 0 \\ \upsilon_{01} e^T & 1 & 0 \\ 0 & \upsilon_{12} e_F & U_{22}^T \end{bmatrix}$$

$$= \begin{bmatrix} U_{00} E_{00} U_{00}^T + \upsilon_{01} \varepsilon_1 e_L e_L^T & \upsilon_{01} \varepsilon_1 e_L & 0 \\ \varepsilon_1 \upsilon_{01} e_L^T & \varepsilon_1 + \upsilon_{12}^2 e_F^T e_F E_{22} & \upsilon_{12} e_F^T E_{22} U_{22}^T \\ 0 & \cdots & \cdots \end{bmatrix}$$

$$\boxed{\alpha_{11} = \varepsilon_1 + \upsilon_{12}^2 e_F^T e_F E_{22}} \Rightarrow ② \; \cancel{\varepsilon_1 = \alpha_{11}}$$

Use the equation for twisted factorization.

$$A = \begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & \upsilon_{12} e_F^T \\ 0 & 0 & U_{22} \end{bmatrix} \begin{bmatrix} D_{00} & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & 0 & E_{22} \end{bmatrix} \begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & \upsilon_{12} e_F^T \\ 0 & 0 & U_{22} \end{bmatrix}^T$$

$$\boxed{\alpha_{11} = \lambda_{10}^2 D_{00} e_L e_L^T + \phi_1 + \upsilon_{12}^2 e_F^T e_F E_{22}} \quad ③$$

Use ①, $\lambda_{10}^2 D_{00} e_L e_L^T = \alpha_{11} - \delta_1$

Use ②, $\upsilon_{12}^2 e_F^T e_F E_{22} = \alpha_{11} - \varepsilon_1$

We can convert ③ to

$$\alpha_{11} = (\alpha_{11} - \delta_1) + \phi_1 + (\alpha_{11} - \varepsilon_1)$$

$$\phi_1 = \delta + \varepsilon_1 - \alpha_{11}$$

2) Given we already have $A = LDL^T = UEU^T$ we just need to calculate ϕ, so its $O(1)$

(3) We need to participate and calculate ϕ_1 for n times therefore the complexity is $O(n)$

$\boxed{\text{Ex 7.1}}$

First we pick $\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$ so

$$\begin{bmatrix} L_{00}^T & \lambda_{10} e_L & 0 \\ 0 & 1 & 0 \\ 0 & v_{12} e_F & U_{22}^T \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$L_{00}^T x_0 + \lambda_{10} e_L x_1 = 0$$

$$x_1 = 1$$

$$v_{12} e_F x_1 + U_{22}^T x_2 = 0$$

This give us

$$L_{00}^T x_0 = -\lambda_{10} e_L \quad ①$$

$$U_{22}^T x_2 = -v_{12} F \quad ②$$

Then the original equation become:

$$\begin{bmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{bmatrix} \begin{bmatrix} D_{00} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$